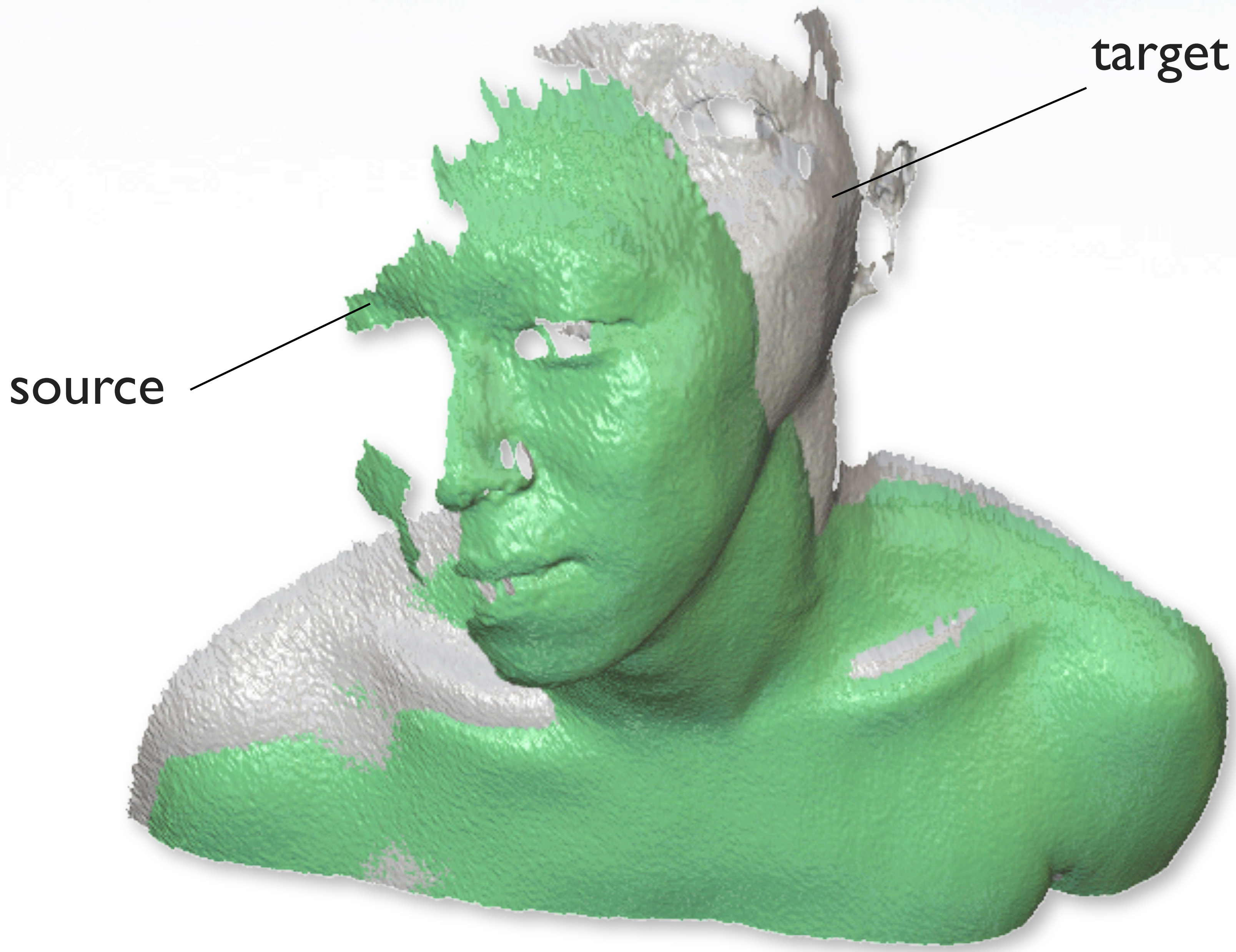
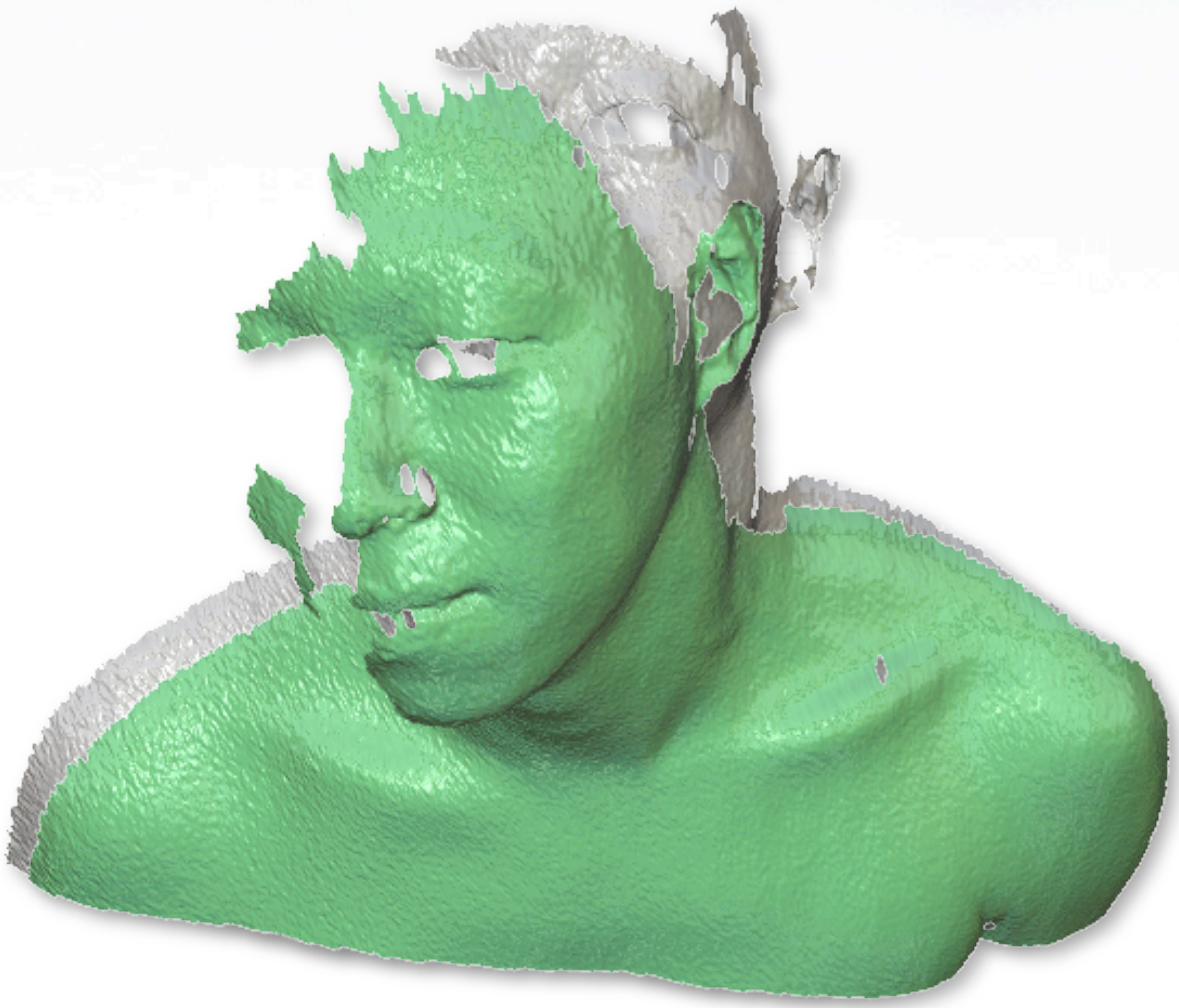
# Introduction to Non-Rigid Registration

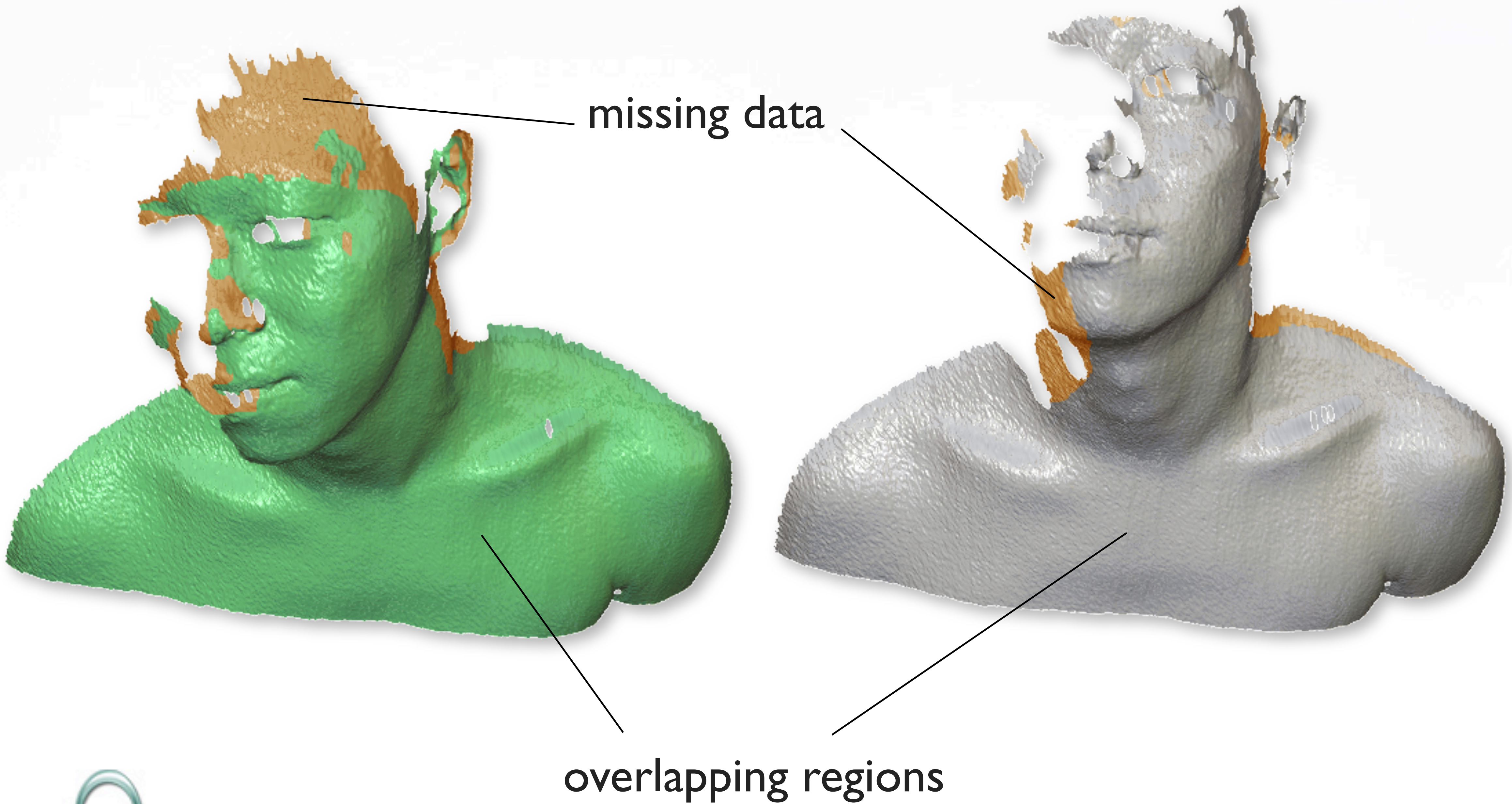
Hao Li

Applied Geometry Group
ETH Zurich

# What is Registration?

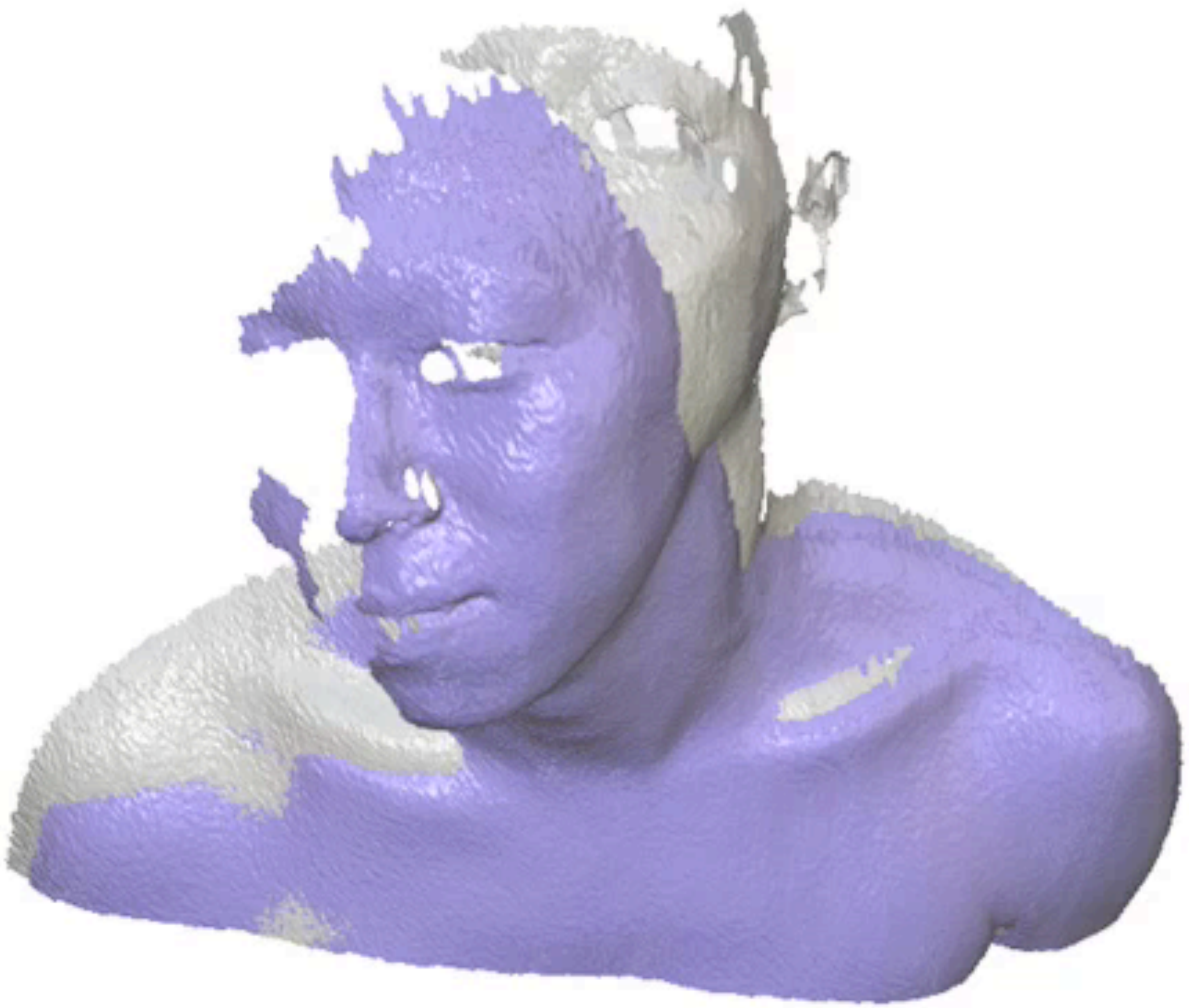




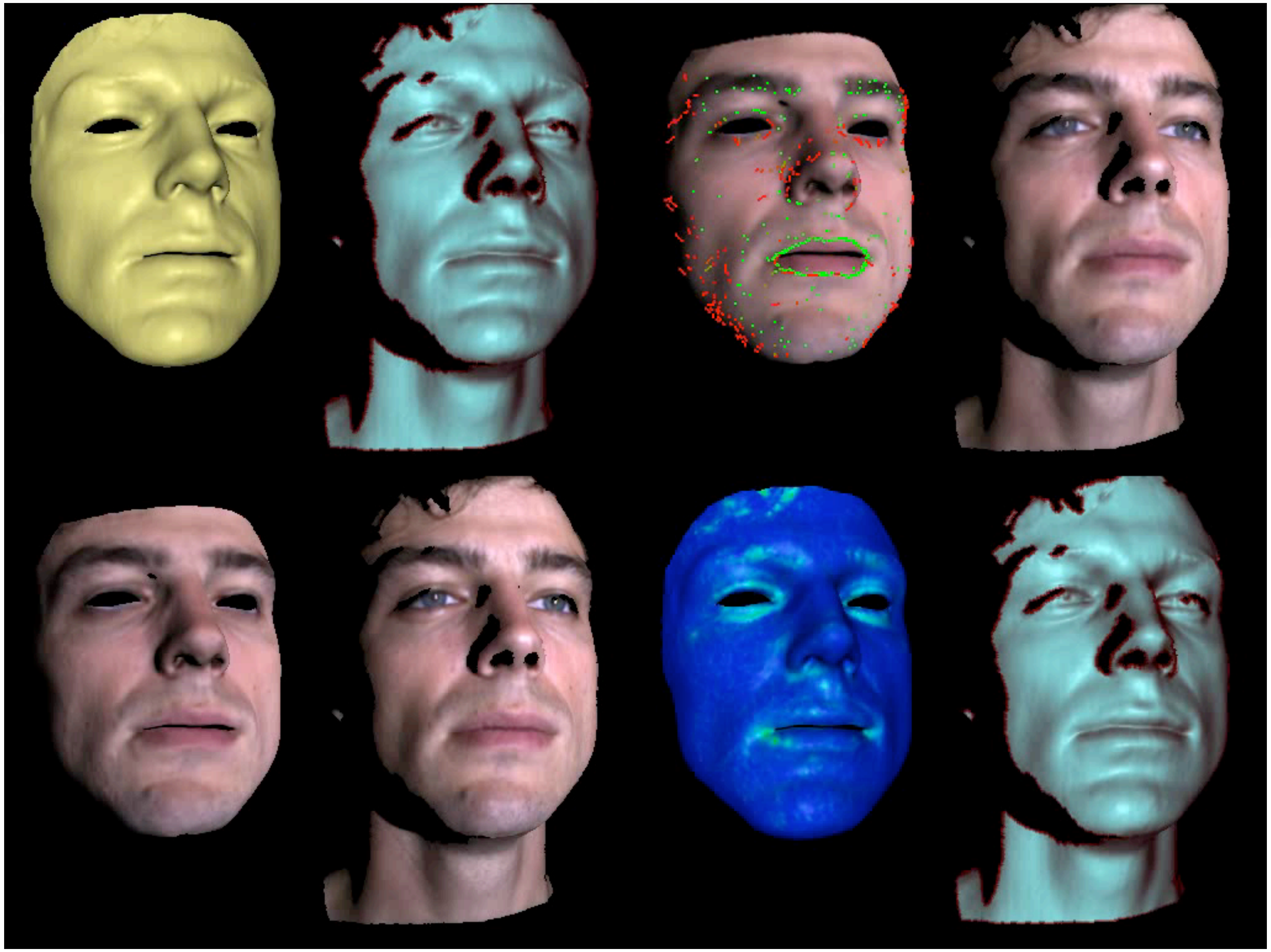

missing data
overlapping regions

initial alignment

non-rigid registration

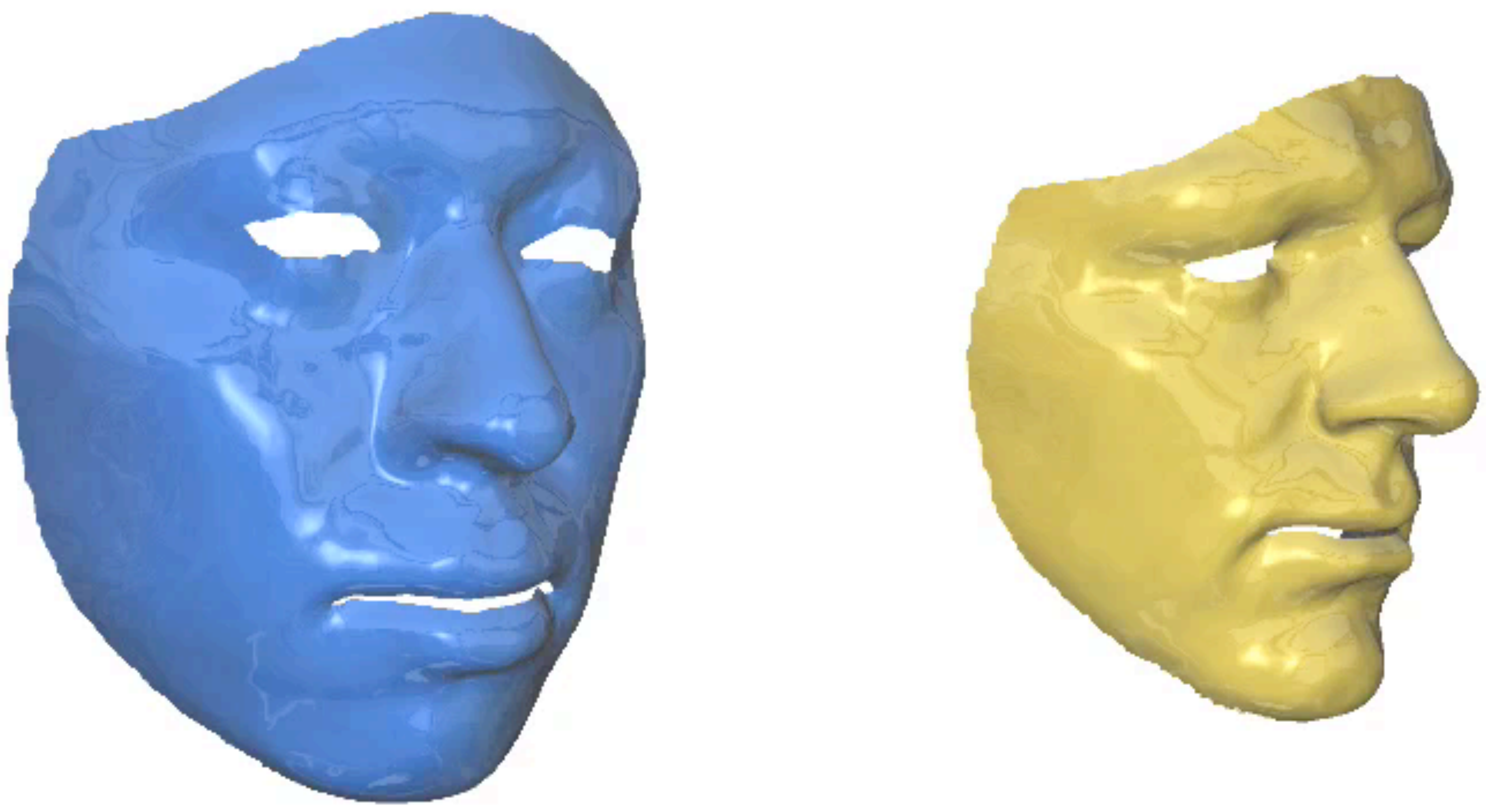


# Template Fitting



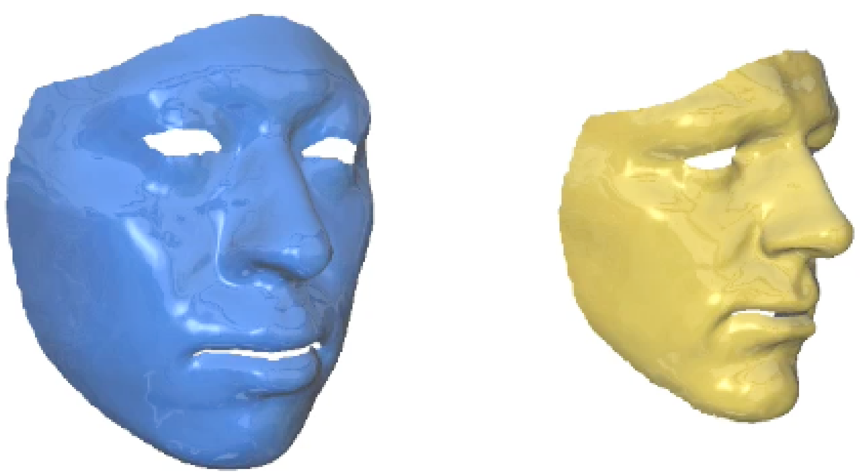
*Collaboration with T. Weise and L. Van Gool*

# Deformation Transfer



*Collaboration with T. Weise and L. Van Gool*

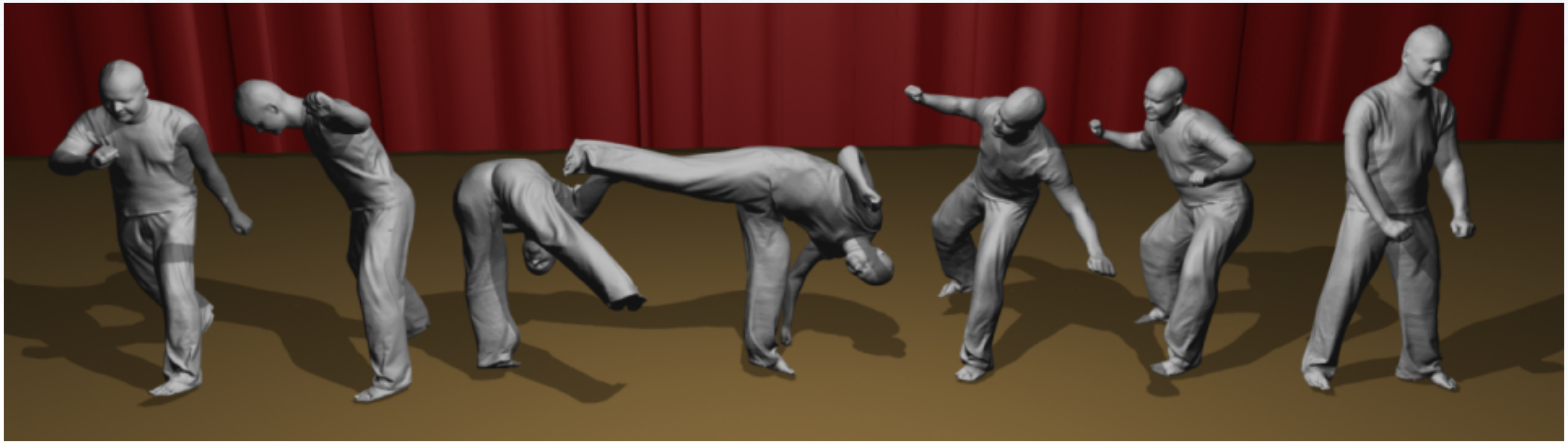
# Correspondence between Shapes



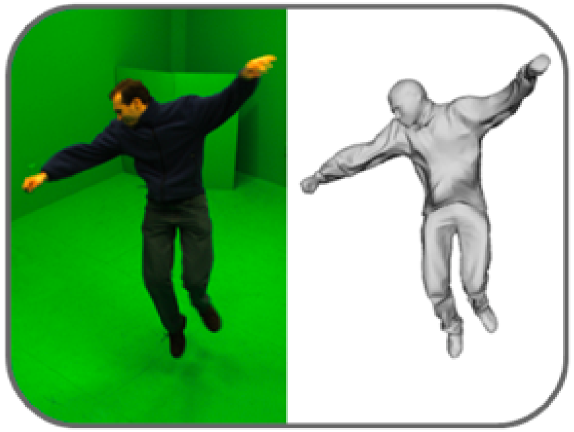
average shape warping

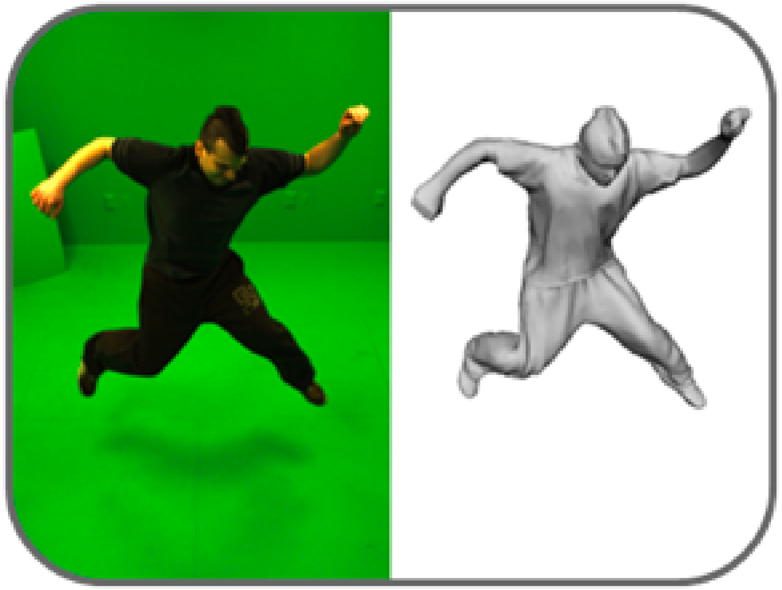
*Collaboration with T. Weise and L. Van Gool*

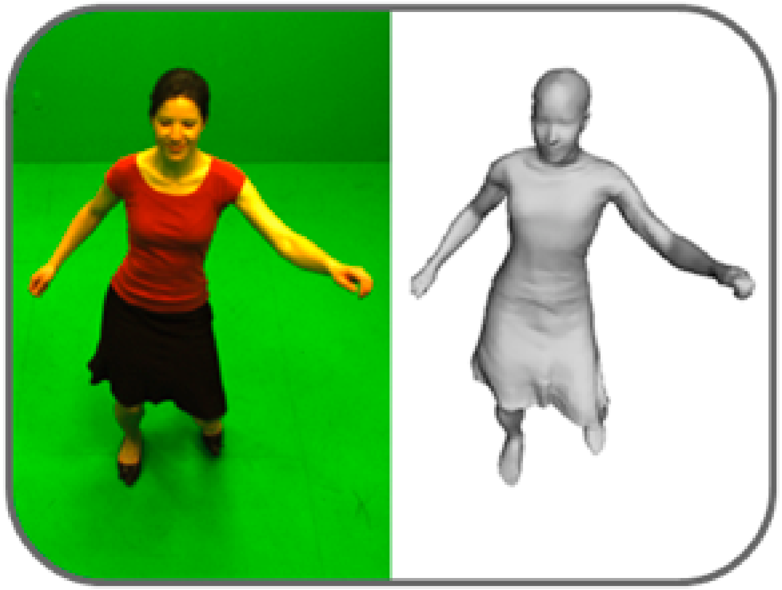
# Performance Capture – More next week



*Source: [De Aguiar et al. 08]*

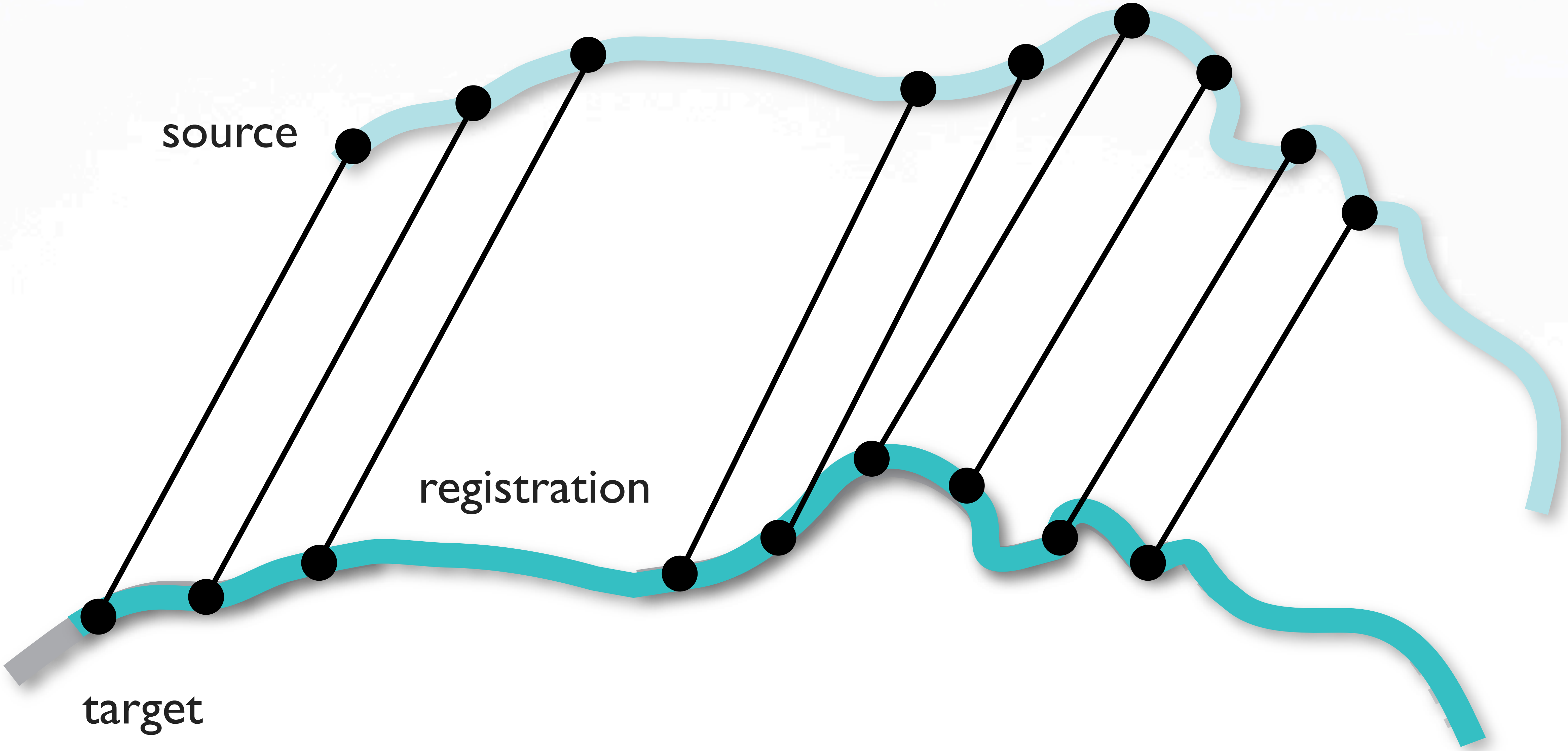






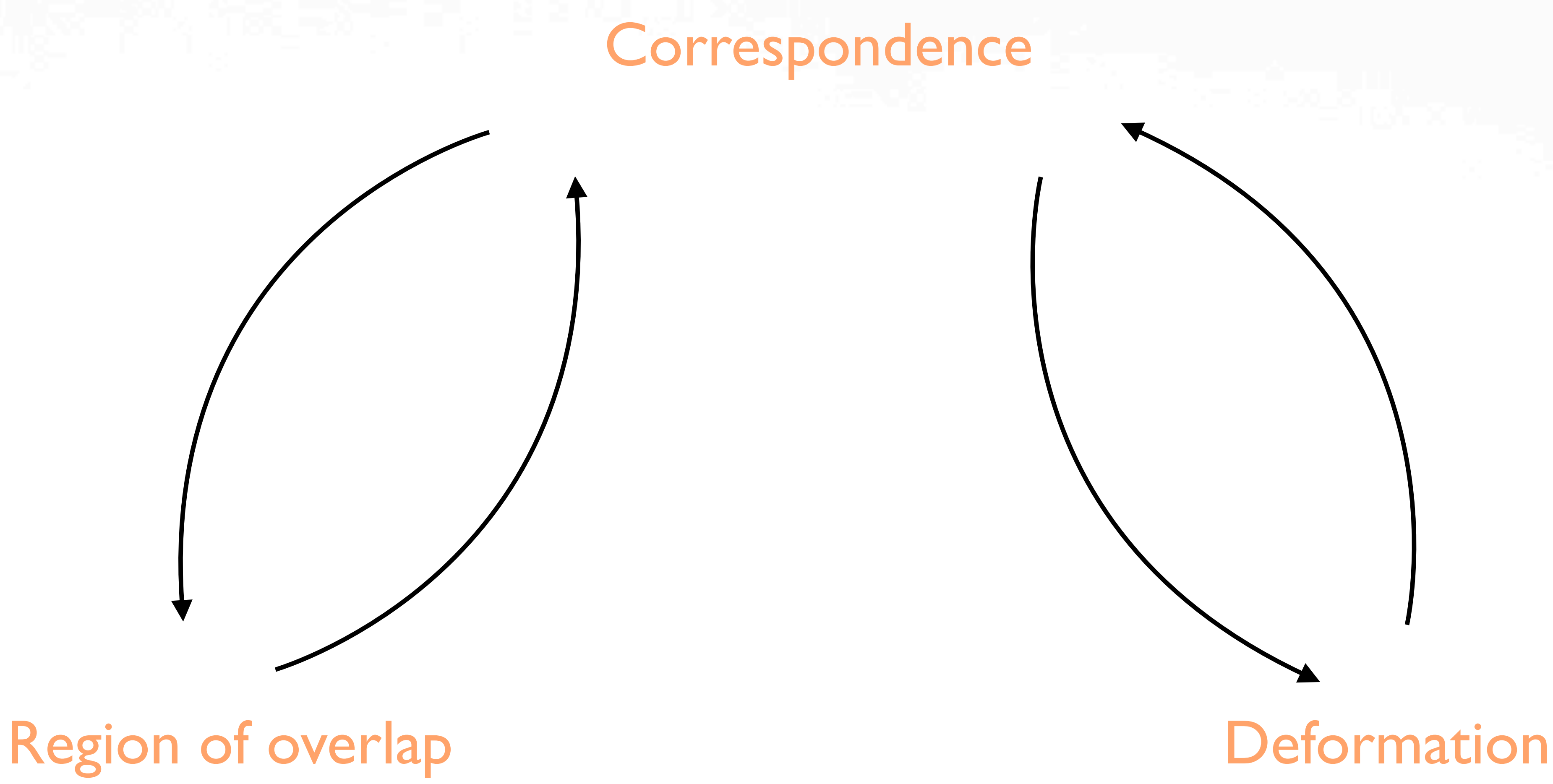
*Source: [Vlasic et al. 08]*

# Ingredients for Registration




Partial Overlap Correspondence Deformation

# Three **Interdependent** Challenges



# Rigid Alignment

# Problem

$$\mathcal{M}_1 \ldots M_n$$

$$(R_1, \mathbf{t}_1) \ldots (R_n, \mathbf{t}_n)$$

$$\tilde{\mathcal{M}}_1 \ldots \tilde{M}_n$$

# Registration Pipeline

scans

↓

**Initial Alignment**

↓

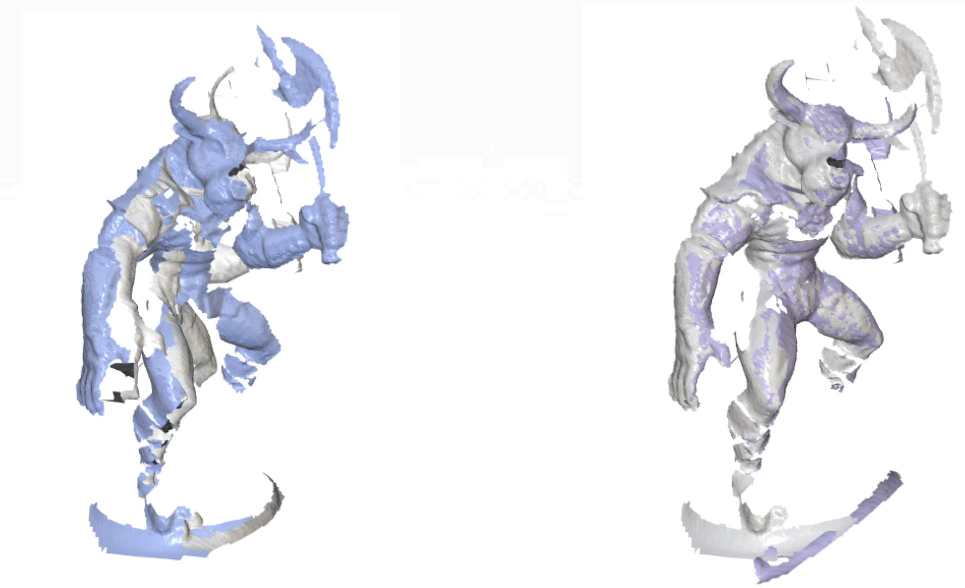
**Pairwise Registration**

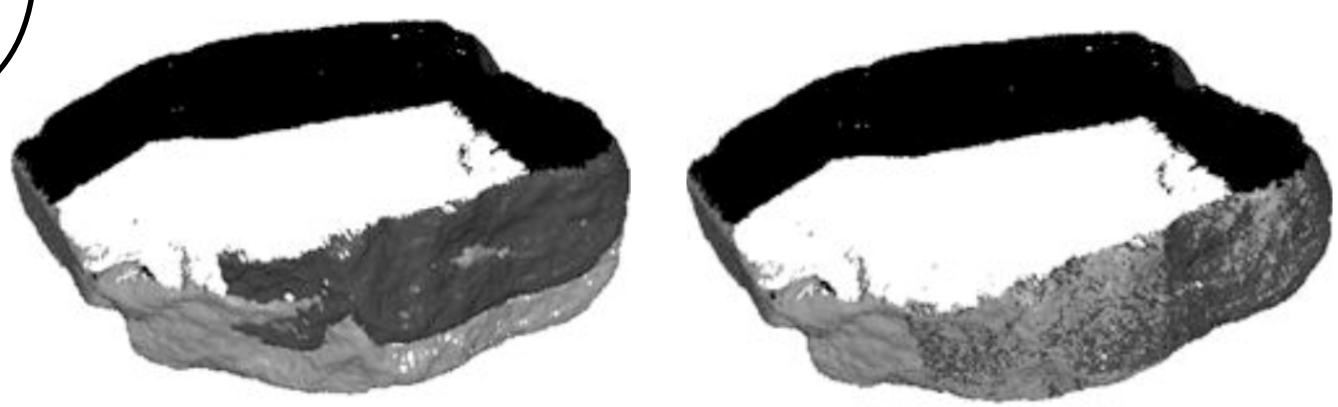
↓

**Global Relaxation**

↓

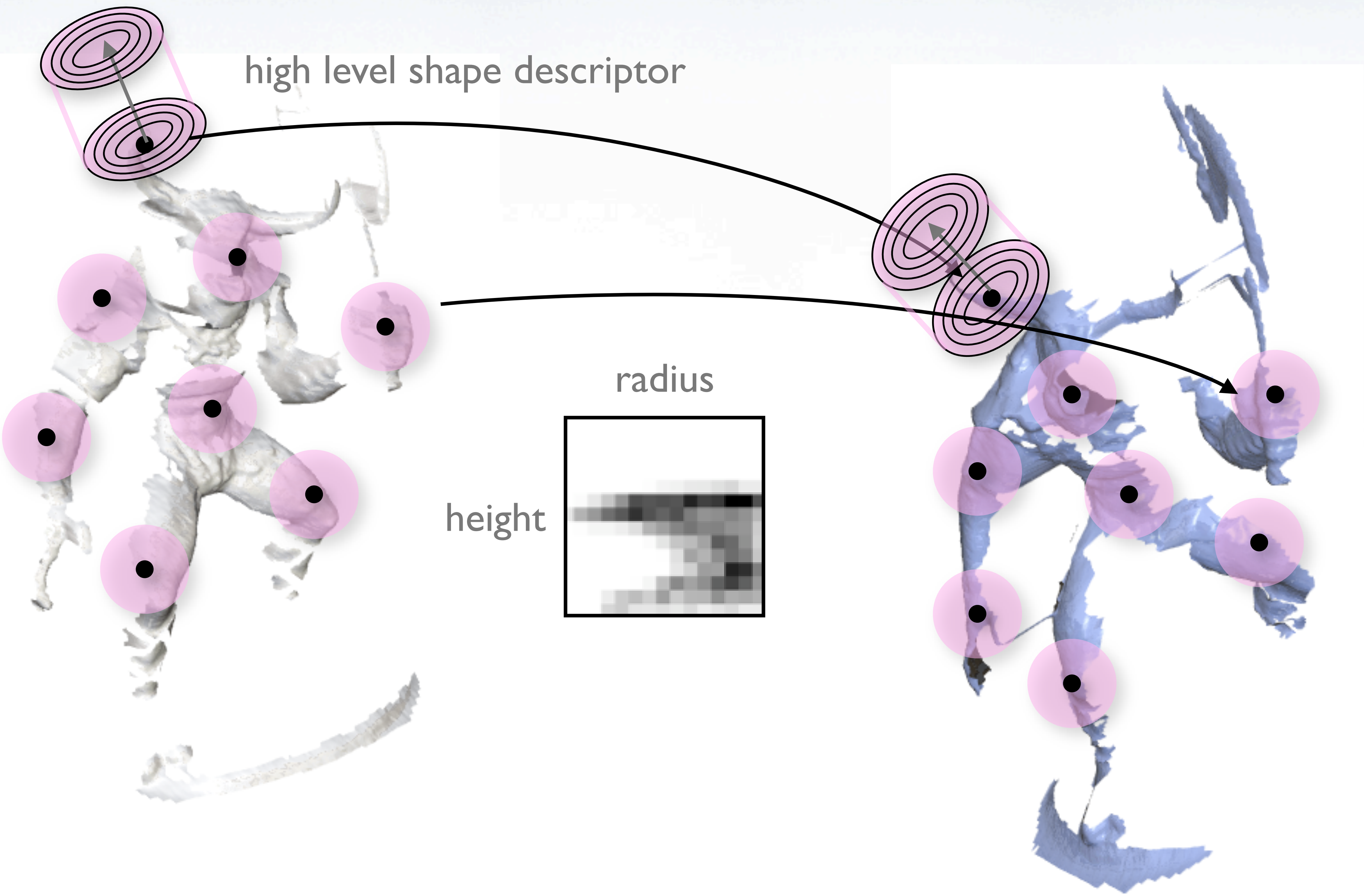
aligned scans





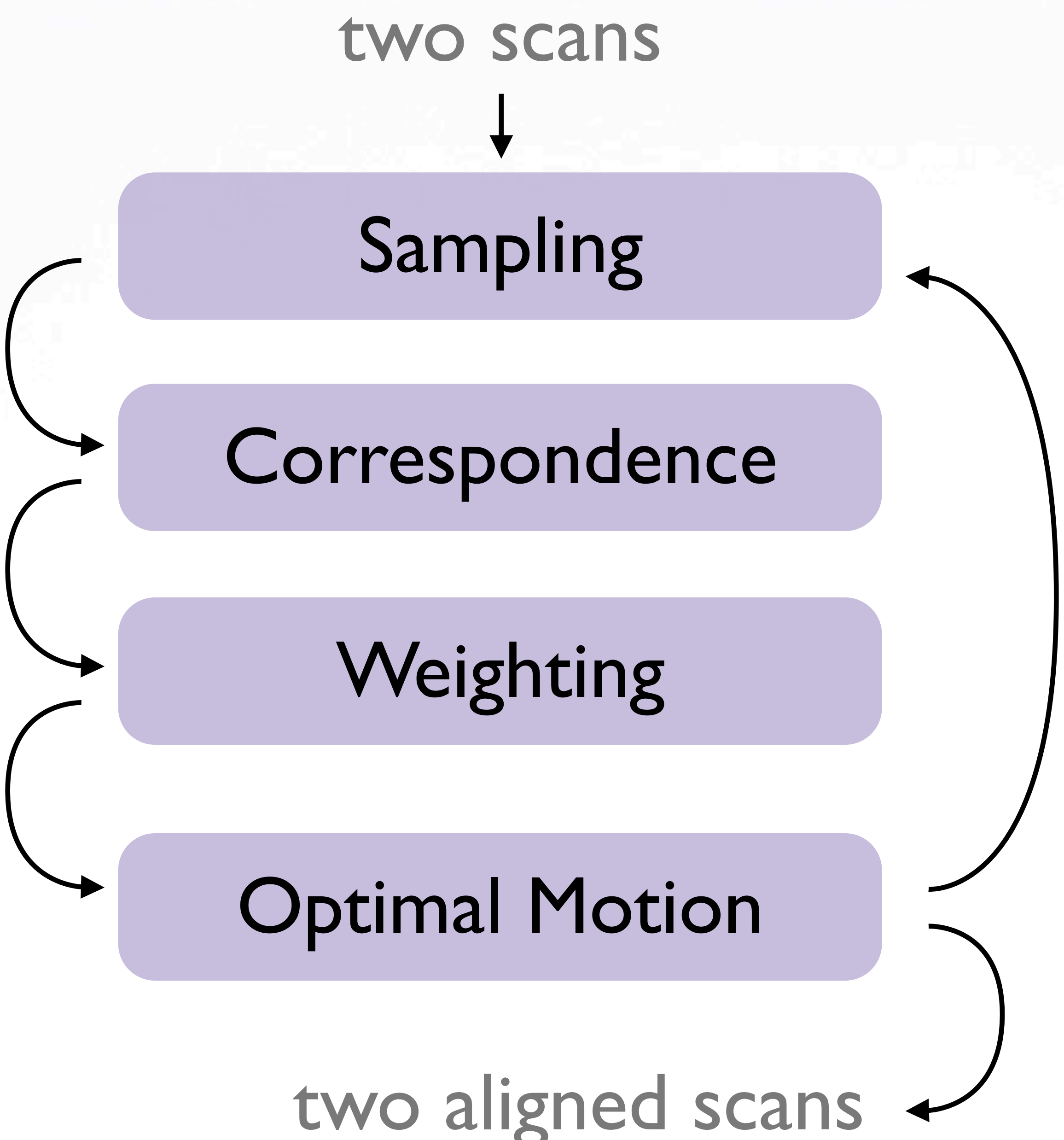
*source: [Pulli etl al. 99]*

# Initial Alignment



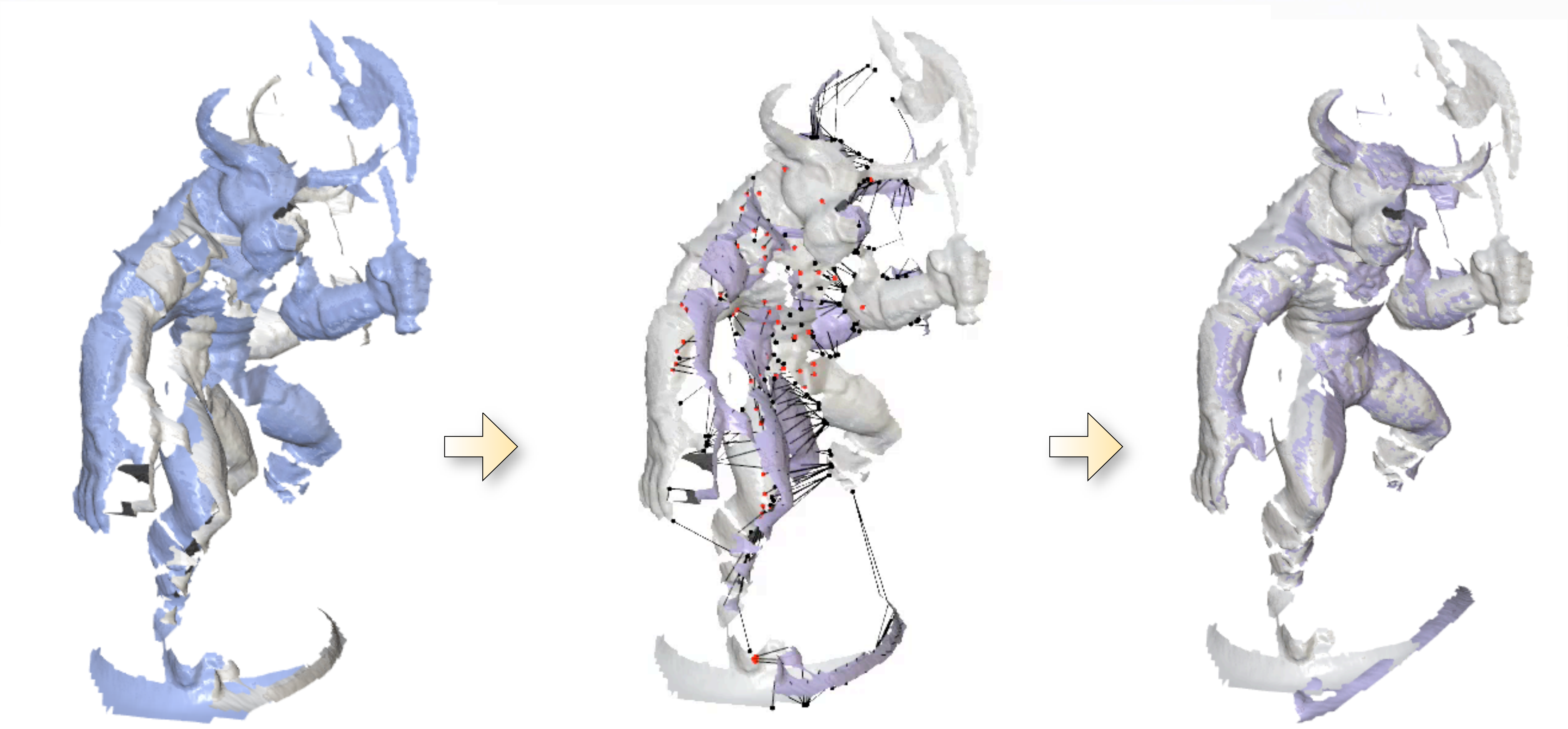

- Motion invariant shape descriptor (Spin images, harmonic shape context)
- Correspondence search (Brute force, branch and bound, RANSAC)

# Iterative Closest Point (ICP)



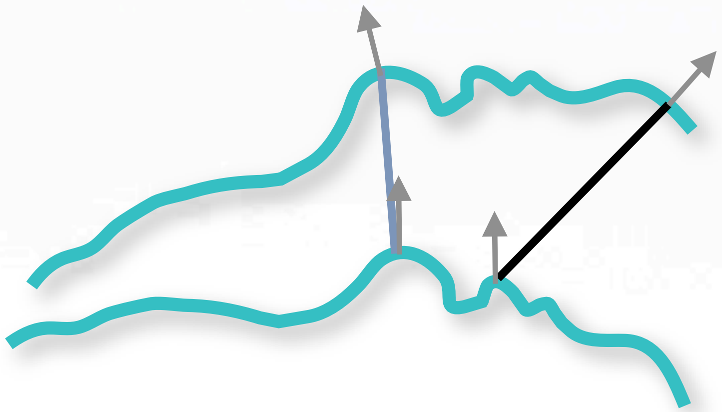

- Uniform, Normal Importance, etc...
- Closest point via *kd* tree
- Correspondence compatibility heuristics
- Different optimization techniques

# ICP Optimization

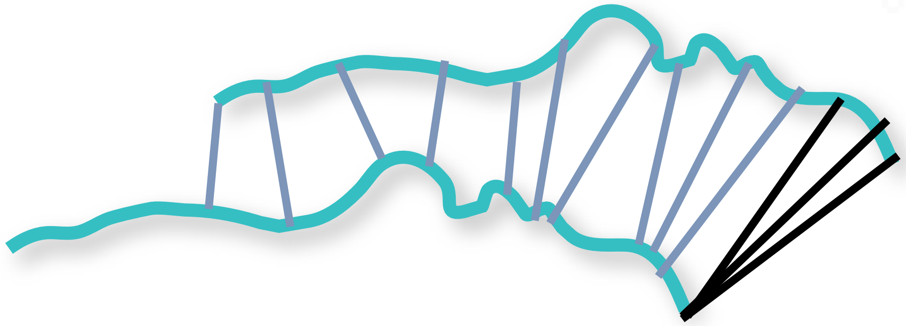


# Effective Weighting Schemes

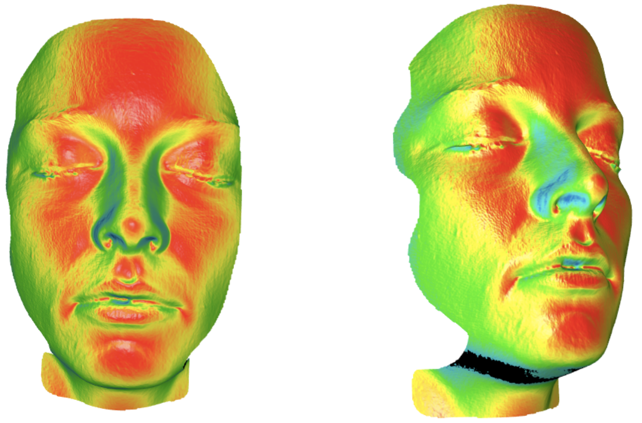
- Normal compatible correspondences



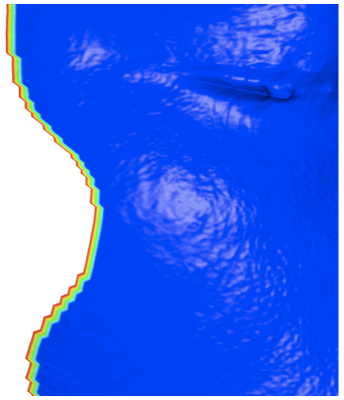
- Prune correspondences to boundaries and points that are too far



- Known surface confidence weights



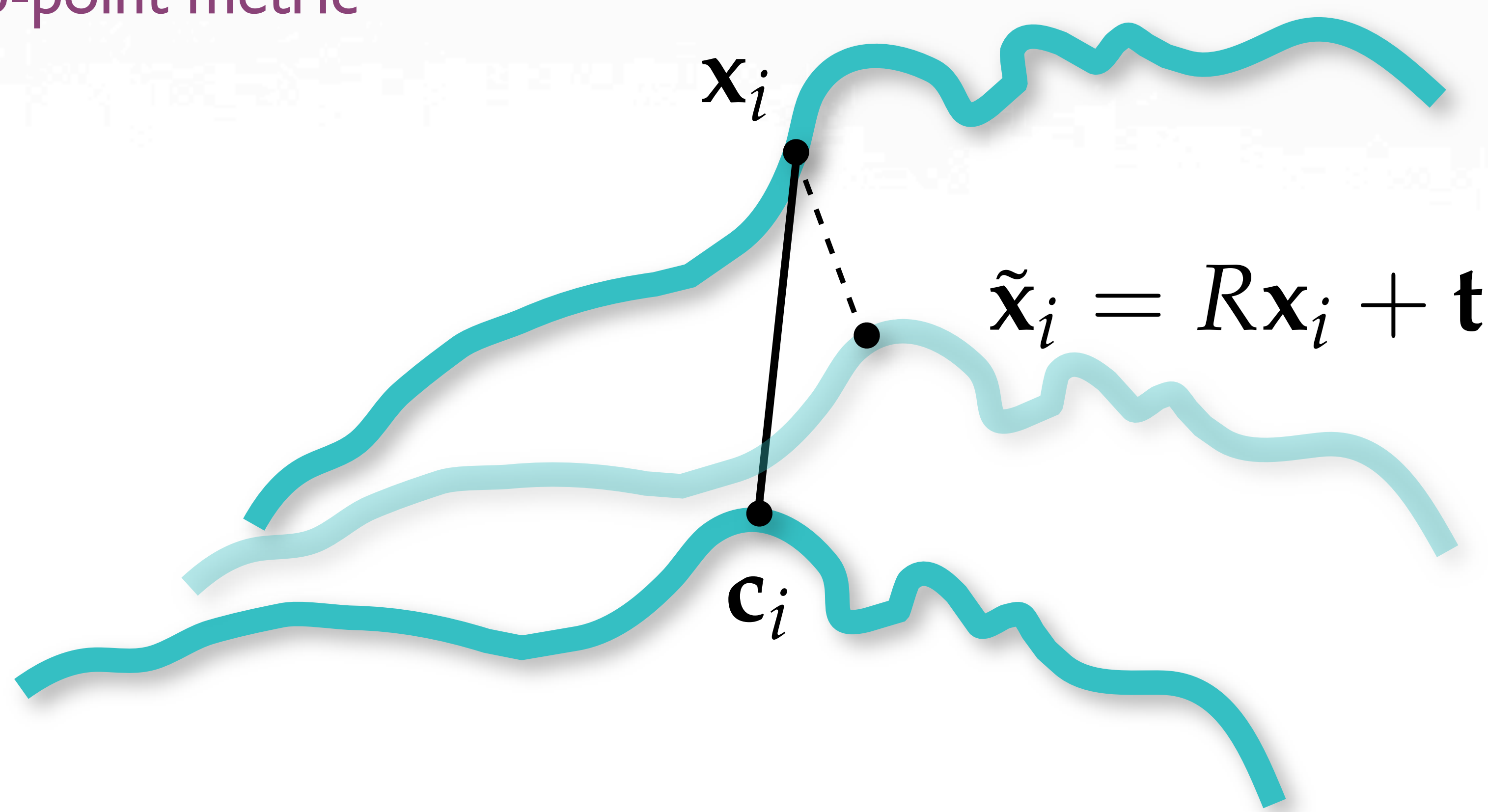
camera orientation



feathering

# Optimization

Point-to-point metric




$$E_{\text{point}} = \sum_i \|(R\mathbf{x}_i + \mathbf{t} - \mathbf{c}_i)\|^2$$

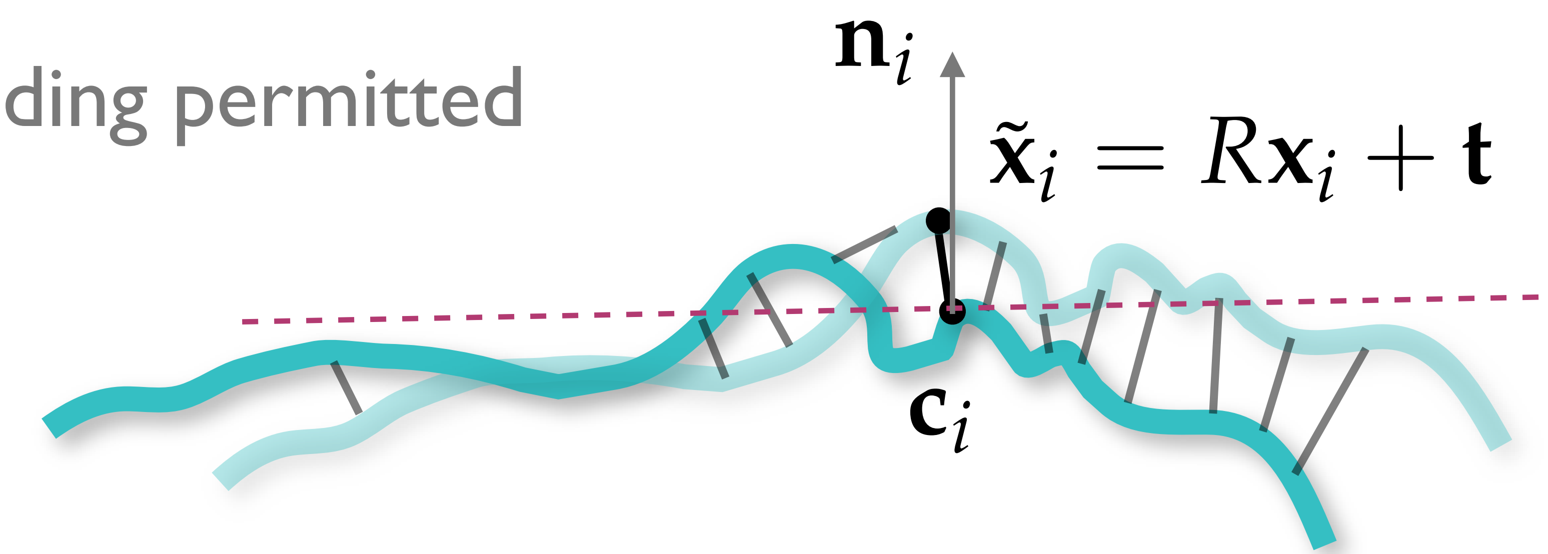
- Closed form solution via quaternions [Horn 87]

# Optimization

[Chen and Medioni '91]

Point-to-plane metric

gliding permitted




$$E_{\text{plane}} = \sum_i \|\mathbf{n}_i^{\mathbf{t}}(R\mathbf{x}_i + \mathbf{t} - \mathbf{c}_i)\|^2$$

# Optimization

$$E_{\text{plane}} = \sum_i \|\mathbf{n}_i^{\mathsf{t}}(R\mathbf{x}_i + \mathbf{t} - \mathbf{c}_i)\|^2 \qquad \sin(\theta) \approx \theta\ ,\ \cos(\theta) \approx 1$$

$$E_{\text{plane}} = \sum_i \|\mathbf{n}_i^{\mathsf{t}}(\mathbf{x}_i - \mathbf{c}_i) + \mathbf{r}^{\mathsf{t}}(\mathbf{x}_i \times \mathbf{n}_i) + \mathbf{n}_i^{\mathsf{t}}\mathbf{t})\|^2$$

$$\begin{bmatrix} -(\mathbf{x}_1 \times \mathbf{n}_1)^{\mathsf{t}} & \mathbf{n}_1^{\mathsf{t}} \\ -(\mathbf{x}_2 \times \mathbf{n}_2)^{\mathsf{t}} & \mathbf{n}_2^{\mathsf{t}} \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} -(\mathbf{x}_1 - \mathbf{c}_1)^{\mathsf{t}}\mathbf{n}_1 \\ -(\mathbf{x}_2 - \mathbf{c}_2)^{\mathsf{t}}\mathbf{n}_2 \\ \vdots \end{bmatrix}$$

overdetermined linear system

# In Practice

$$E_{\text{tot}} = E_{\text{plane}} + \lambda E_{\text{point}} \qquad \lambda \approx 0.1$$

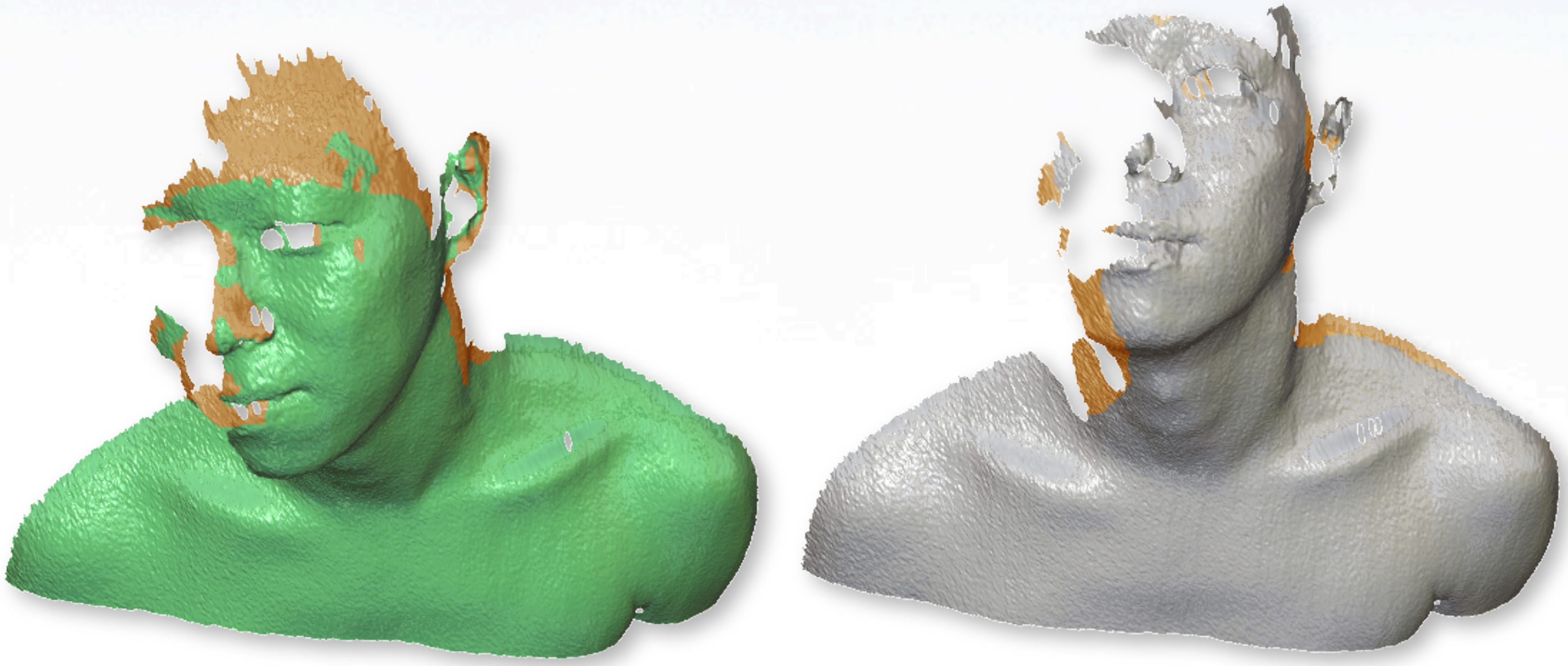
$$E_{\text{plane}} = \sum_i \|\mathbf{n}_i^{\mathbf{t}}(R\mathbf{x}_i + \mathbf{t} - \mathbf{c}_i)\|^2$$

$$E_{\text{point}} = \sum_i \|(R\mathbf{x}_i + \mathbf{t} - \mathbf{c}_i)\|^2$$

- Eplane alone can exhibit oscillations
- Favor Eplane when convergence is slow

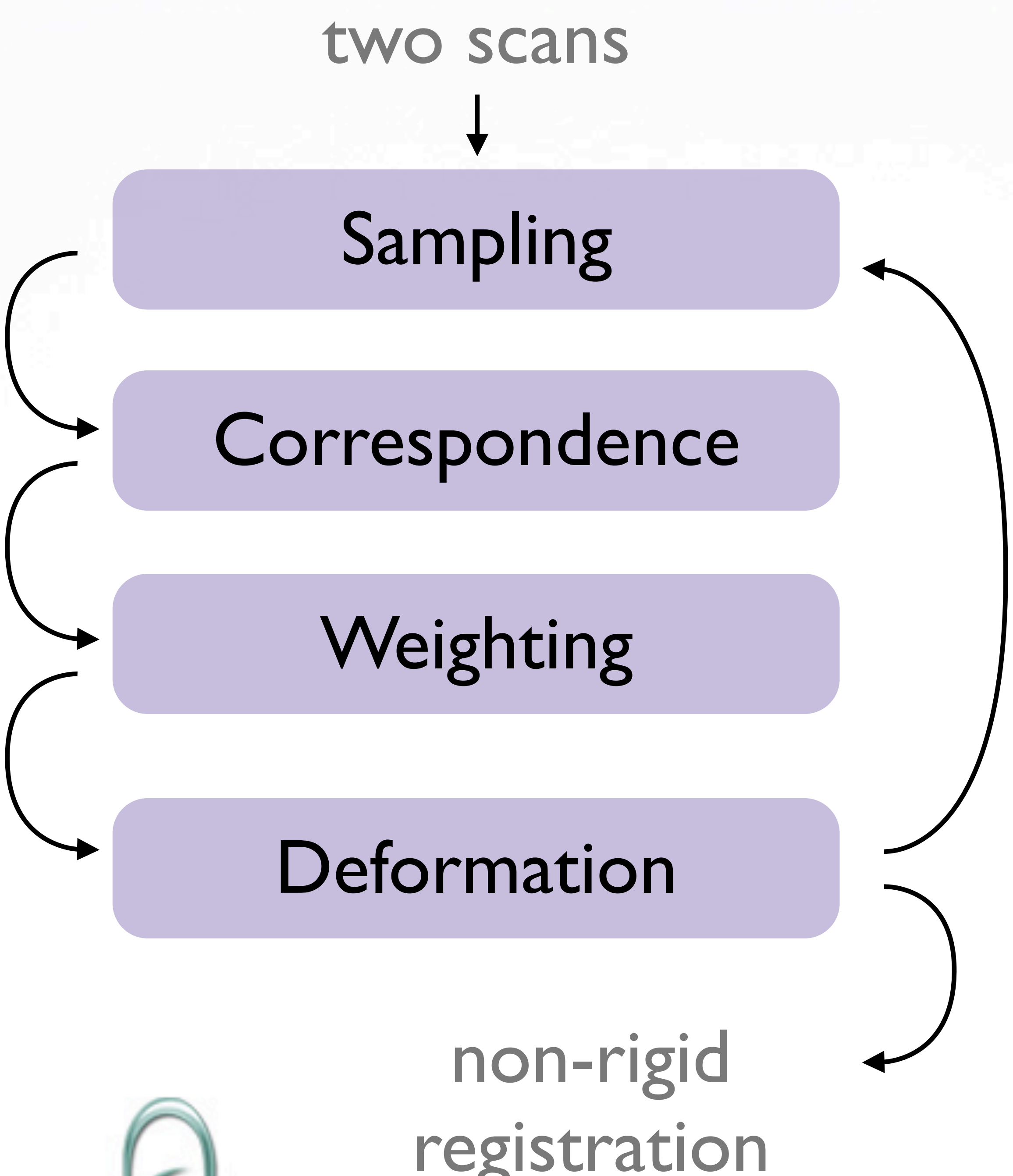
# Non-Rigid Registration

# Problem



- Partial overlap unknown
- Correspondences not given
- No prior about deformation
- Interdependence of problems

# Decoupled Approach




Correspondence must be robust w.r.t. underlying deformation

In general: Non-linear problem

$$E_{\text{tot}} = \alpha_{\text{fit}} E_{\text{fit}} + \alpha_{\text{reg}} E_{\text{reg}}$$

# Non-Linear Optimization

$$E_{\text{tot}} = \alpha_{\text{fit}} E_{\text{fit}} + \alpha_{\text{reg}} E_{\text{reg}} \qquad E_{\text{tot}} = \|\mathbf{f}(\mathbf{x})\|^2$$

1st order Taylor

$$\|\mathbf{f}(\mathbf{x}^{k+1})\|^2 \approx \|\mathbf{f}(\mathbf{x}^k) + J_{\mathbf{f}}(\mathbf{x}^{k+1} - \mathbf{x}^k)\|^2$$

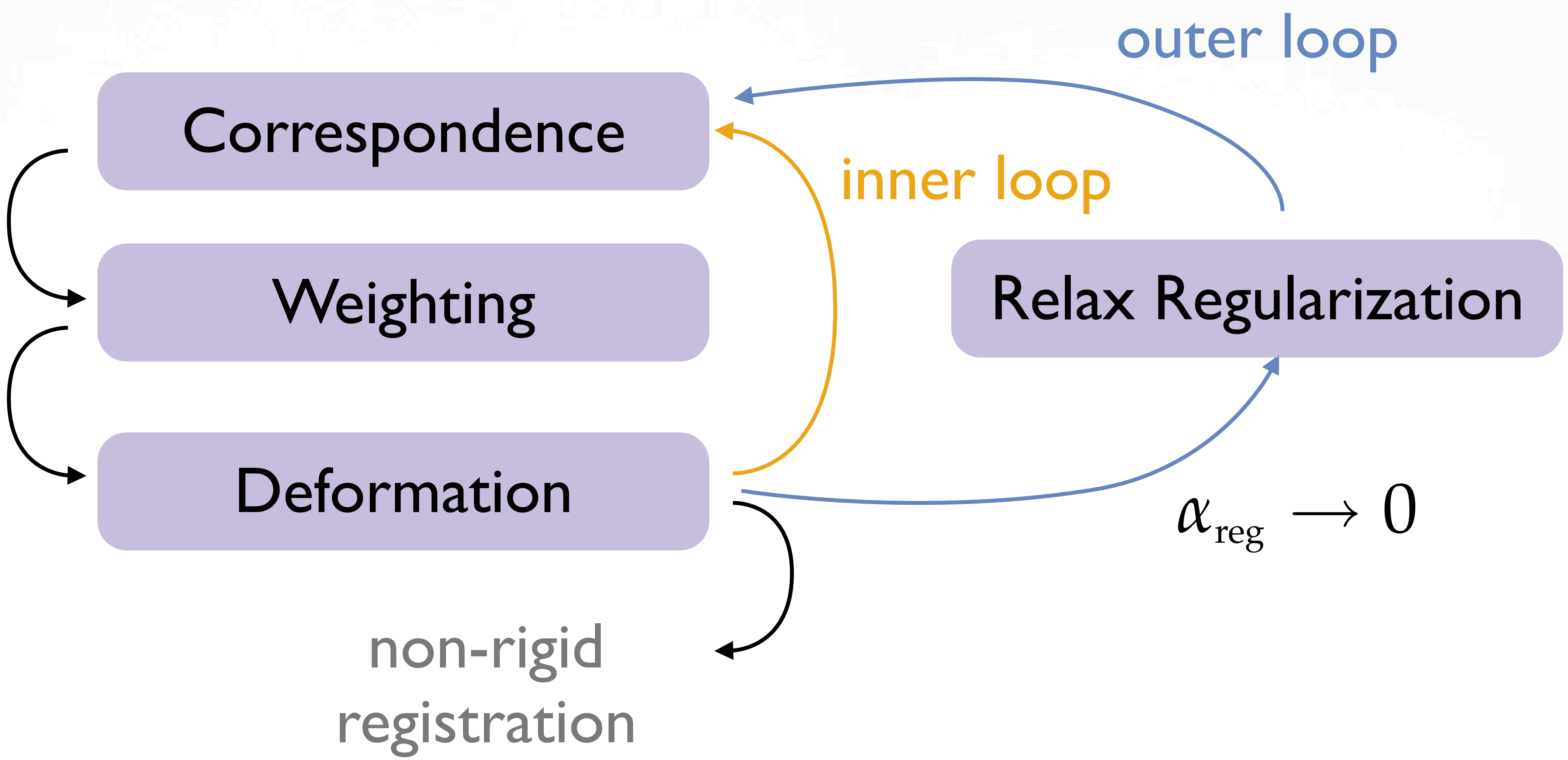
$$\|\mathbf{f}(\mathbf{x}^{k+1})\|^2 \approx \|\mathbf{f}(\mathbf{x}^k) + J_{\mathbf{f}}\Delta\mathbf{x}^k\|^2$$

Gauss-Newton

$$\Delta\mathbf{x}^k_{\min} = \arg\min_{\Delta\mathbf{x}^k} E_{\text{tot}} \qquad J_{\mathbf{f}}^{\mathsf{t}} J_{\mathbf{f}} \Delta\mathbf{x}^k_{\min} = -J_{\mathbf{f}}^{\mathsf{t}} \mathbf{f}(\mathbf{x}^k)$$

- Use direct solver with sparse Cholesky factorization
- Extension to Levenberg-Marquardt
- Other techniques: Quasi-Newton, ...

# Scheduled Regularization




- Energy landscape smoothing (prevents local minima)
- Other technique: multi-resolution

# Deformation Models

# Linear Regularization Methods

- Smooth Displacement

$$E_{\text{displ}} = \sum_{(\mathbf{x}_i,\mathbf{x_j})\in\text{edges}} \|\mathbf{d_i} - \mathbf{d_j}\|^2 / \|\mathbf{x_i} - \mathbf{x_j}\|^2$$

- Smooth Affine Transforms

$$E_{\text{affine}} = \sum_{(\mathbf{x}_i,\mathbf{x_j})\in\text{edges}} \|T_i - T_j\|_F^2 \quad , \quad T \in \mathbb{R}^{4\times4}$$

- Linear Variational Techniques [Botsch and Sorkine 07]

$$E_{\text{memb}} = \int_\Omega \|\mathbf{x}_u\|^2 + \|\mathbf{x}_v\|^2 \mathrm{d}u\mathrm{d}v$$

$$E_{\text{plate}} = \int_\Omega \|\mathbf{x}_{uu}\|^2 + 2\|\mathbf{x}_{vv} + \|\mathbf{x}_{vv}\|^2 \mathrm{d}u\mathrm{d}v$$

# Embedded Deformation

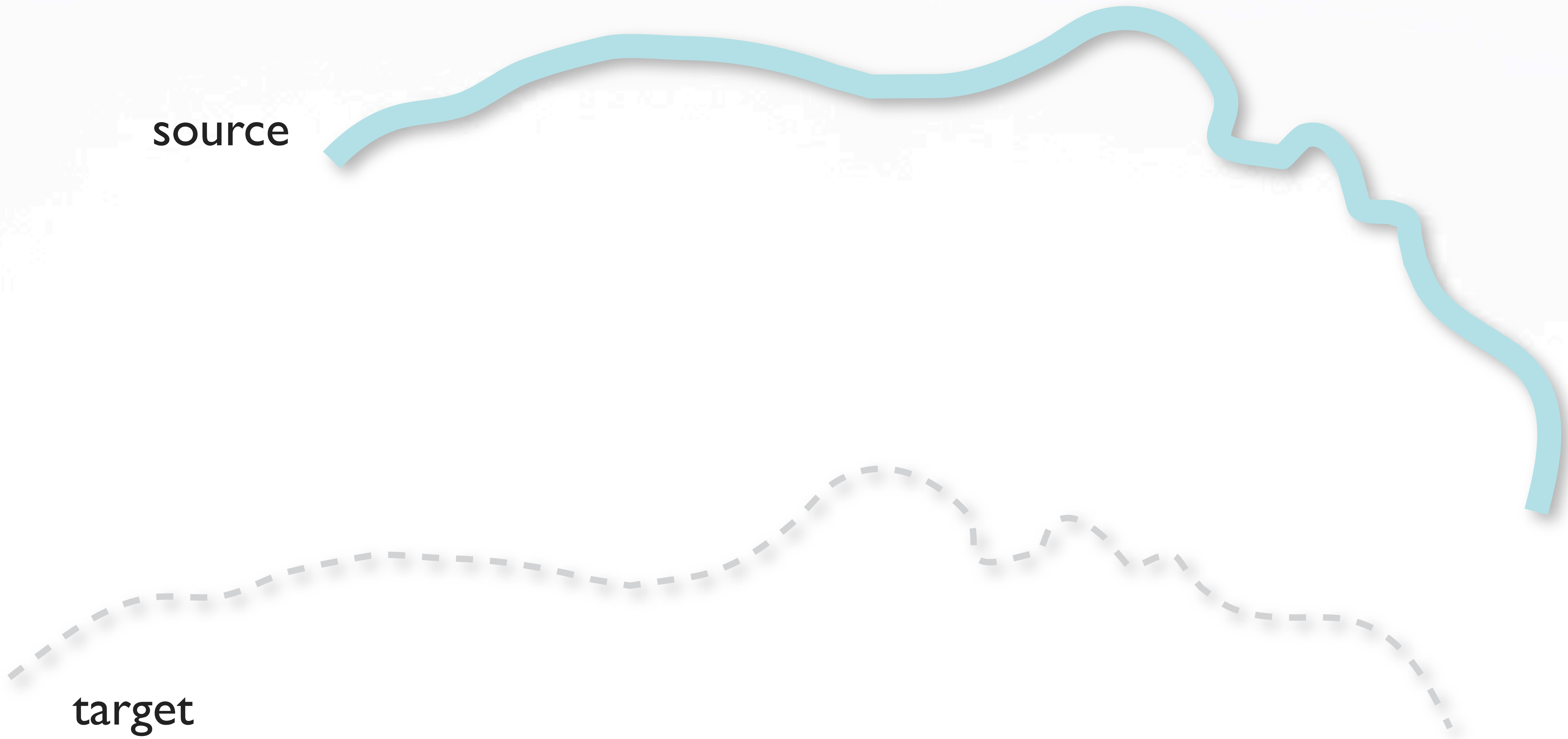
[Sumner et al. '07]

- Efficiency
- Generality
- Natural Deformations
- Detail Preservation

Maximize local rigidity

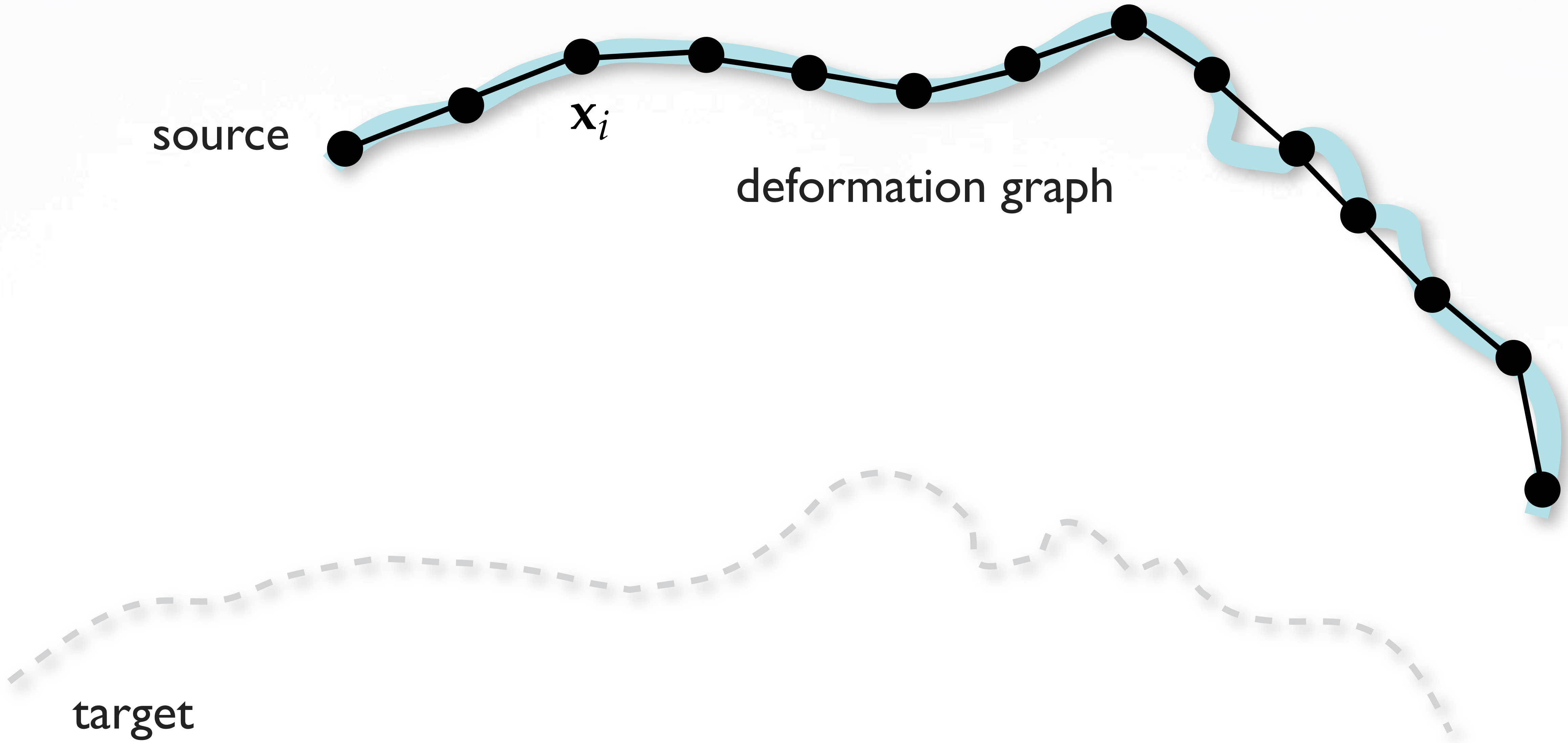
# Embedded Deformation

[Sumner et al. ‘07]



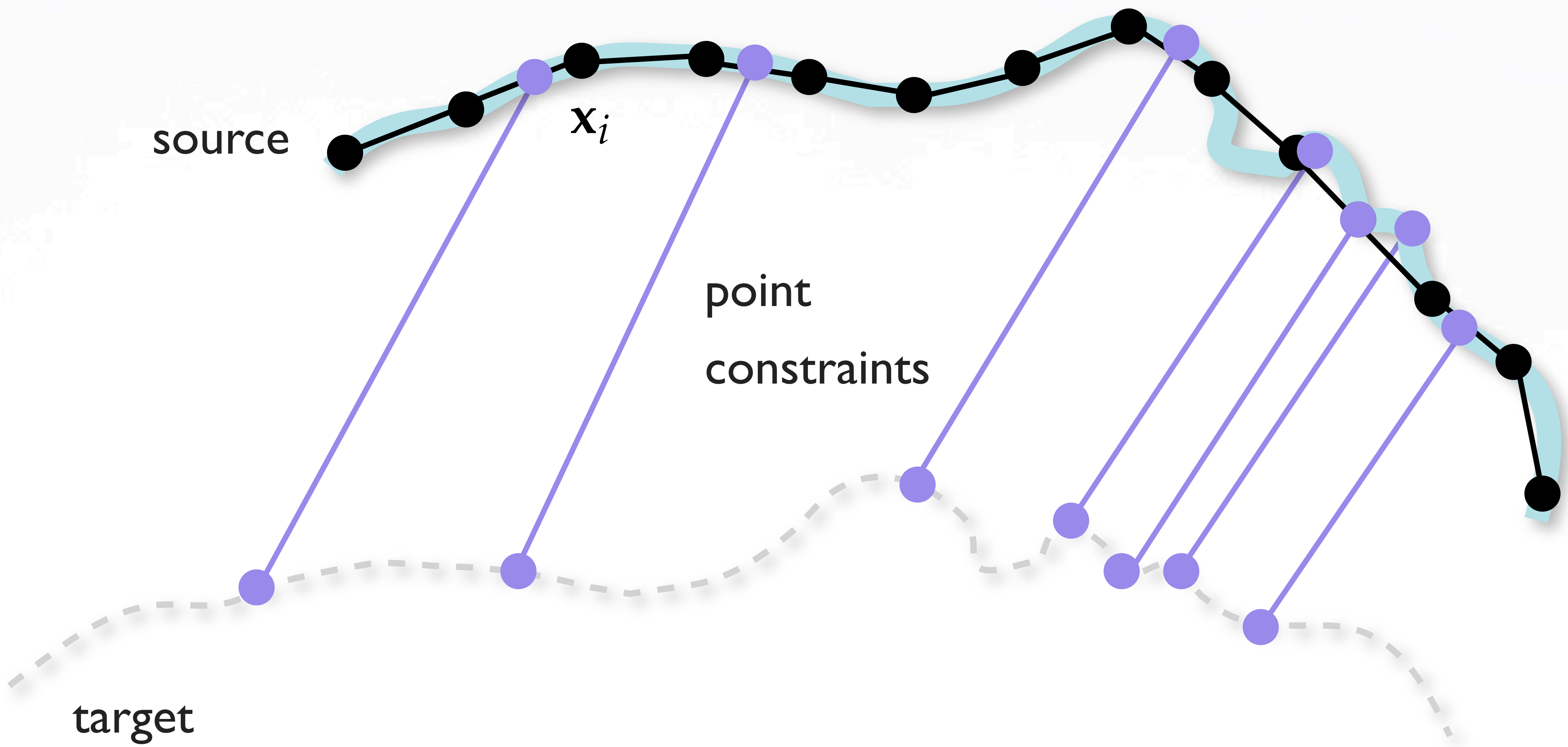
# Embedded Deformation

[Sumner et al. '07]



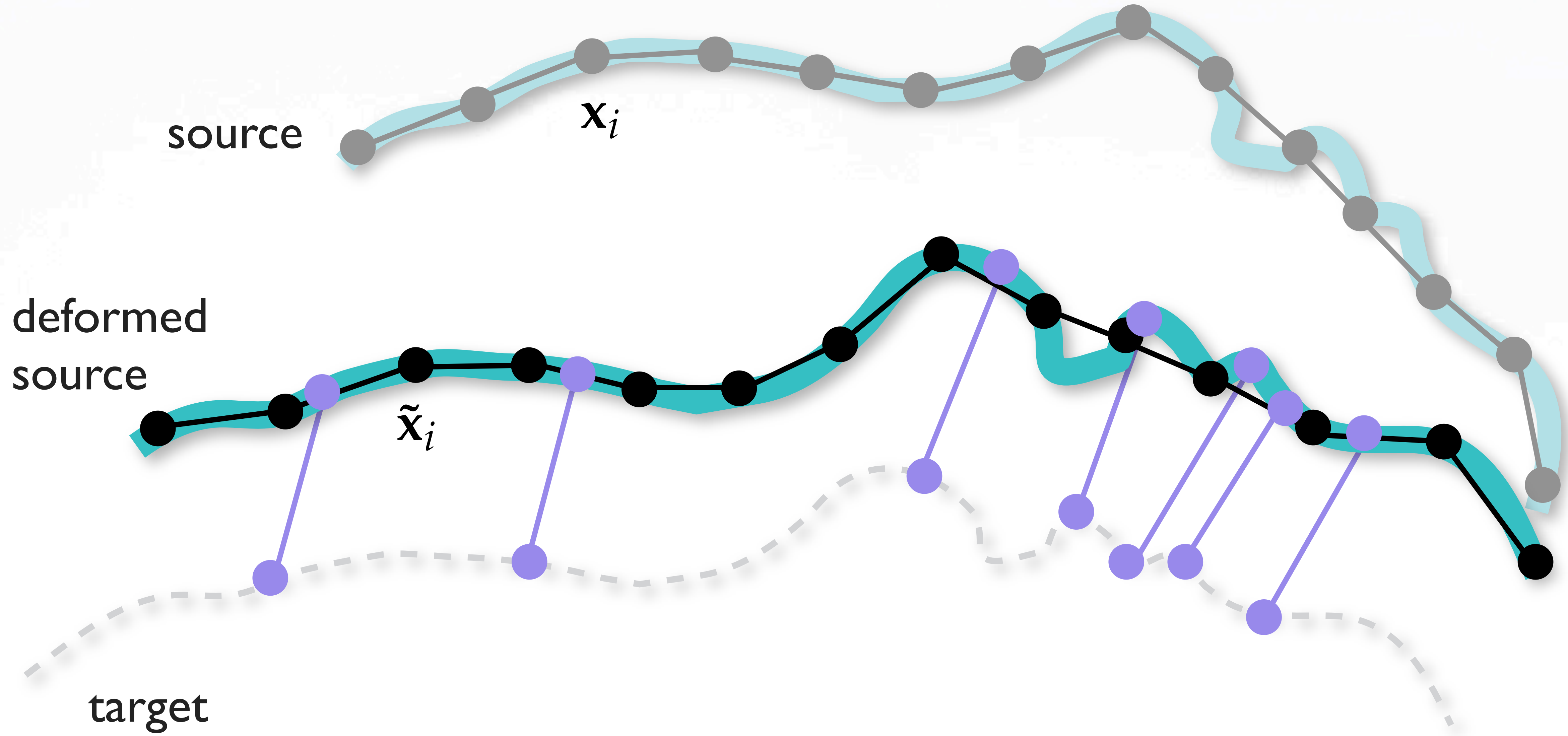
# Embedded Deformation

[Sumner et al. '07]



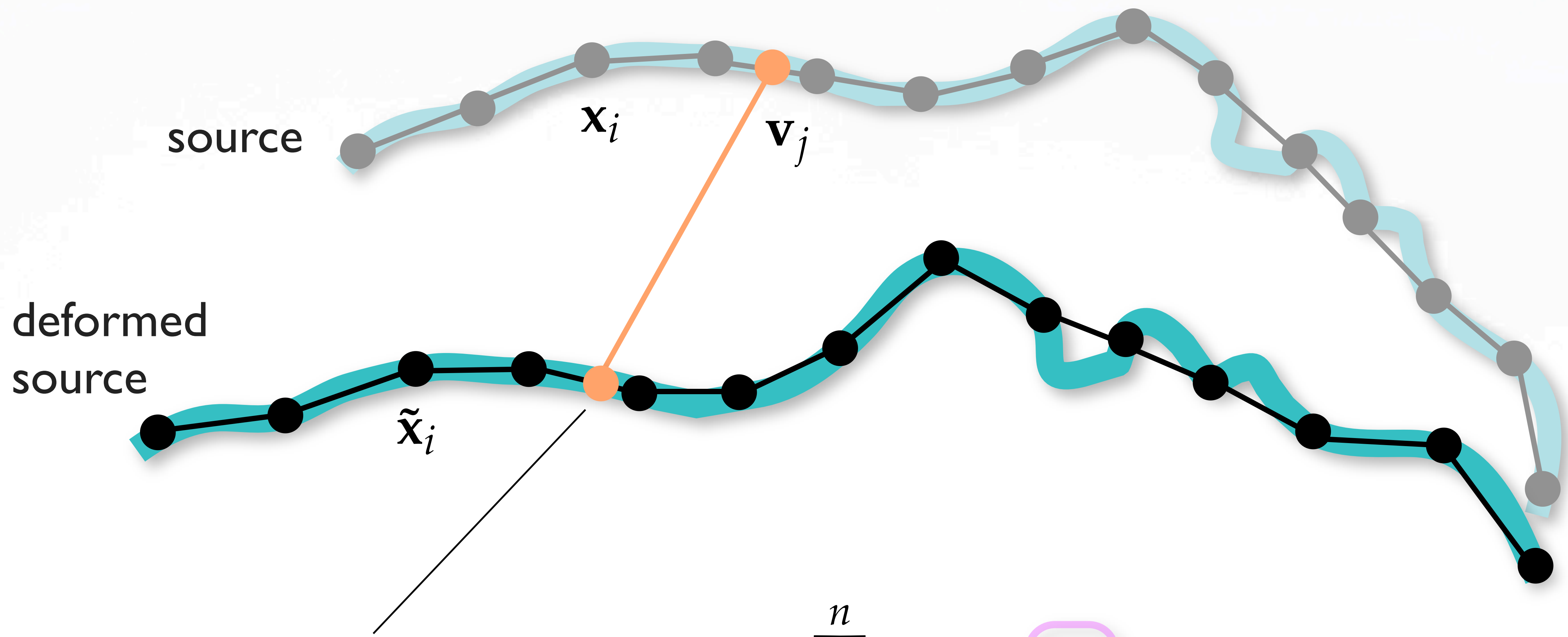
# Embedded Deformation

[Sumner et al. '07]



# Embedded Deformation

[Sumner et al. '07]

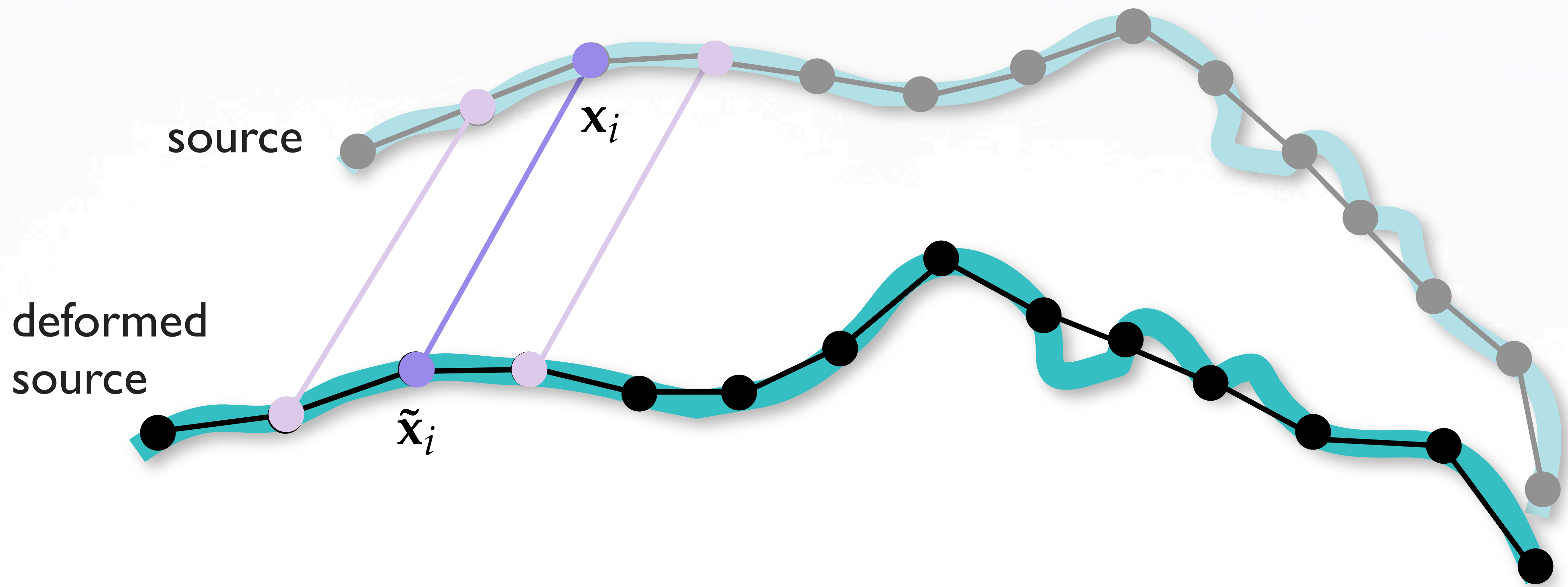



$$\tilde{\mathbf{v}}_j = \Phi_{\text{affine}}(\mathbf{v}_j) = \sum_{i=1}^{n} w_i(\mathbf{v}_j)\left[\boxed{A_i}(\mathbf{v}_j - \mathbf{x}_i) + \mathbf{x}_i + \mathbf{b}_i\right]$$

$E_{\text{rigid}}$ measures deviation from rigid motion

# Embedded Deformation

[Sumner et al. '07]




$E_{\text{smooth}}$ regularizes the deformation locally

$E_{\text{rigid}}$ measures deviation from rigid motion

# Correspondence Search

# Non-Rigid ICP

$\alpha_{\text{smooth}} \longrightarrow 0 \quad \alpha_{\text{rigid}} \longrightarrow 0$

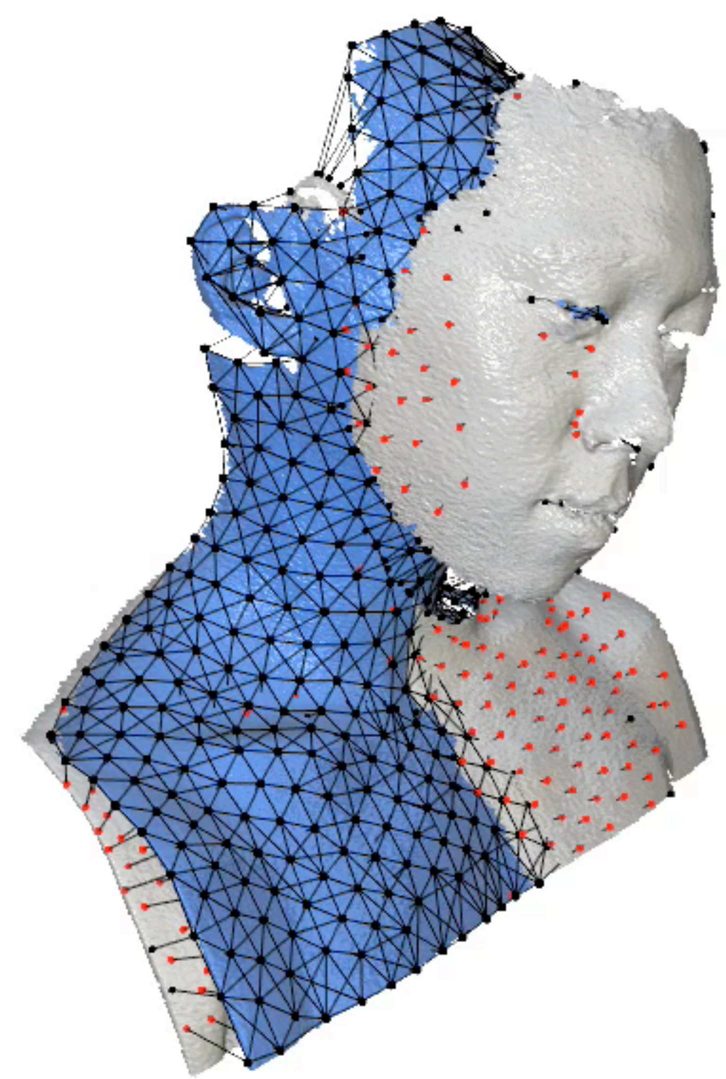
Relax Regularization

Correspondence

Weighting

Deformation

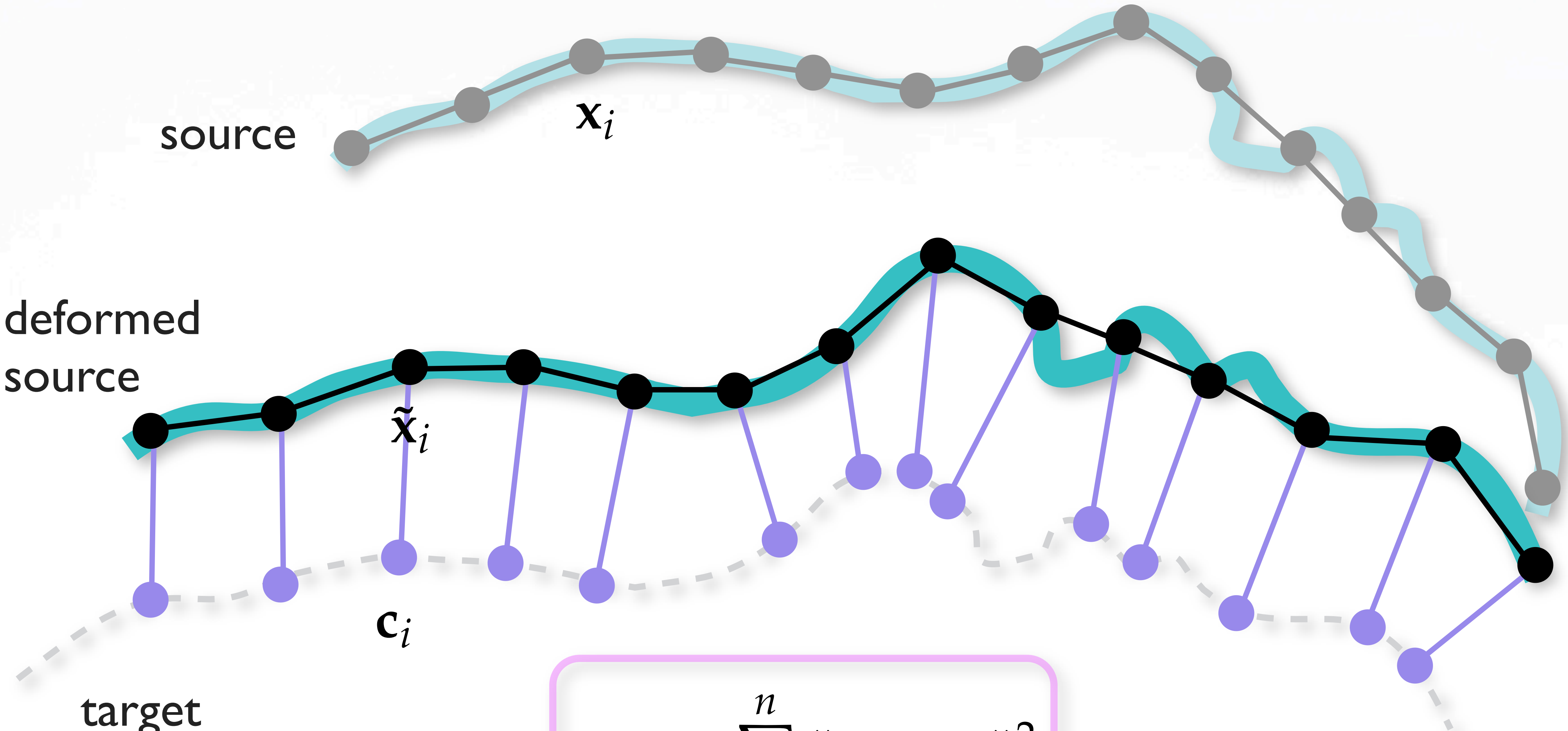
non-rigid registration



- Example with Embedded Deformation Model

# Coupled Optimization

[Li et al. '08]



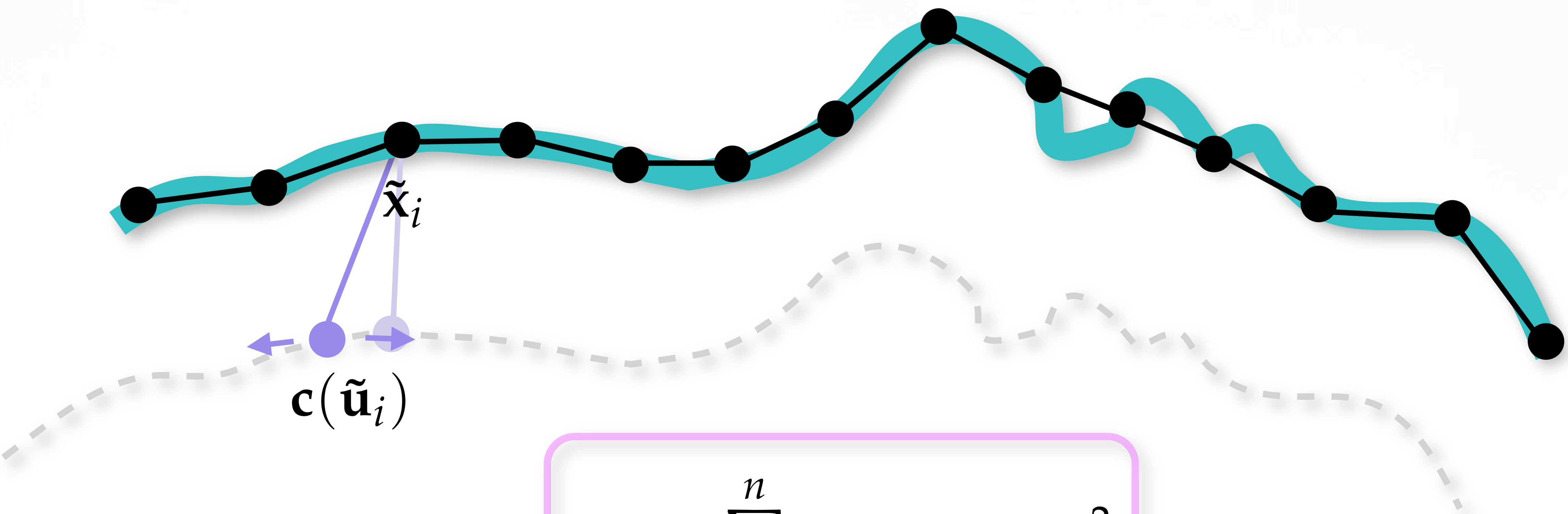

$$E_{\text{fit}} = \sum_{i=1}^{n} \|\tilde{\mathbf{x}}_i - \mathbf{c}_i\|_2^2$$

# Coupled Optimization

[Li et al. '08]

$\mathbf{c}(\tilde{\mathbf{u}}_i)$

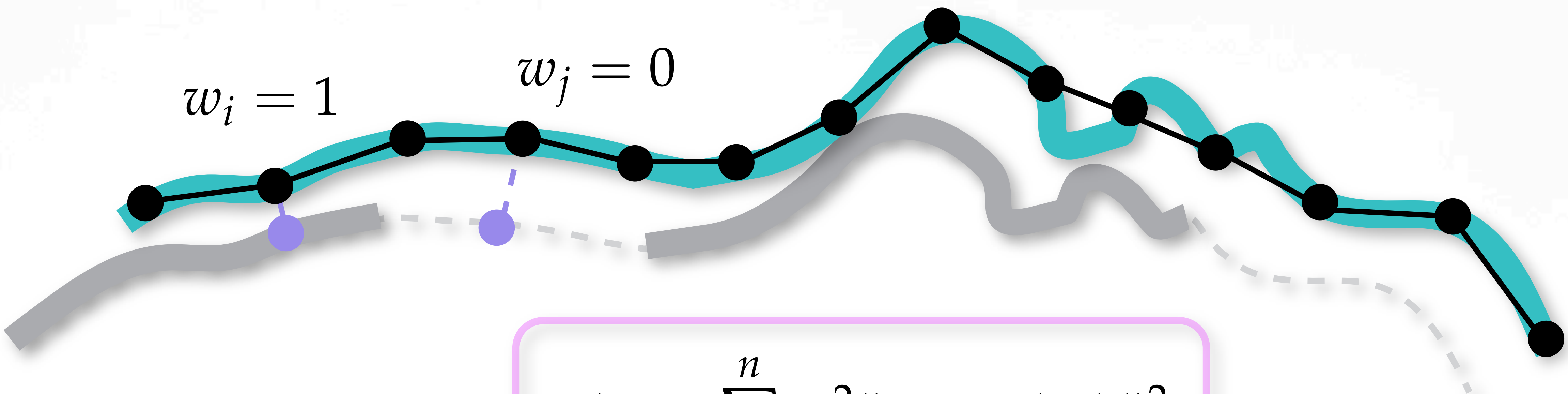
$\tilde{\mathbf{u}}_i = (\tilde{u}_i, \tilde{v}_i)$ optimization variable




$$E_{\text{fit}} = \sum_{i=1}^{n} \|\tilde{\mathbf{x}}_i - \mathbf{c}(\tilde{\mathbf{u}}_i)\|_2^2$$

# Coupled Optimization

[Li et al. '08]




$$E^*_{\text{fit}} = \sum_{i=0}^{n} w_i^2 \|\tilde{\mathbf{x}}_i - \tilde{\mathbf{c}}(\mathbf{u}_i)\|_2^2$$

# Coupled Optimization

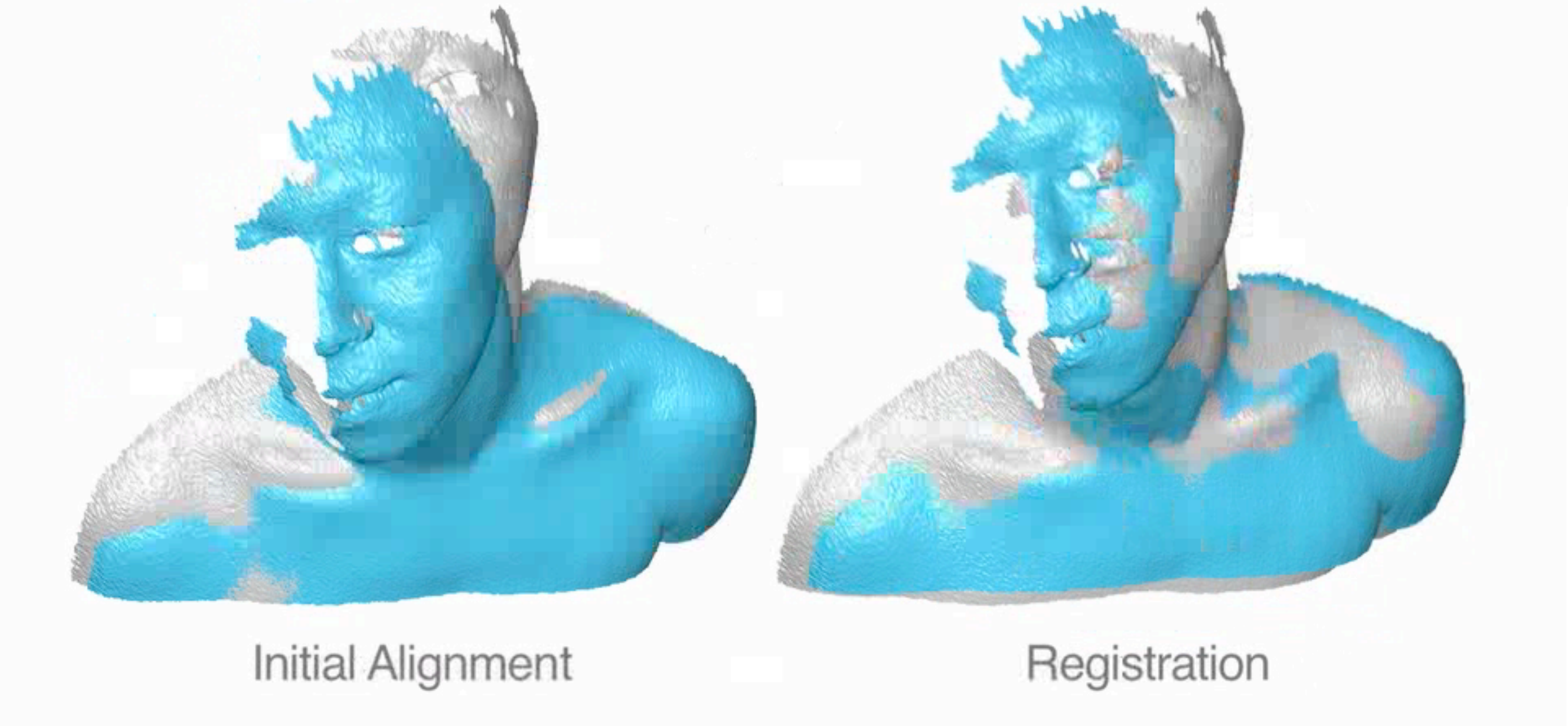
[Li et al. '08]

$$E = \boxed{\alpha_{\mathrm{rigid}} E_{\mathrm{rigid}} + \alpha_{\mathrm{smooth}} E_{\mathrm{smooth}}} + \boxed{\alpha_{\mathrm{fit}} E^*_{\mathrm{fit}}} + \boxed{\alpha_{\mathrm{conf}} E_{\mathrm{conf}}}$$

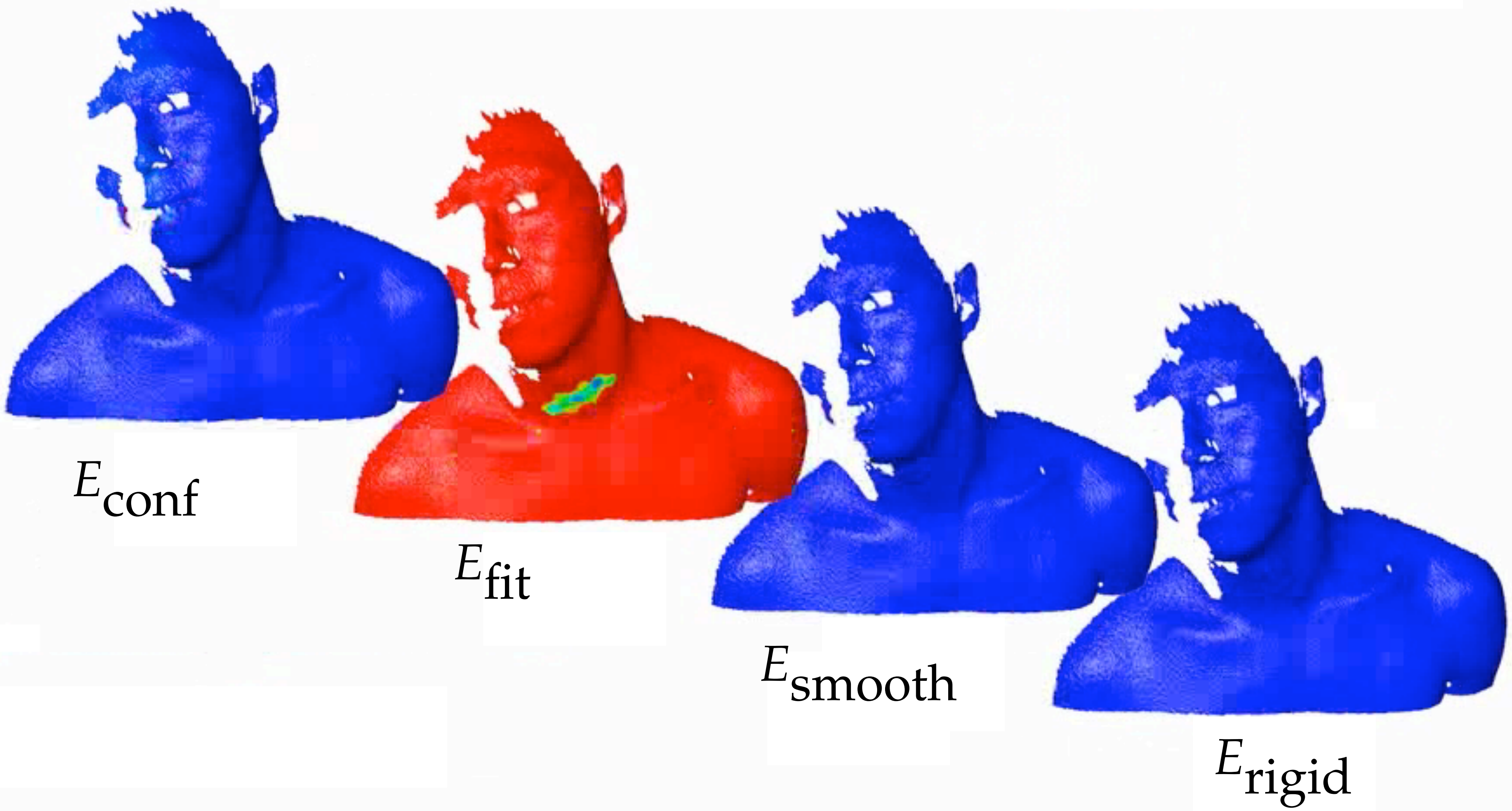
- Minimize deformation energy
- Minimize alignment error
- Maximize regions of overlap

# Optimization

219 iterations 2 min 19 s



# Energy Term Visualization

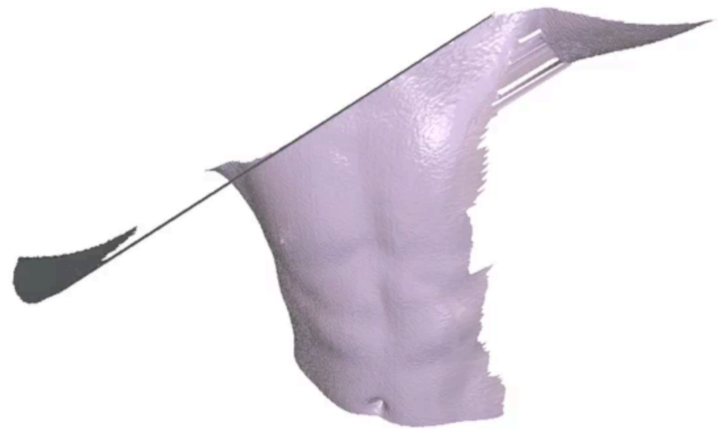


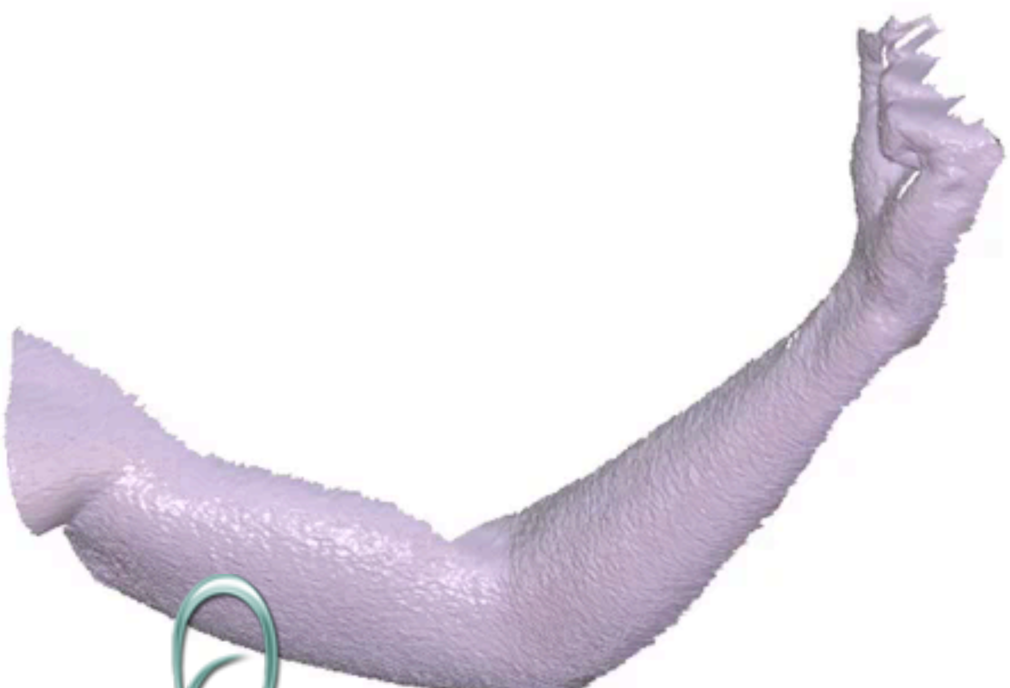
# In Progress

# Limitations of Template-based Methods

- Requires template with all details
- Still sparse motion capture
- No distinction of transient and persistent data

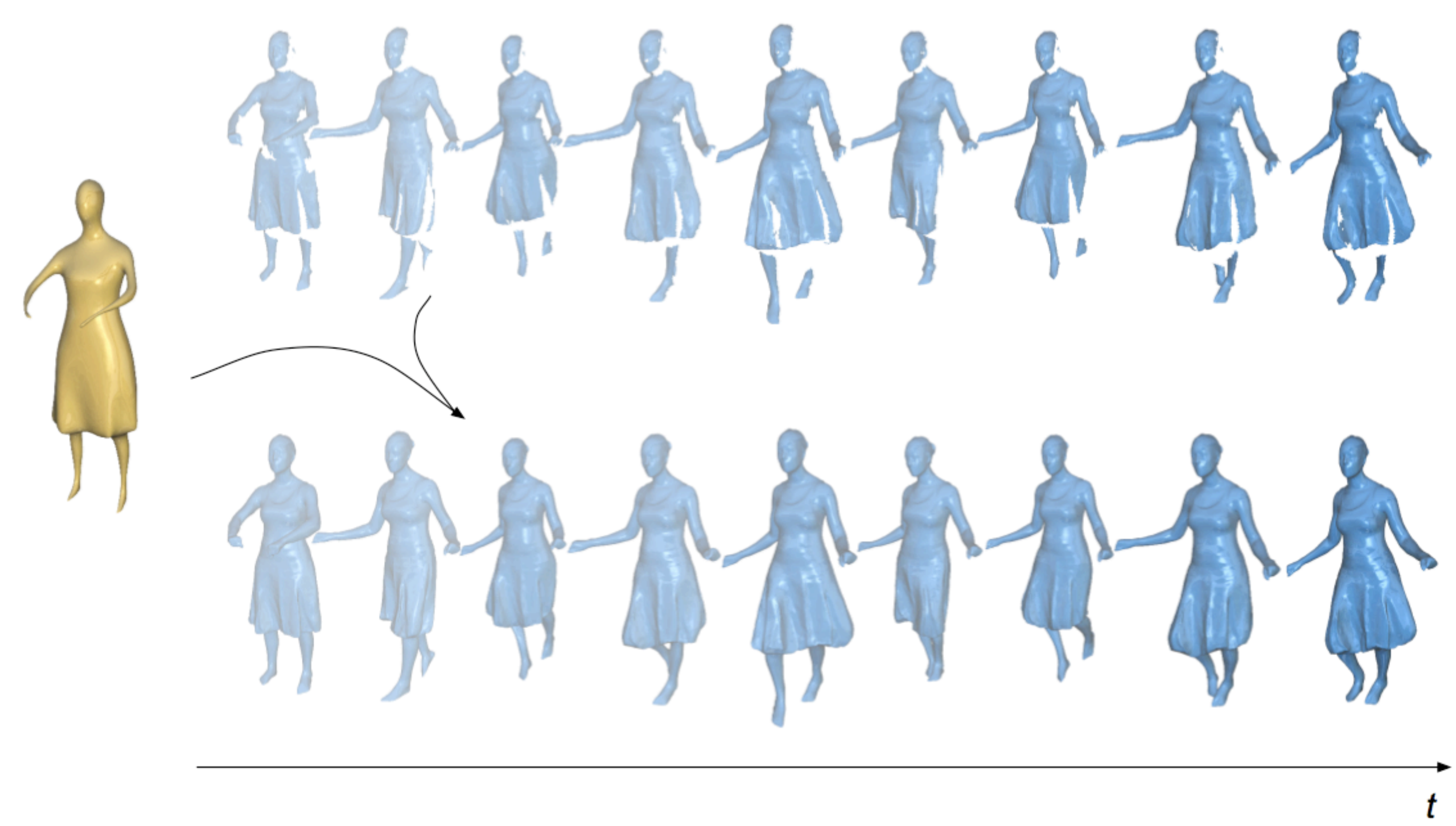
# Capturing Deformable Shapes





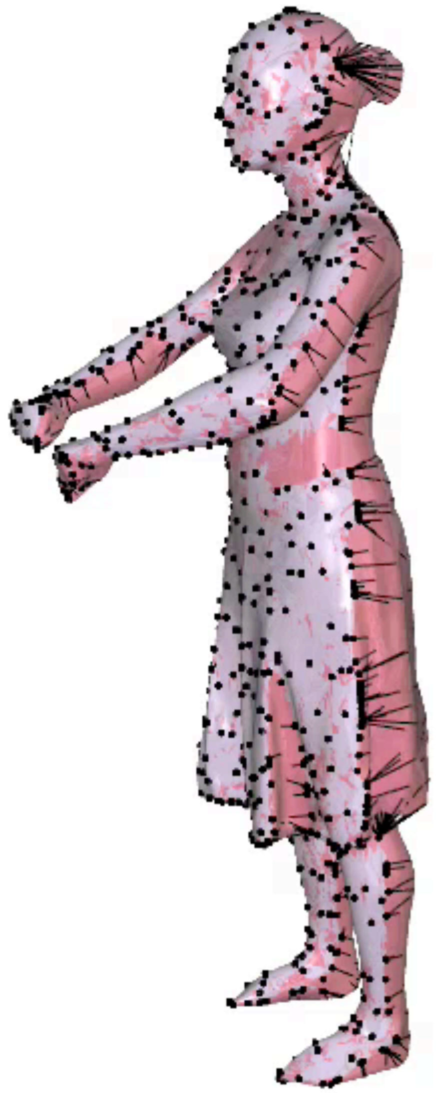
*Data provided with T. Weise and L. Van Gool*

# Geometry and Motion Reconstruction




*data provided by Stanford and MPI Saarbrücken* input data

template fitting



*data provided by Stanford and MPI Saarbrücken*

hao@inf.ethz.ch

# ?