



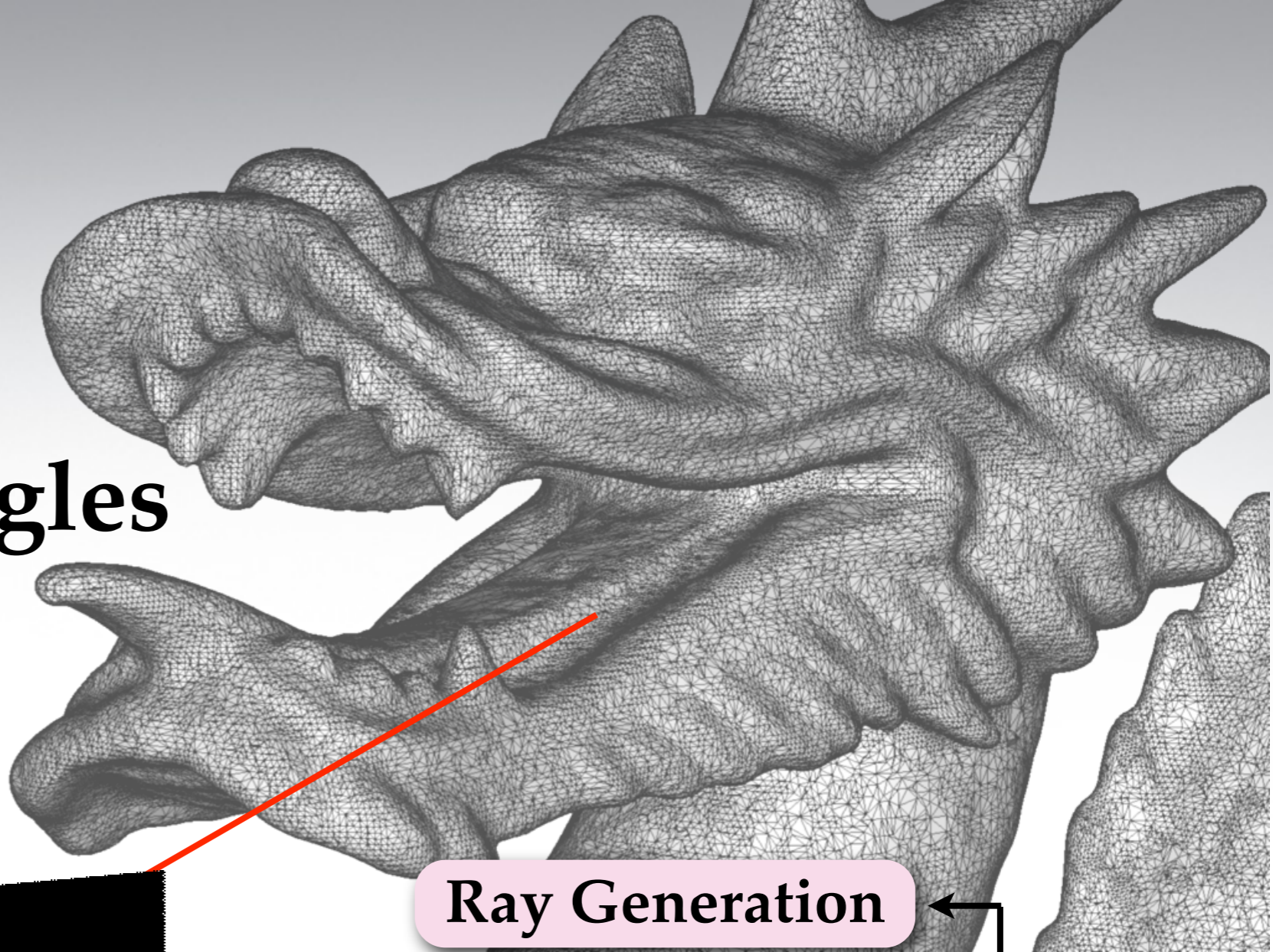


Scans: XYZRGB



Rendering: University of Utah  
Model: Stanford University

**5.5 million triangles**



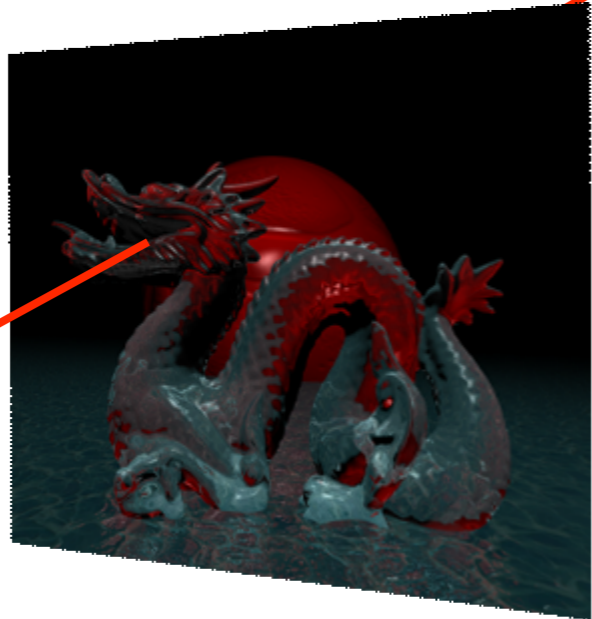
**Ray Generation**



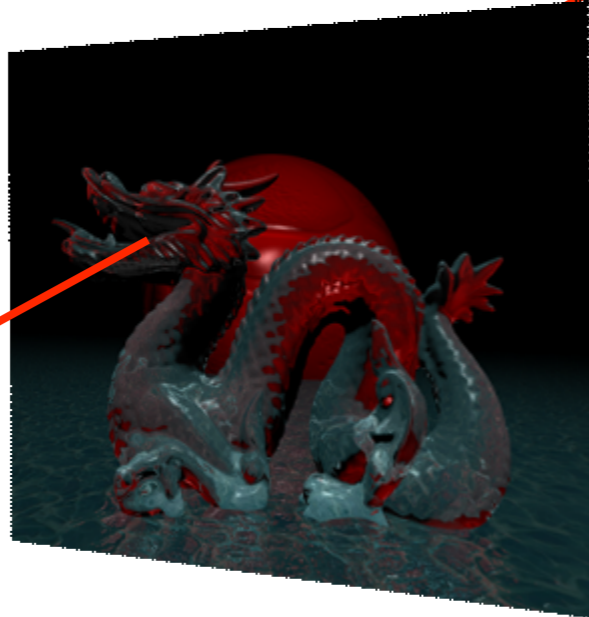
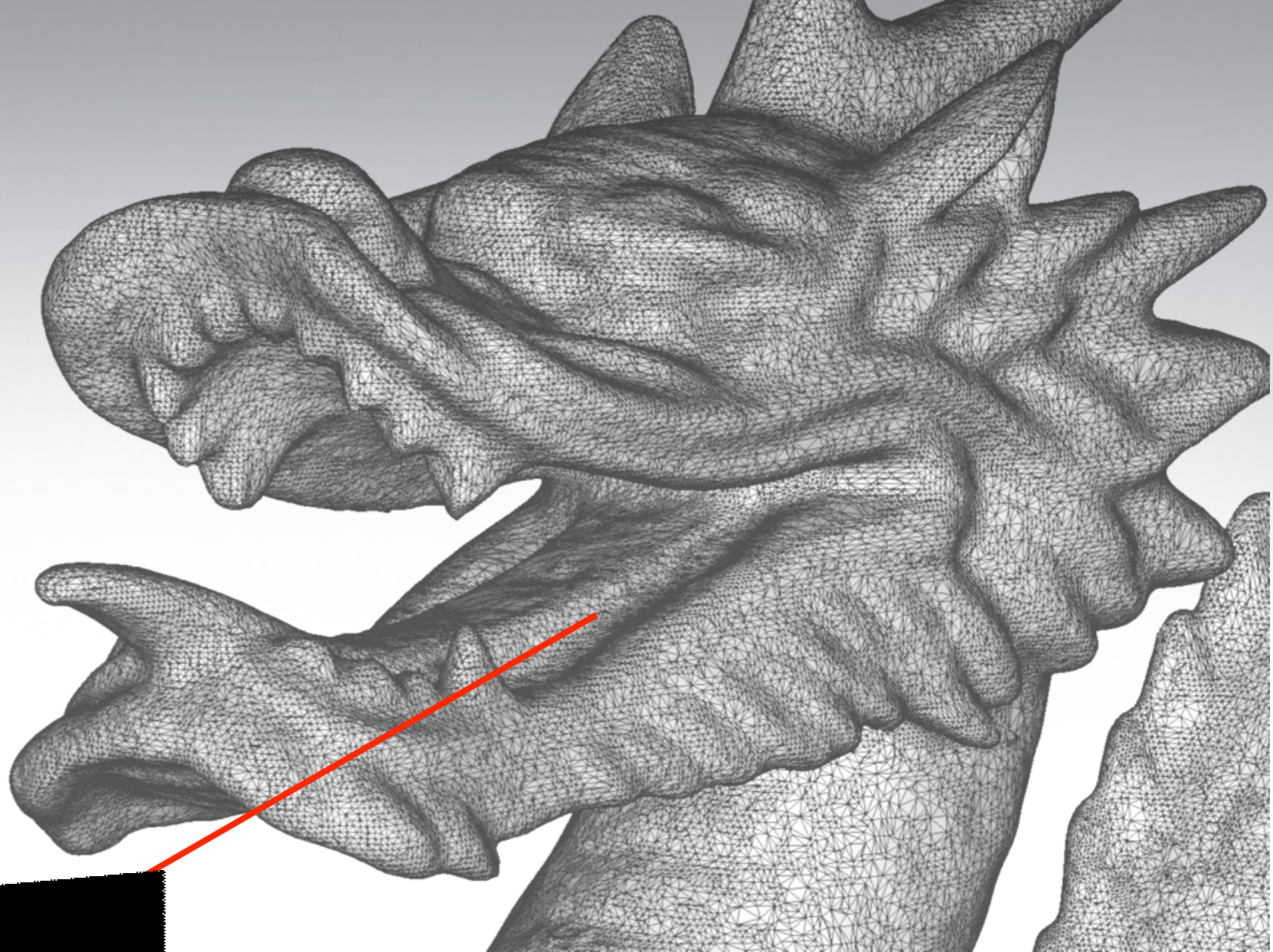
**Intersection**



**Shading**



half-line  $pv$



half-line  $pv$

**Brute-force**

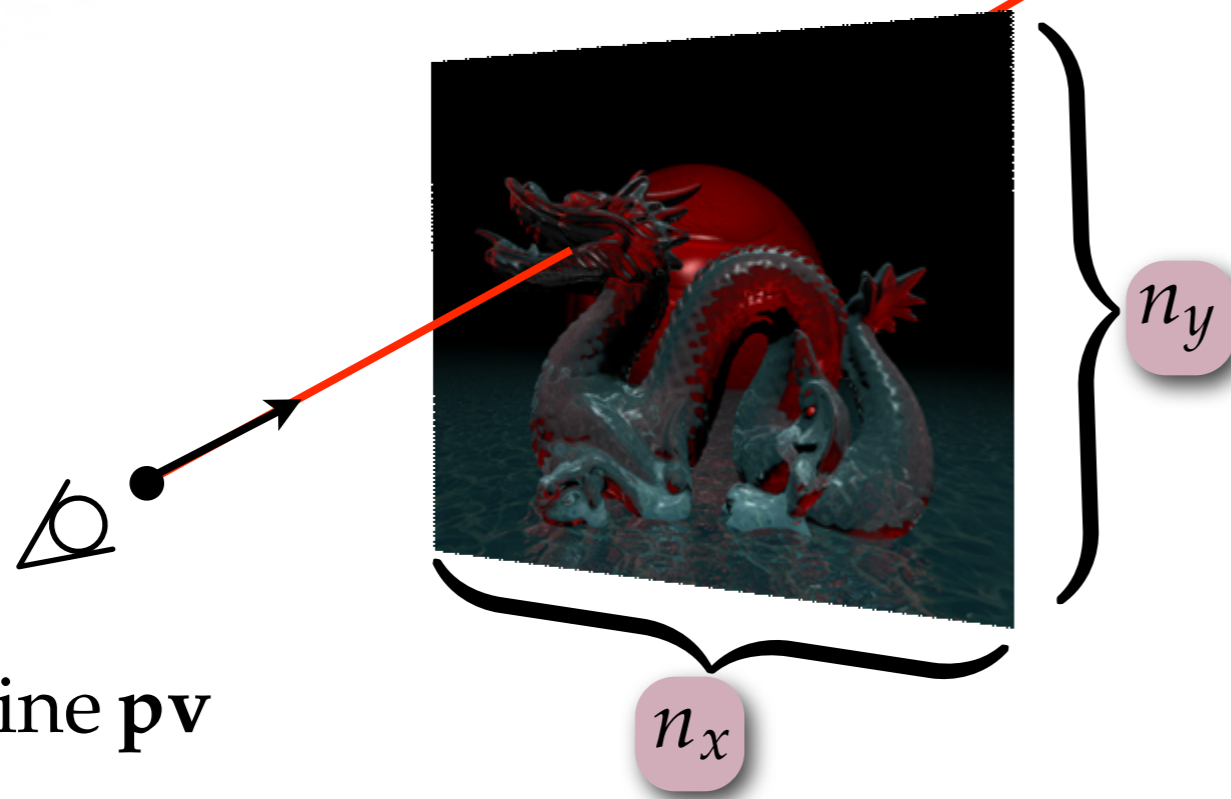
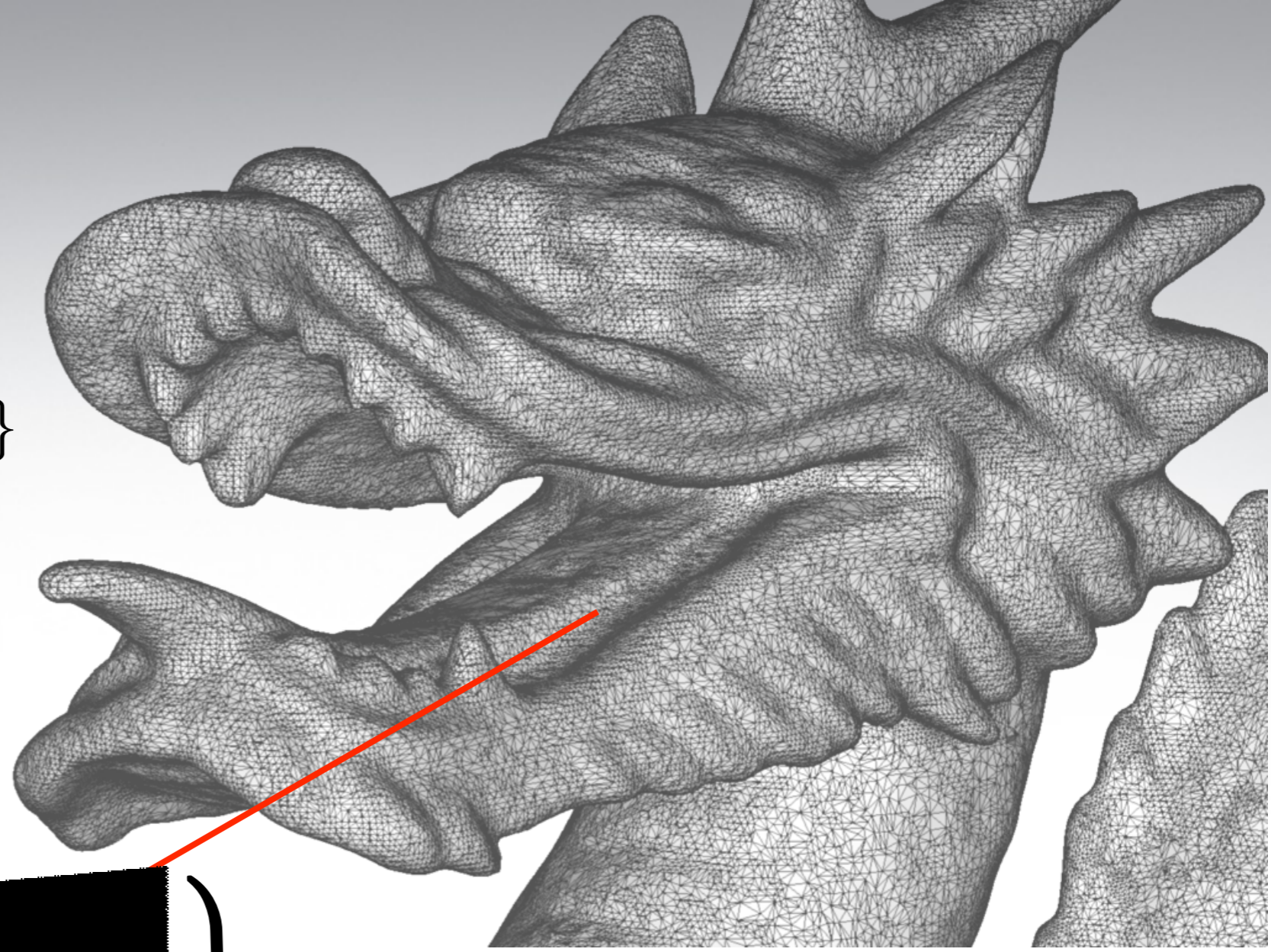
for all rays

for all primitives

intersect ray primitive

return closest intersection

$$n_t = \#\{\text{triangles } x_i x_j x_k\}$$



half-line pv

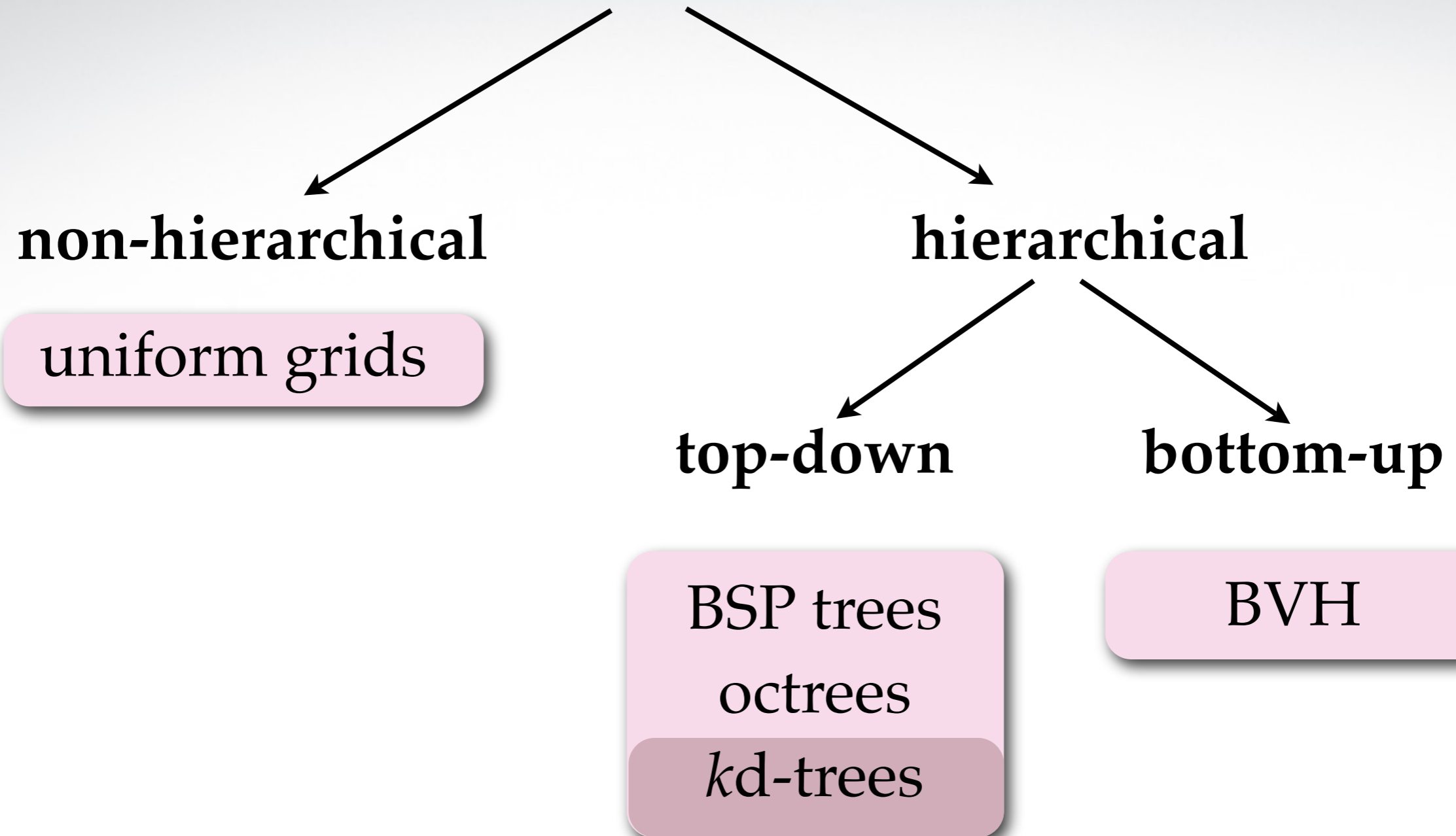
Complexity

$$O(n_x n_y n_t)$$

**“95 % computing time is spent on  
intersection computations”**

*Whitted, 1980*

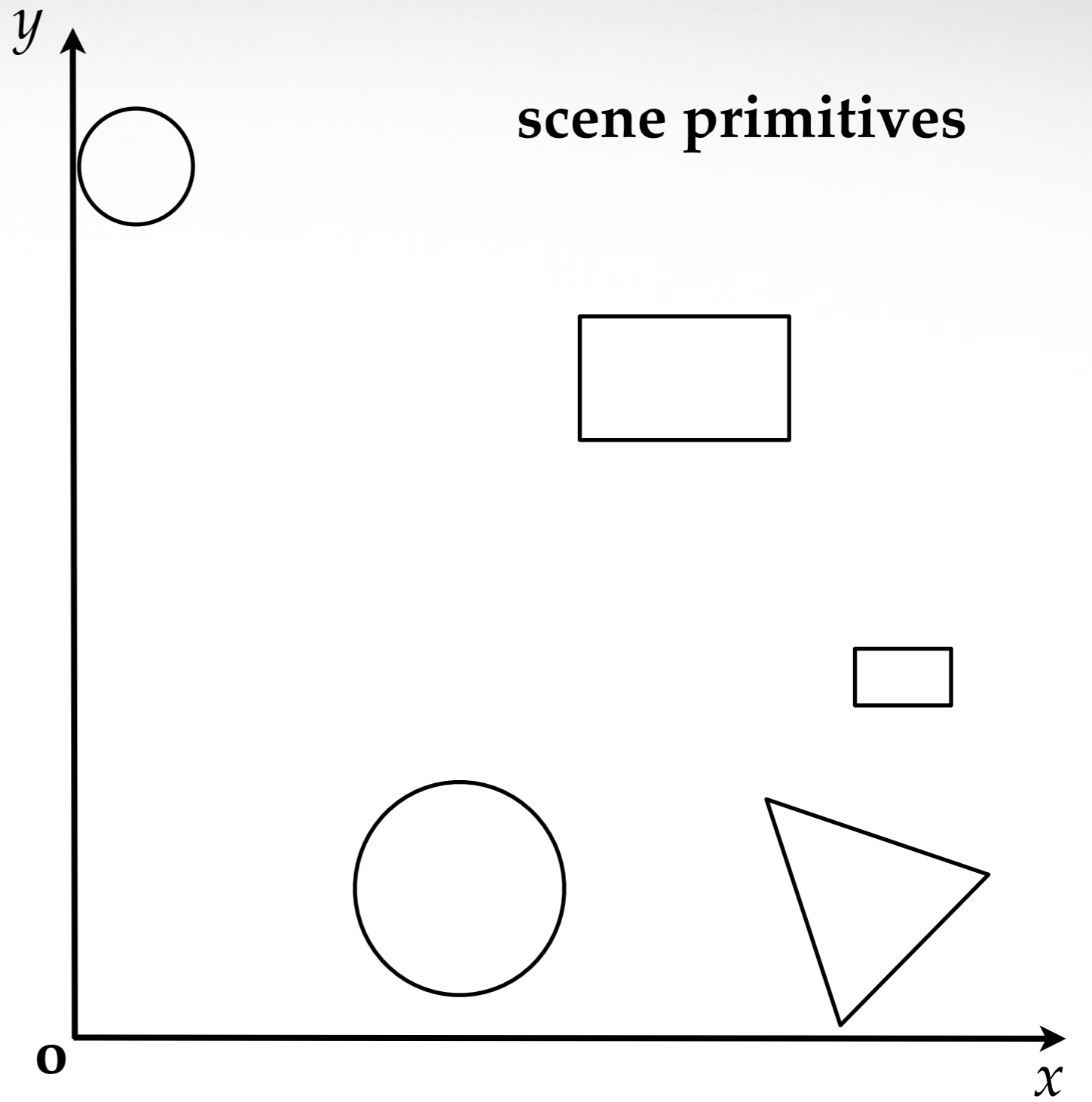
# Acceleration methods for ray-surface intersections

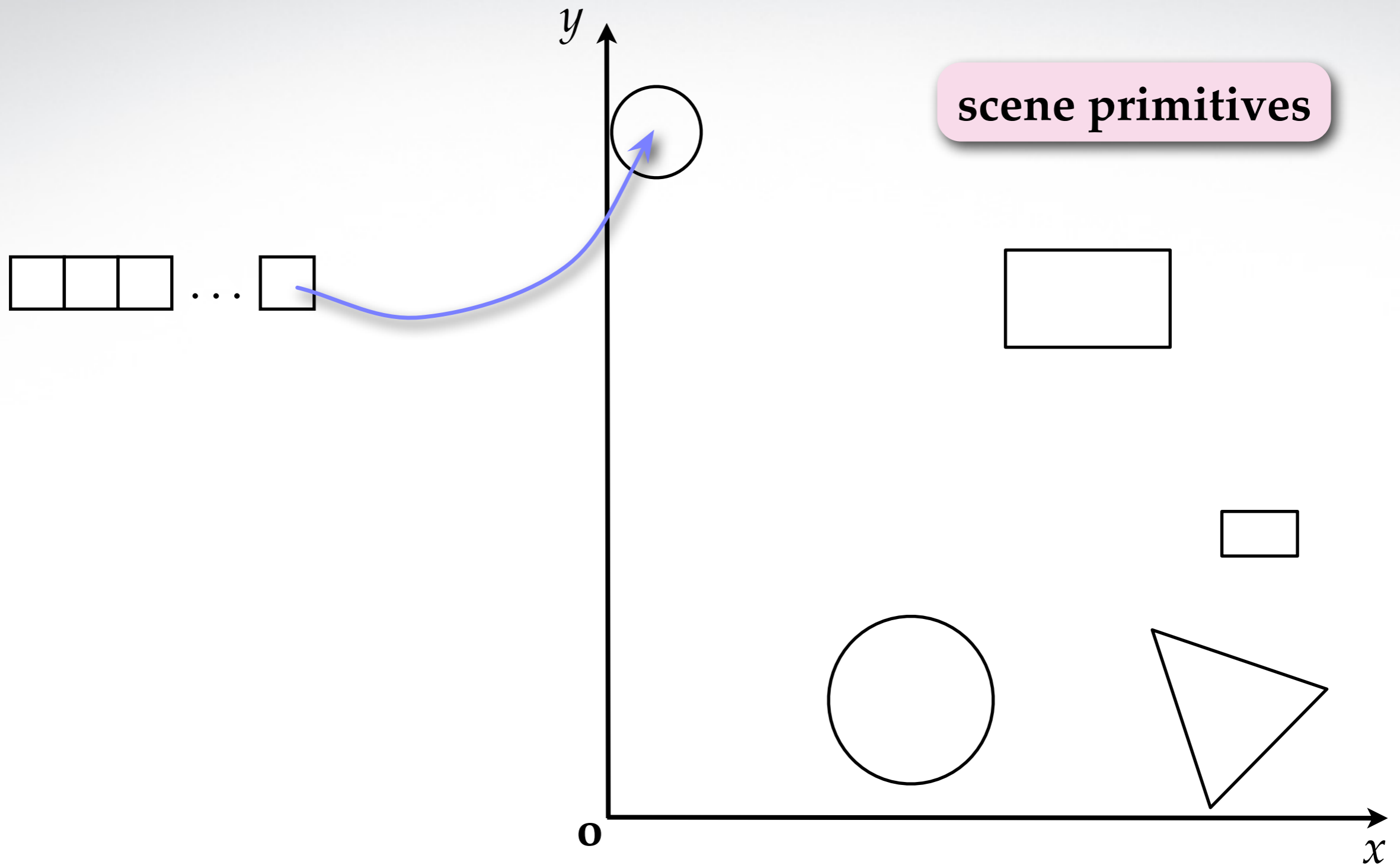


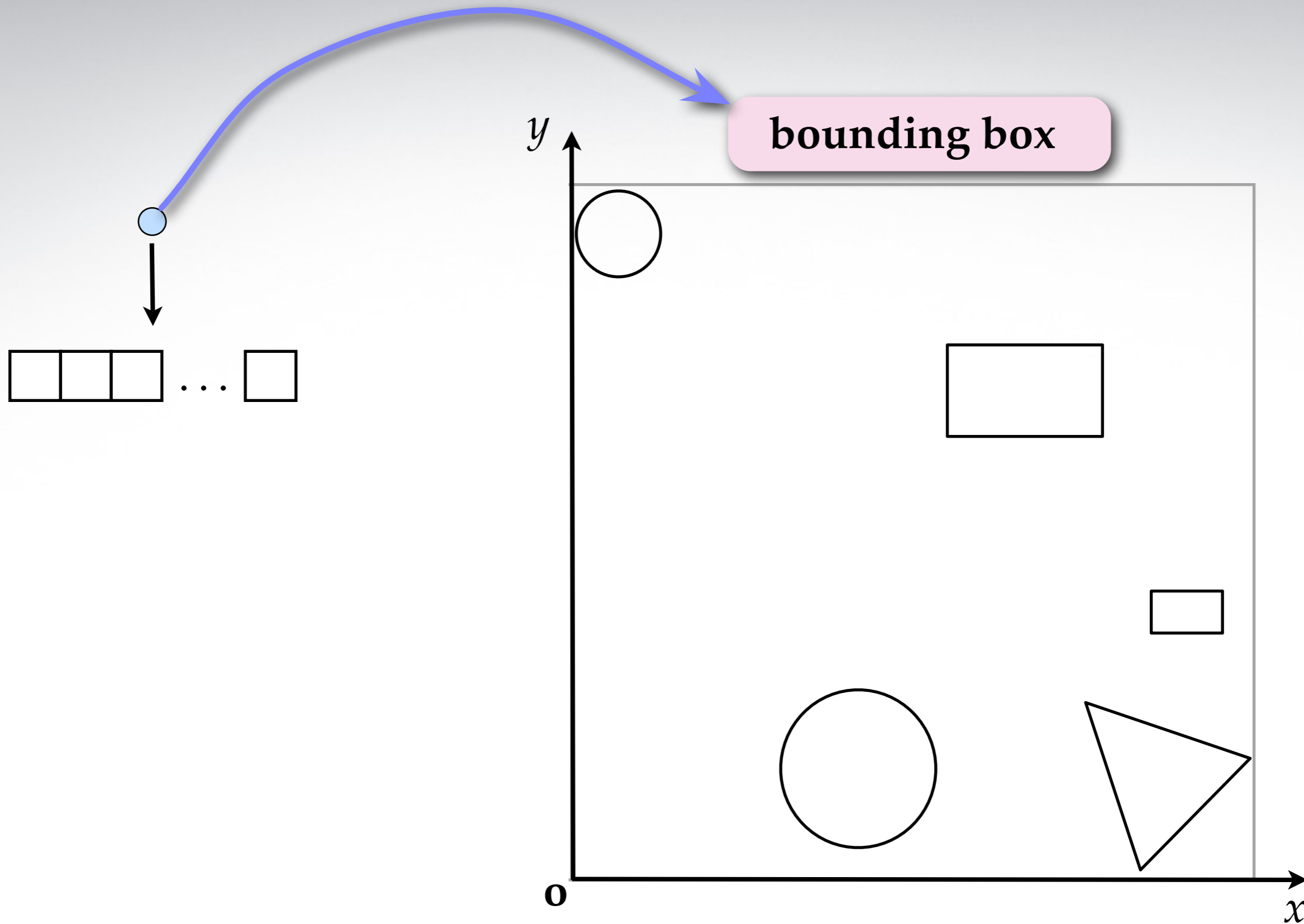


# The *kd*-tree ( $k=2$ )

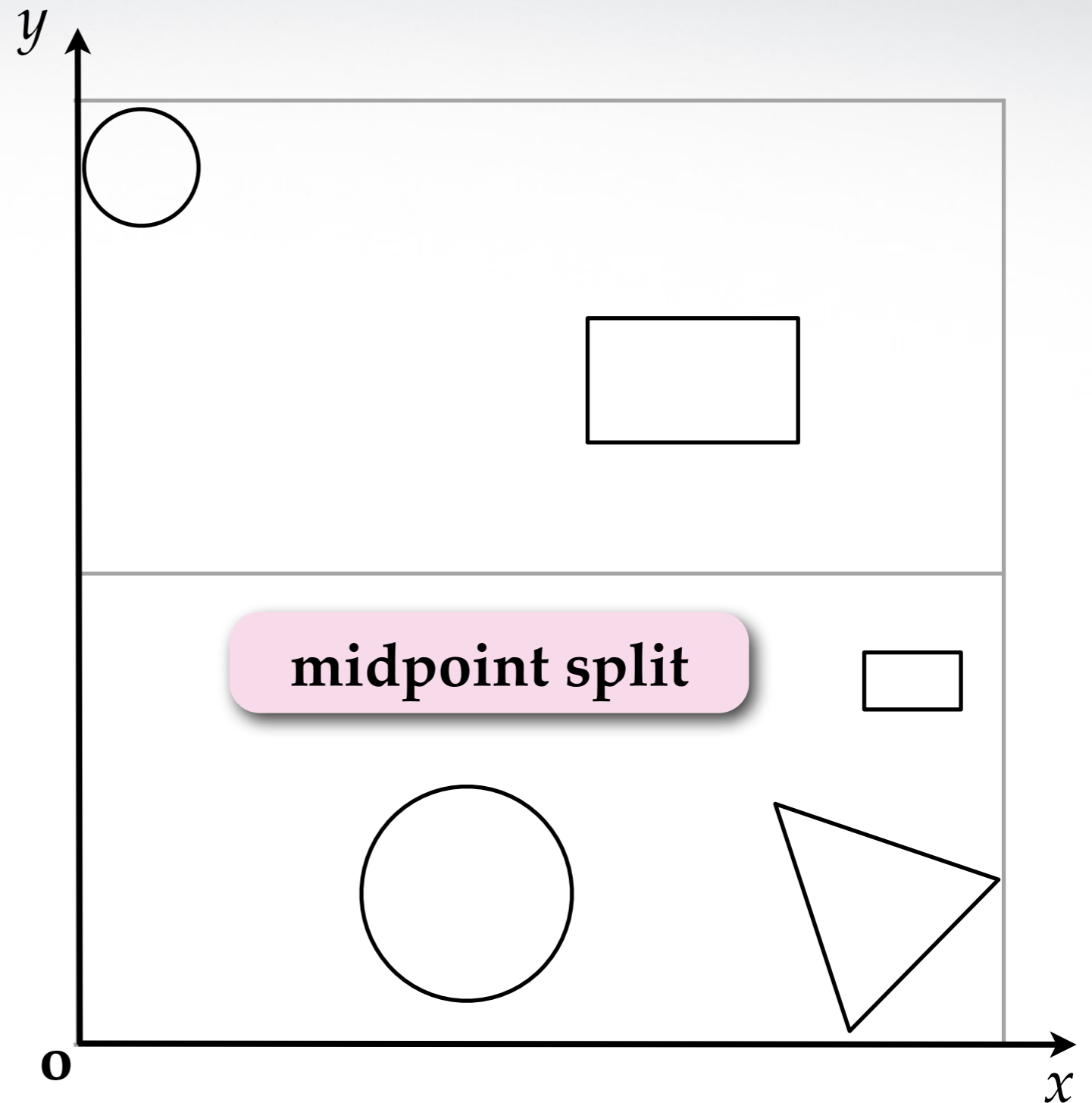
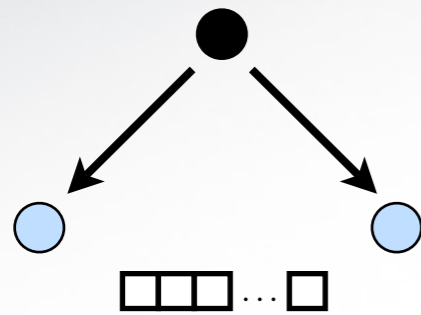


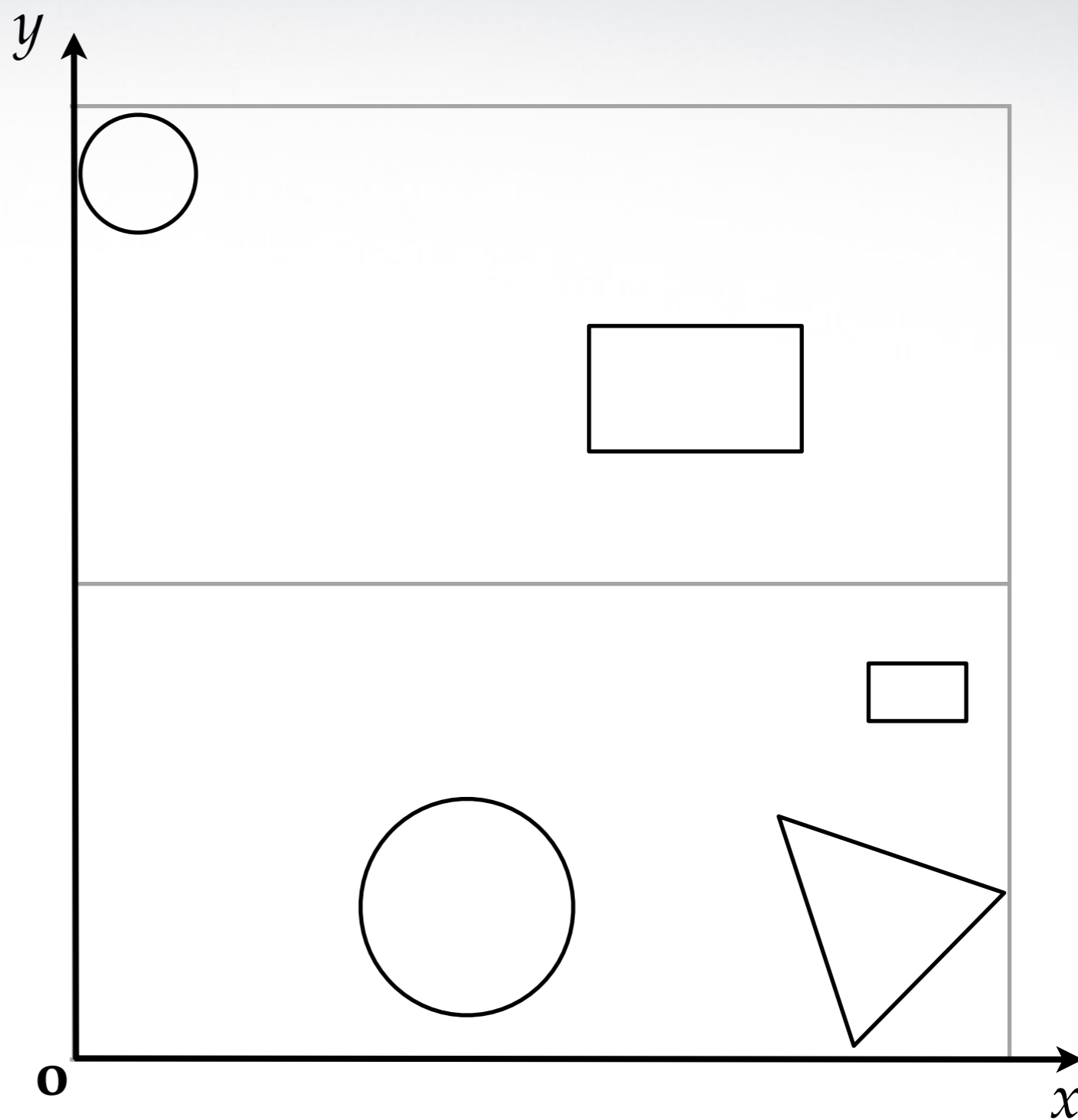
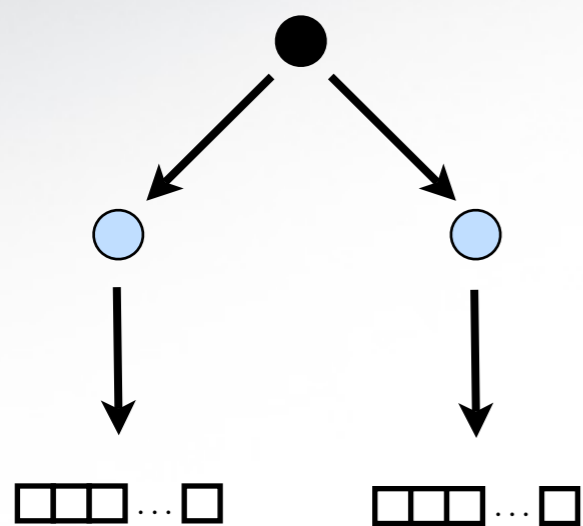


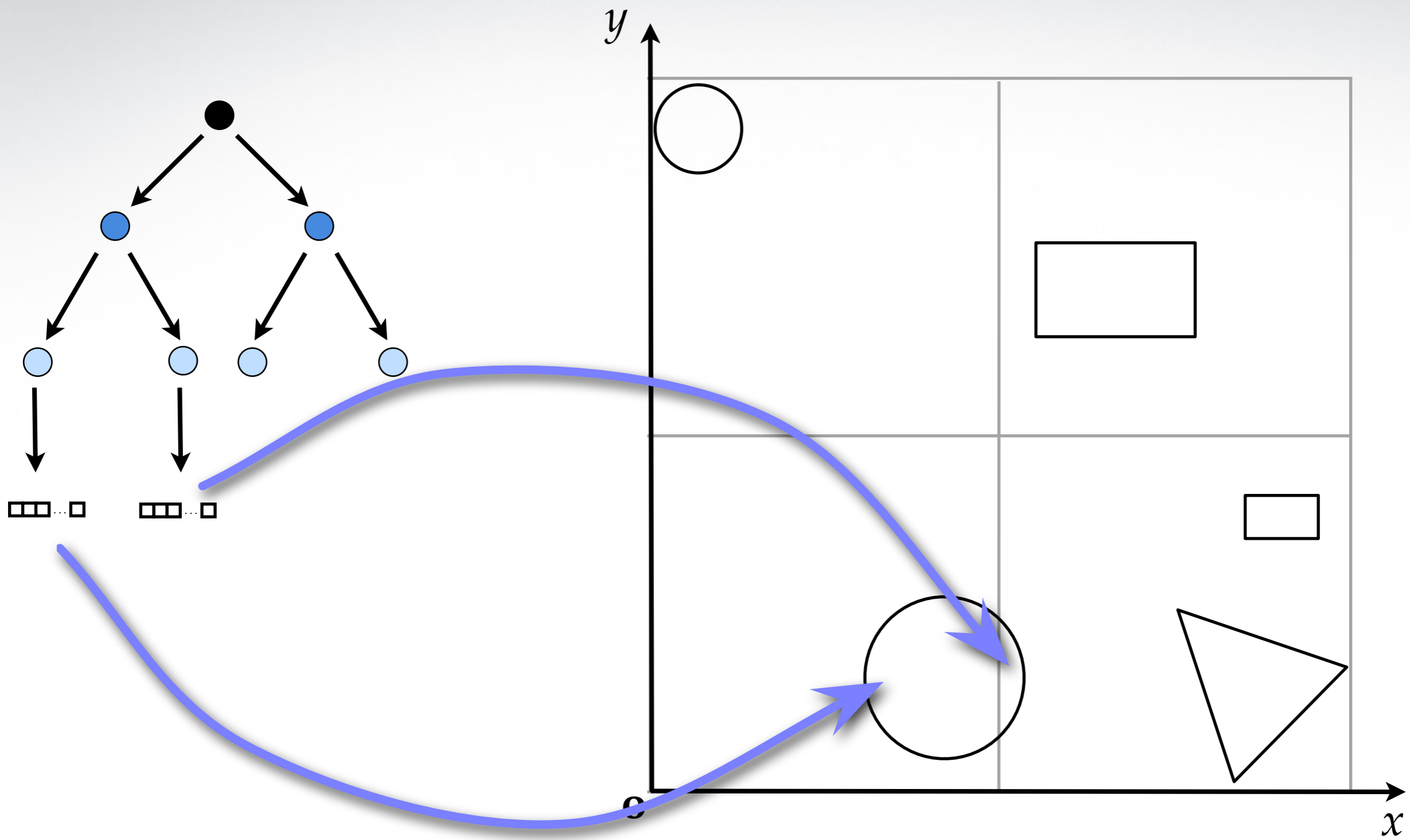


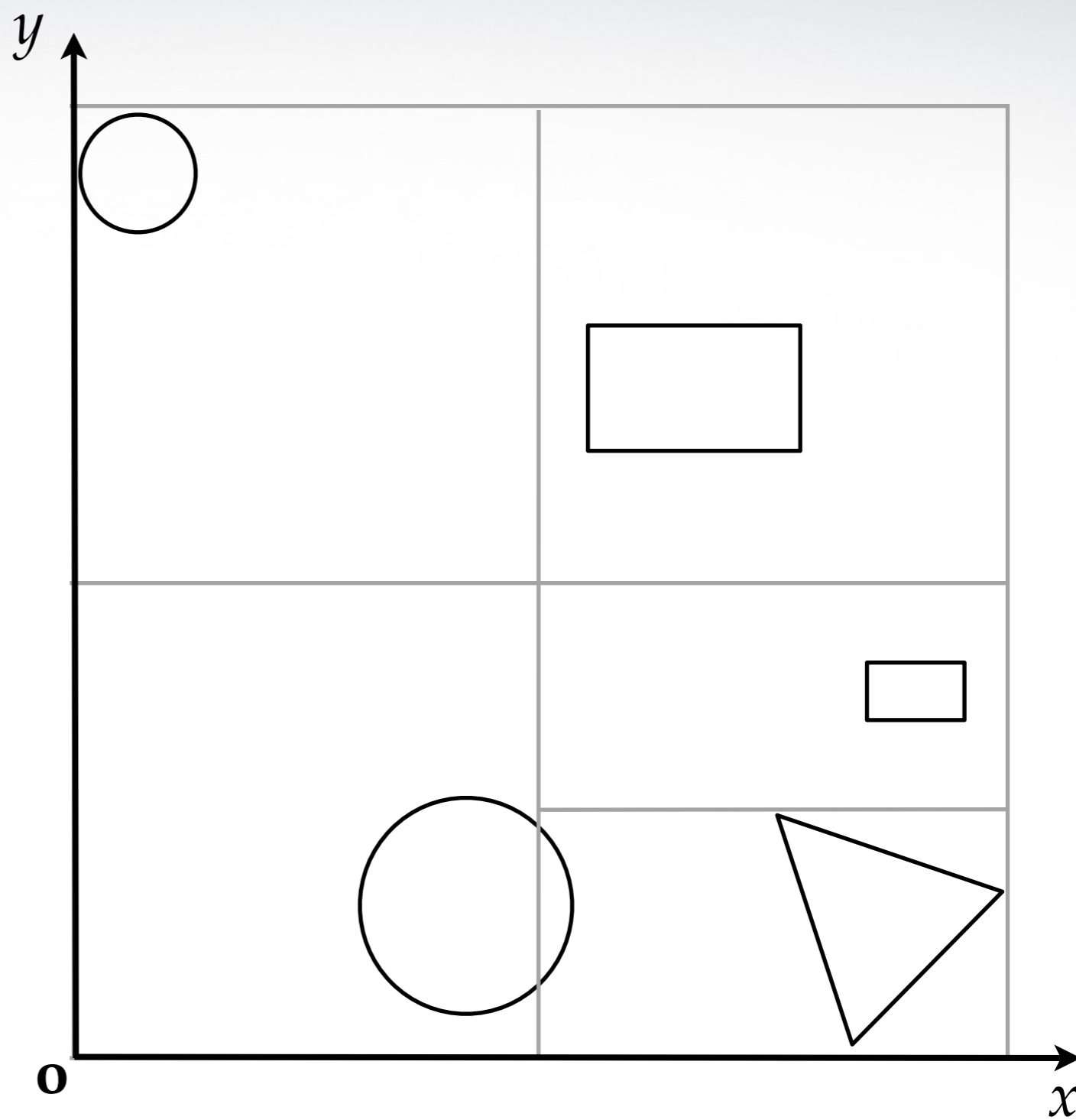
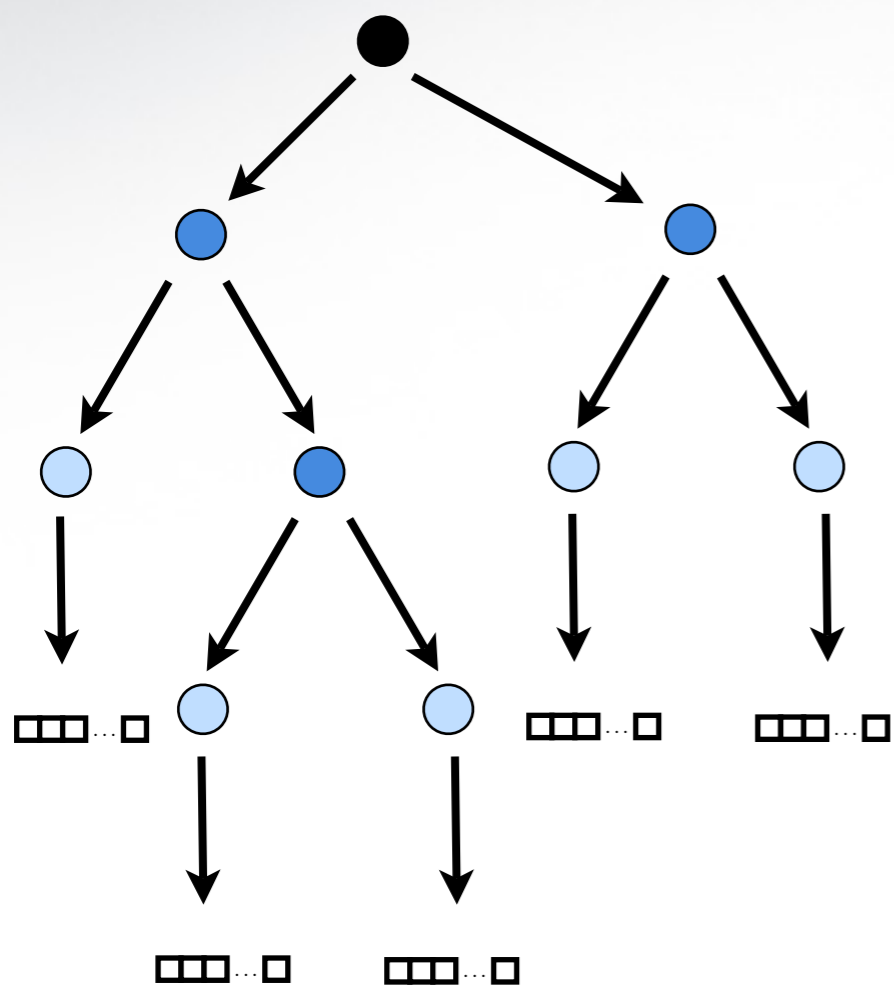


# Goal: Reduce number of intersections





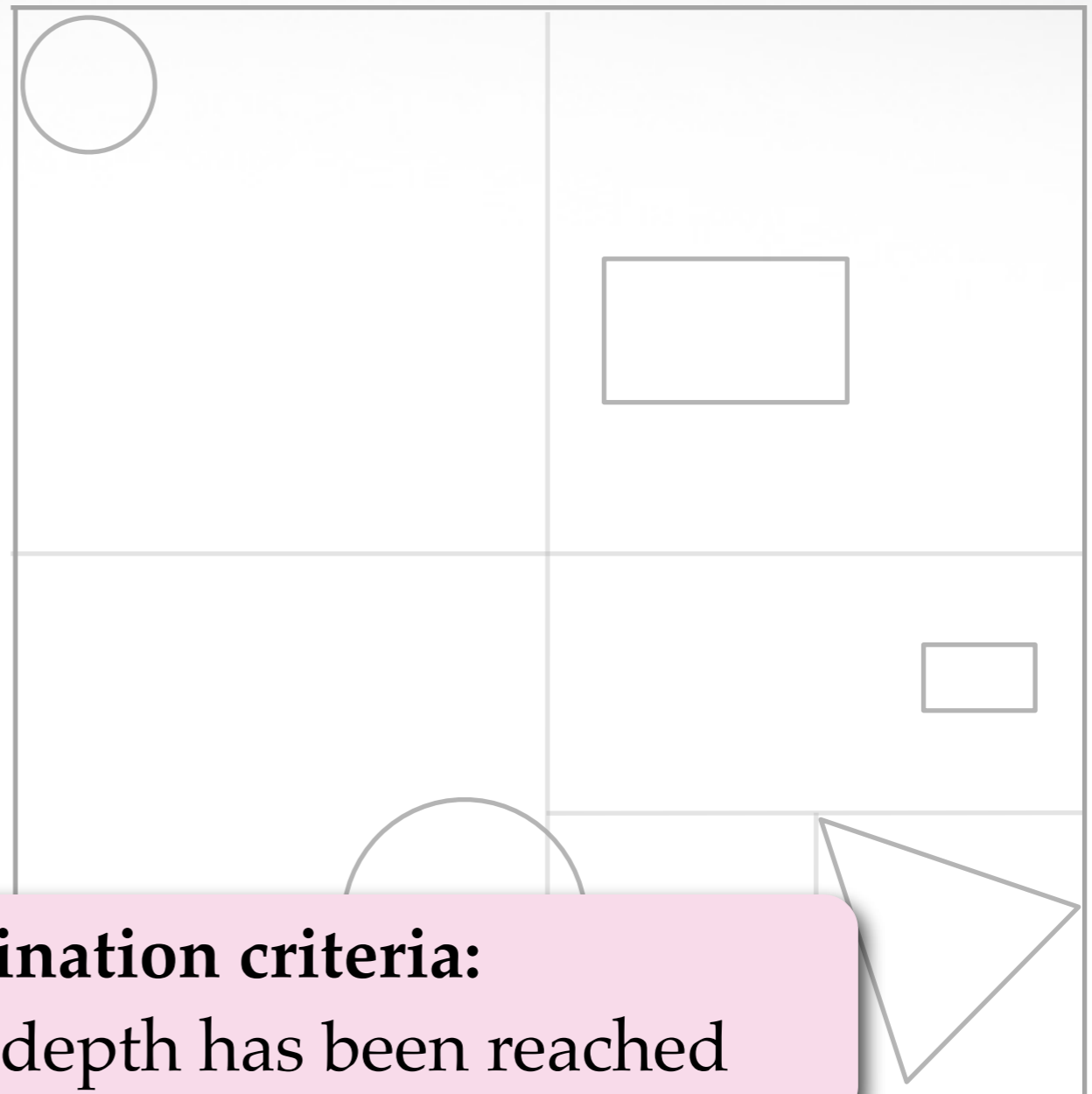
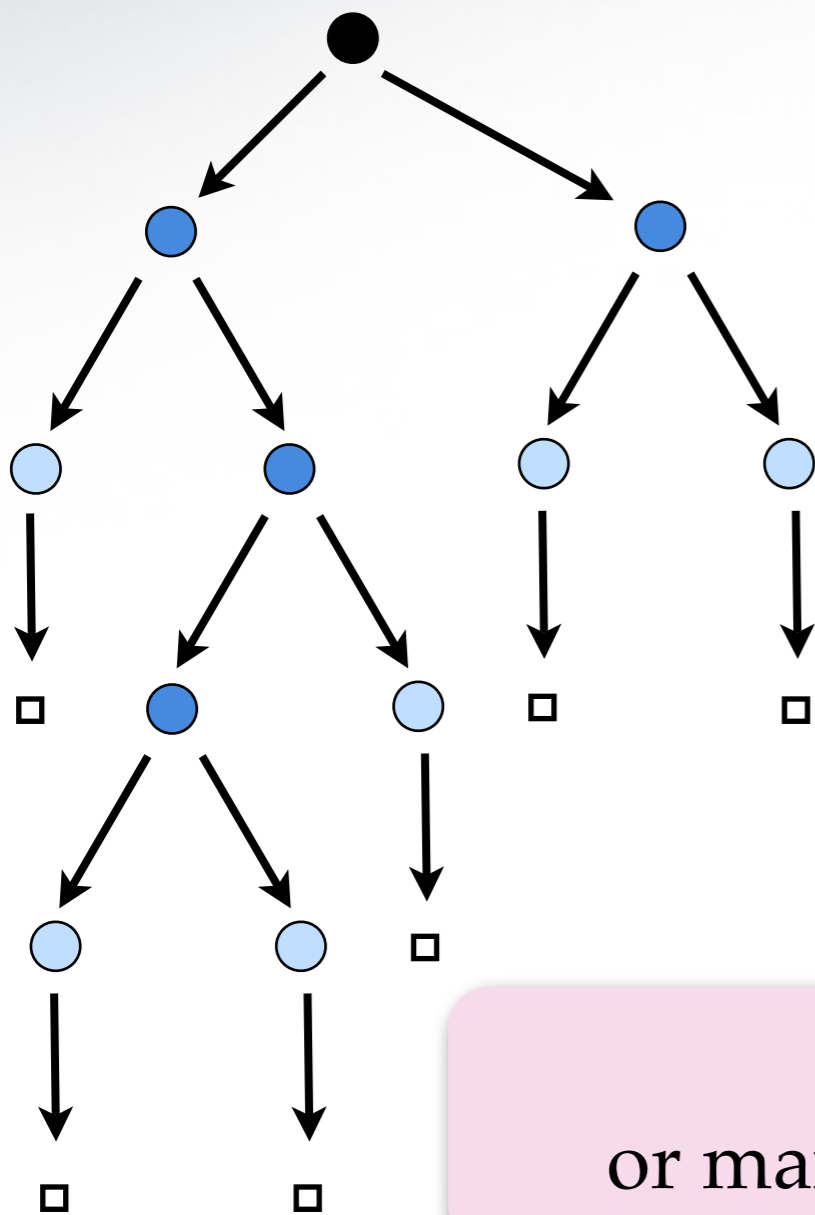






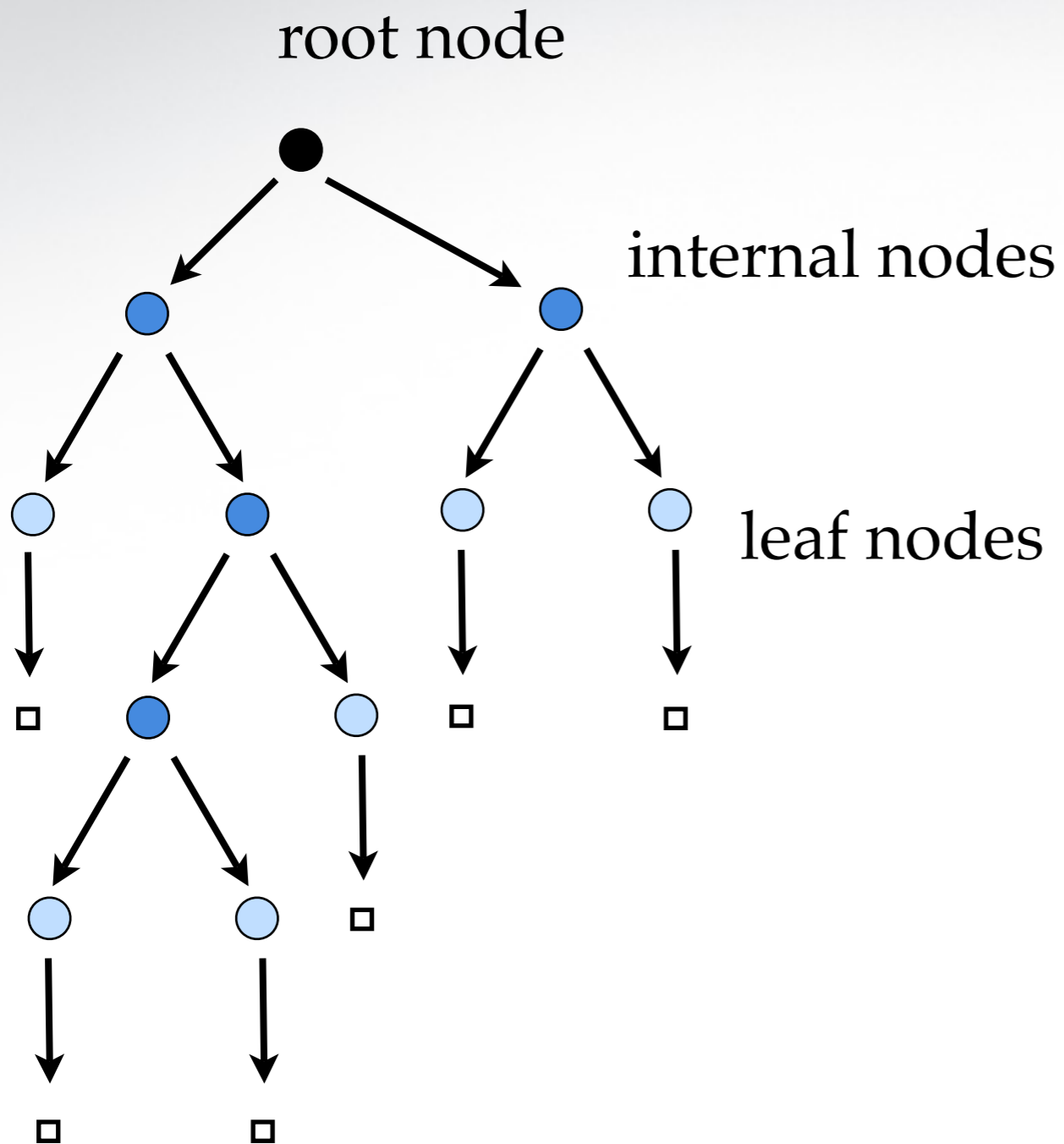






**Termination criteria:**  
or maximum depth has been reached

# binary tree



```
Node * rootNode  
  
Node {  
    ... // data  
  
    Node * leftChildNode  
    Node * rightChildNode  
}
```

All the data we need:

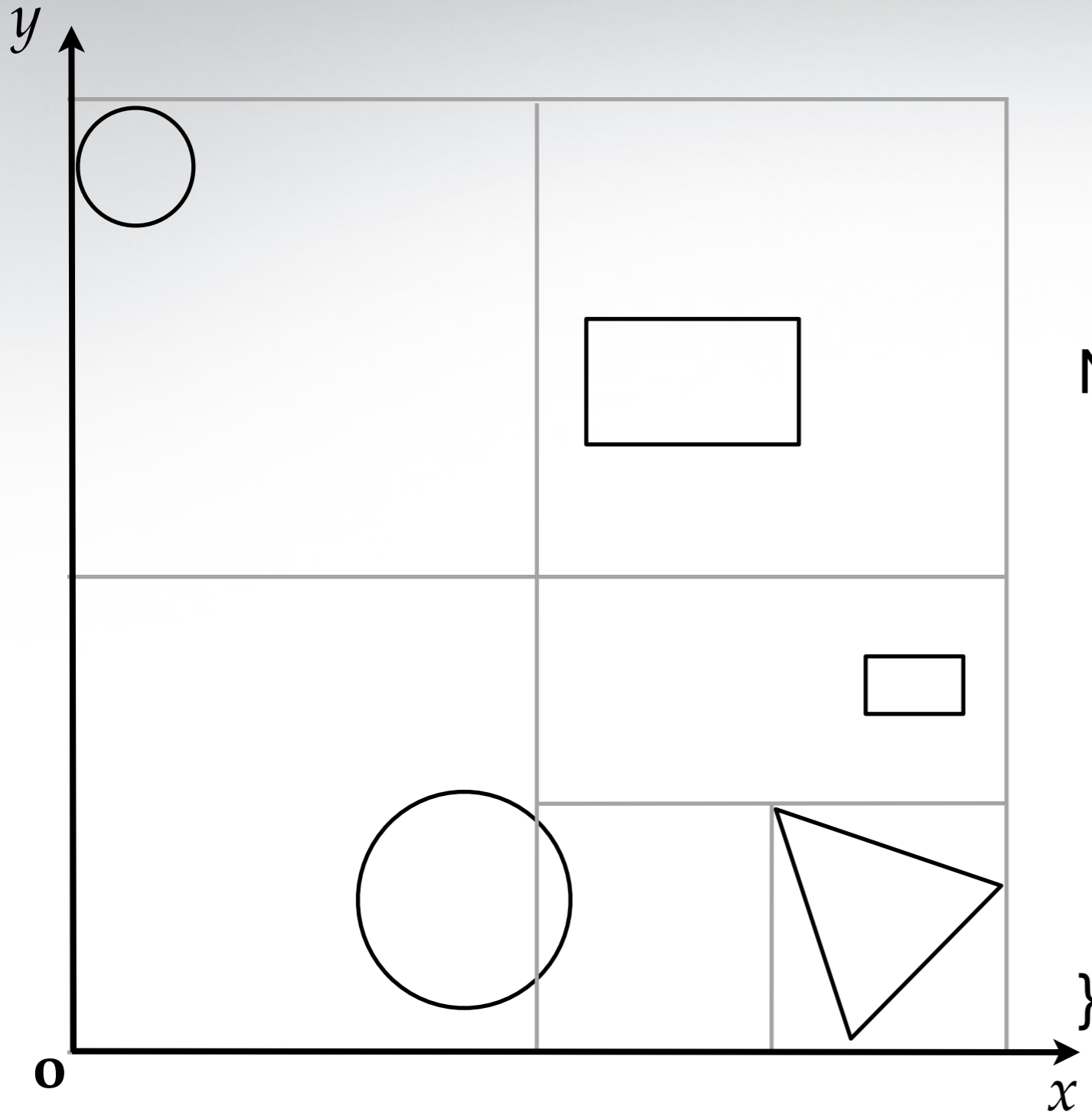
Node \* rootNode

Node {

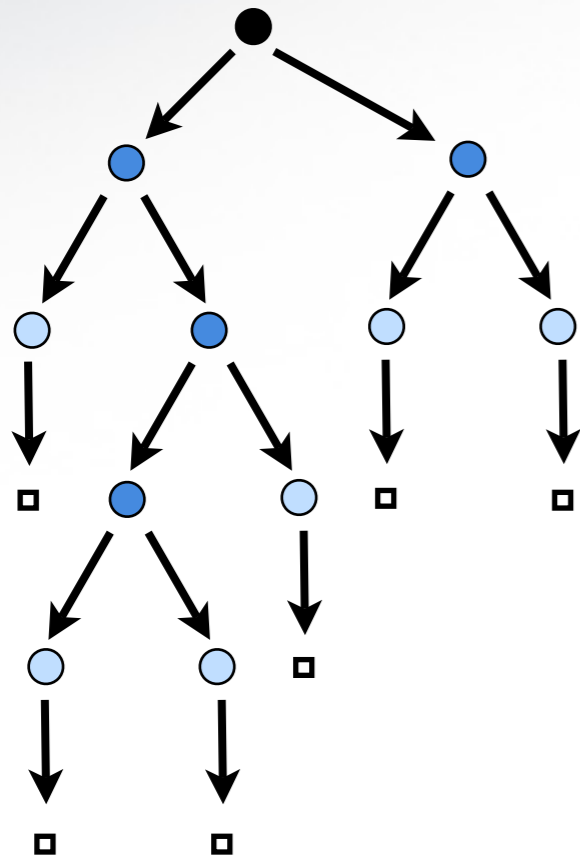
AABB boundingBox  
axis splittingAxis  
double splittingCoordinate  
list pointersToPrimitives(if leaf)

Node \* leftChildNode

Node \* rightChildNode



How do we build it?

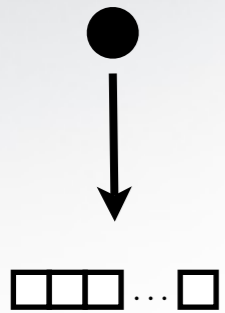


procedure buildKDTree(Scene, Node, ...)

initScene(scene, rootNode)

subdivideCell(scene, node, ...) //recursive

# Initialization



procedure initKDTree(Scene, Node)

```
rootNode → boundingBox = computeBB(...)
           splittingAxis = X
           splittingCoordinate = midPoint(...)
           leftChildNode = NULL
           rightChildNode = NULL
```

for each primitive P in scene

```
rootNode → pointersToPrimitives.add(P)
```



# Recursion

```
procedure subdivideCell(Scene, Node, ...)
```

```
if(!terminateConstruction(node, ...))
```

```
    currentDepth++
```

```
    computeSplittingCoordinate(node)
```

```
    ... // create children
```

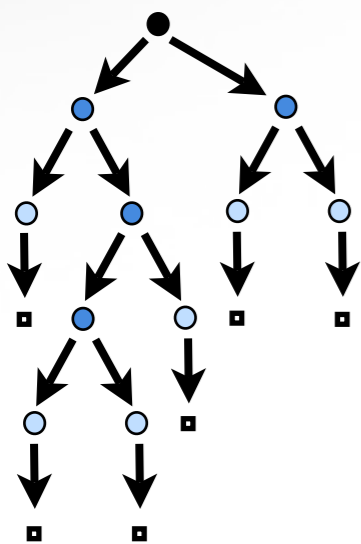
```
    ... // initialize children
```

```
        node→leftChildNode.splittingAxis=nextAxis(node)
```

```
    ... // move primitives to children
```

```
    subdivideCell(scene, node→leftChildNode, ...)
```

```
    subdivideCell(scene, node→rightChildNode, ...)
```



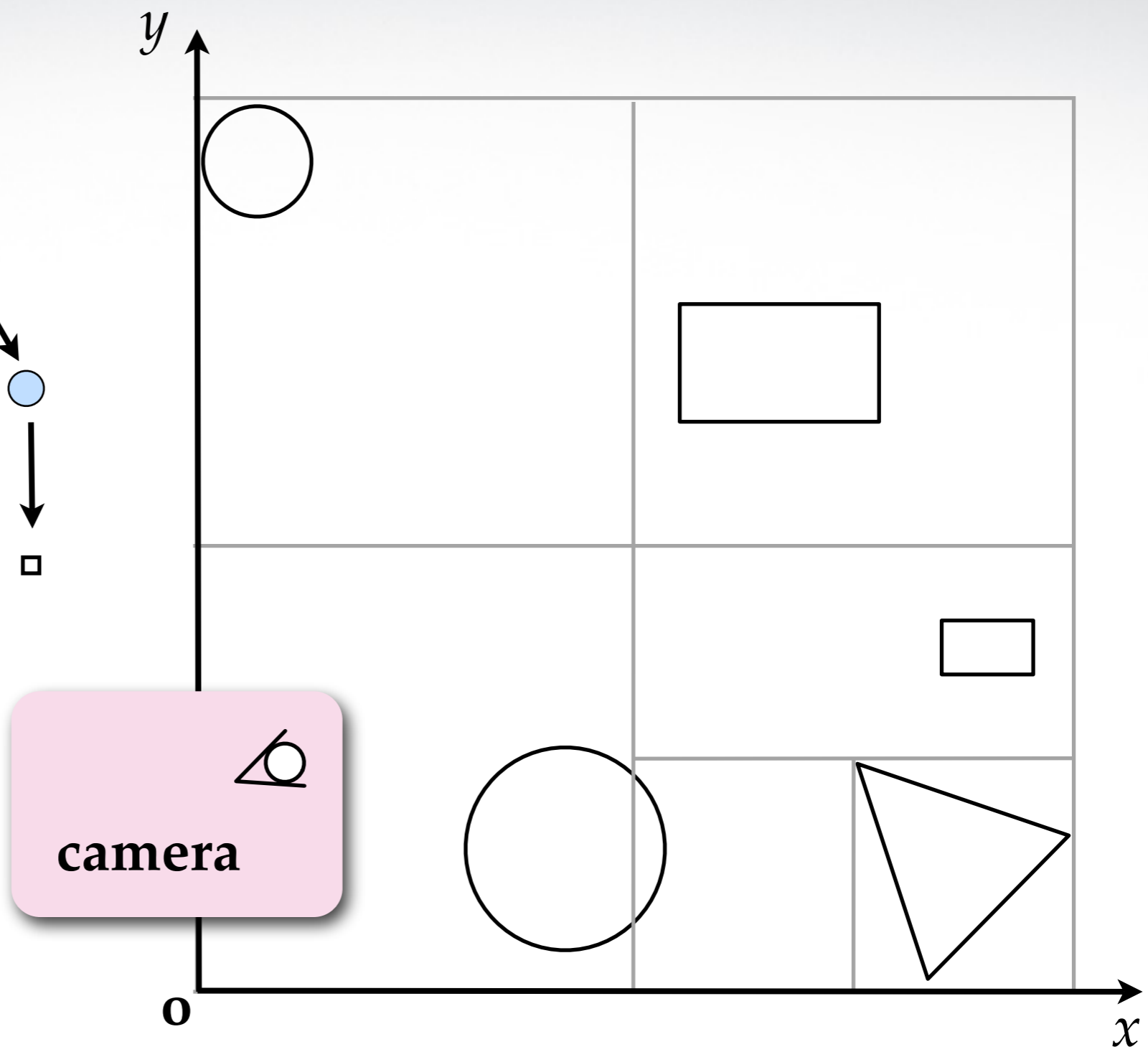
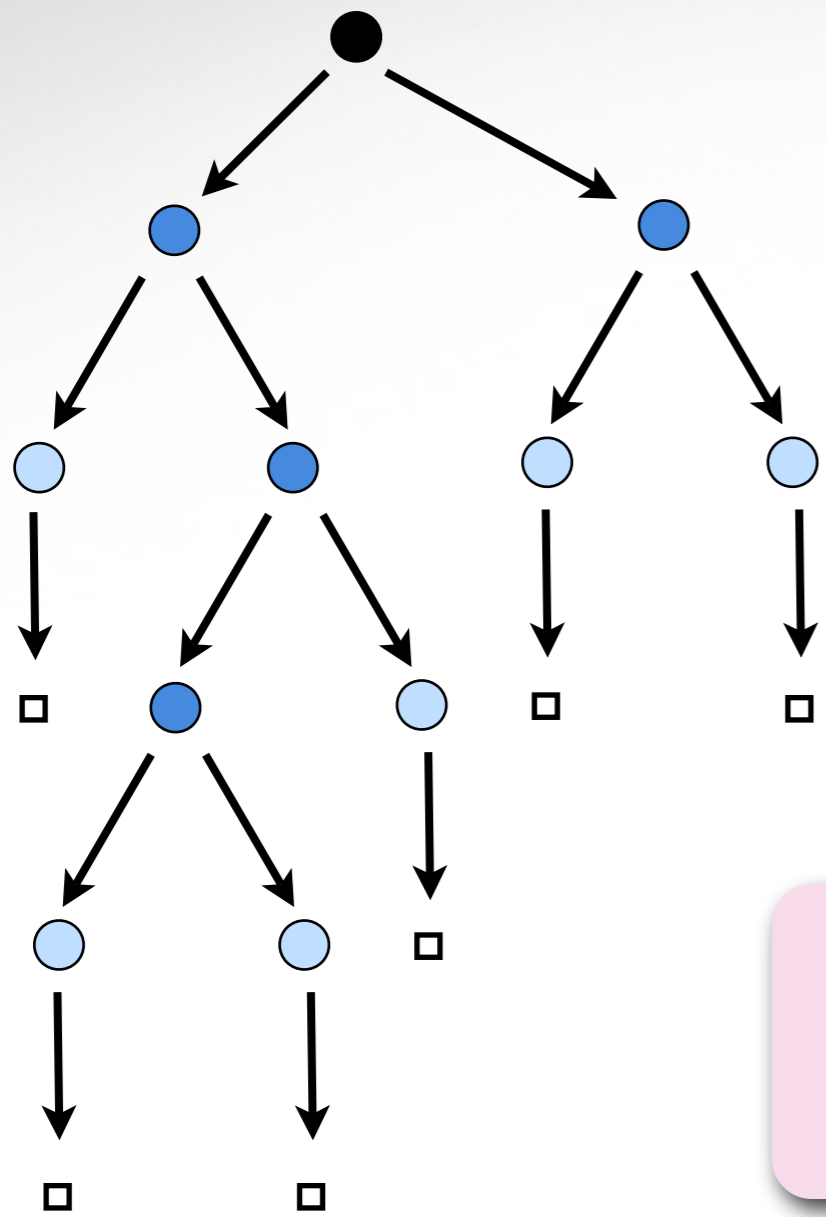
# Termination

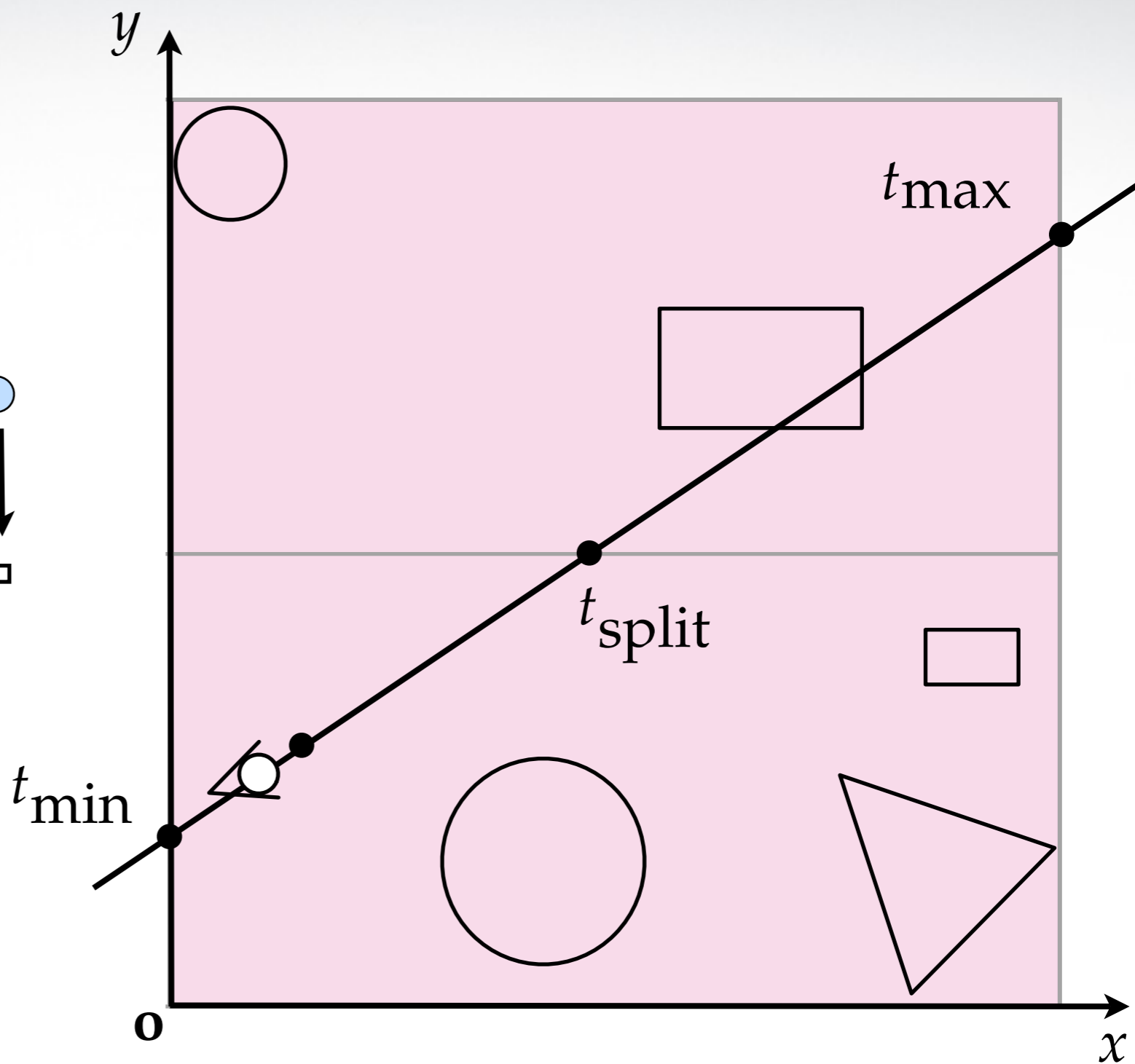
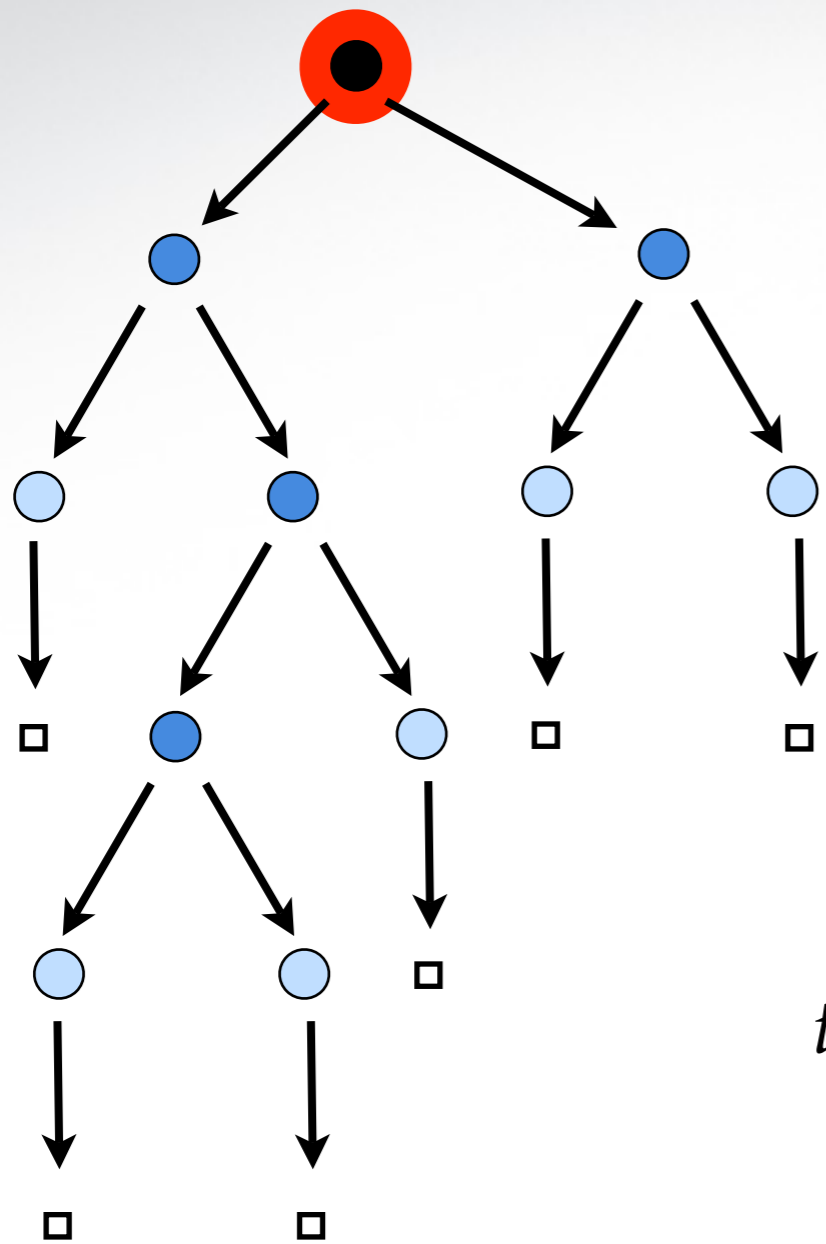
function terminateConstruction(node, ...)

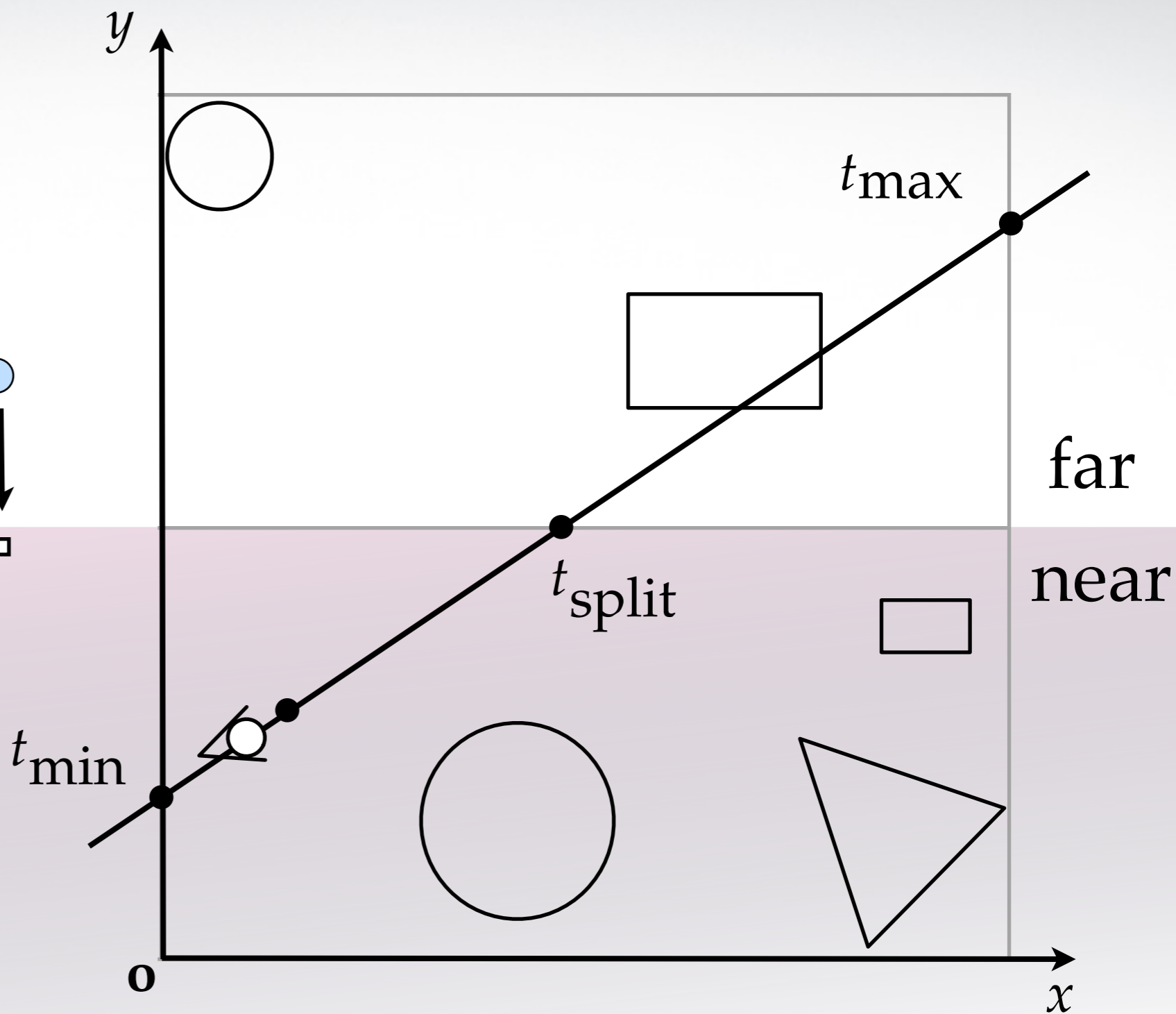
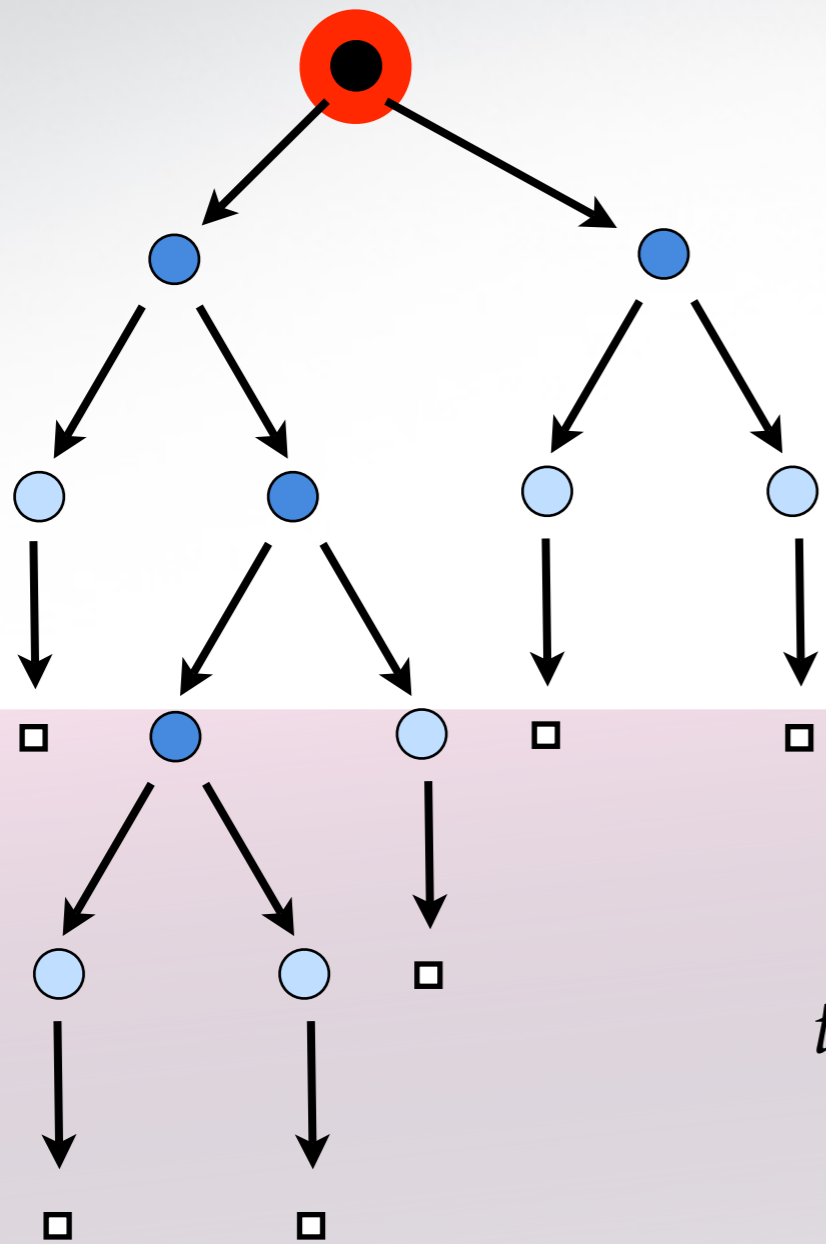
```
if (currentDepth == maxDepth) or  
    ... // too few primitives in node  
    return true  
else  
    return false
```

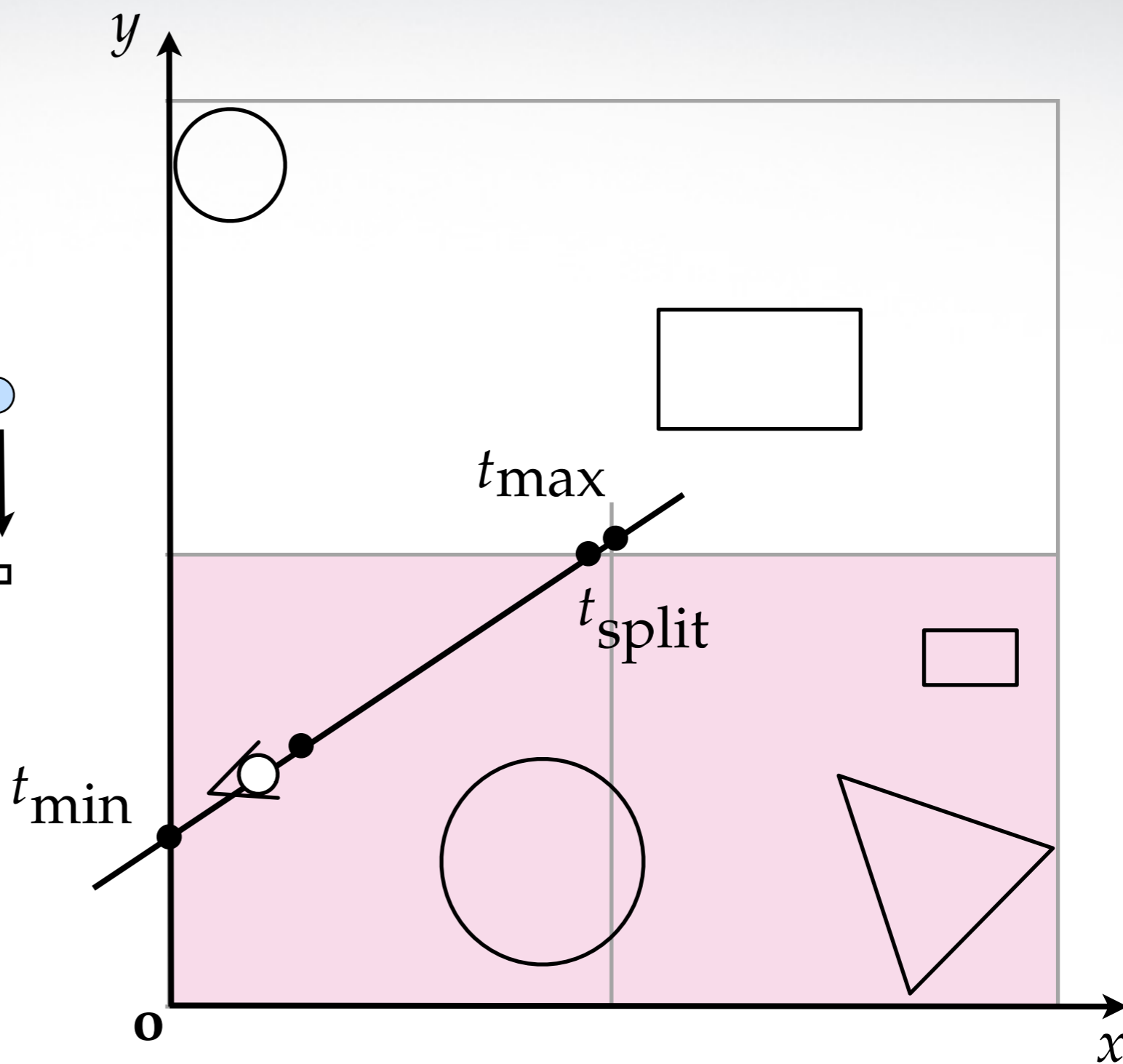
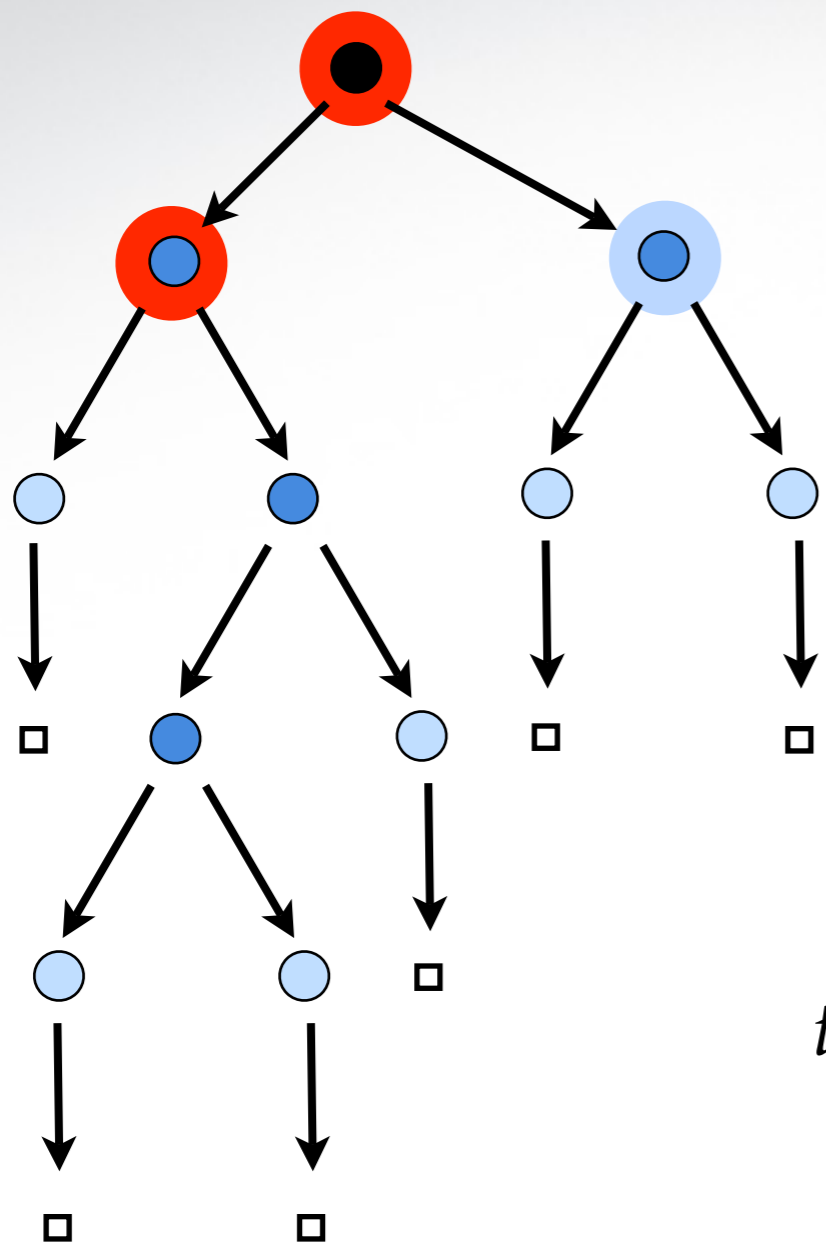
The cells of the *kd*-tree localize the **relevant primitives** for the intersections

Traversing the *kd*-tree  
is fast...

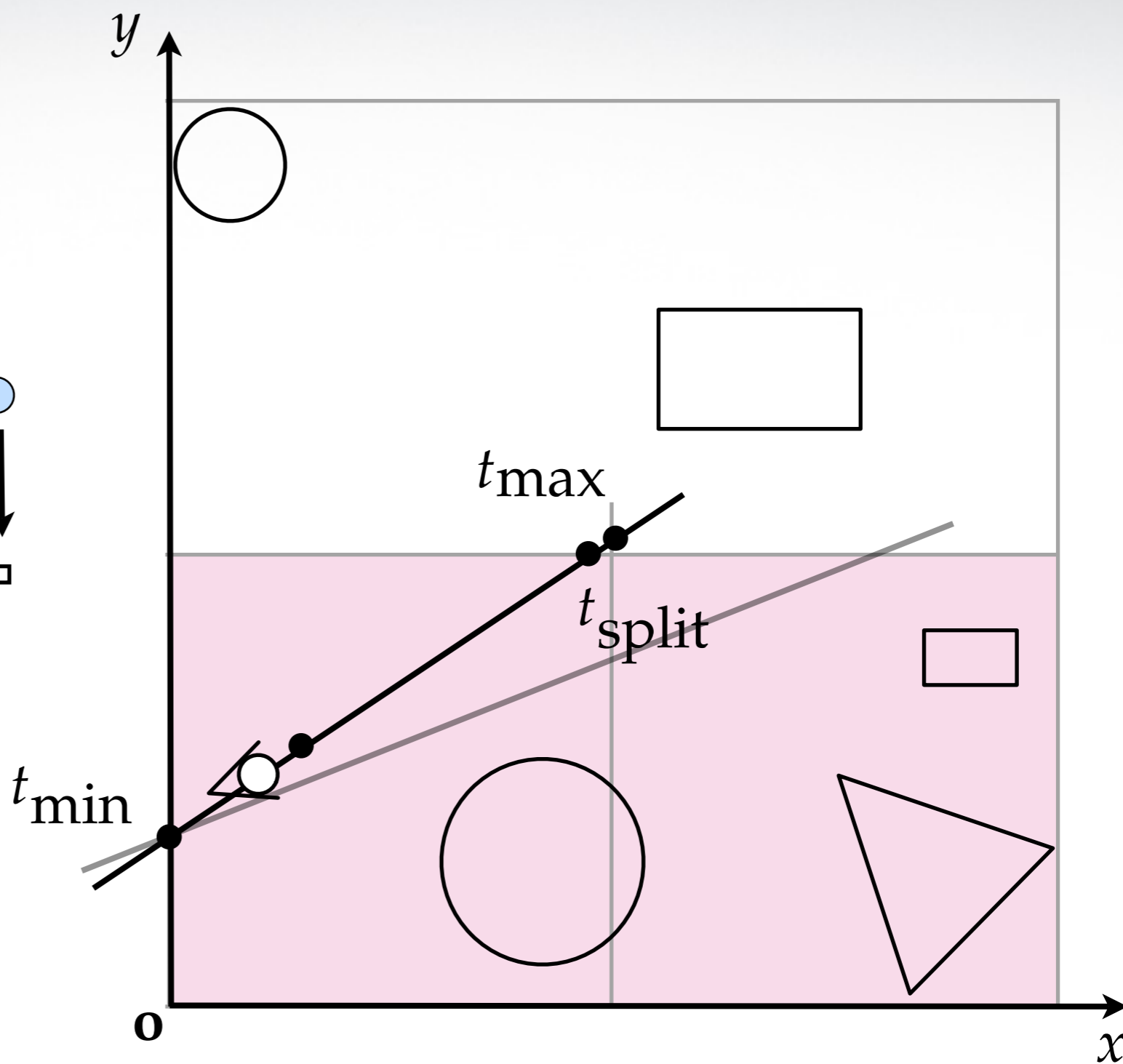
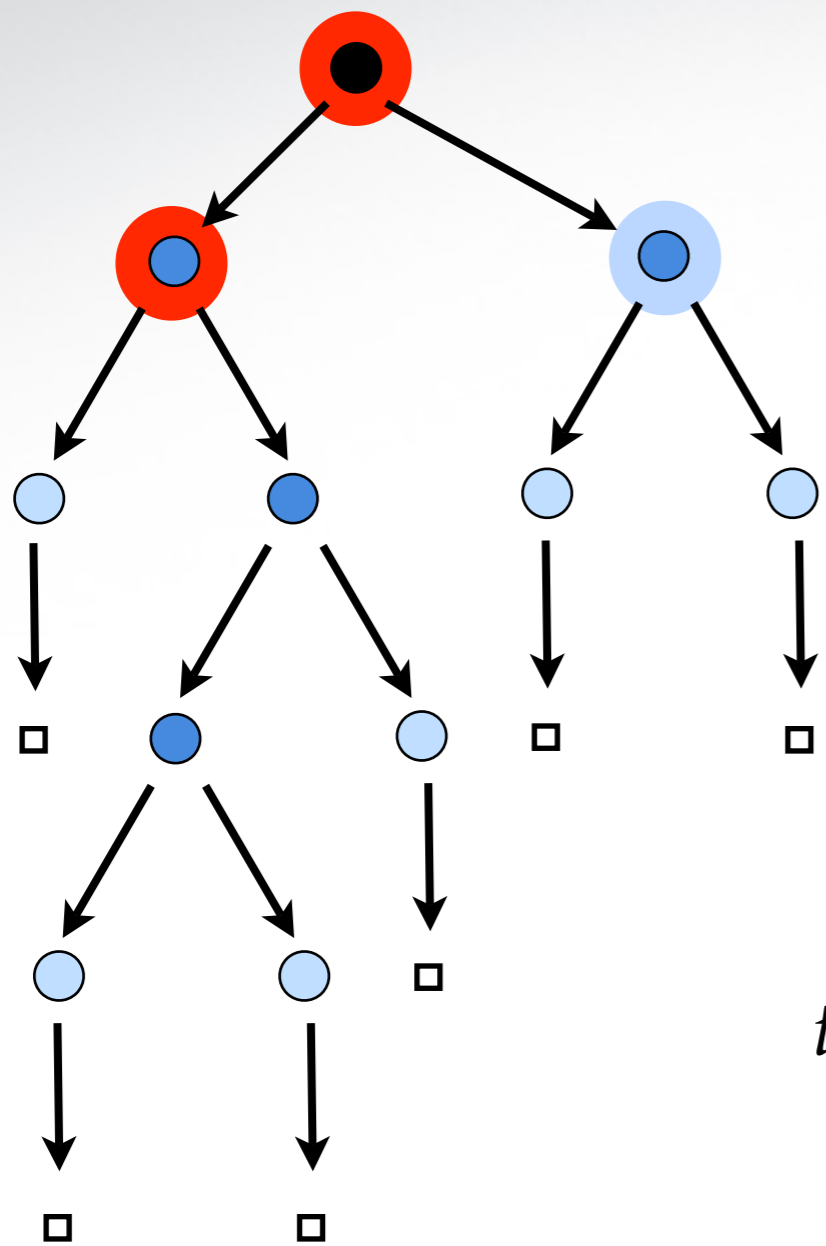


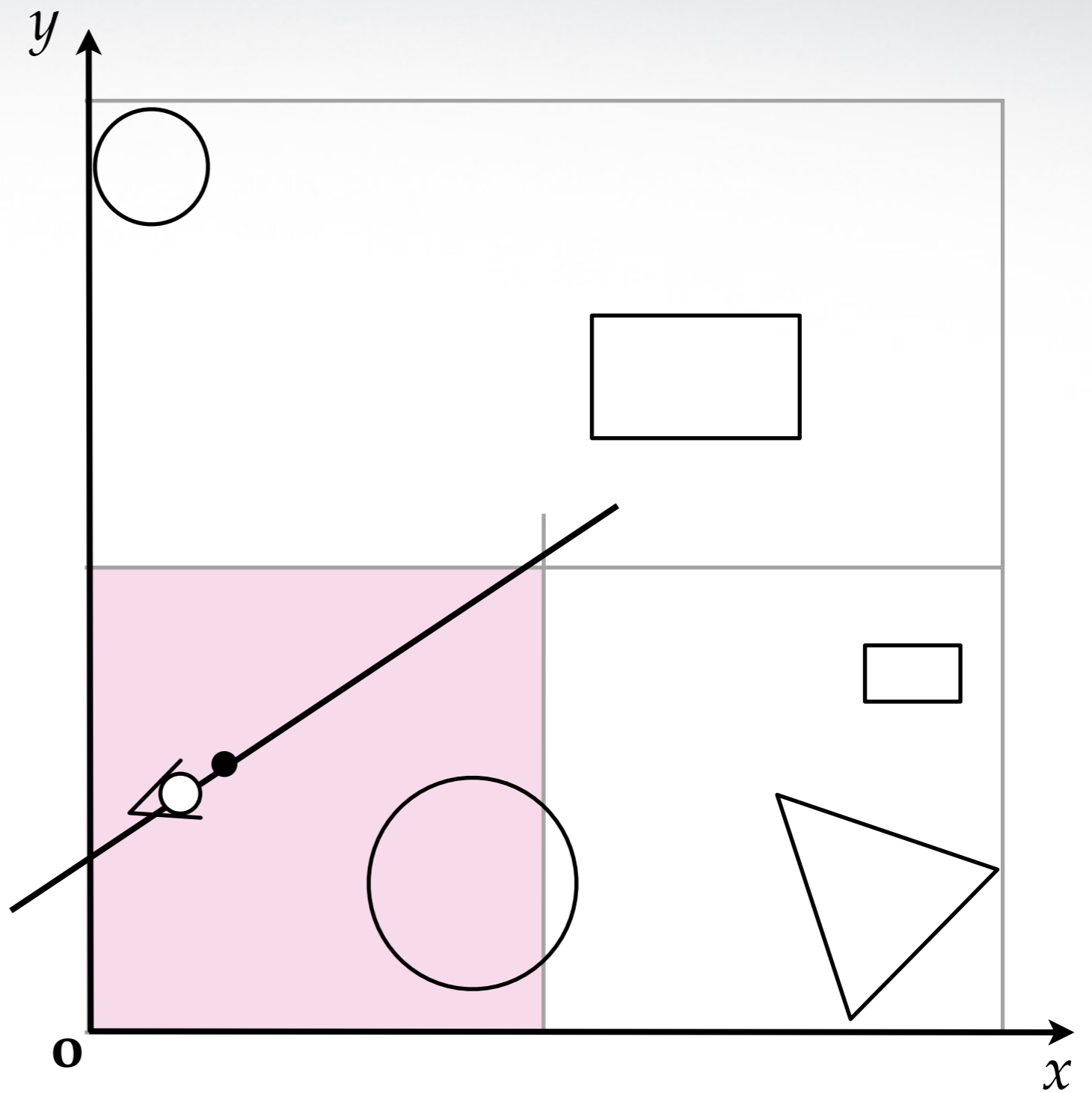
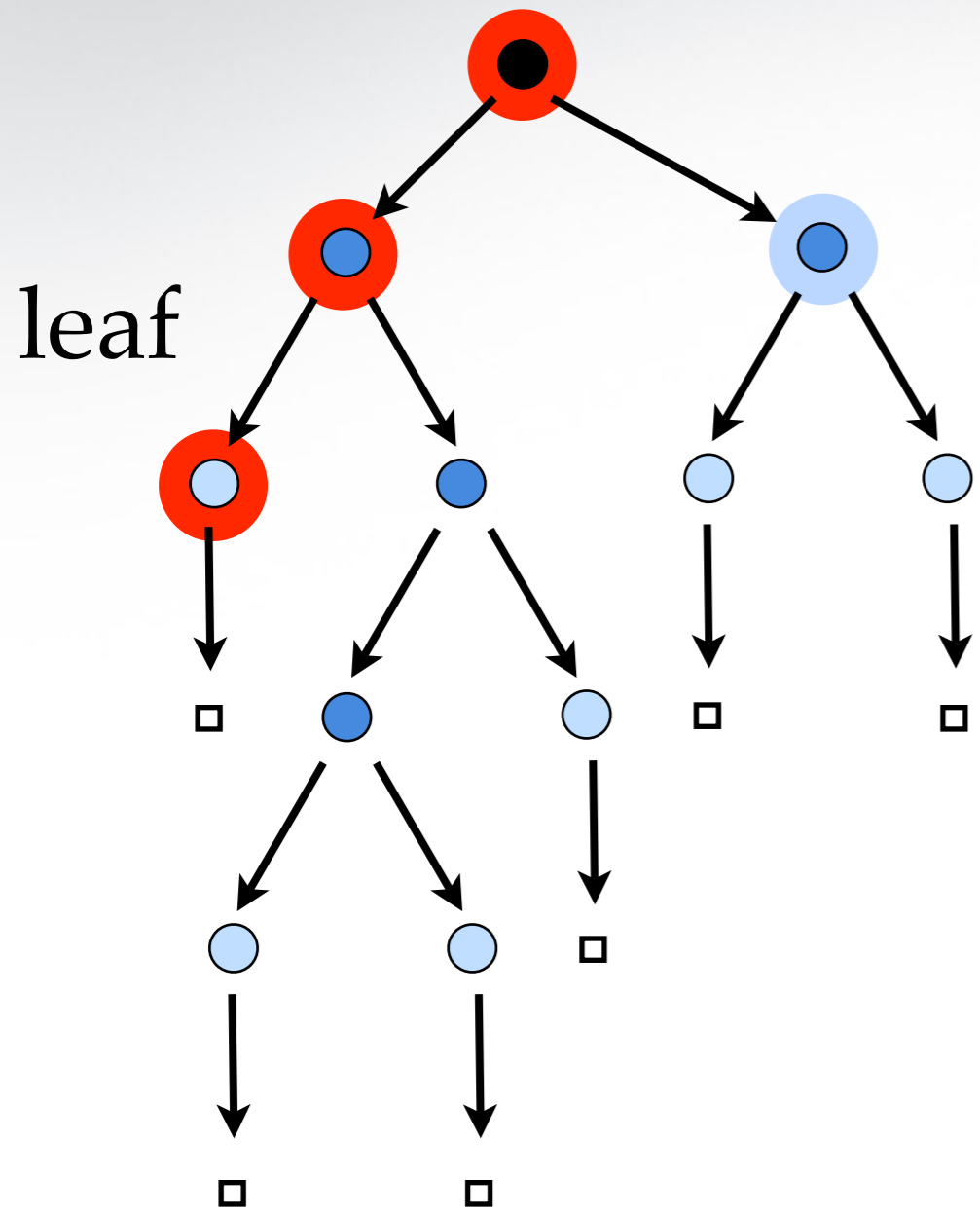


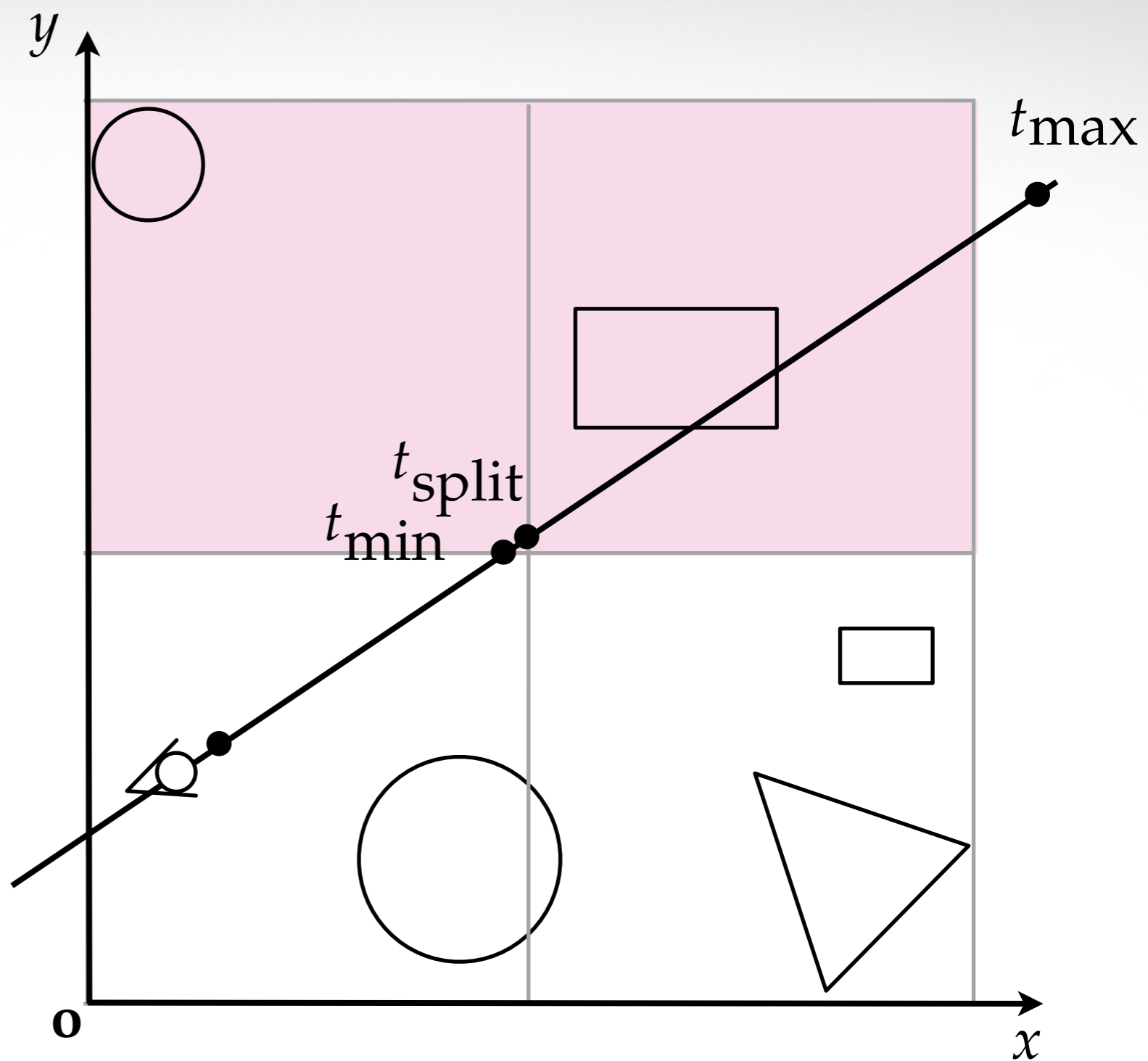
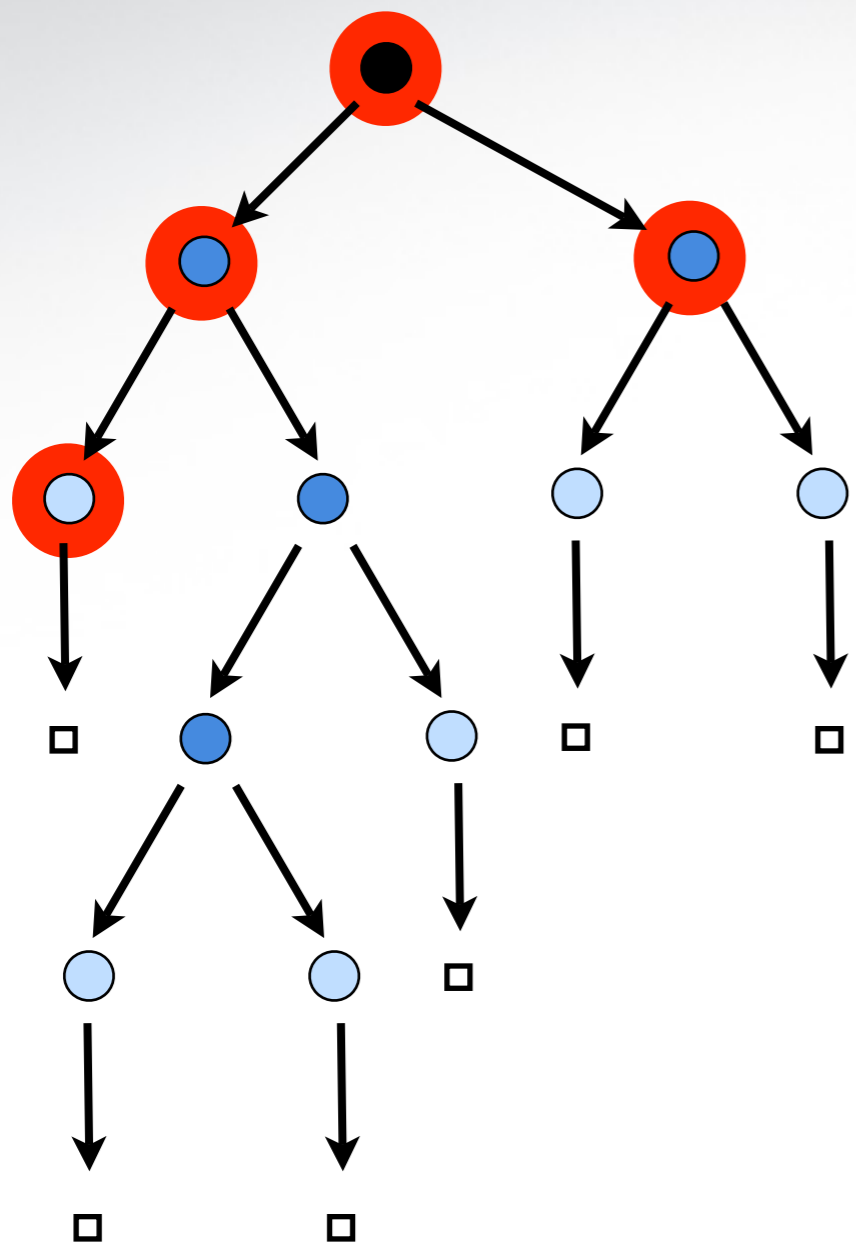


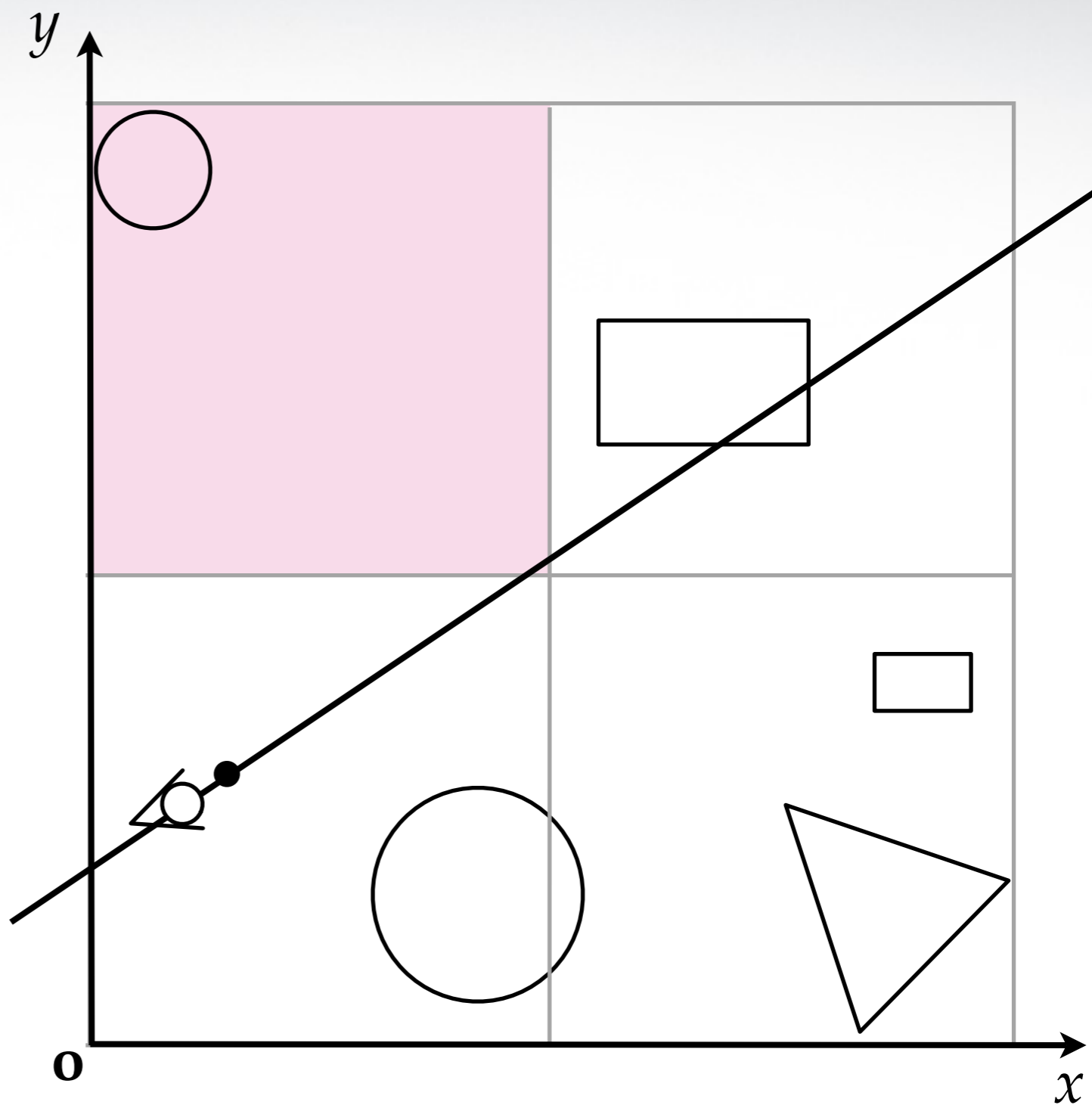
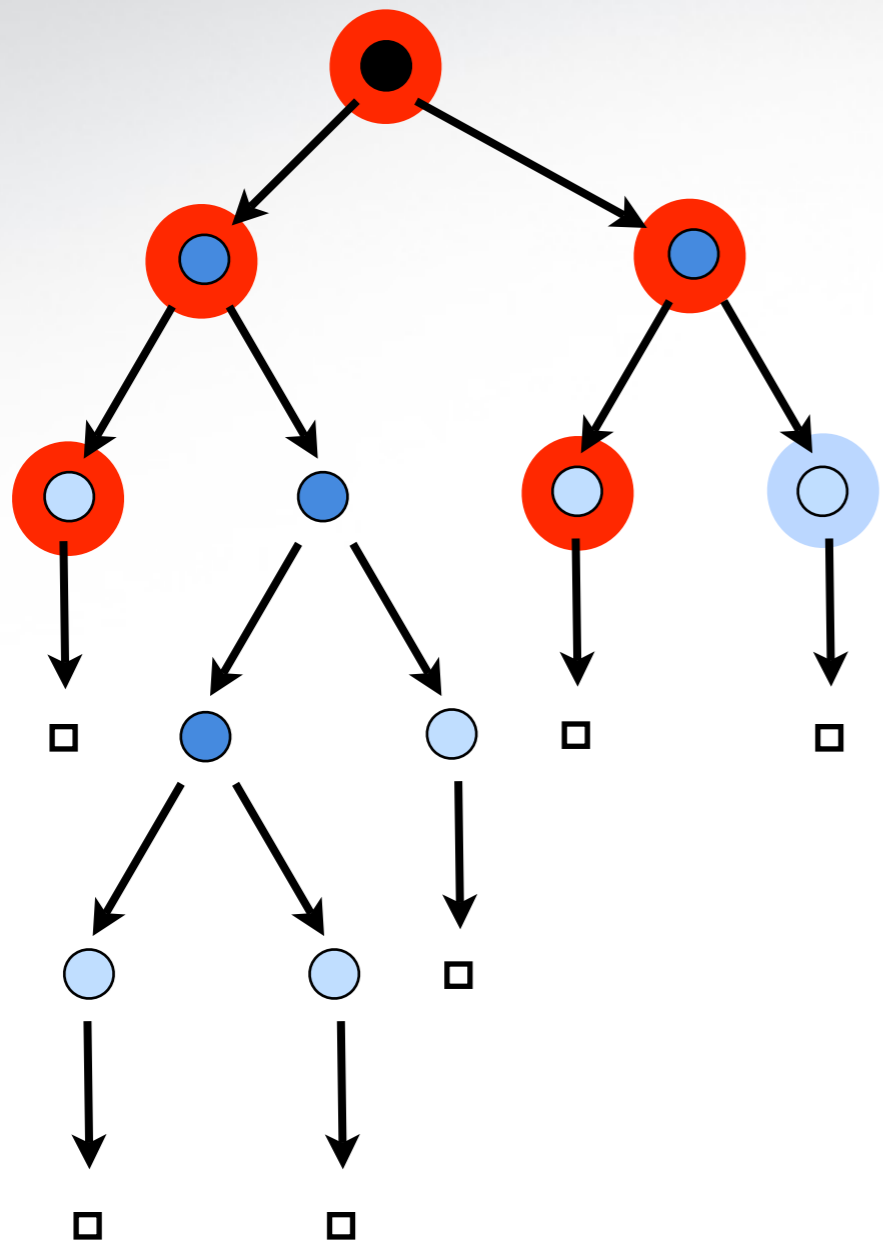




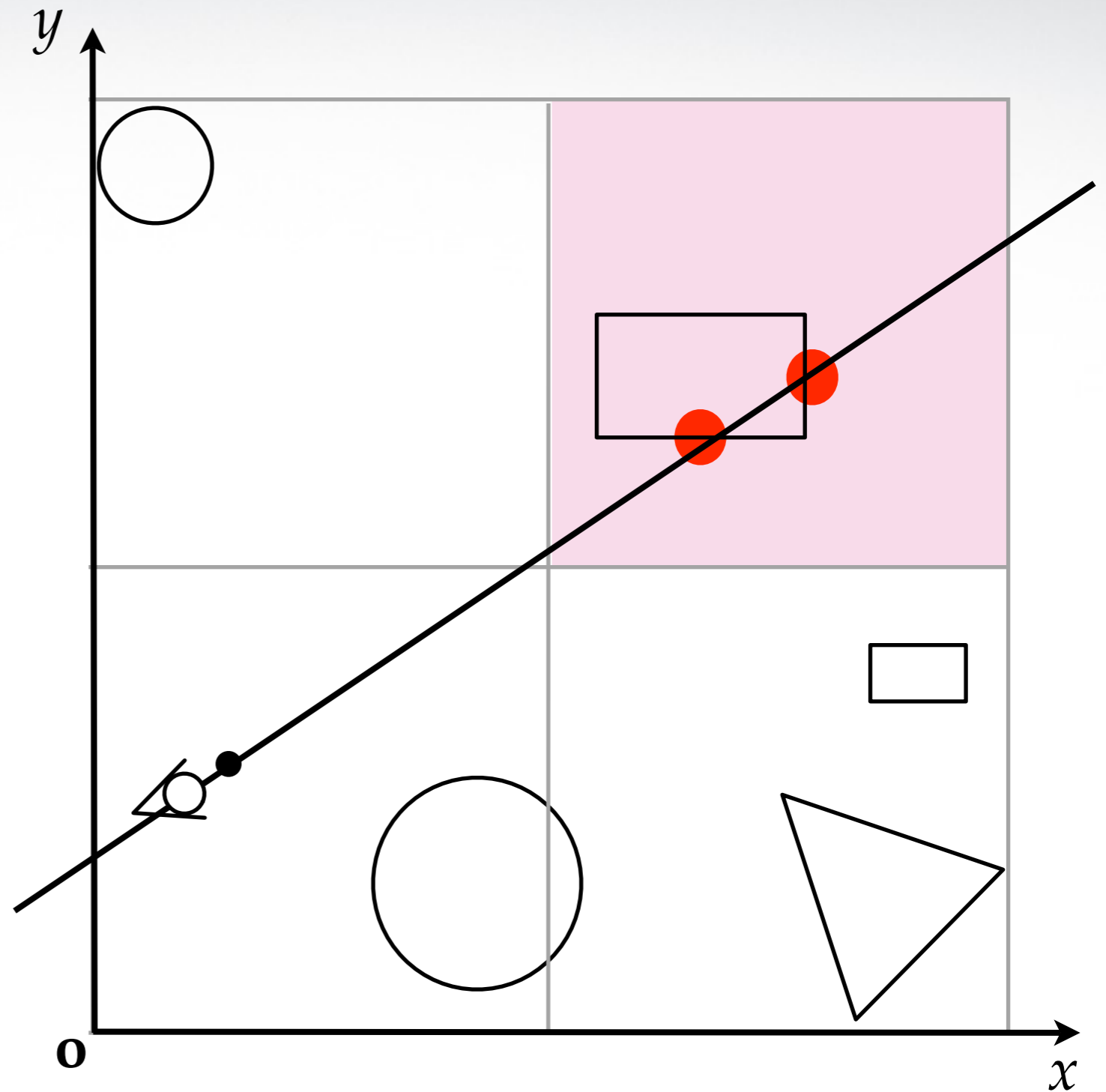
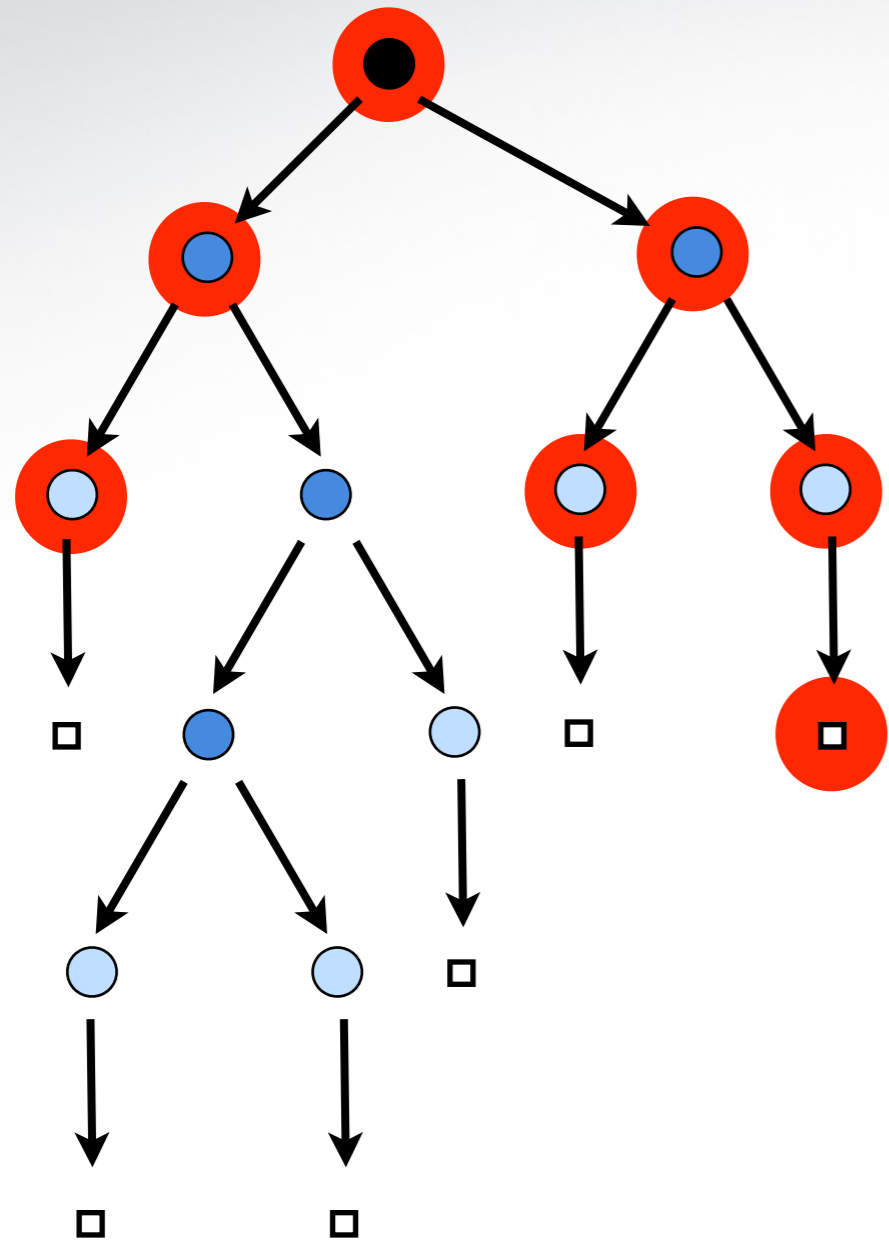








# Intersections found!

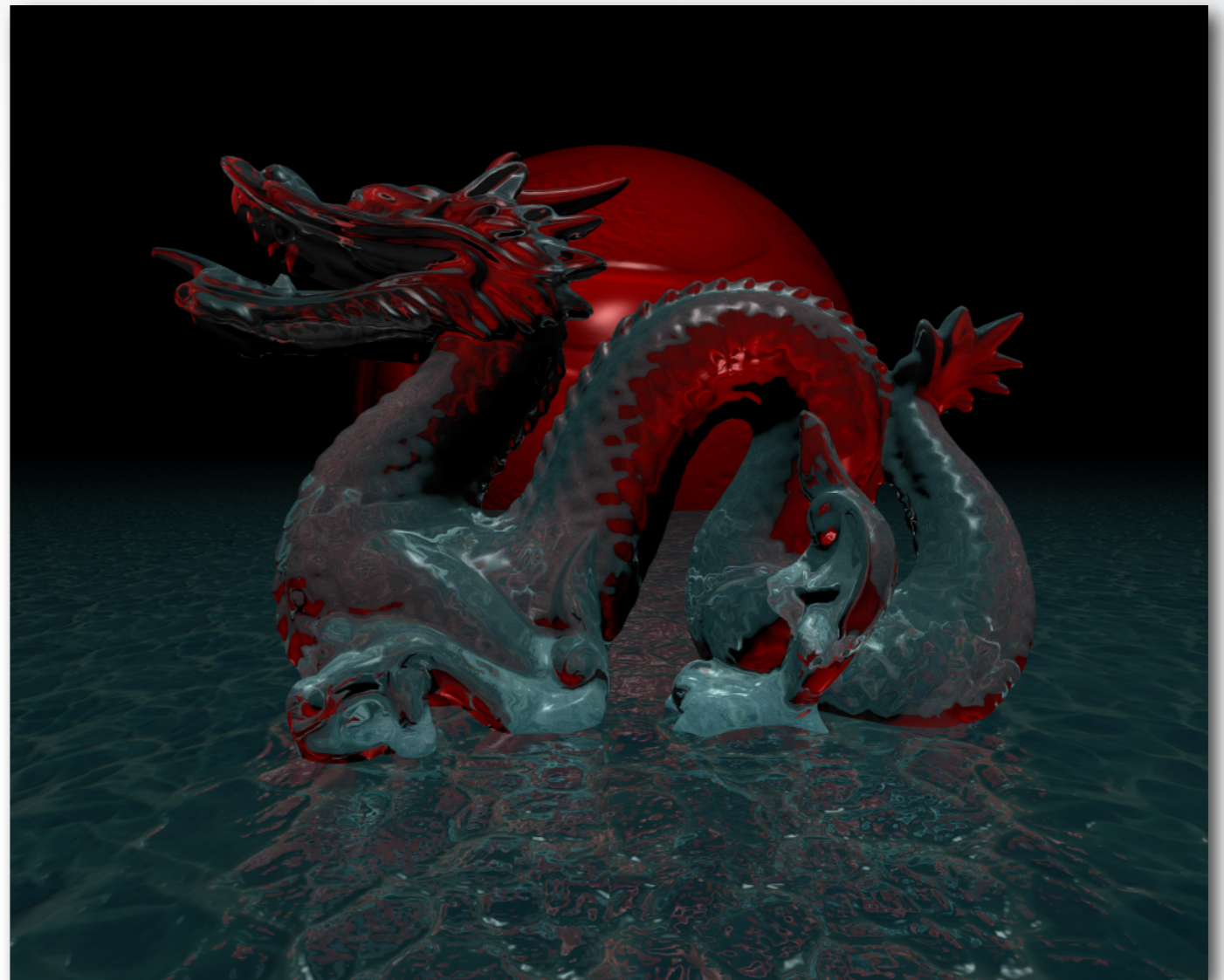


# Average Speed-up

$$O(n_x n_y n_t)$$

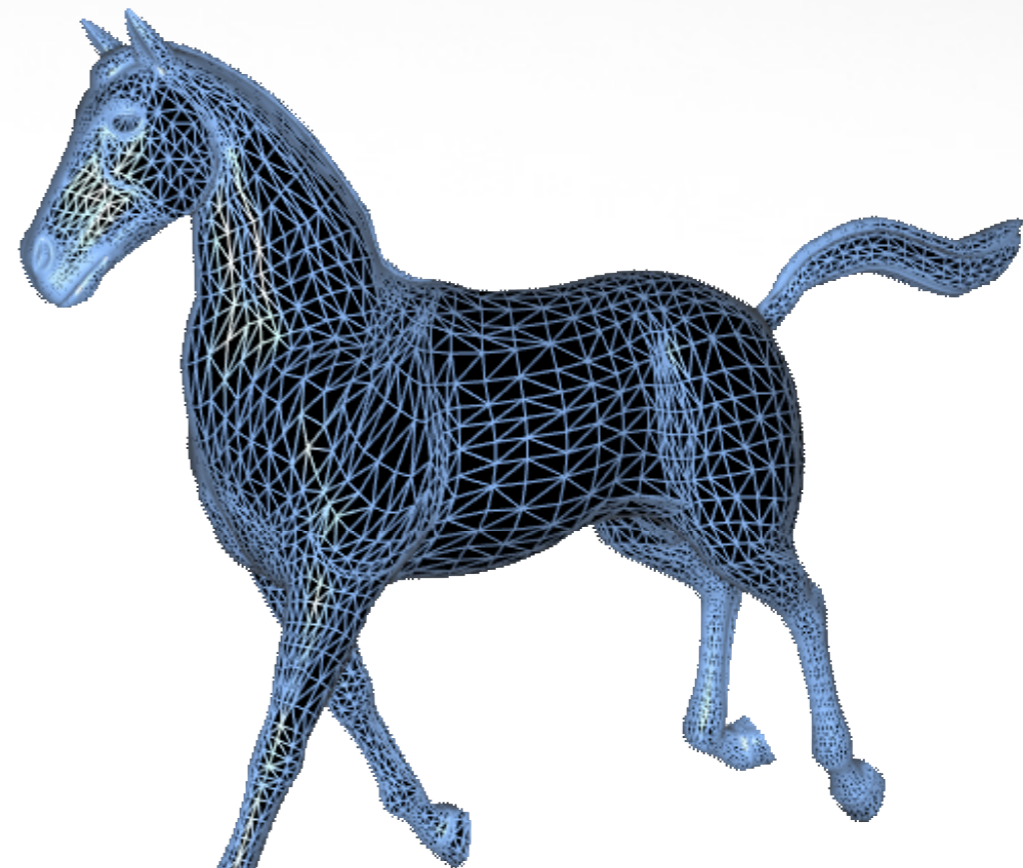
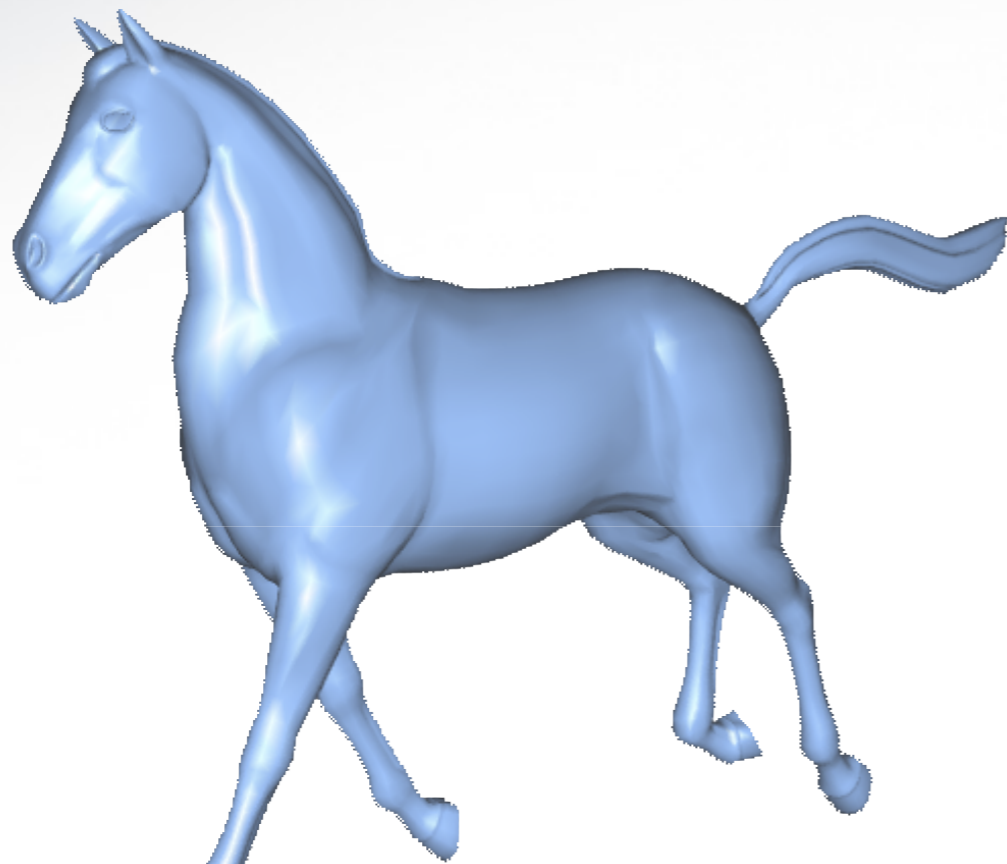


$$O(n_x n_y \log(n_t))$$

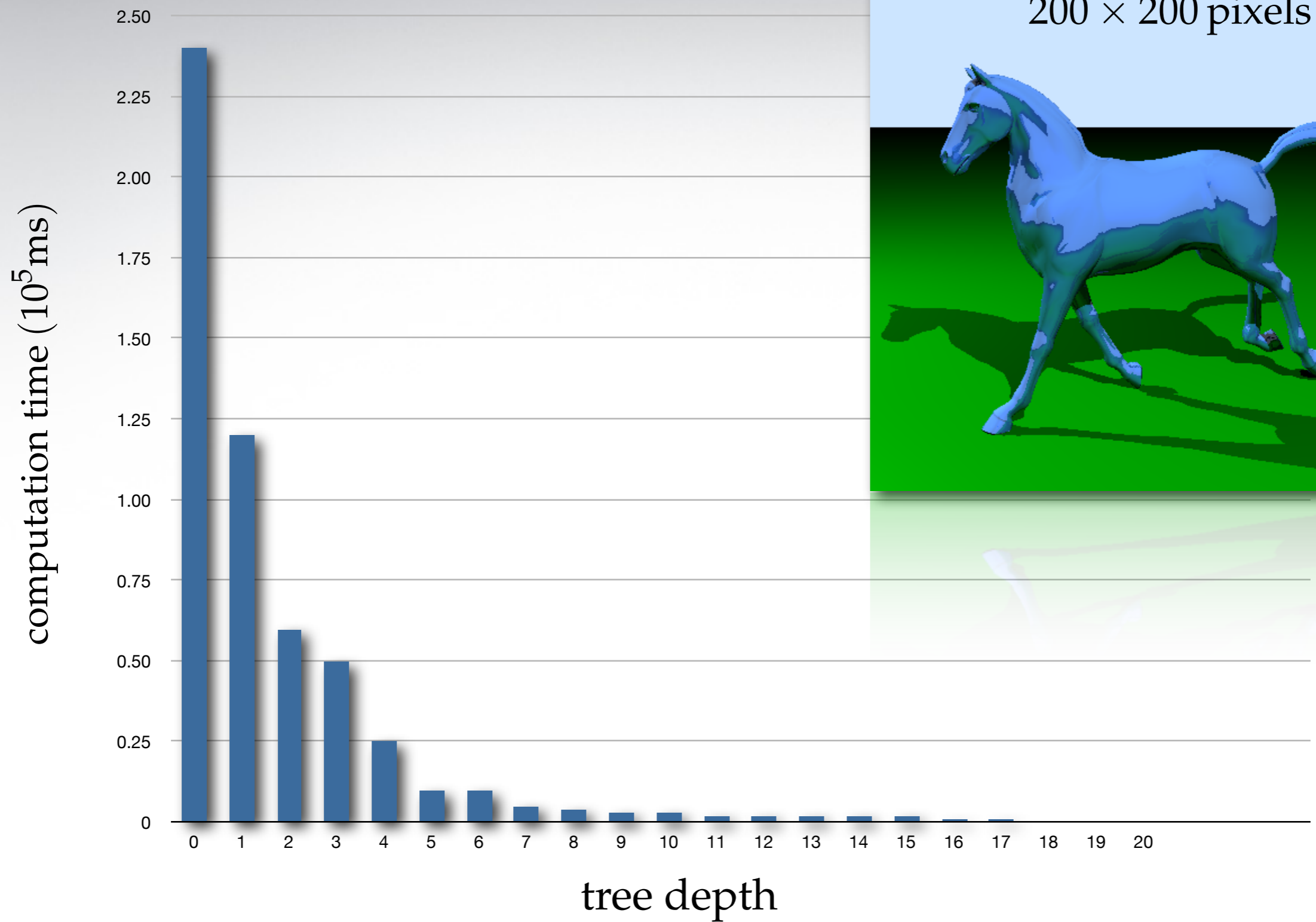


in other words...

16843 faces







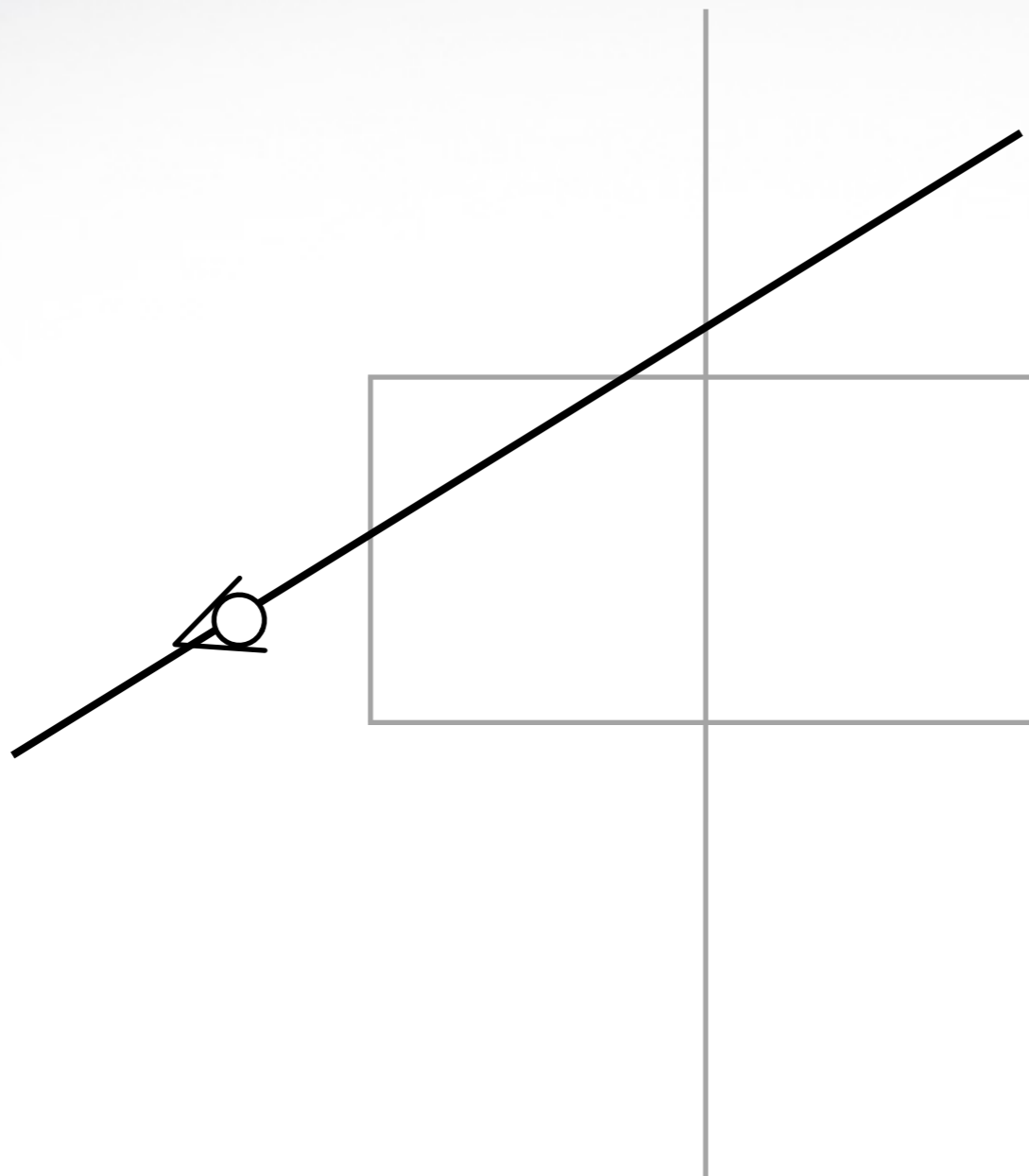
one more thing...

# Pseudo code for traversal [Havran 01]

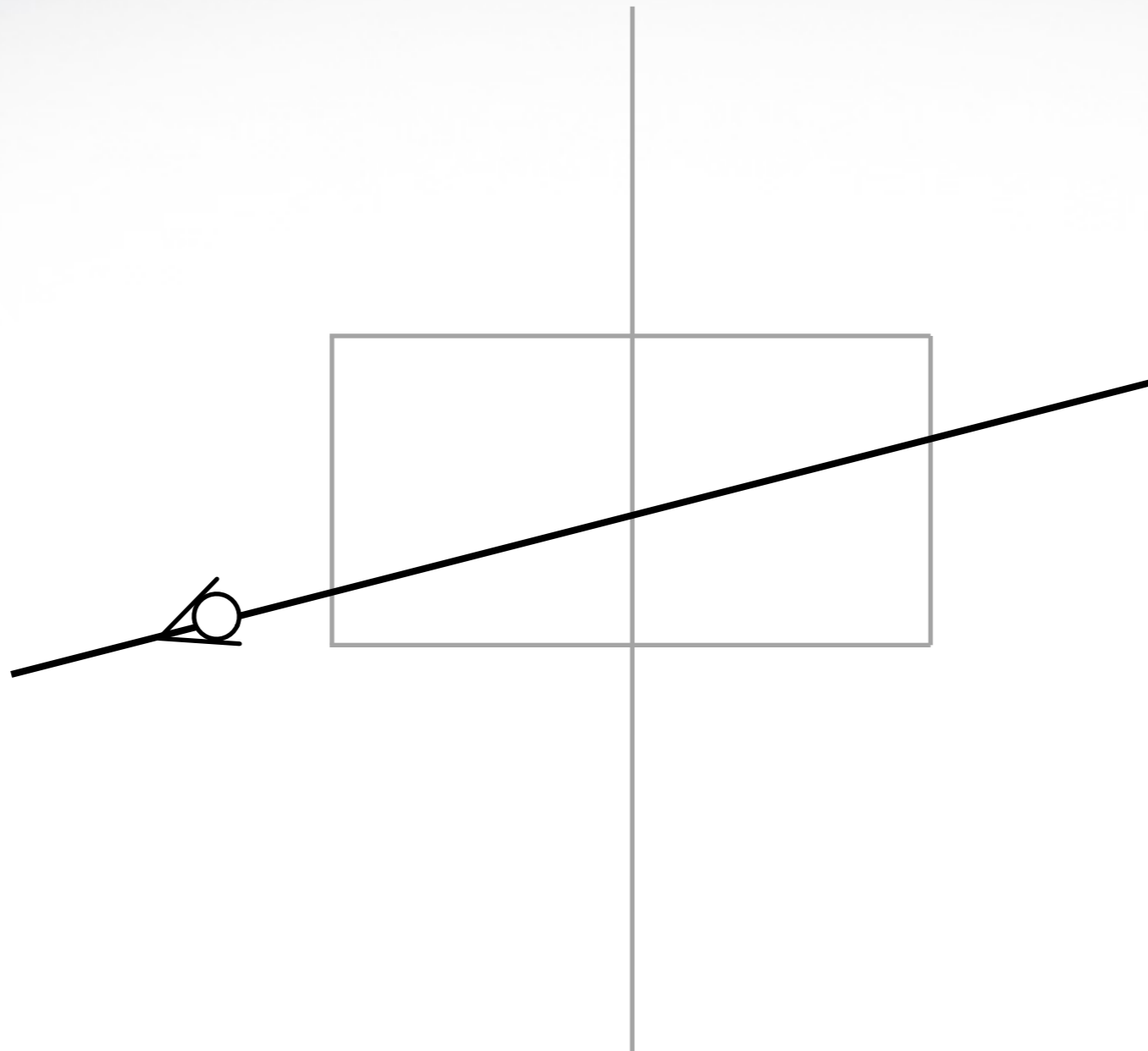
```
function rayTreeIntersection(Ray ray, Node node, double tMin, double tMax)

if node is empty
    return NULL
else
    if node is leaf
        intersect ray with all primitives in node
        return closest primitive
    else
        compute tSplit
        nearNode is child of node of near side
        farNode is child of node of far side
        if (tSplit > tMax)
            return rayTreeIntersection(ray, nearNode, tMin, tMax)
        else if (tSplit < tMin)
            return rayTreeIntersection(ray, node containing tMin and tMax, tMin, tMax)
        else
            intersectedPrimitive = rayTreeIntersection(ray, nearNode, tMin, tSplit)
            if (intersectedPrimitive not NULL)
                return intersectedPrimitive
            else
                return rayTreeIntersection(ray, farNode, tSplit, tMax)
```

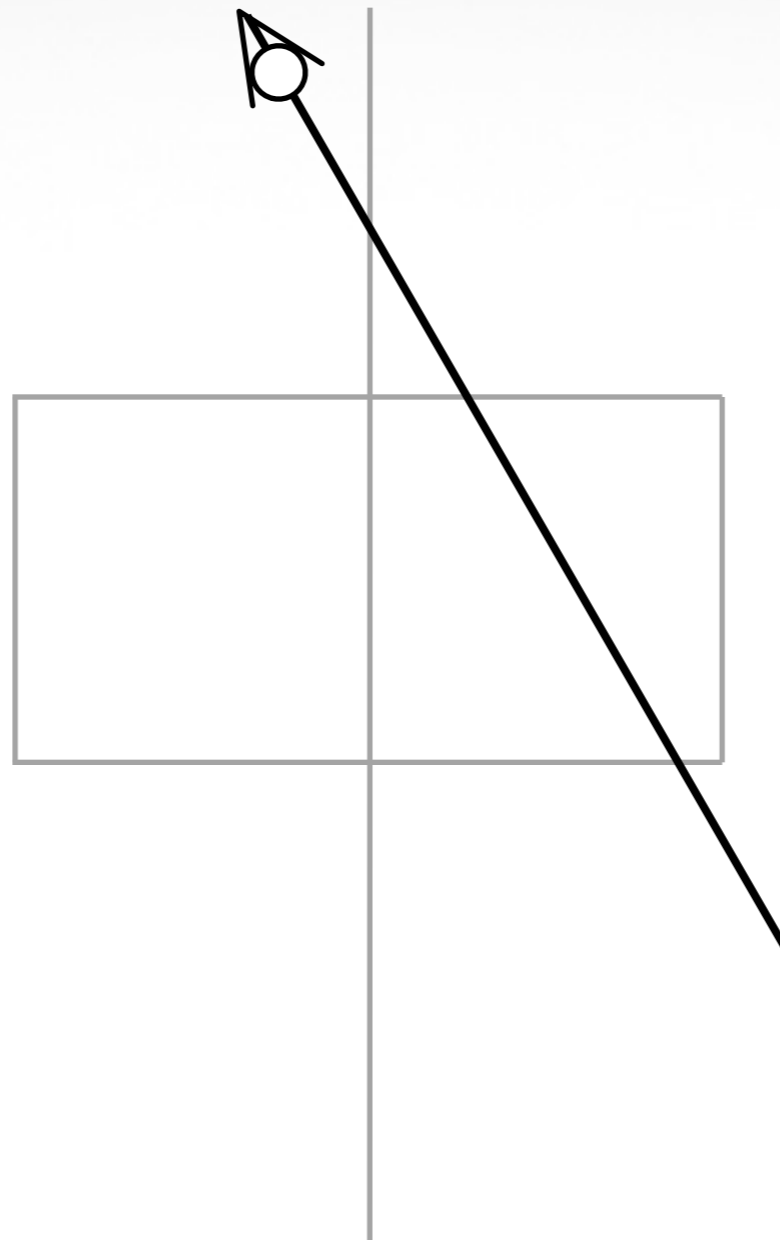
Hint: case distinctions



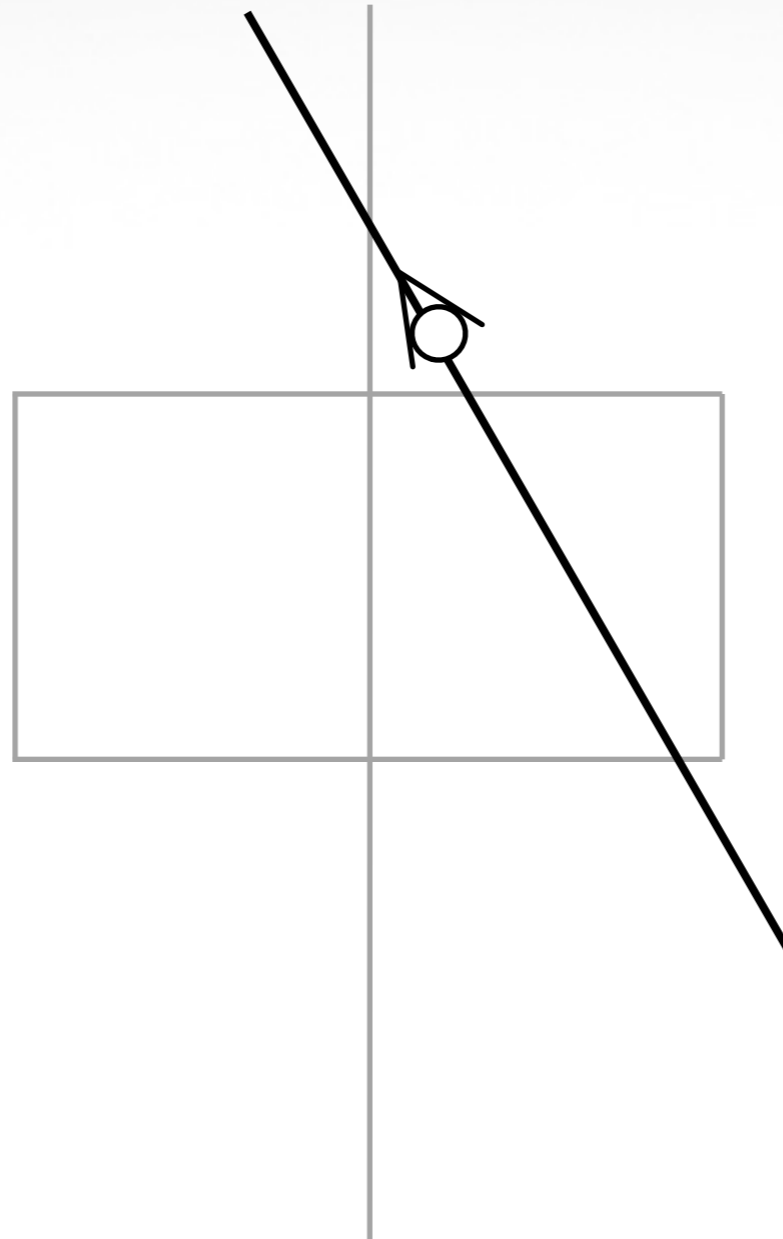
Hint: case distinctions



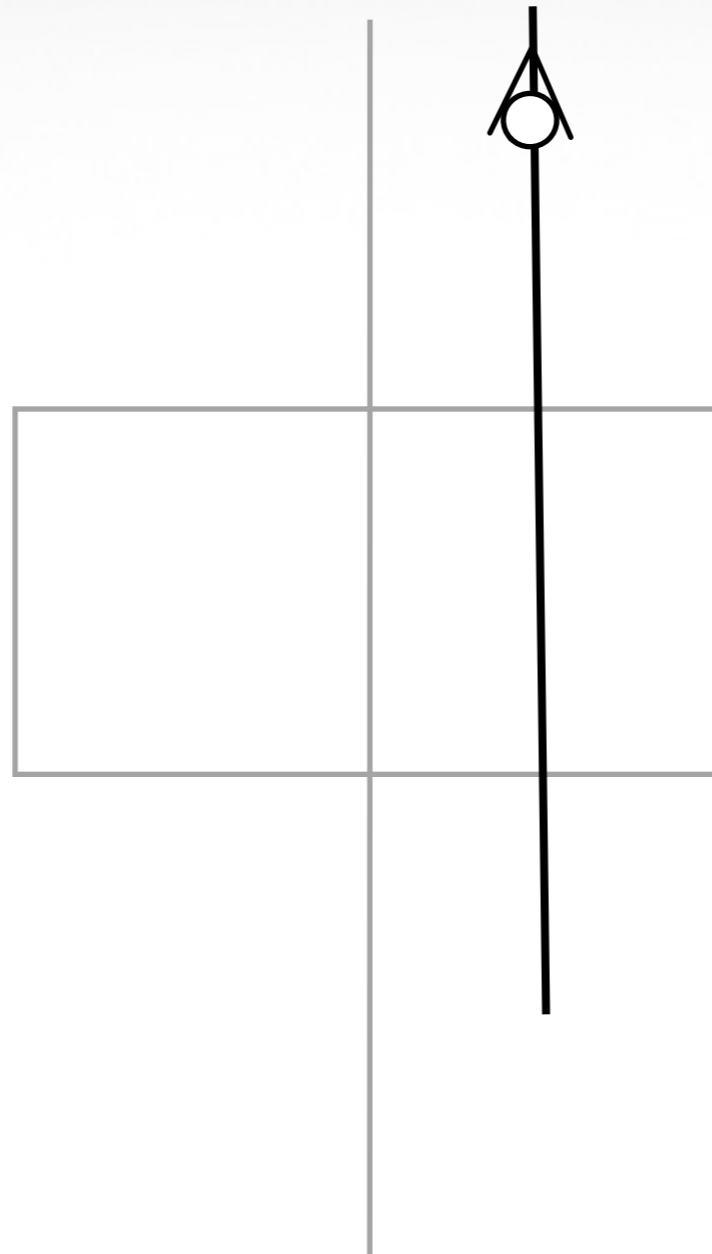
Hint: case distinctions



Hint: case distinctions



Hint: case distinctions





# Further Readings

- Heuristic Ray Shooting Algorithms [Havran 2001]
- Realtime Ray Tracing [Ingo Wald 2004]
- Multidimensional Binary Search  
Trees Used for Associative Searching [Bentley 1975]

?

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