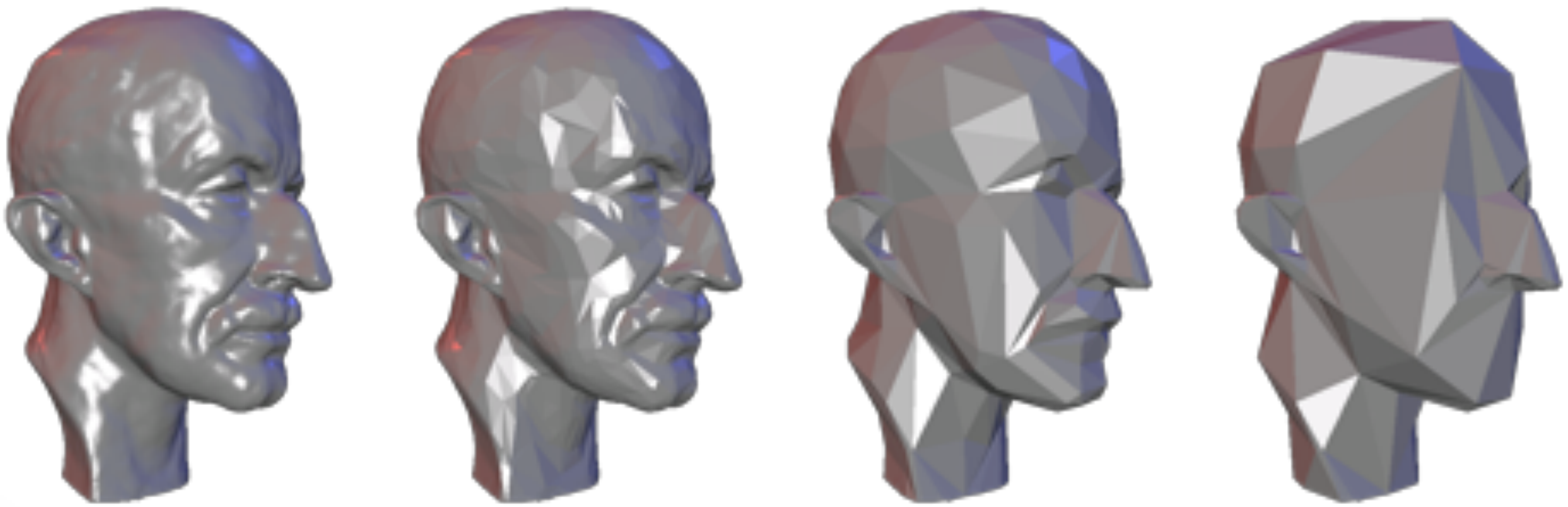
9.2 **Decimation**



Hao Li

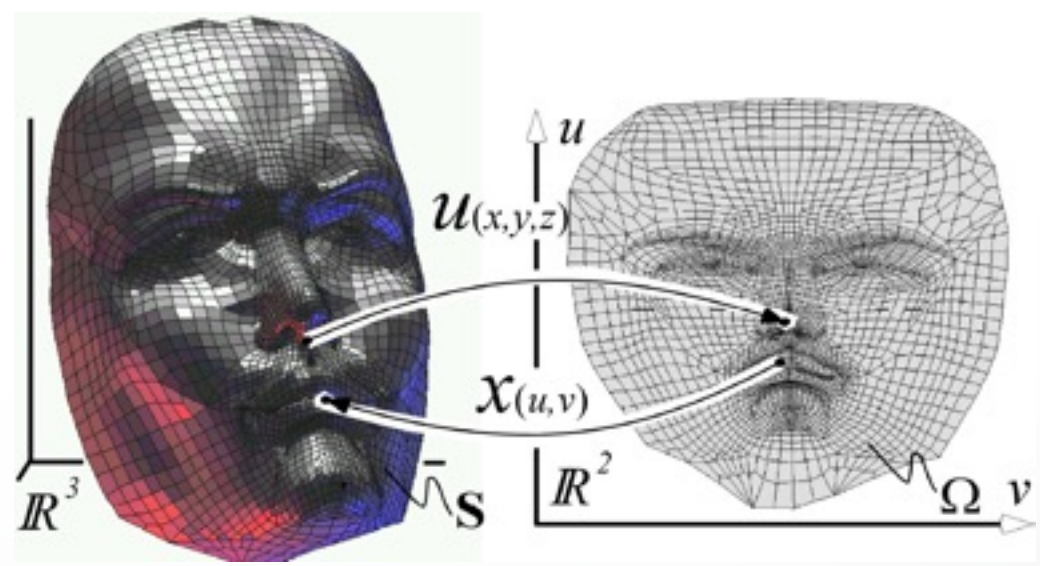
http://cs621.hao-li.com

Last Time

Parameterization

- isometric $$\mathbf{I}(u,v) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
- conformal $$\mathbf{I}(u,v) = s(u,v) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
- equiareal $$\det(\mathbf{I}(u,v)) = 1$$

$$\mathbf{I} = \begin{pmatrix} \mathbf{x}_u^T \mathbf{x}_u & \mathbf{x}_u^T \mathbf{x}_v \\ \mathbf{x}_u^T \mathbf{x}_v & \mathbf{x}_v^T \mathbf{x}_v \end{pmatrix}$$



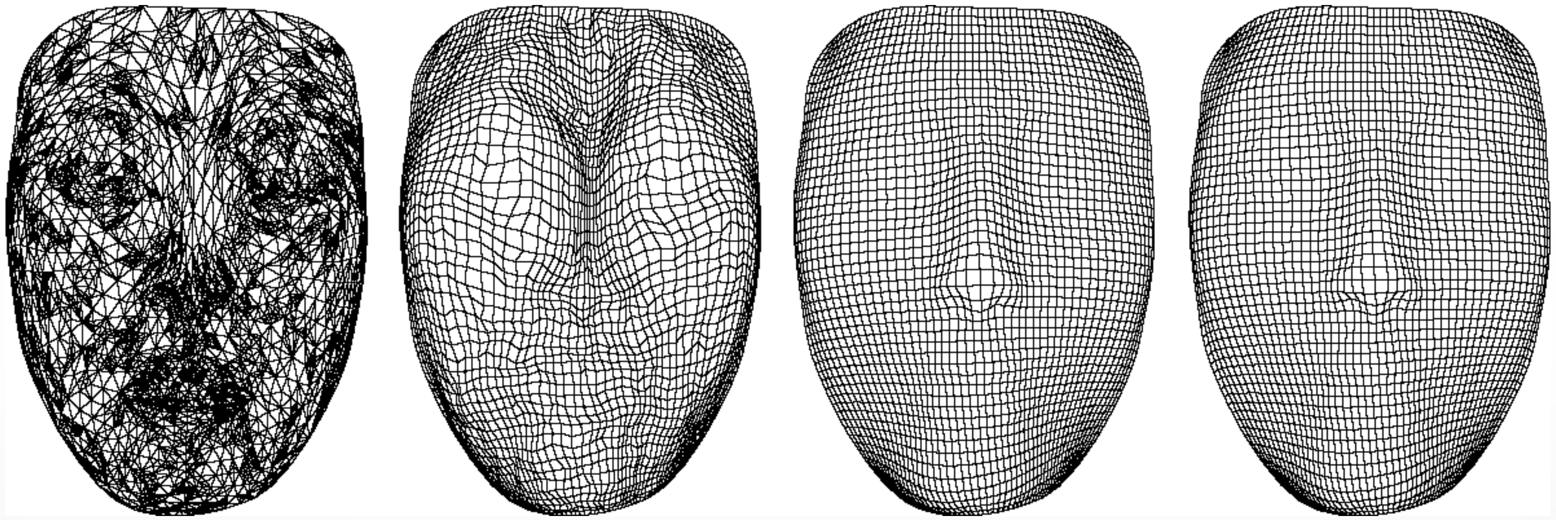
Last Time

Harmonic Maps

- minimize Dirichlet energy: $\int_\Omega \|\nabla \mathbf{x}\|^2 = \int_\Omega \|\mathbf{x}_u\|^2 + \|\mathbf{x}_v\|^2 \, \mathrm{d}u \, \mathrm{d}v$
- Euler-Lagrange PDE $\Delta \mathbf{x}(u,v) = 0$

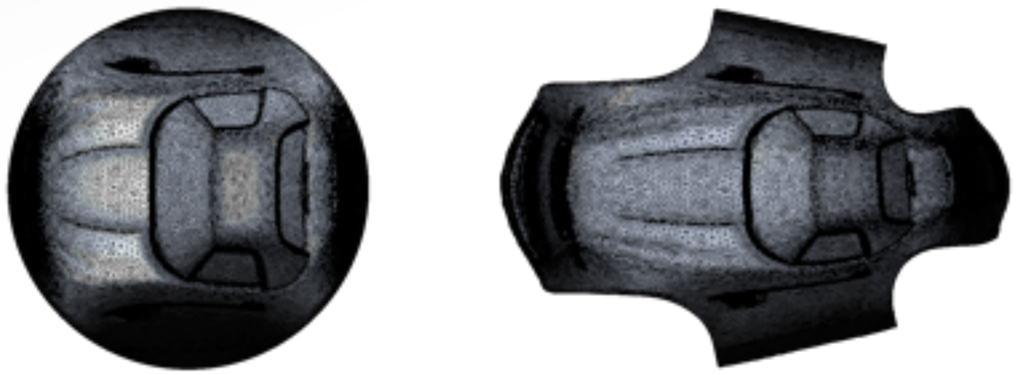
Discrete Harmonic Maps

Convex Combination Maps

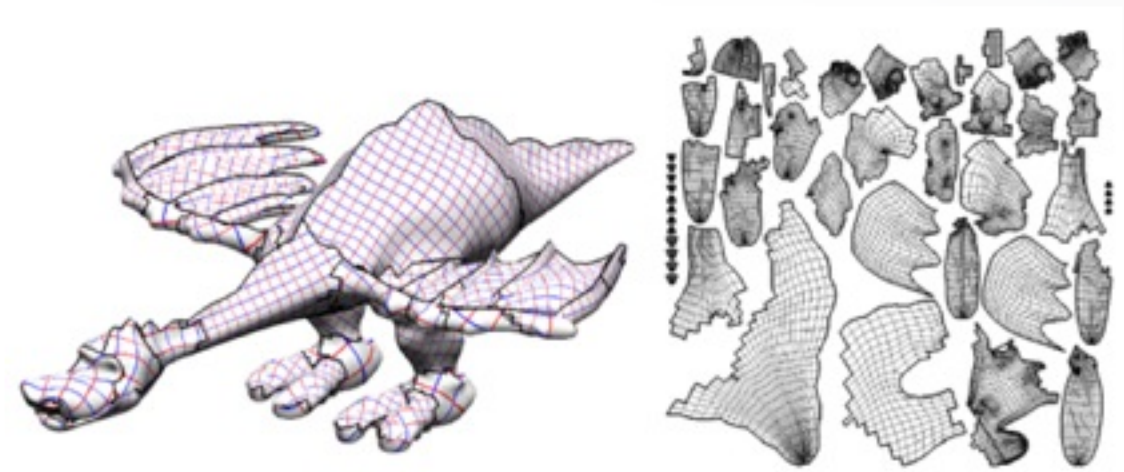


original mesh | uniform weights | cotan weights | mean value

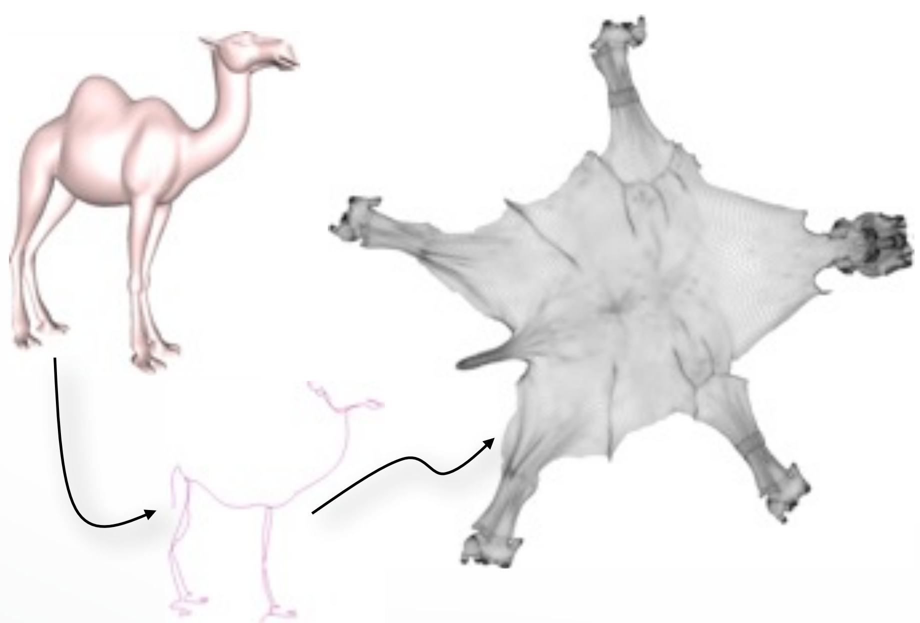
Last Time



fixed vs. open boundaries



texture atlases



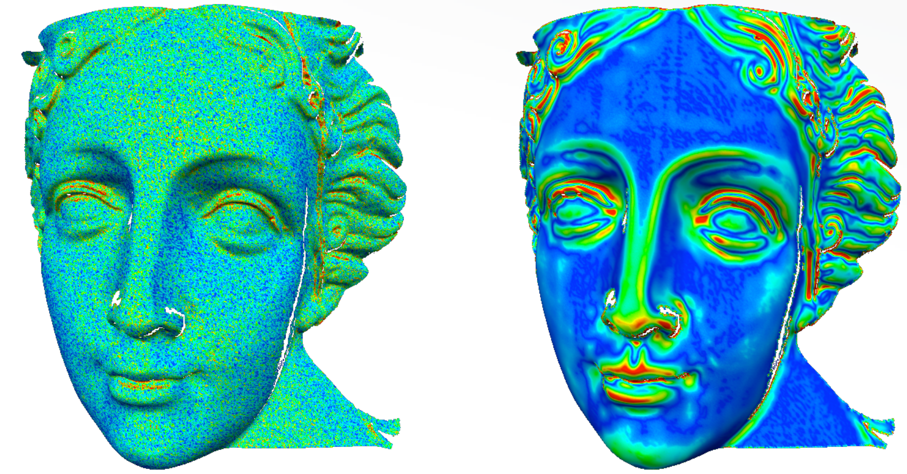
cutting the mesh → disk topology

constrained parameterization

Mesh Optimization

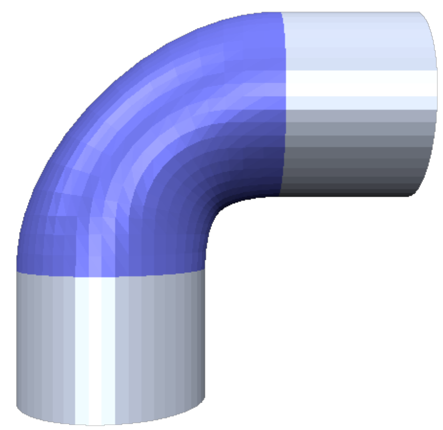
Smoothing

- Low geometric noise



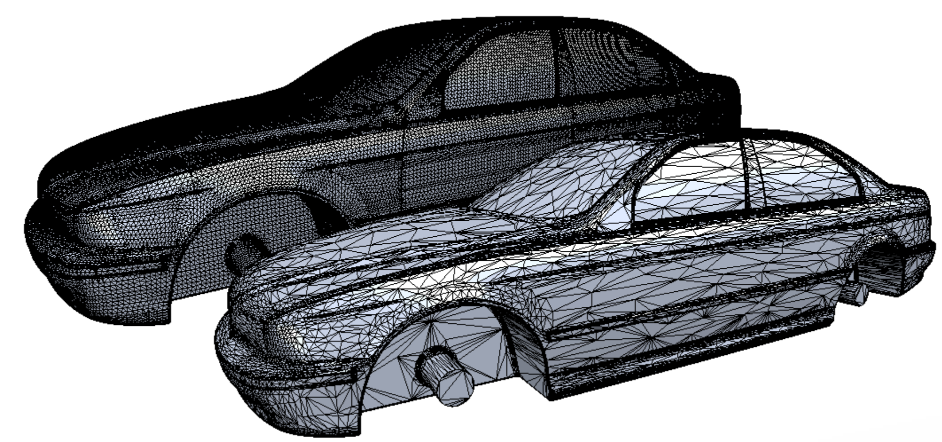
Fairing

- Simplest shape



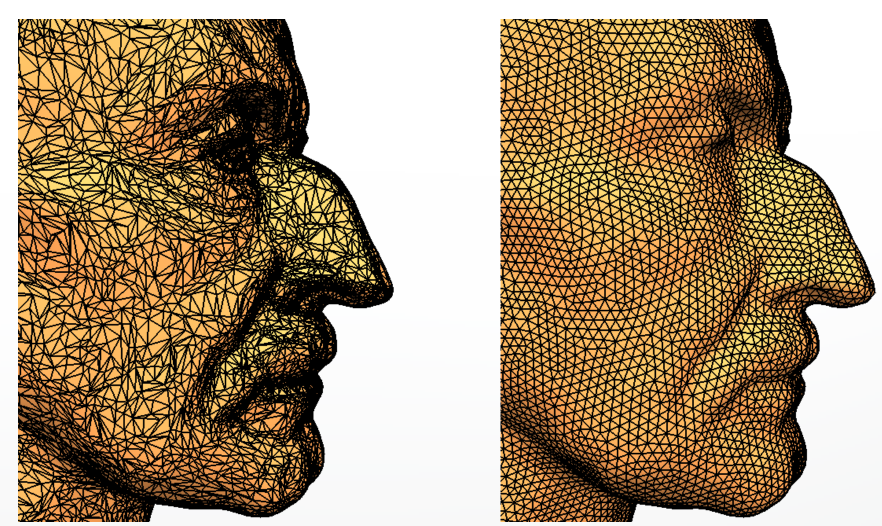
Decimation

- Low complexity



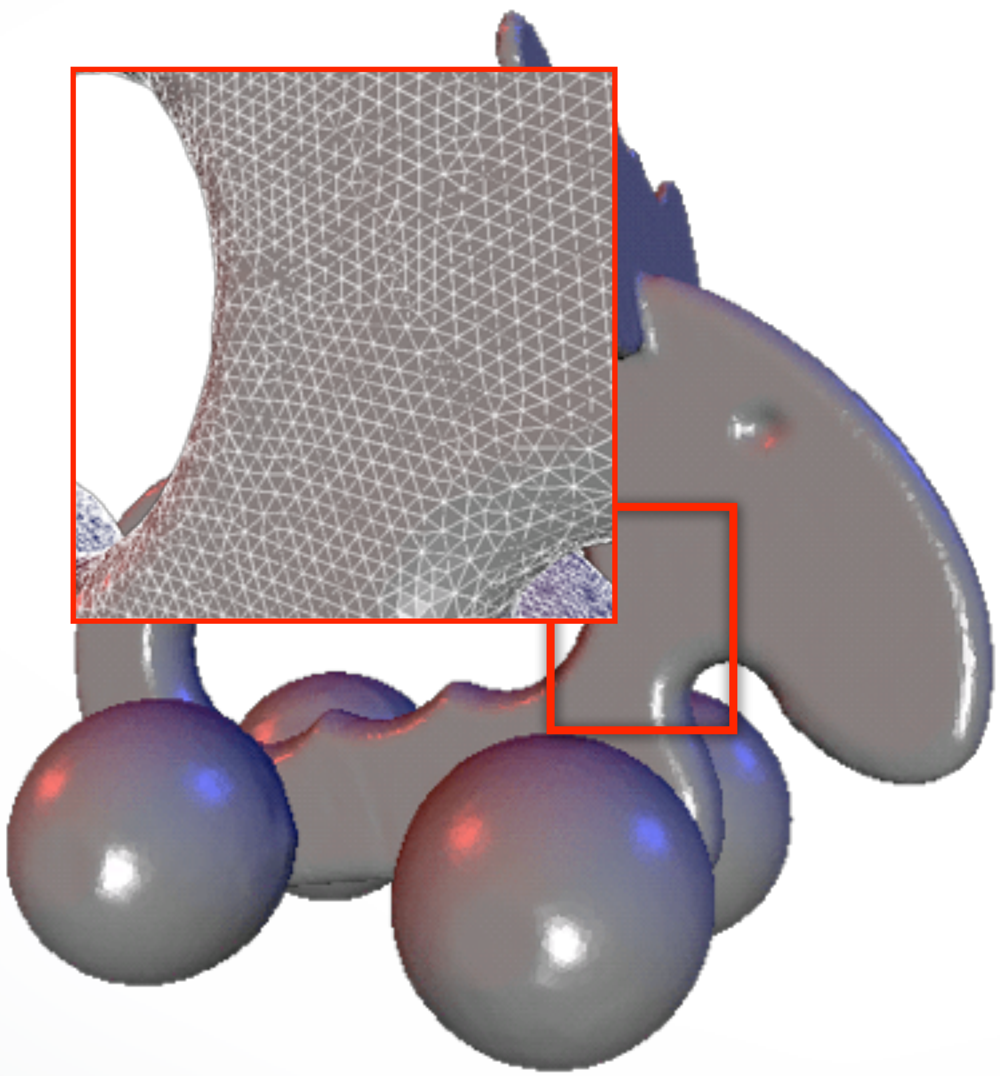
Remeshing

- Triangle Shape

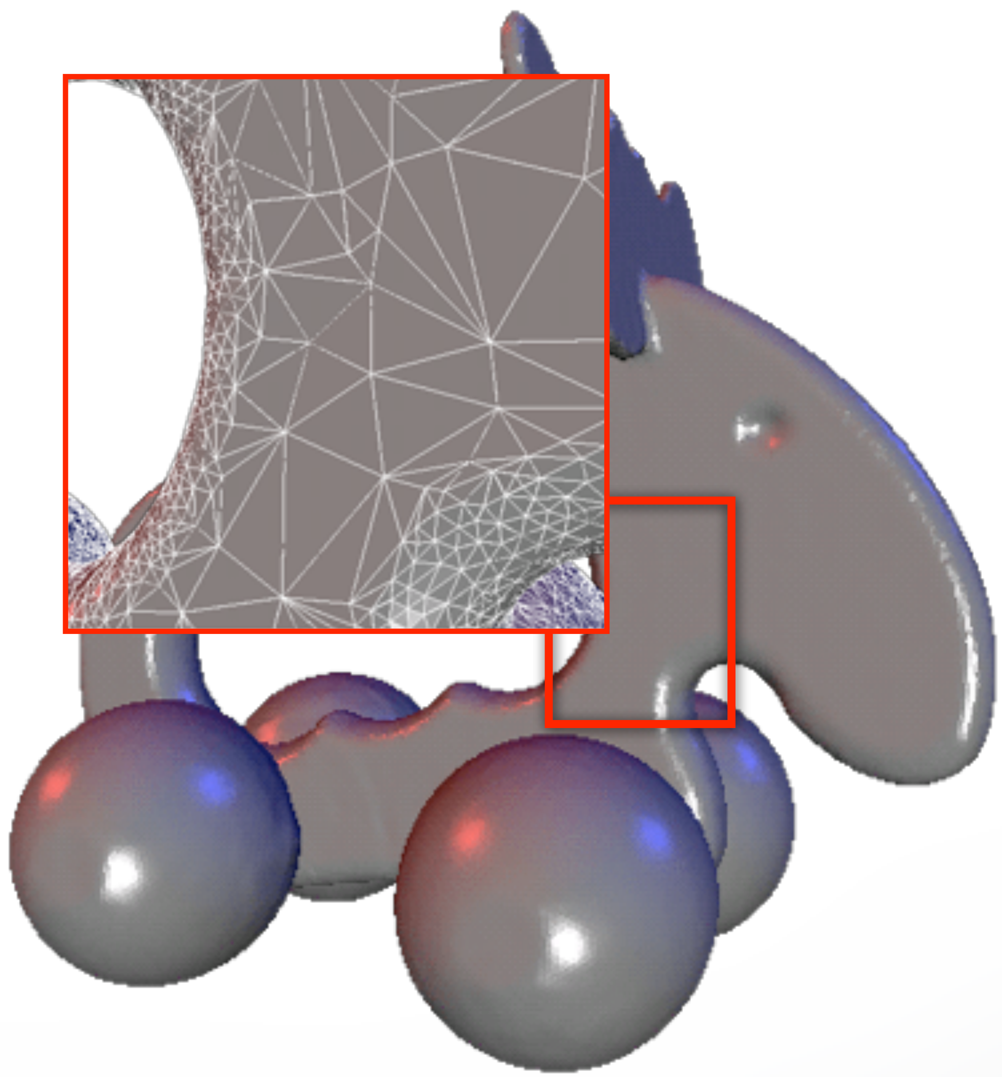


Mesh Decimation

Oversampled 3D scan data



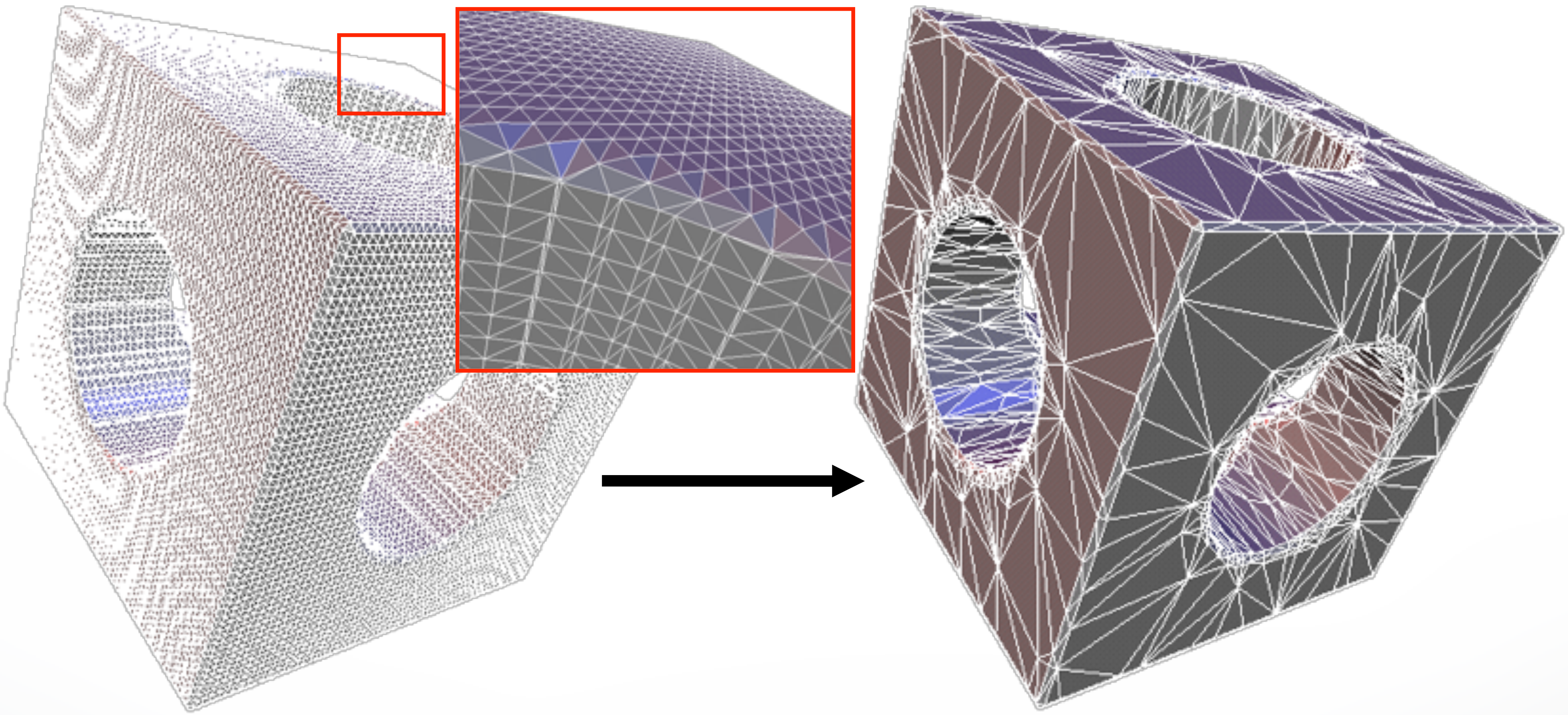
~150k triangles



~80k triangles

Mesh Decimation

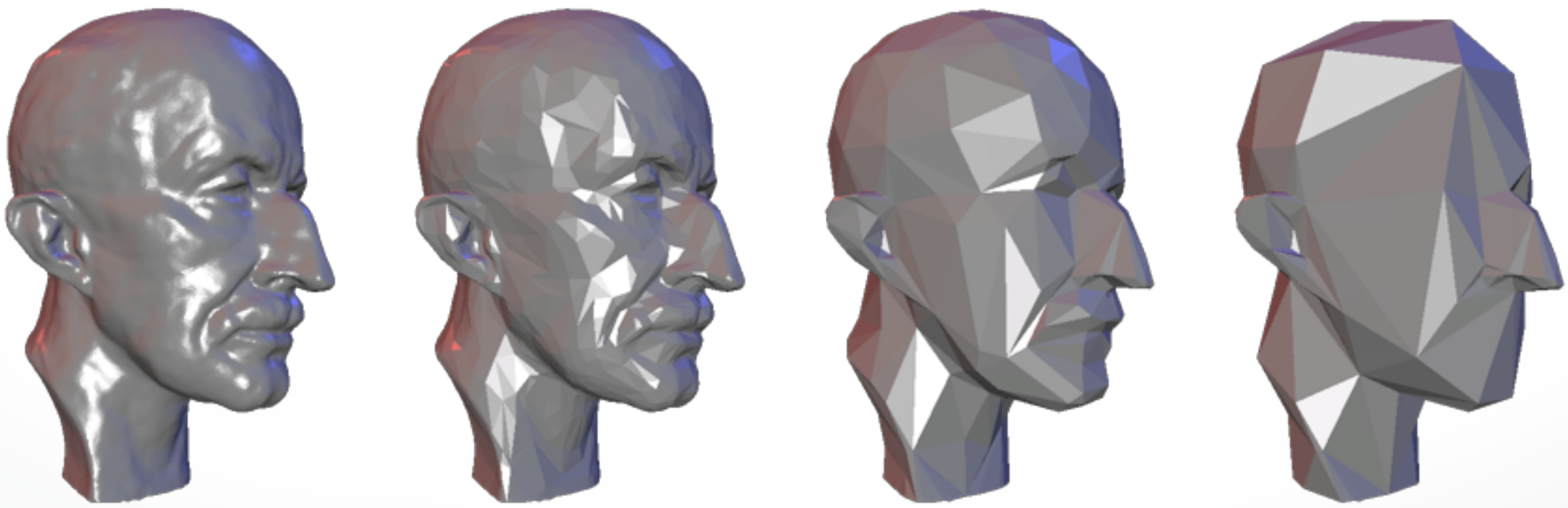
Over tesselation: e.g., Iso-surface extraction



Mesh Decimation

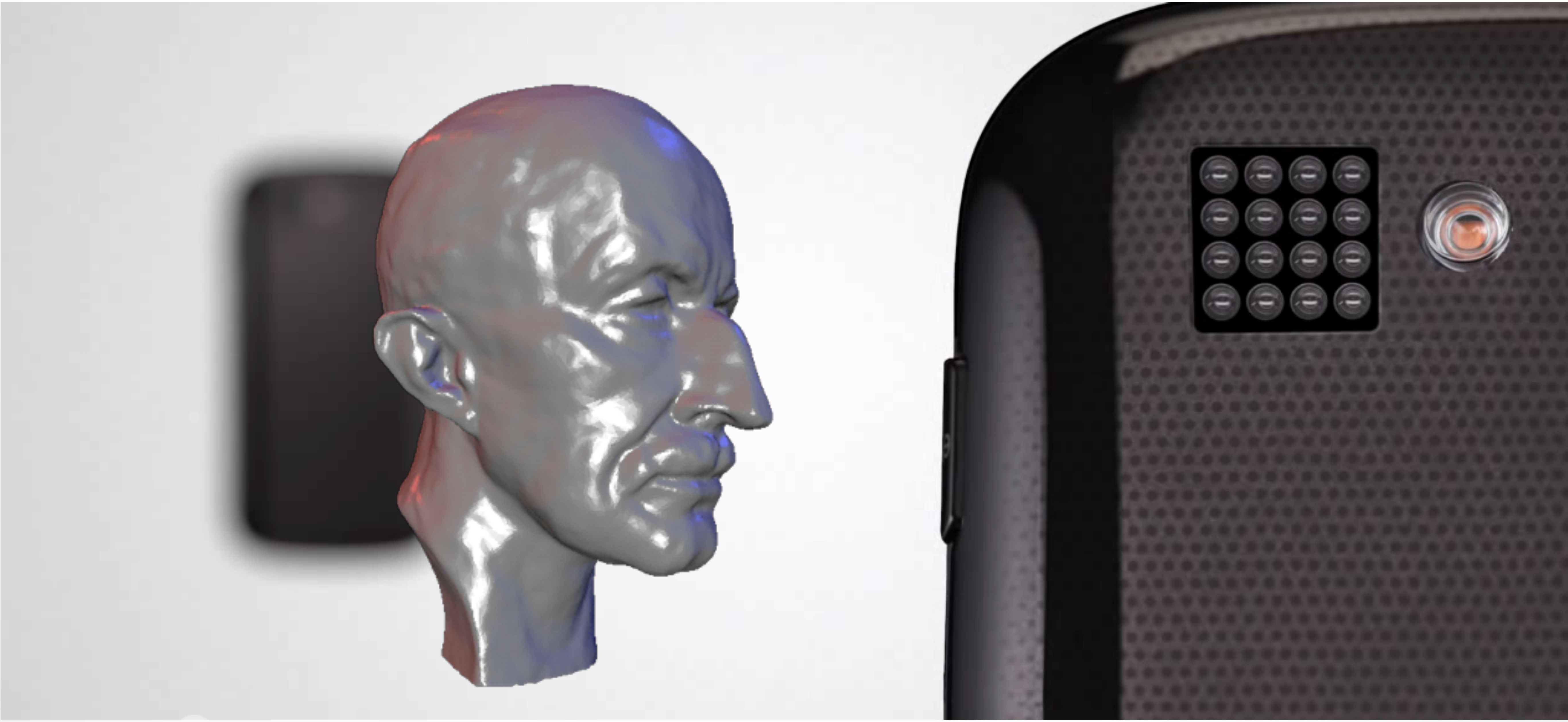
Multi-resolution hierarchies for

- efficient geometry processing
- level-of-detail (LOD) rendering



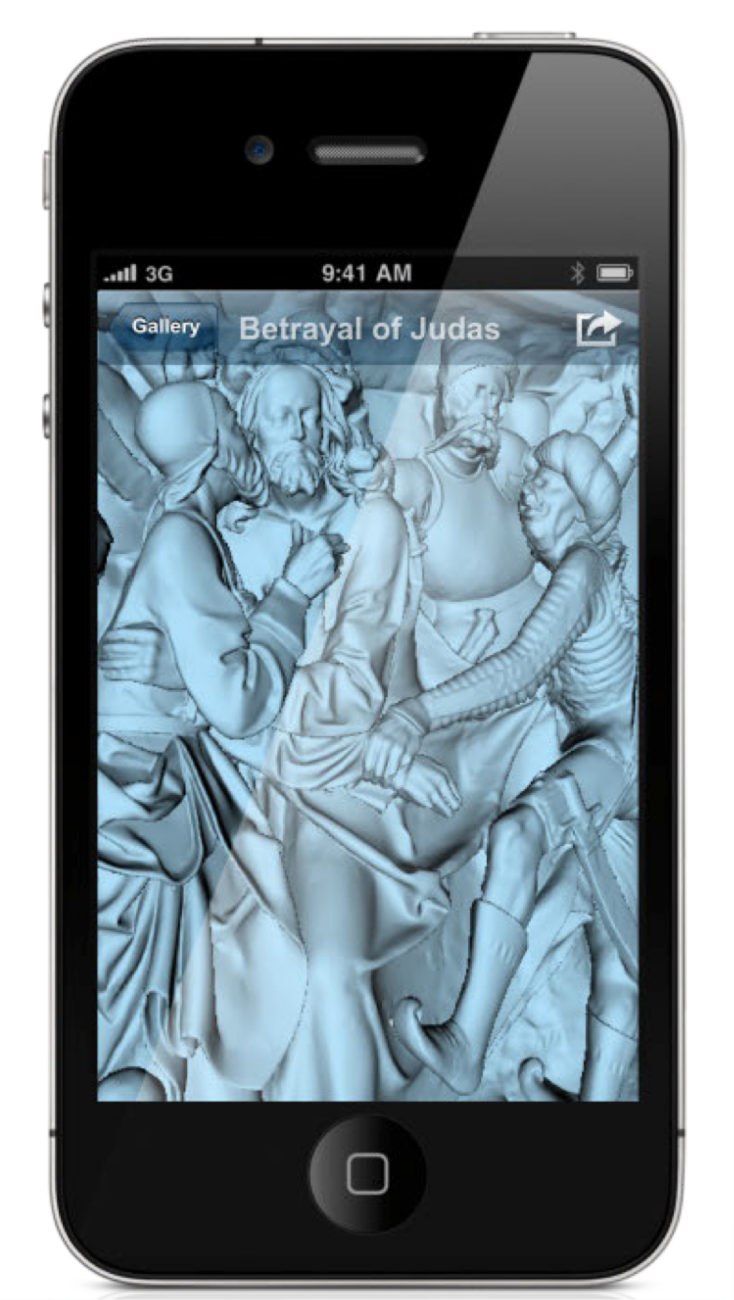
Mesh Decimation

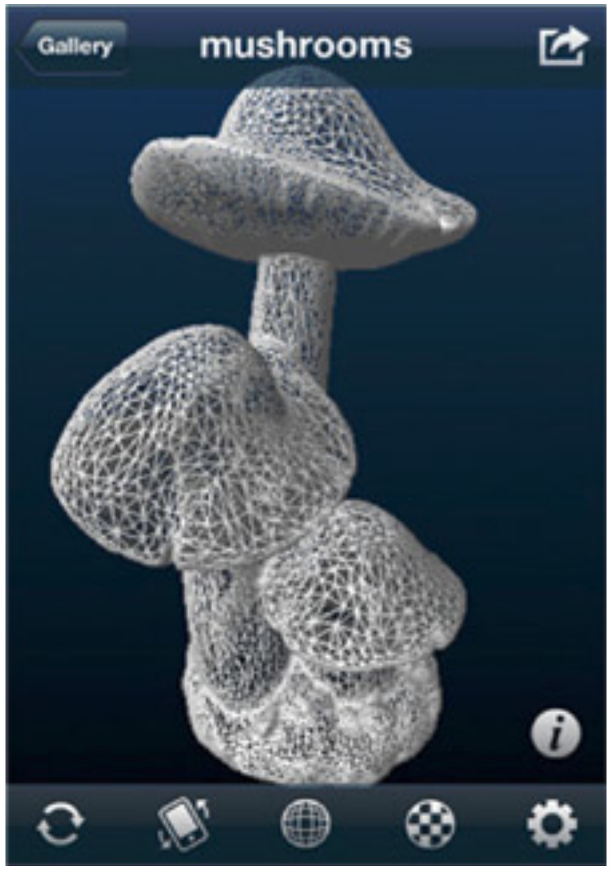
Adaptation to hardware capabilities



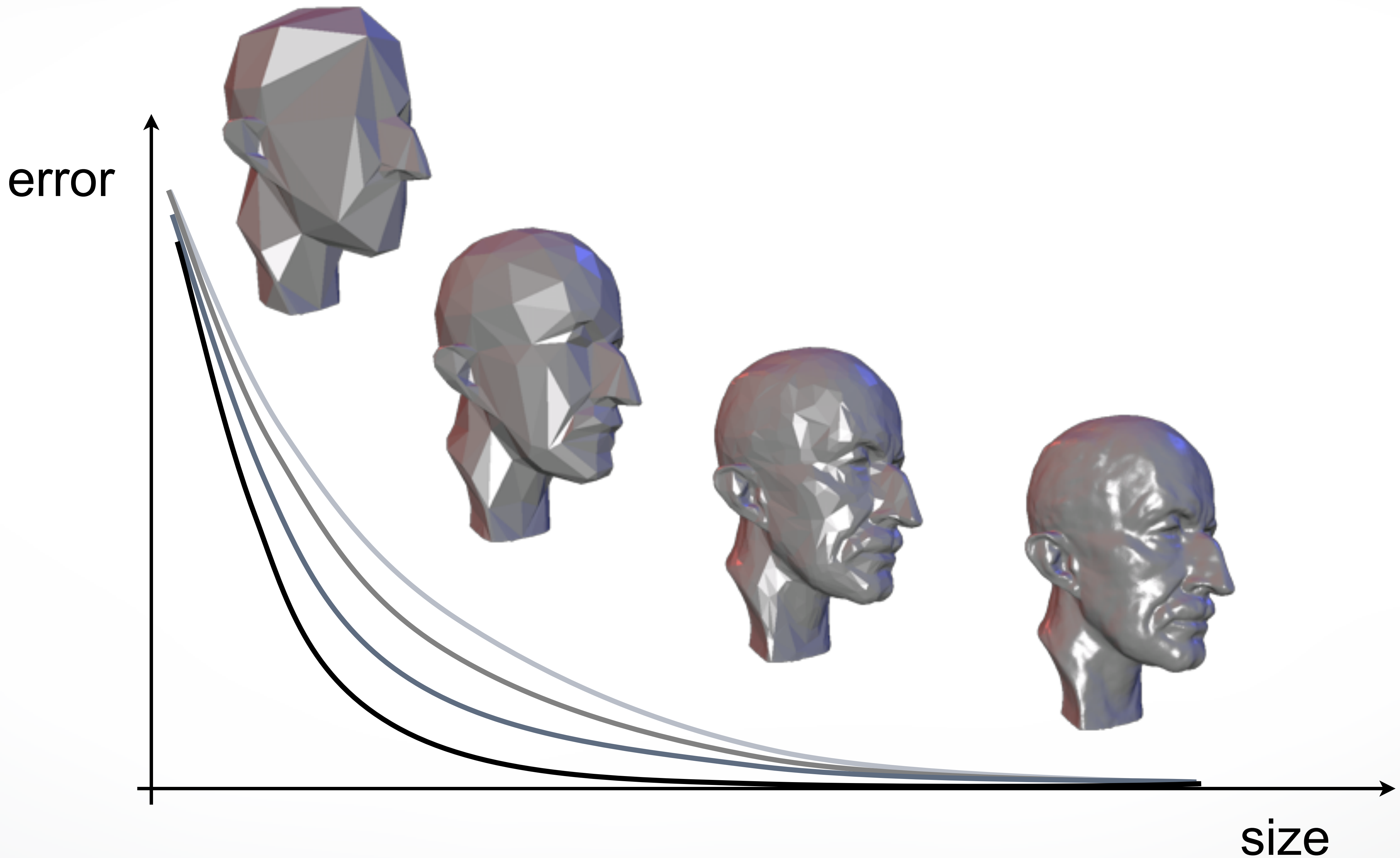
Mesh Decimation

Adaptation to hardware capabilities





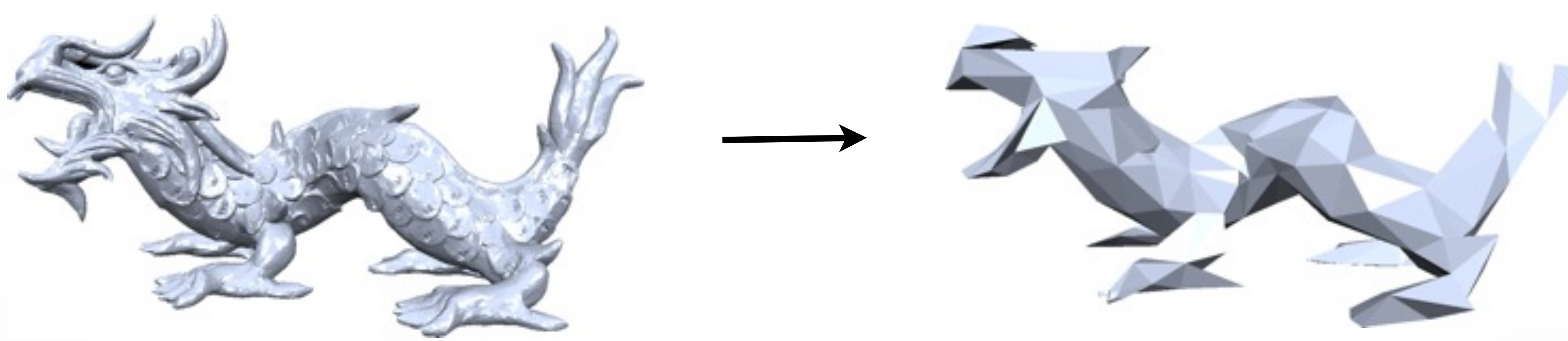
Size-Quality Tradeoff



Problem Statement

Given $\mathcal{M} = (\mathcal{V}, \mathcal{F})$, find $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

- $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or
- $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal



$\mathcal{M}$ $\mathcal{M}'$

Problem Statement

Given $\mathcal{M} = (\mathcal{V}, \mathcal{F})$, find $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

- $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or
- $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal

NP hard

- Look for sub-optimal solution

Respect additional fairness criteria

- Normal deviation, triangle shape, colors,…

Outline

Mesh Decimation methods

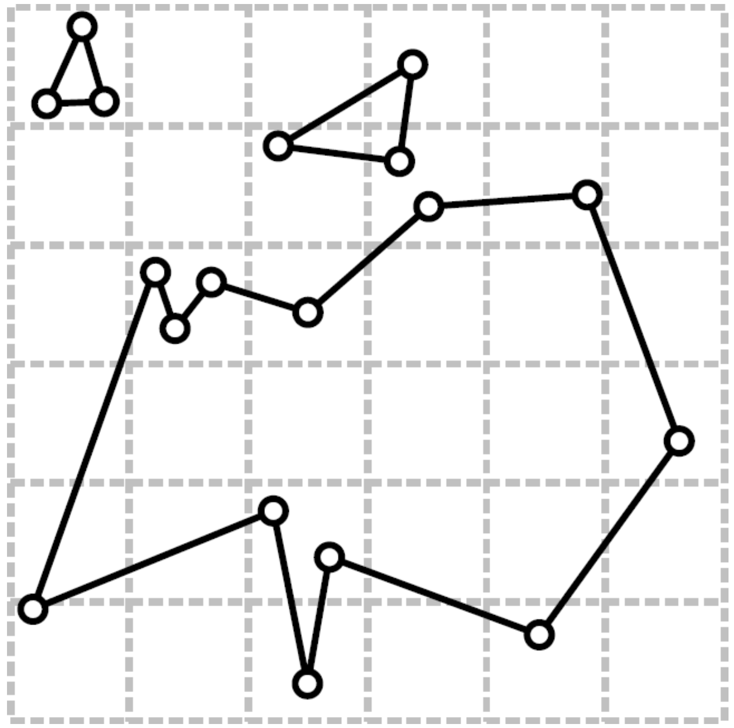
- **Vertex Clustering**
- Iterative Decimation

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes

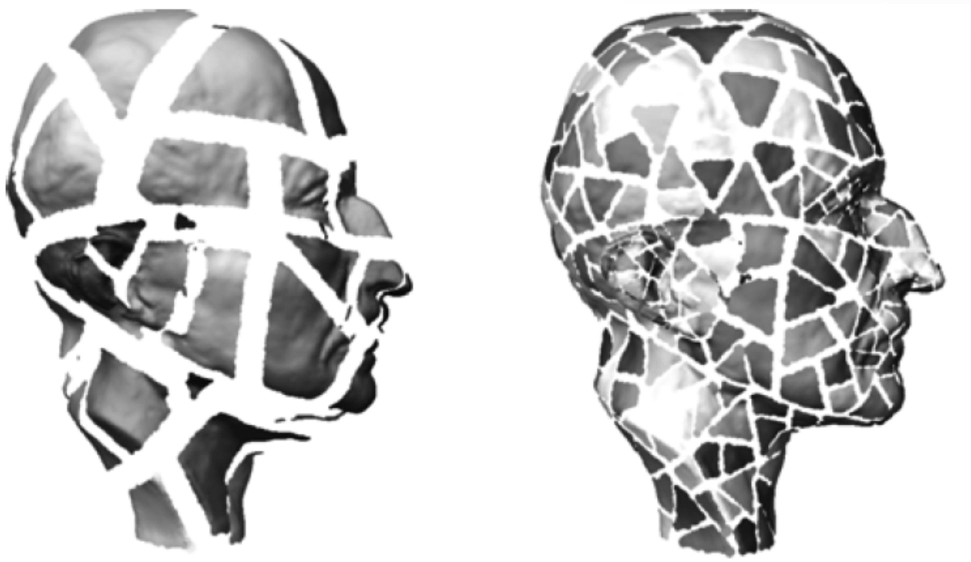
Vertex Clustering

- **Cluster Generation**
  - Uniform 3D grid
  - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

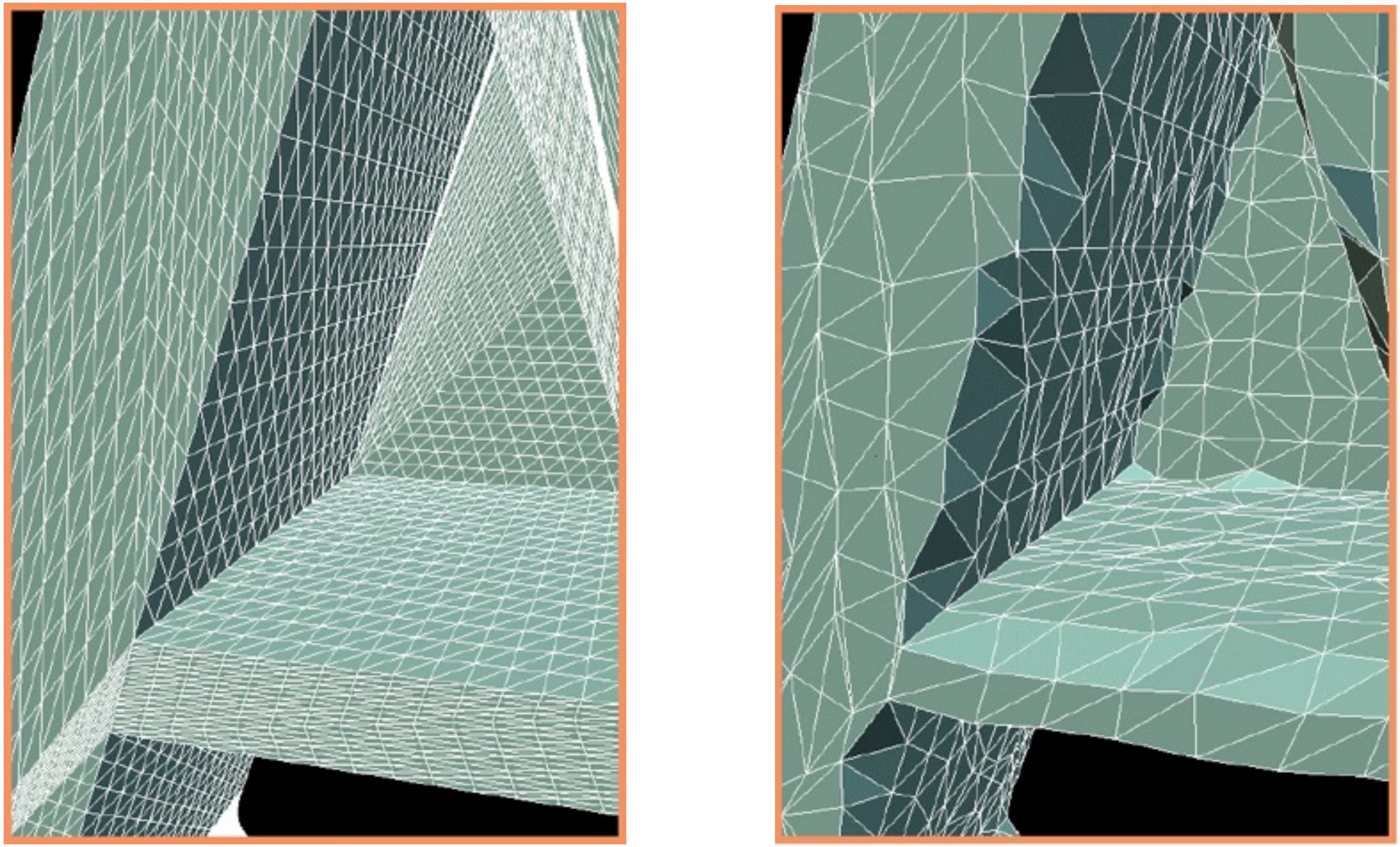
- **Cluster Generation**
  - Hierarchical approach
  - Top-down or bottom-up
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

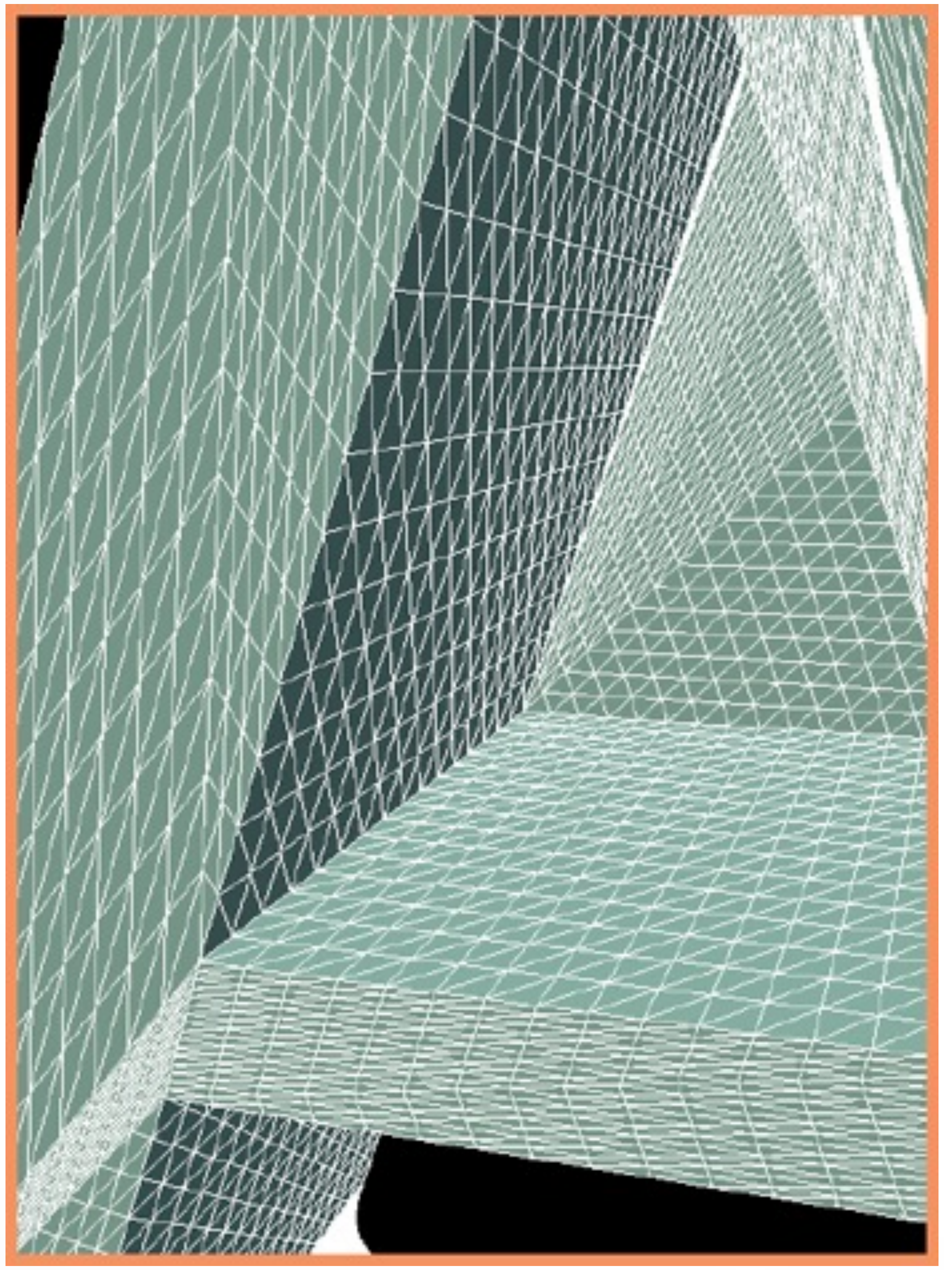
- Cluster Generation
- **Computing a representative**
  - Average/median vertex position
  - Error quadrics
- Mesh generation
- Topology changes

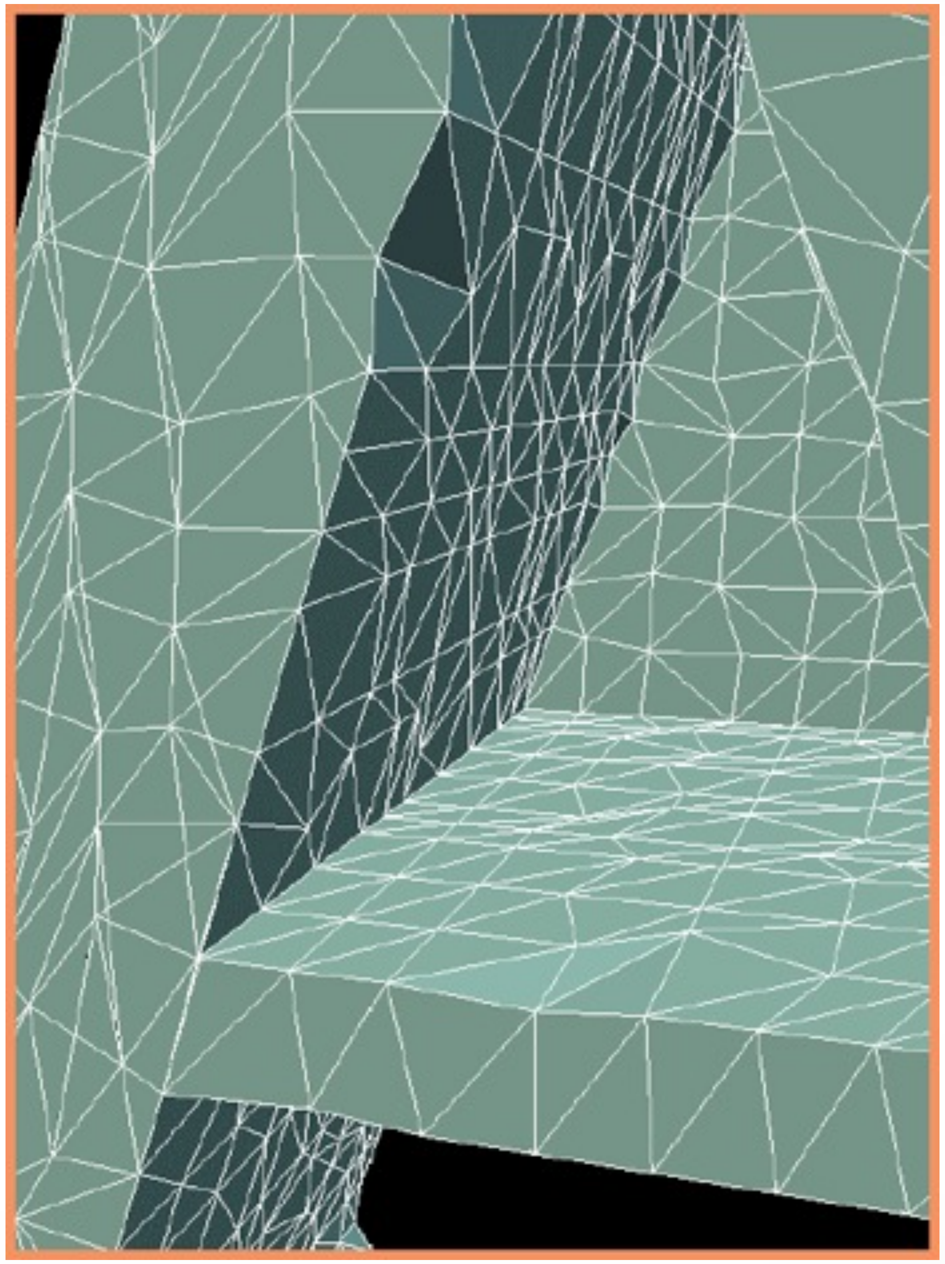
Computing a Representative



average vertex position → low pass filter

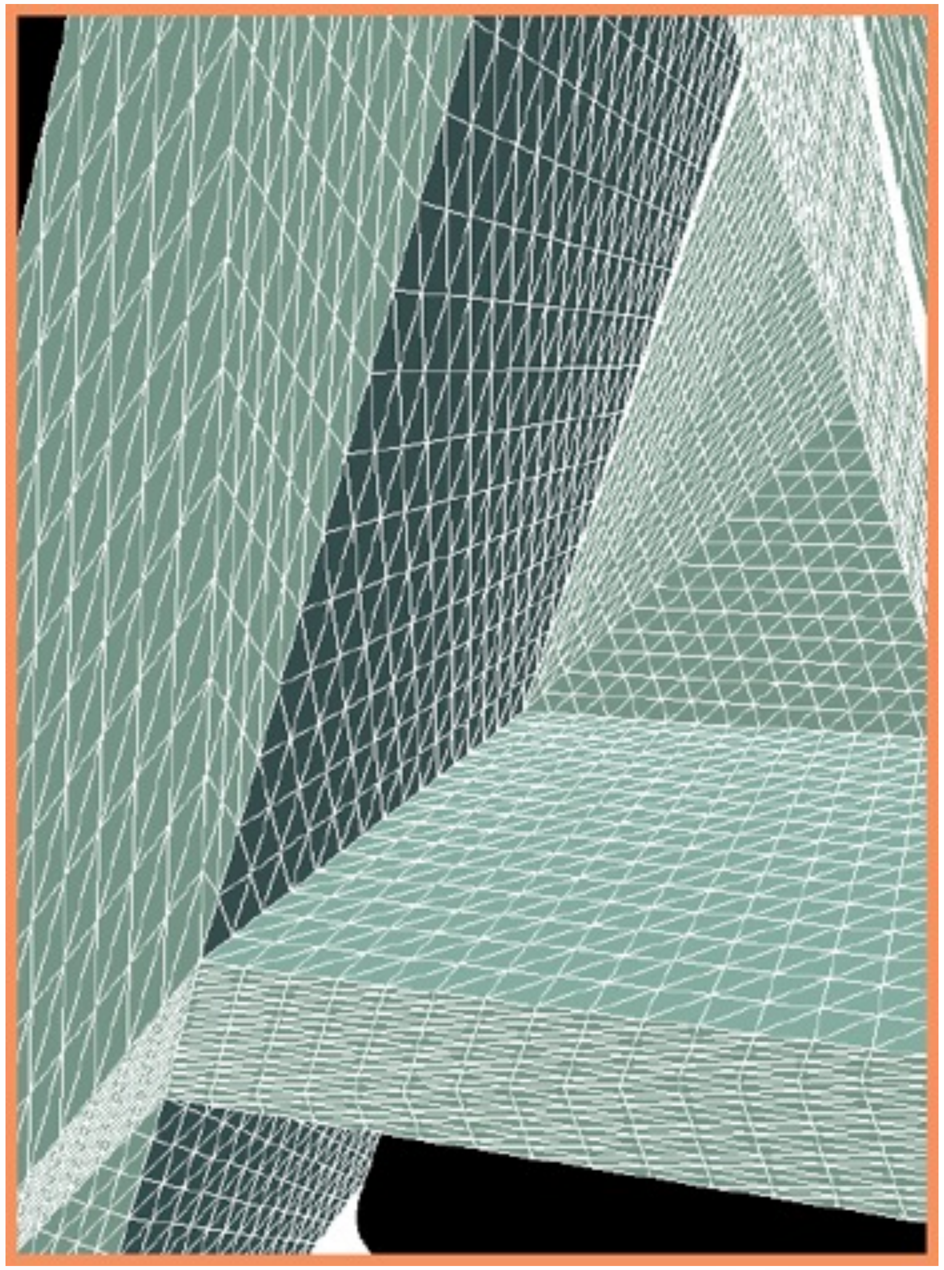
Computing a Representative

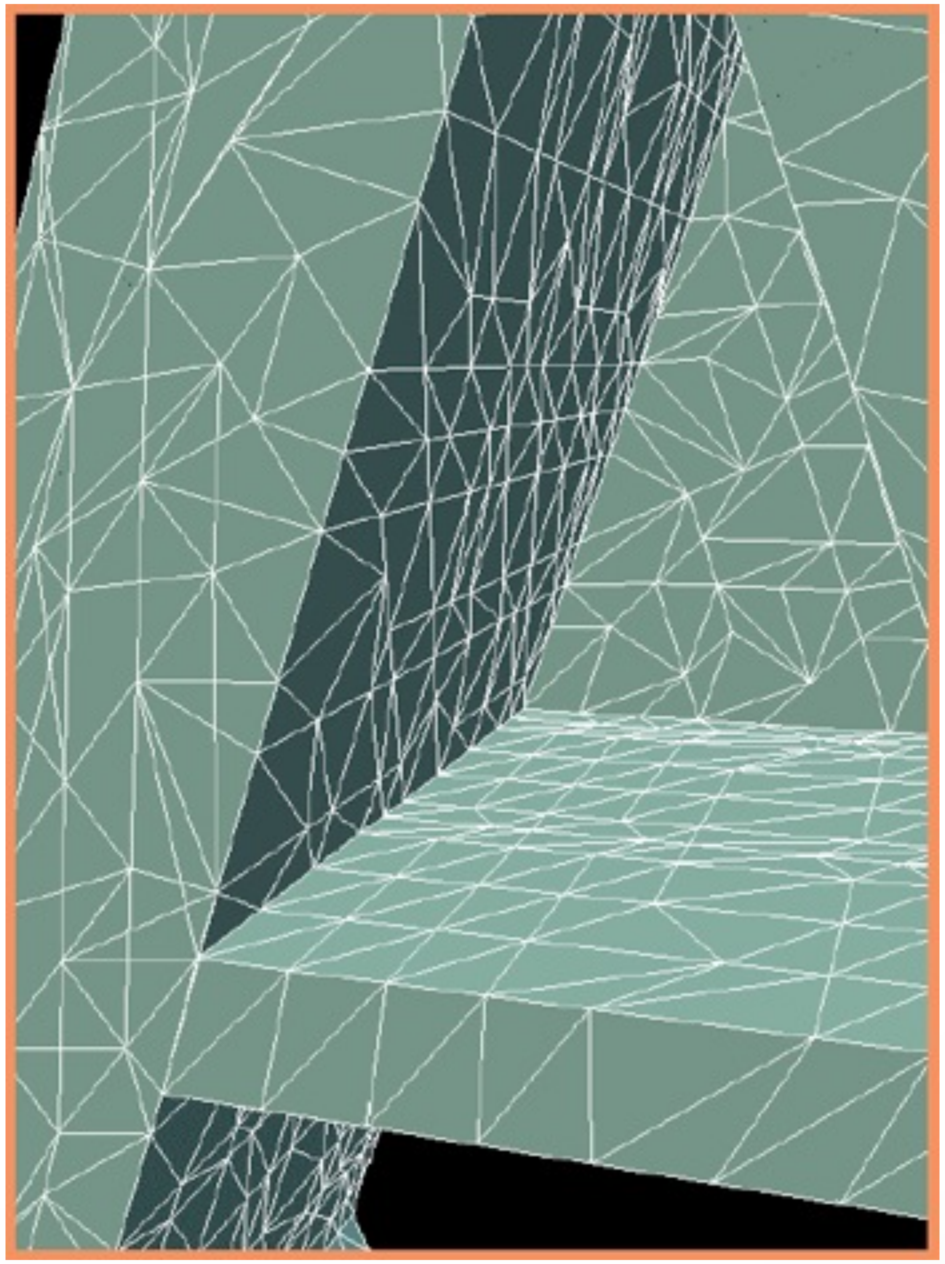




median vertex position → sub-sampling

Computing a Representative





error quadrics → feature preservation

Error Quadrics

Squared distance to plane

$$\mathbf{p} = (x, y, z, 1)^T, \quad \mathbf{q} = (a, b, c, d)^T$$

$$\mathrm{dist}(\mathbf{q}, \mathbf{p})^2 = (\mathbf{q}^T \mathbf{p})^2 = \mathbf{p}^T \left(\mathbf{q}\mathbf{q}^T\right) \mathbf{p} =: \mathbf{p}^T \mathbf{Q}_{\mathbf{q}} \mathbf{p}$$

Error Quadrics

Squared distance to plane

$$\mathbf{p} = (x, y, z, 1)^T, \quad \mathbf{q} = (a, b, c, d)^T$$

$$\text{dist}(\mathbf{q}, \mathbf{p})^2 = (\mathbf{q}^T \mathbf{p})^2 = \mathbf{p}^T \left(\mathbf{q}\mathbf{q}^T\right) \mathbf{p} =: \mathbf{p}^T \mathbf{Q}_\mathbf{q} \mathbf{p}$$

$$\mathbf{Q}_\mathbf{q} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

Sum of distances to vertex planes

$$\sum_i \mathrm{dist}(\mathbf{q}_i, \mathbf{p})^2 =$$

Error Quadrics

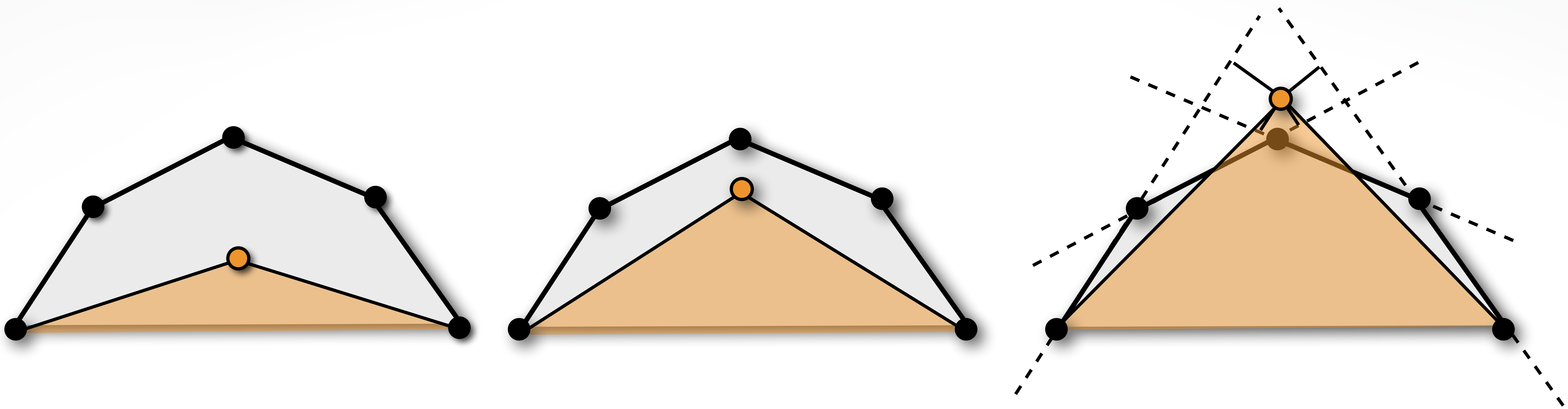
Sum of distances to vertex planes

$$\sum_i \mathrm{dist}(\mathbf{q}_i, \mathbf{p})^2 = \sum_i \mathbf{p}^T \mathbf{Q}_{\mathbf{q}_i} \mathbf{p} = \mathbf{p}^T \left(\sum_i \mathbf{Q}_{\mathbf{q}_i} \right) \mathbf{p} =: \mathbf{p}^T \mathbf{Q}_{\mathbf{p}} \mathbf{p}$$

Point that minimizes the error

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{p}^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Comparison



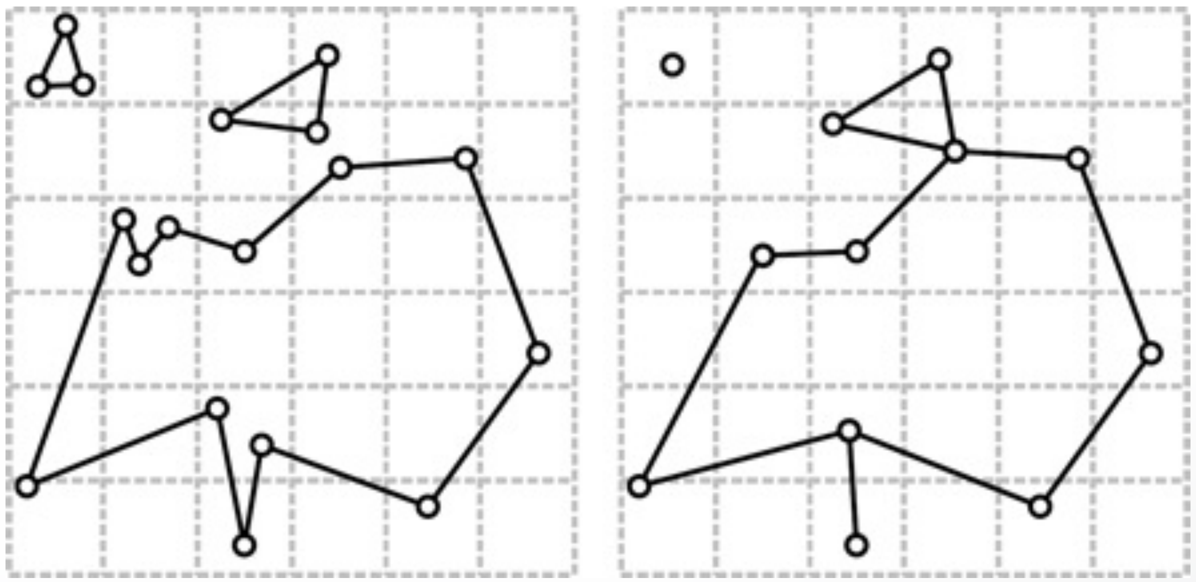
average   median   error quadric

Vertex Clustering

- Cluster Generation
- Computing a representative
- **Mesh generation**
  - Clusters $\mathbf{p} \Leftrightarrow \{\mathbf{p}_0, \ldots, \mathbf{p}_n\}, \mathbf{q} \Leftrightarrow \{\mathbf{q}_0, \ldots, \mathbf{q}_n\}$
  - Connect $(\mathbf{p}, \mathbf{q})$ if there was an edge $(\mathbf{p}_i, \mathbf{q}_j)$
- Topology changes

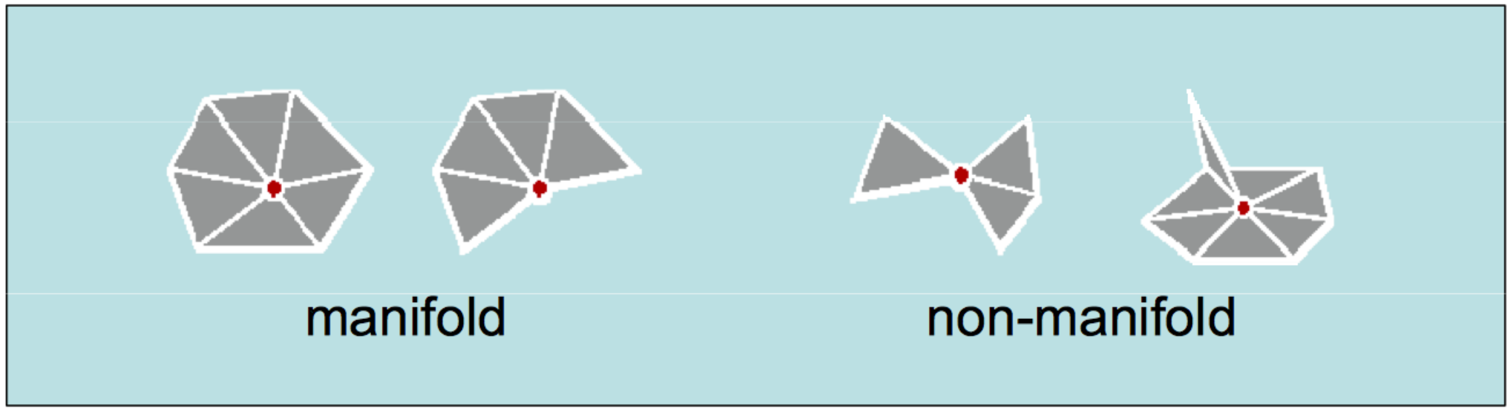
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- **Topology changes**
  - If different sheets pass through on cell
  - Can be non-manifold



Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- **Topology changes**
    - If different sheets pass through on cell
    - Can be non-manifold

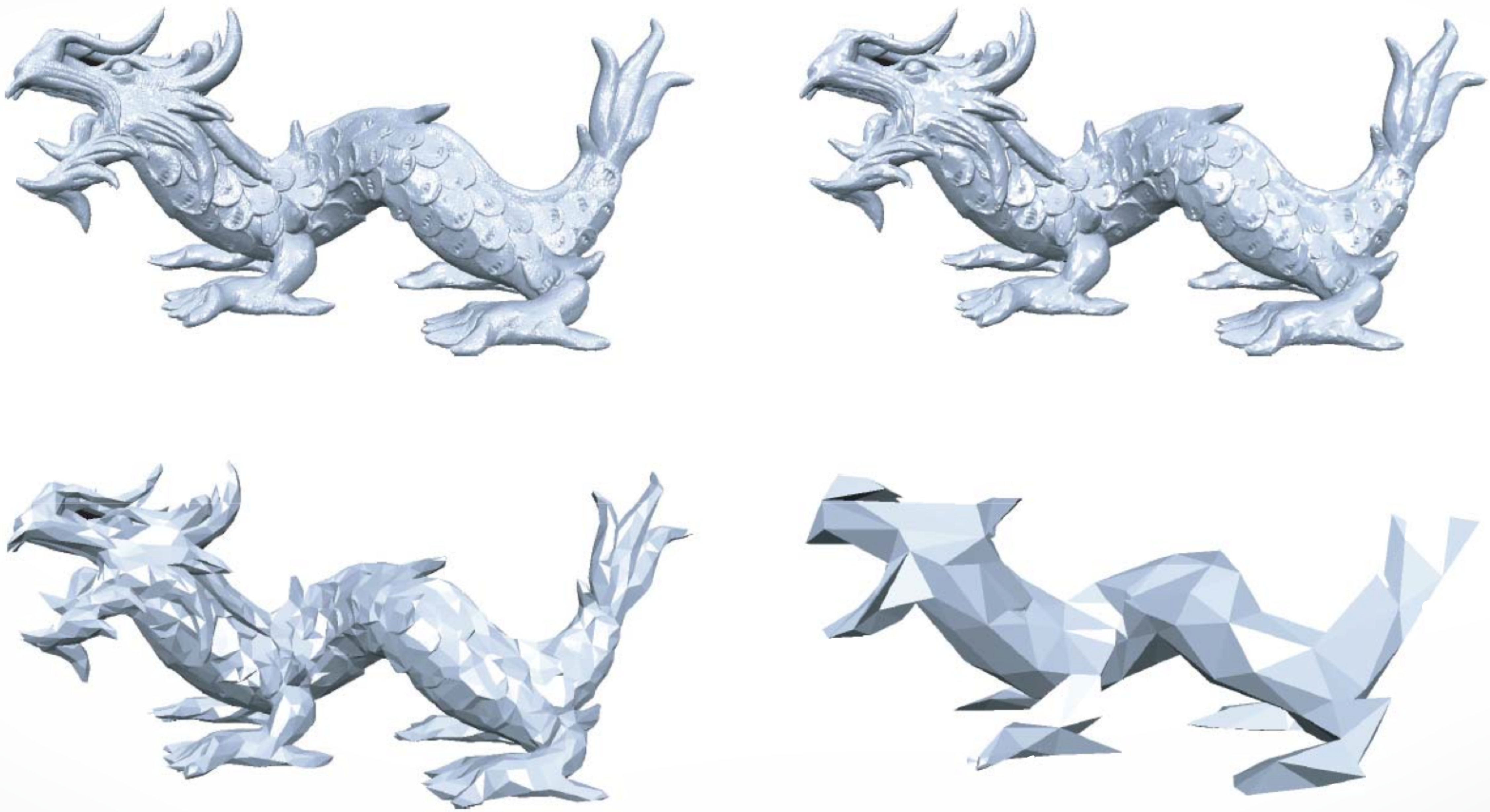


Outline

Mesh Decimation methods

- Vertex Clustering
- **Iterative Decimation**

Example



Incremental Decimation

- **General Setup**
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

General Setup

```
Repeat:
  pick mesh region
  apply decimation operator
Until no further reduction possible
```

Greedy Optimization

```
For each region
  evaluate quality after decimation
  enqueue(quality, region)

Repeat:
  pick best mesh region
  apply decimation operator
  update queue
Until no further reduction possible
```

Global Error Control

```
For each region
  evaluate quality after decimation
  enqueue(quality, region)

Repeat:
  pick best mesh region
  if error < ε
     apply decimation operator
     update queue
Until no further reduction possible
```

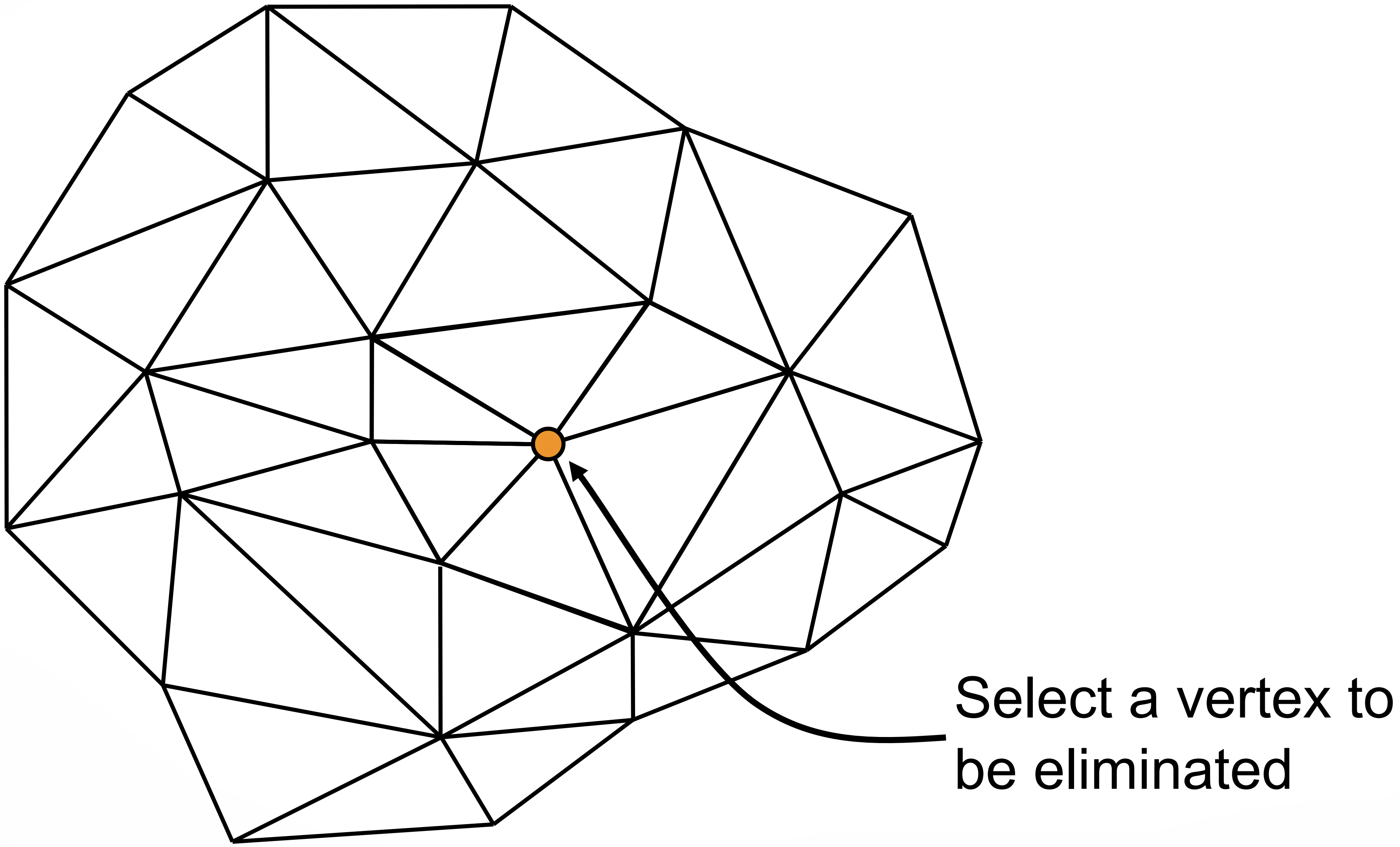
Incremental Decimation

- General Setup
- **Decimation operators**
- Error metrics
- Fairness criteria
- Topology changes

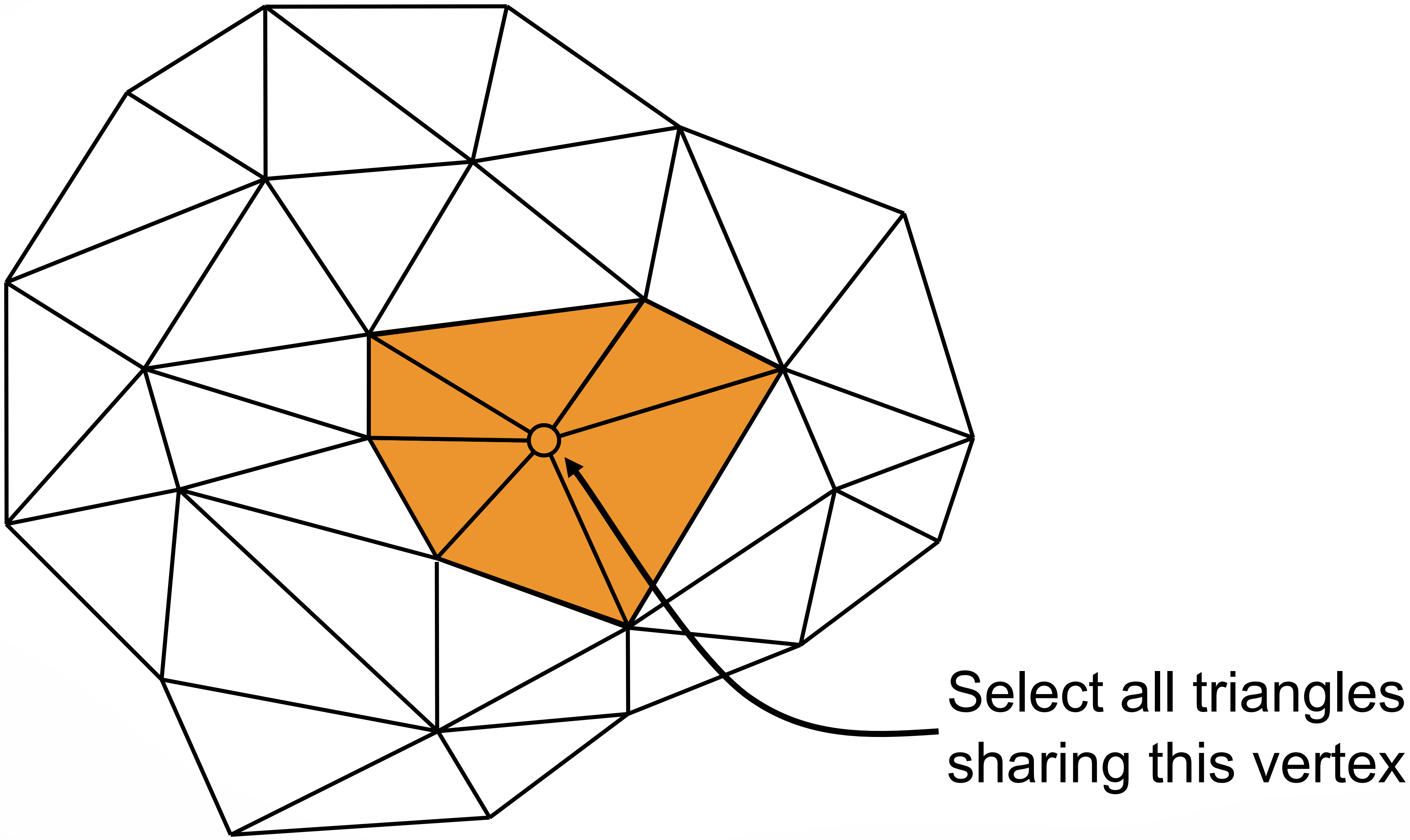
Decimation Operators

- What is a “region”?
- What are the DOFs for re-triangulation?
- Classification
  - topology-changing vs. topology-preserving
  - subsampling vs. filtering
  - inverse operation → progressive meshes

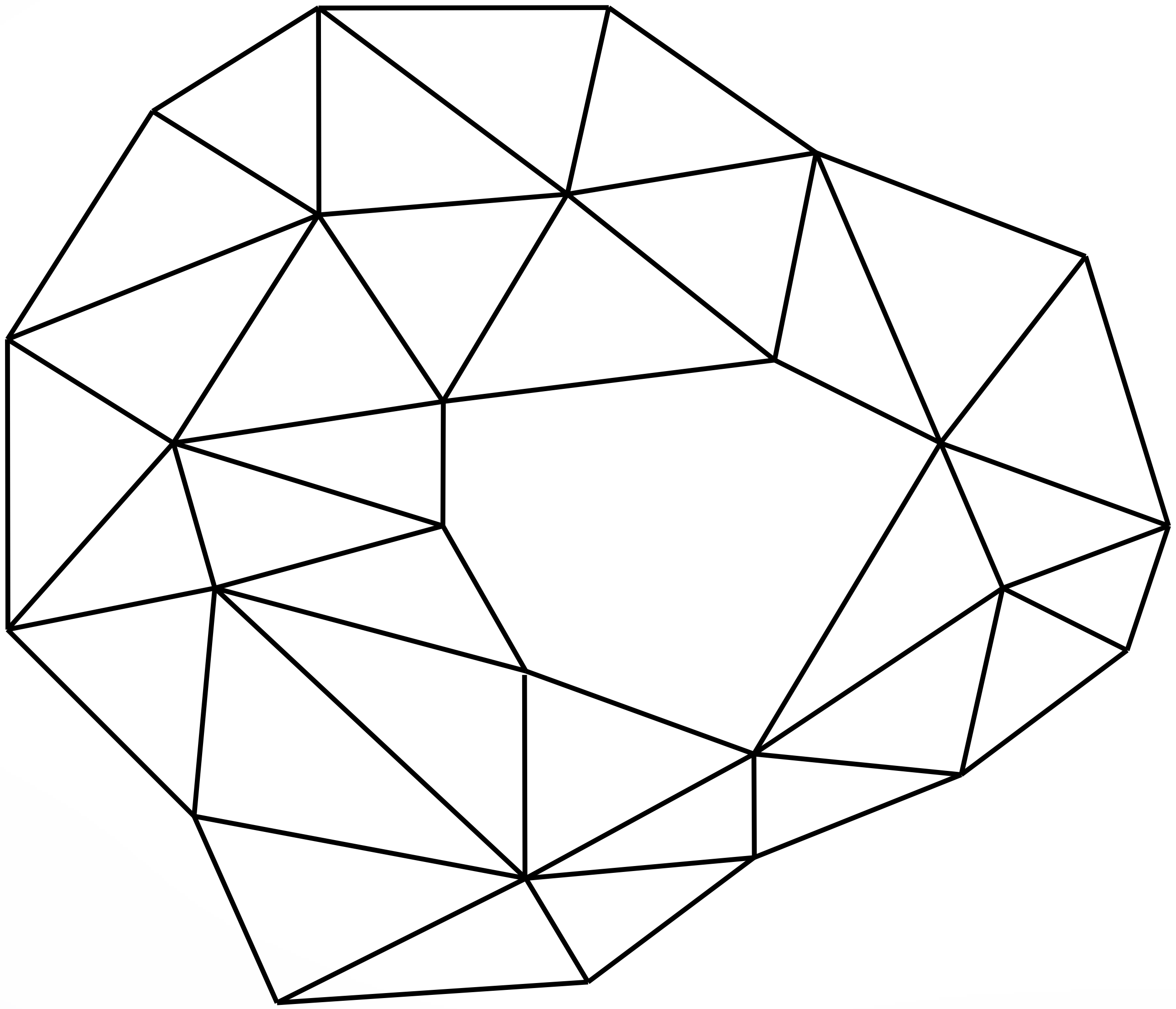
Vertex Removal



Vertex Removal

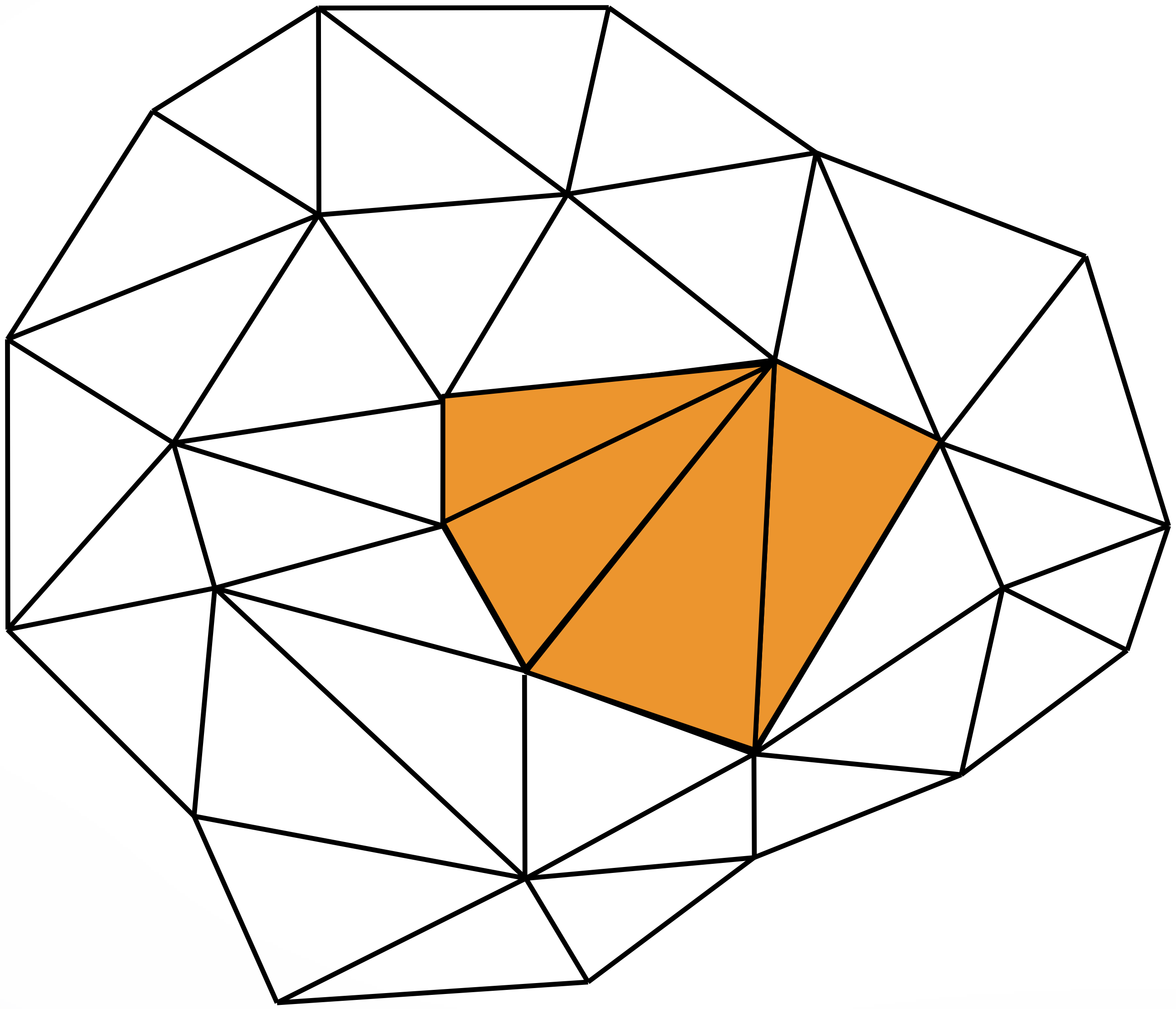


Vertex Removal



Remove the selected triangles, creating a hole

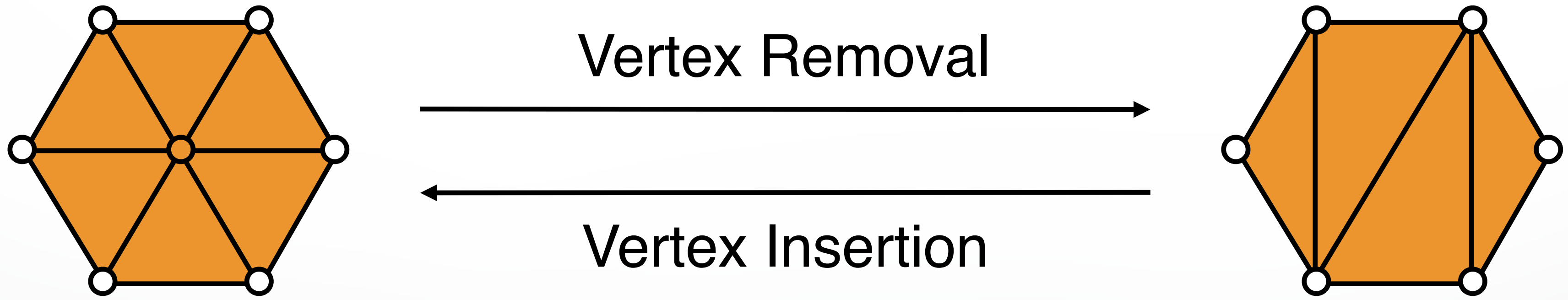
Vertex Removal



Fill the hole
with triangles

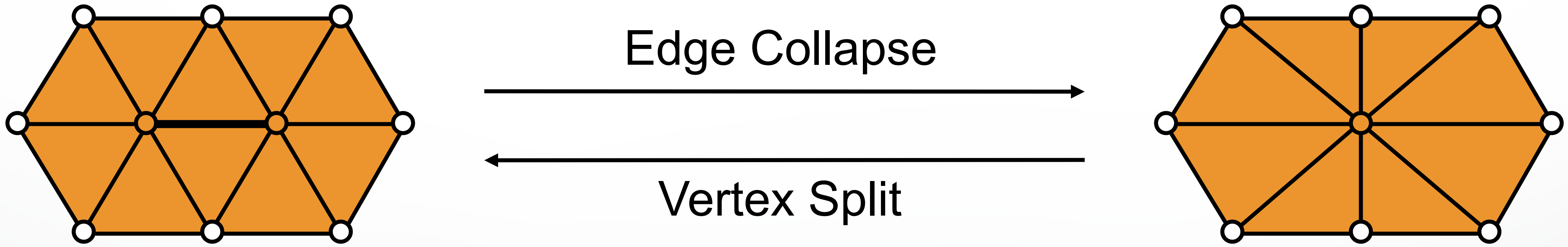
Decimation Operators

- Remove vertex
- Re-triangulate hole
  - Combinatorial DOFs
  - Sub-sampling



Decimation Operators

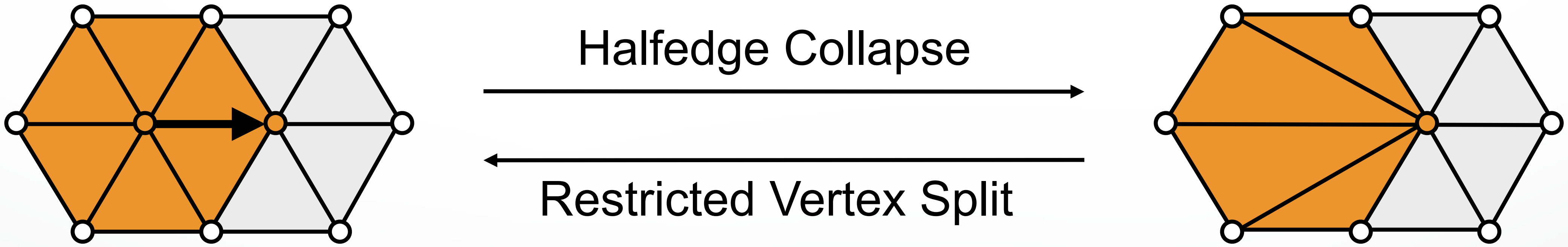
- Merge two adjacent triangles
- Define new vertex position
  - Continuous DOF
  - Filtering



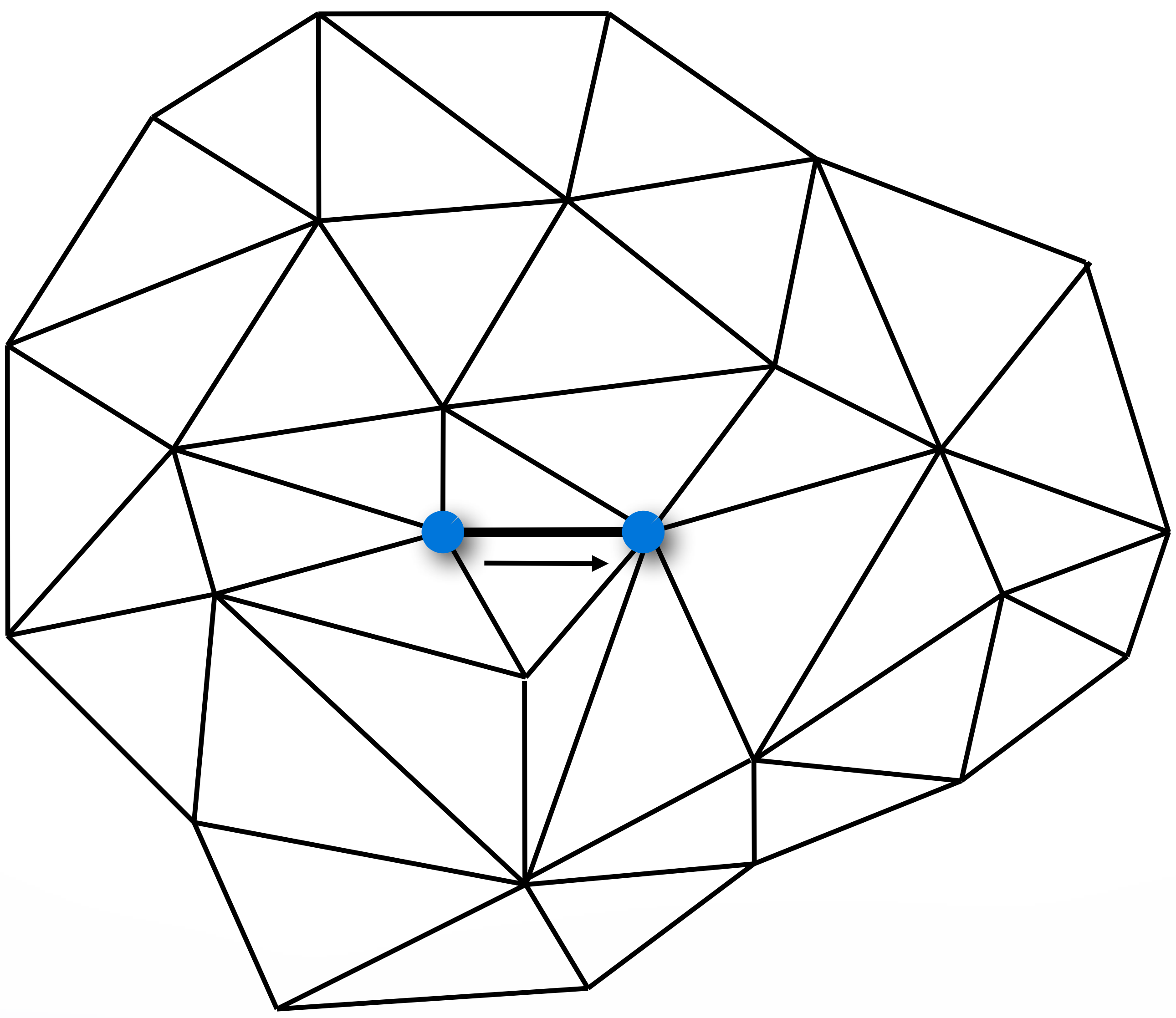
Decimation Operators

- Collapse edge into one end point
  - Special vertex removal
  - Special edge collapse
- No DOFs
  - One operator per half-edge
  - Sub-sampling

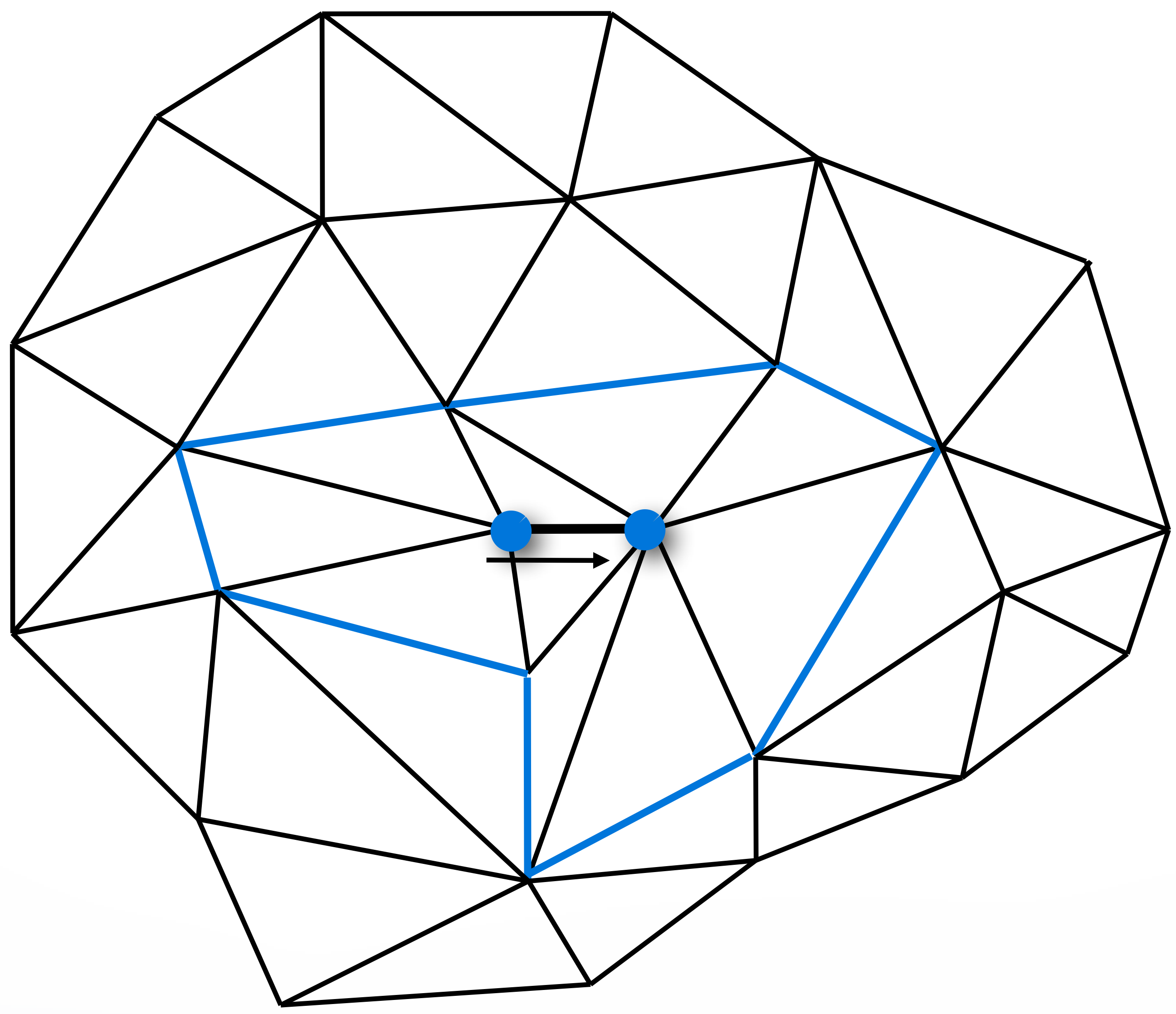
H. Hoppe: Progressive Meshes



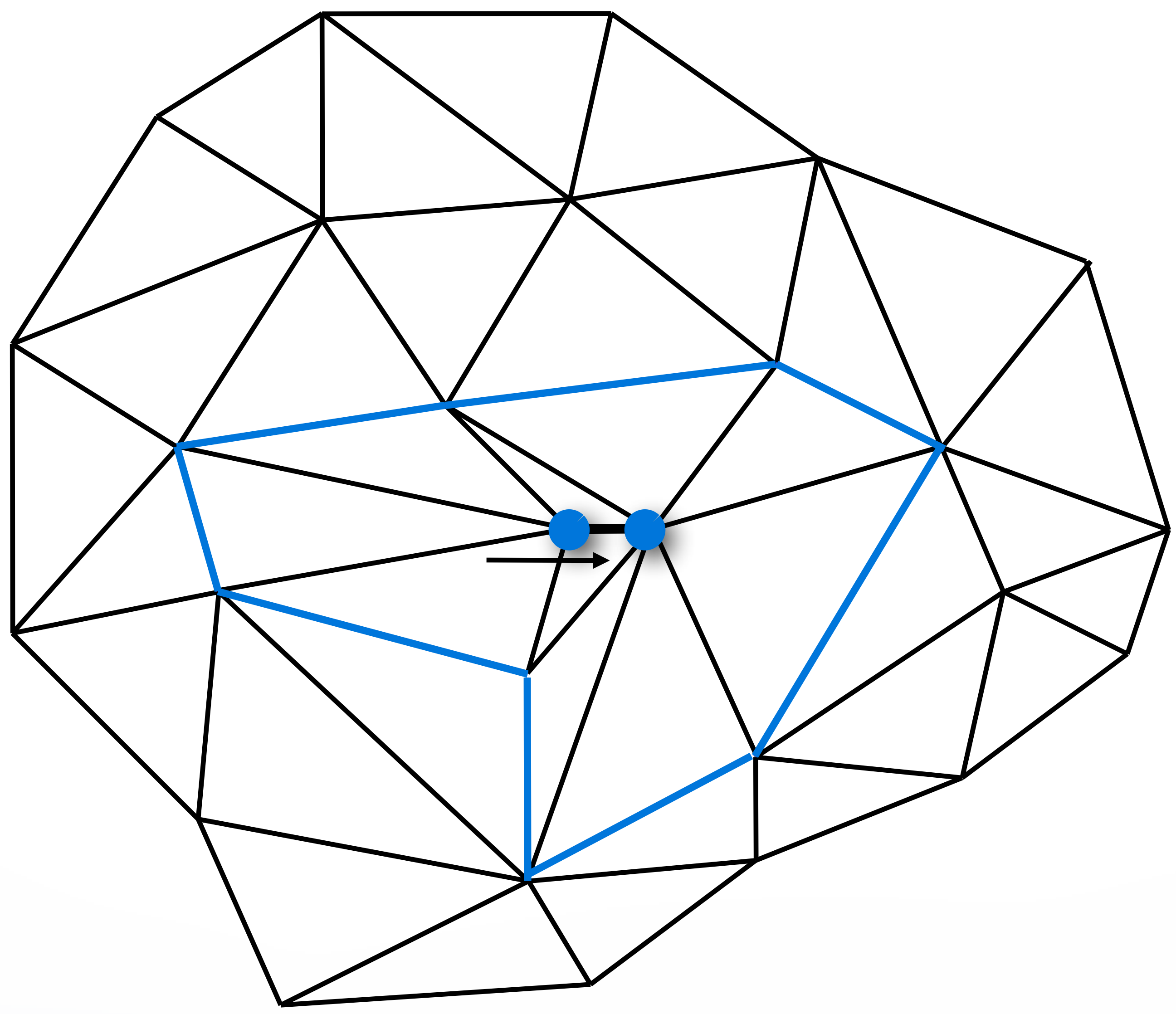
Edge Collapse



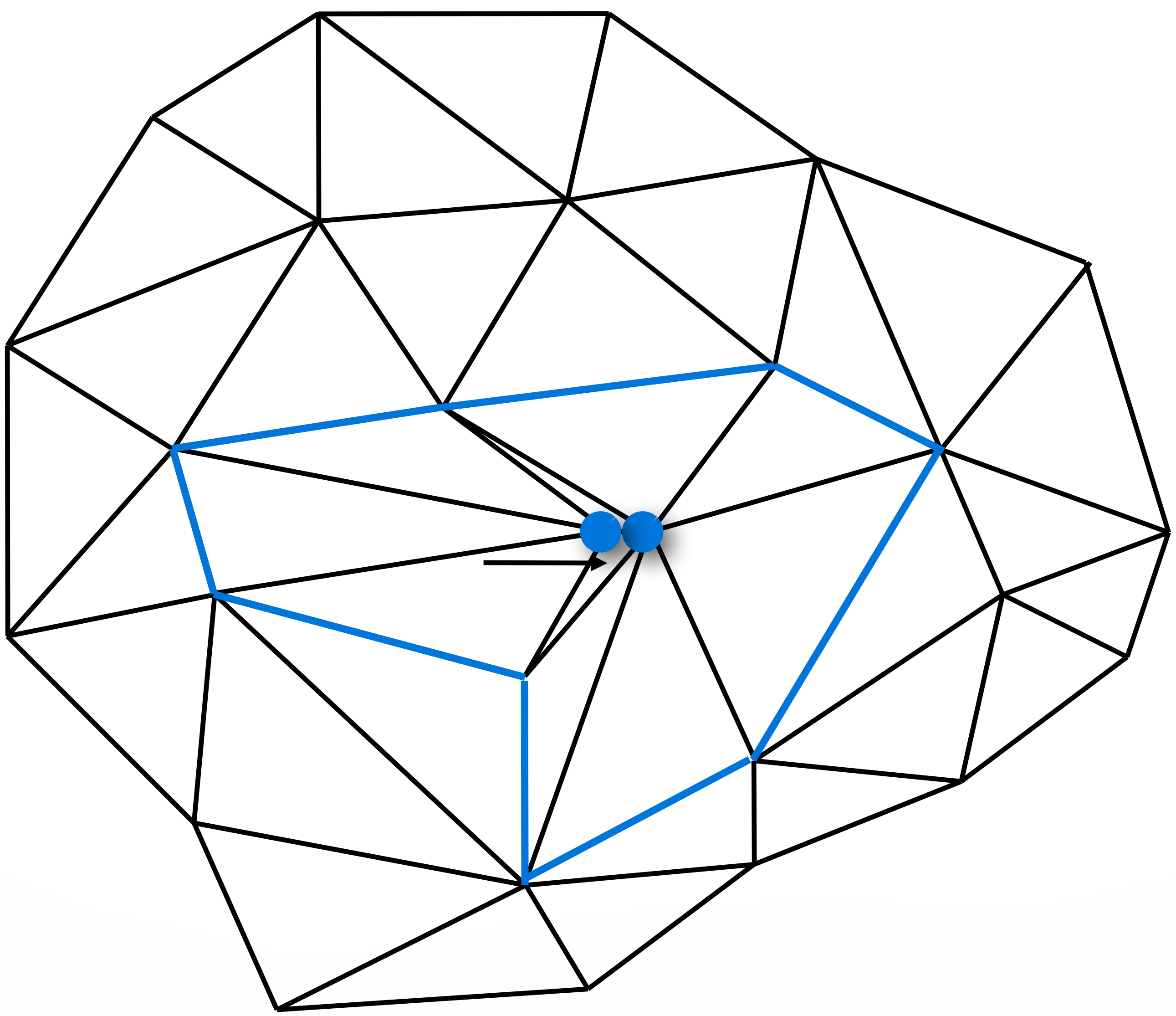
Edge Collapse



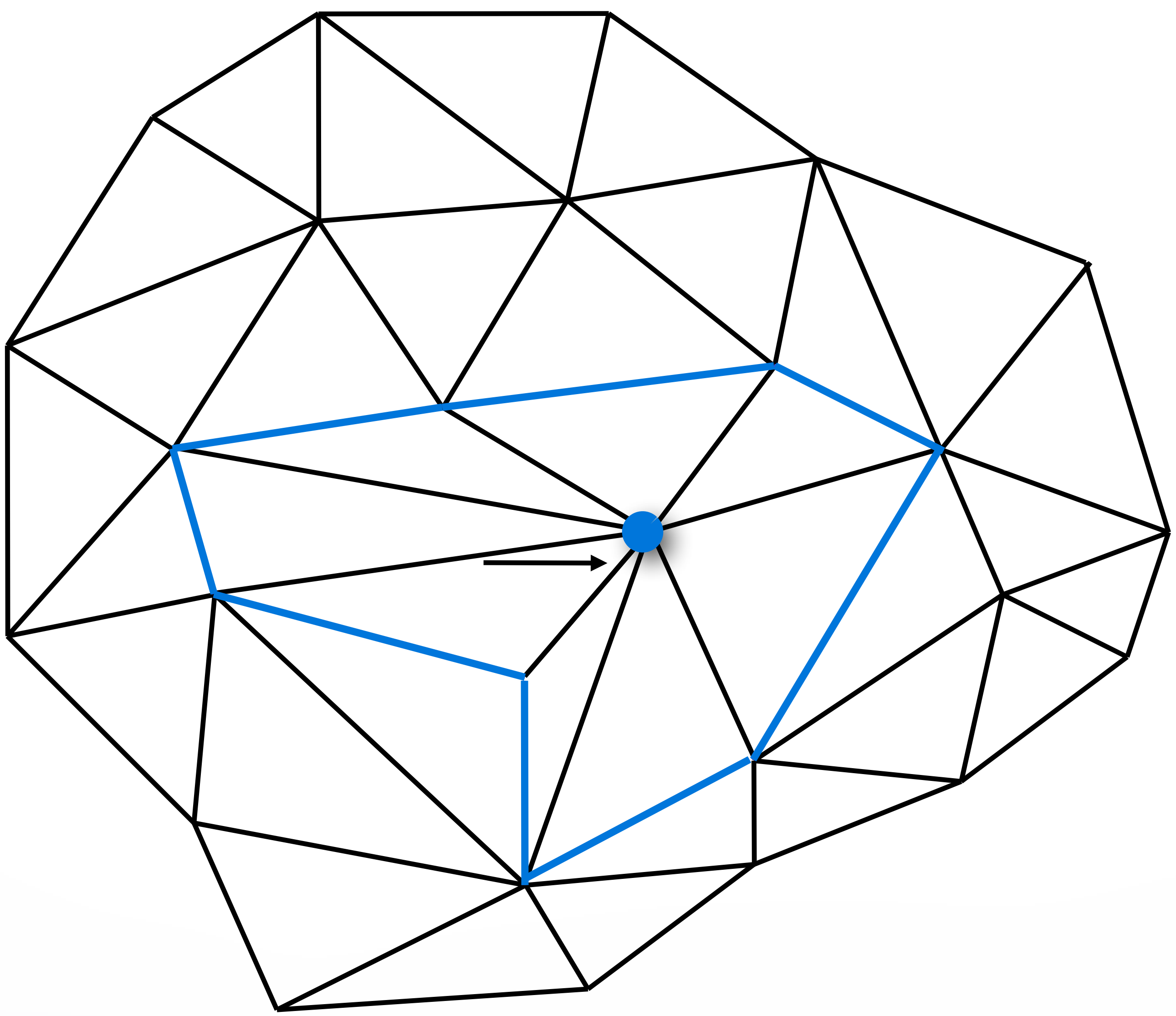
Edge Collapse



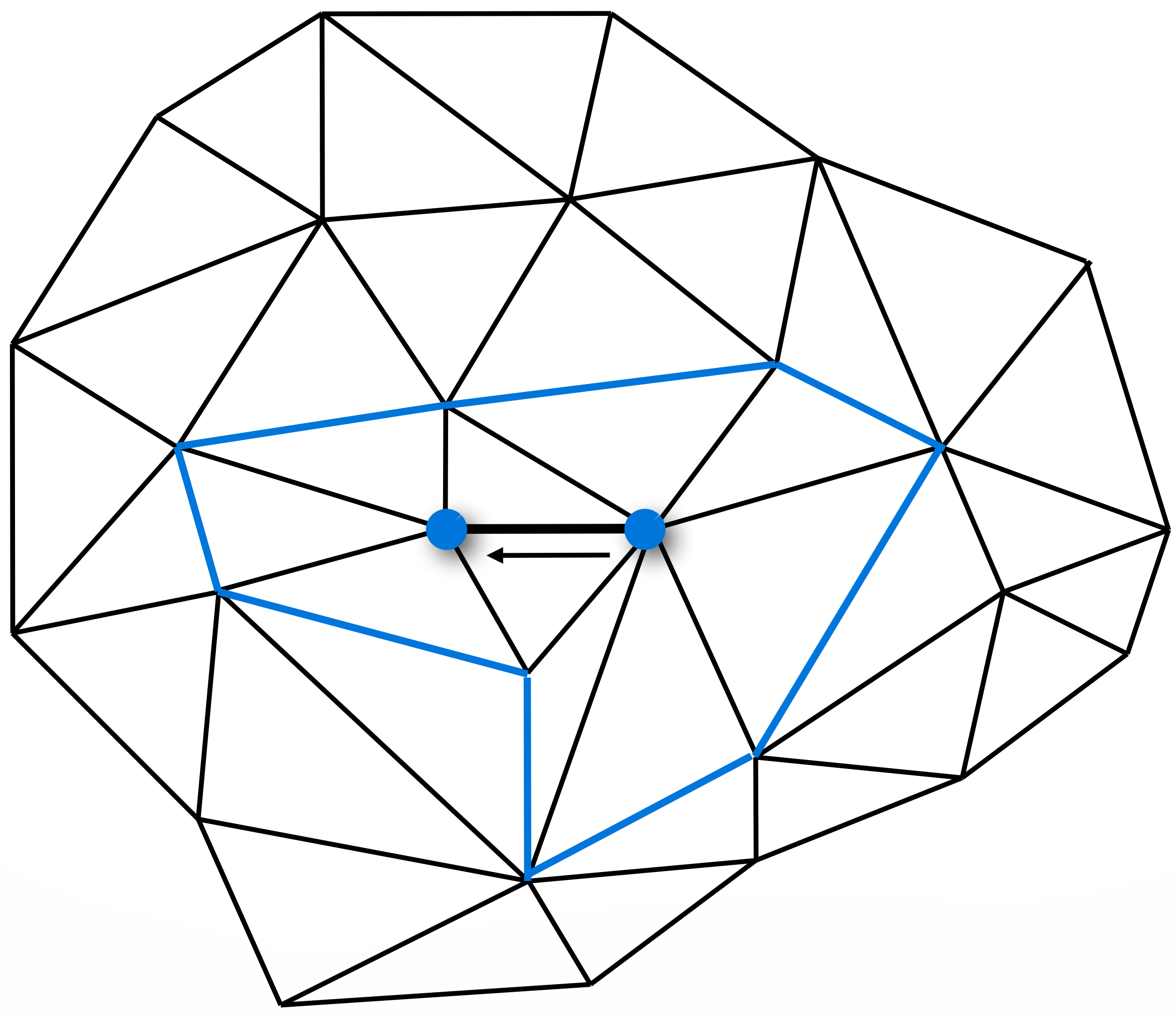
Edge Collapse



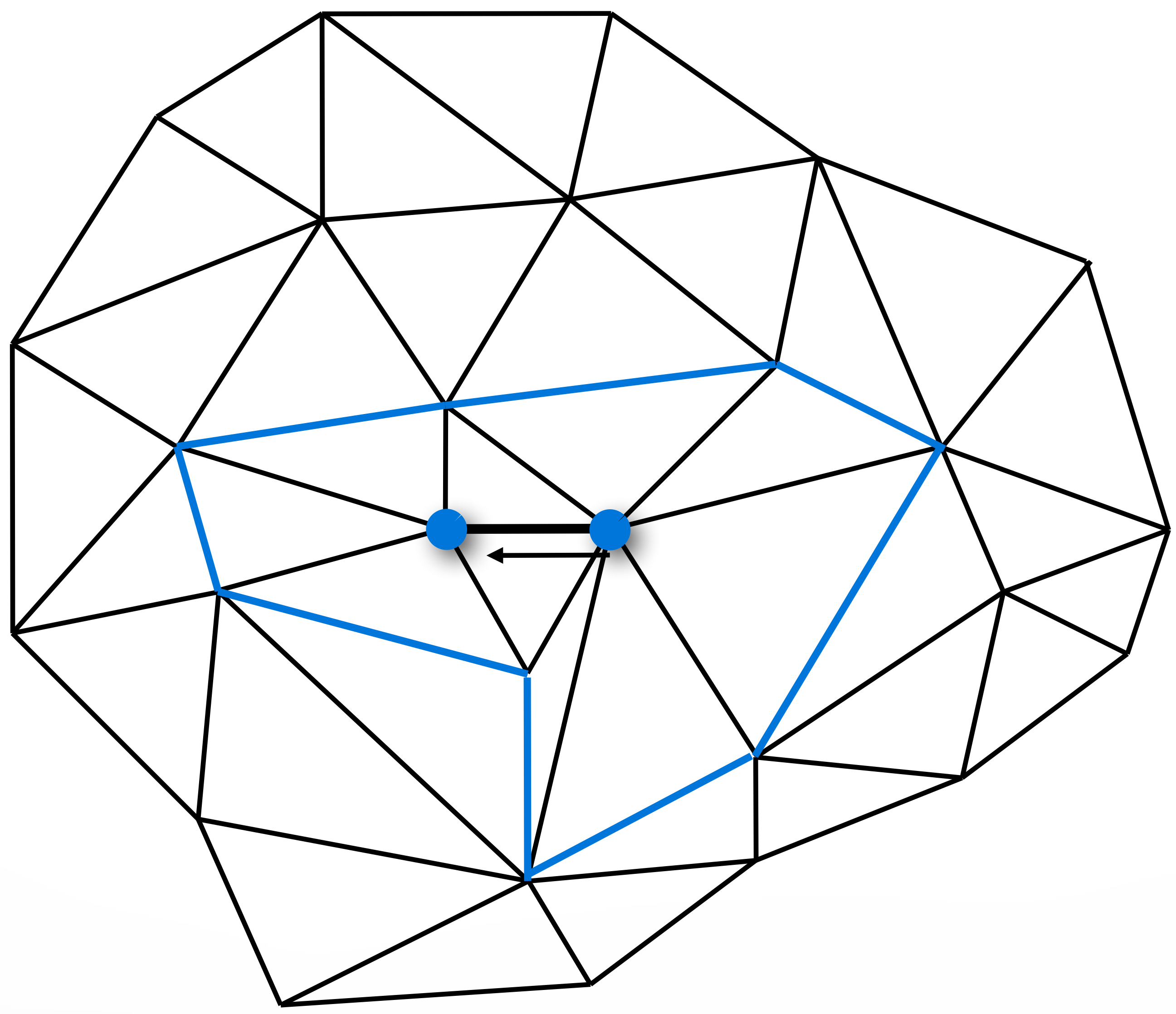
Edge Collapse



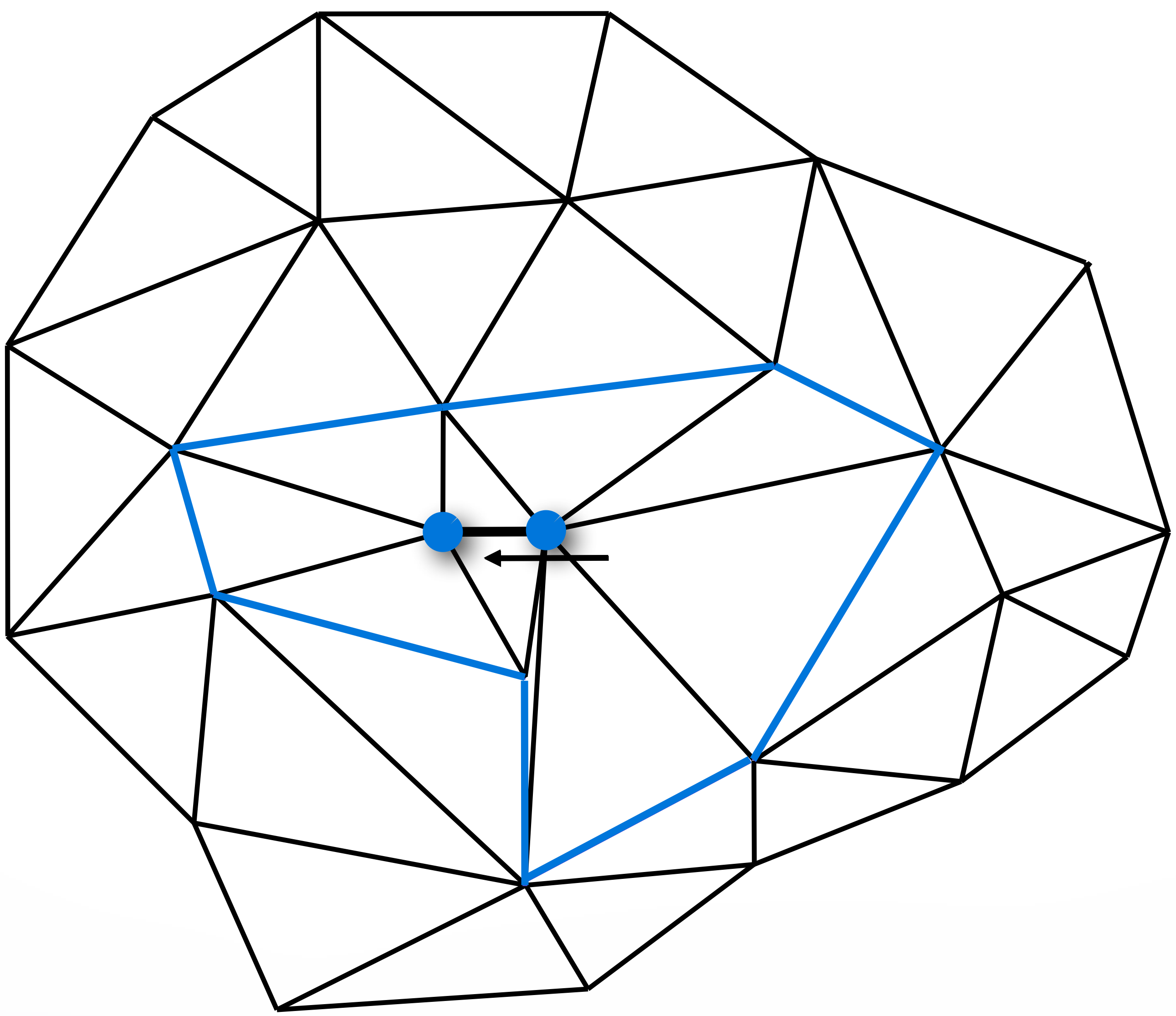
Edge Collapse



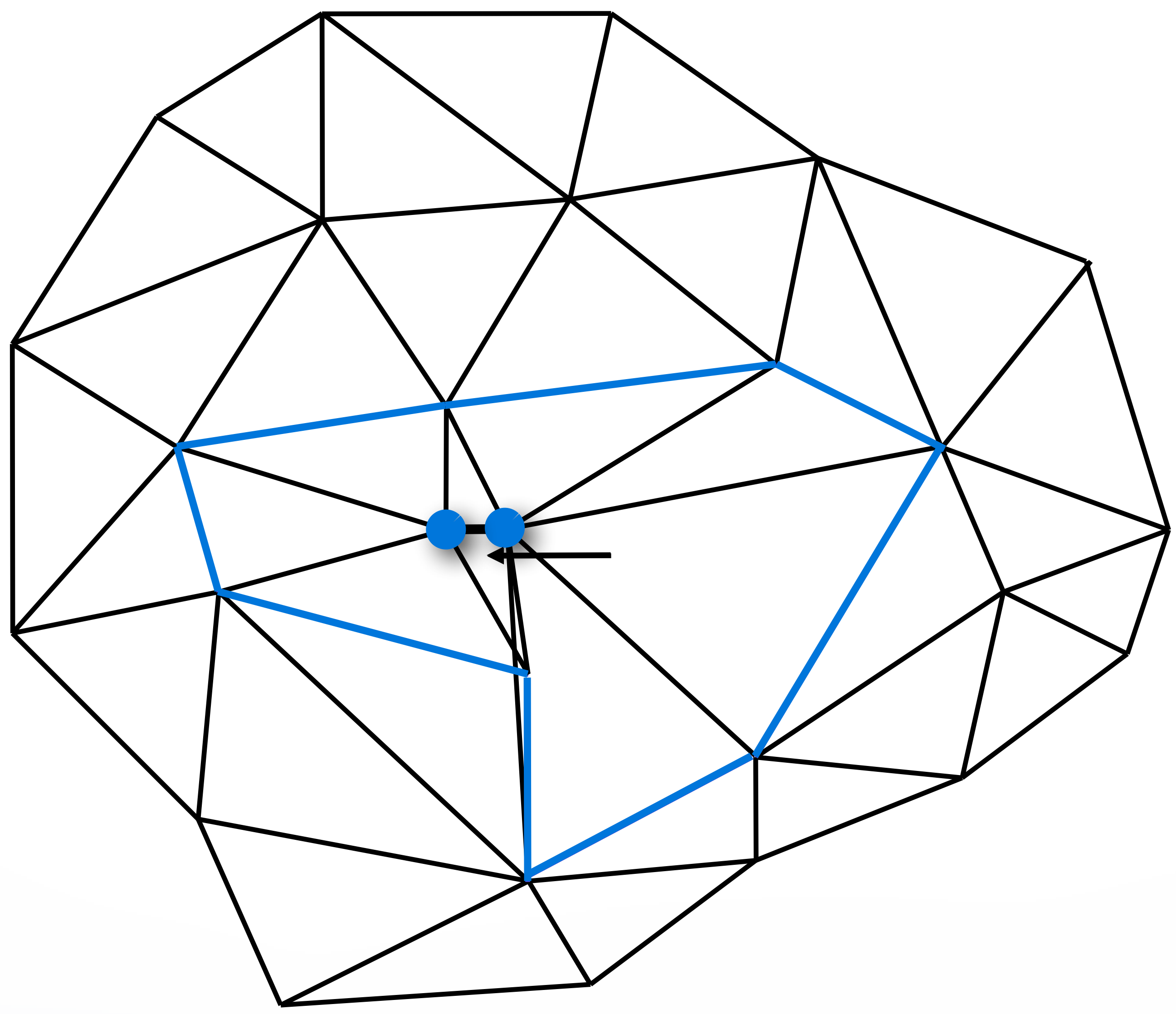
Edge Collapse



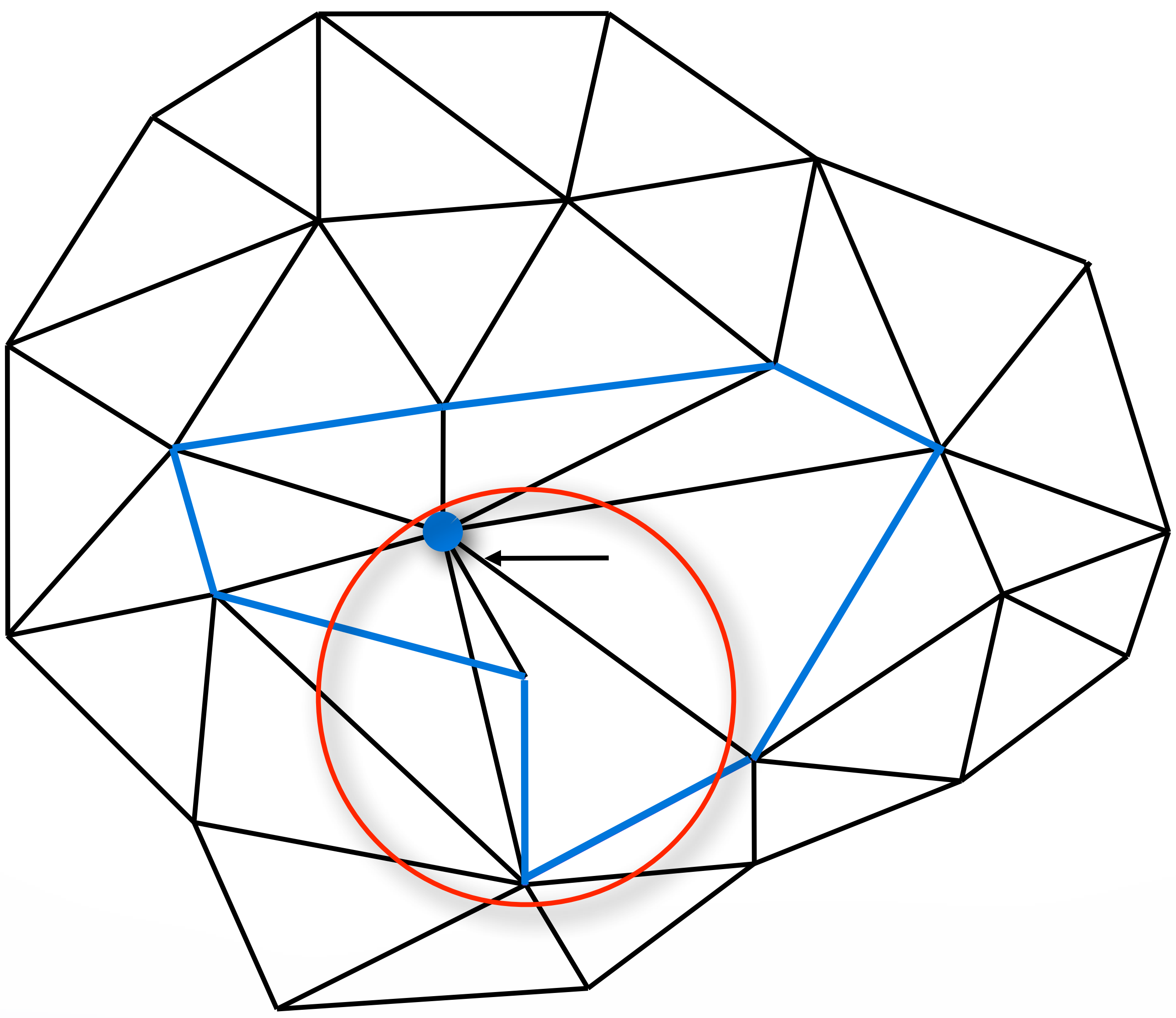
Edge Collapse



Edge Collapse

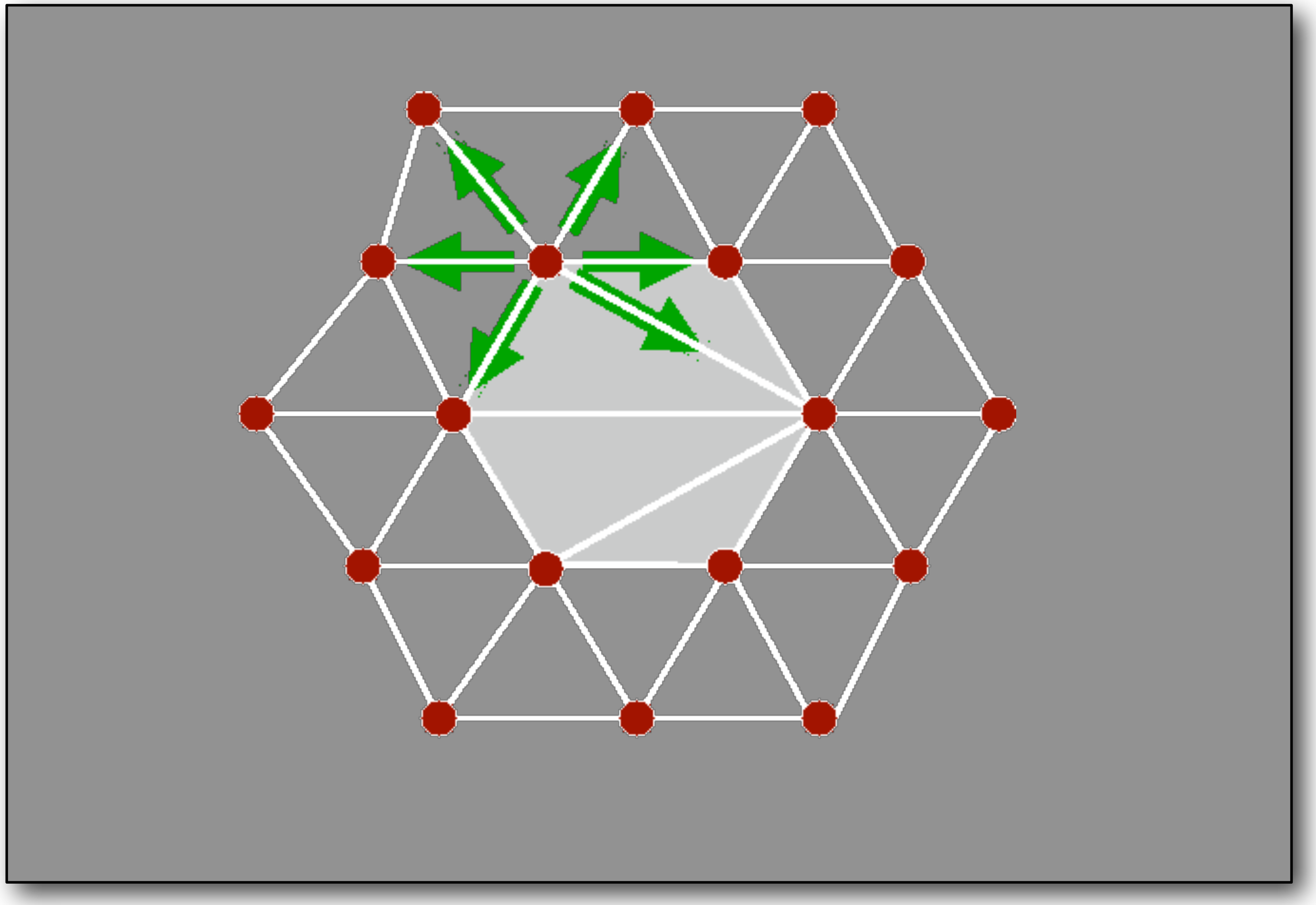


Edge Collapse (Flip!)



Application: **Progressive Meshes**

Priority Queue Updating



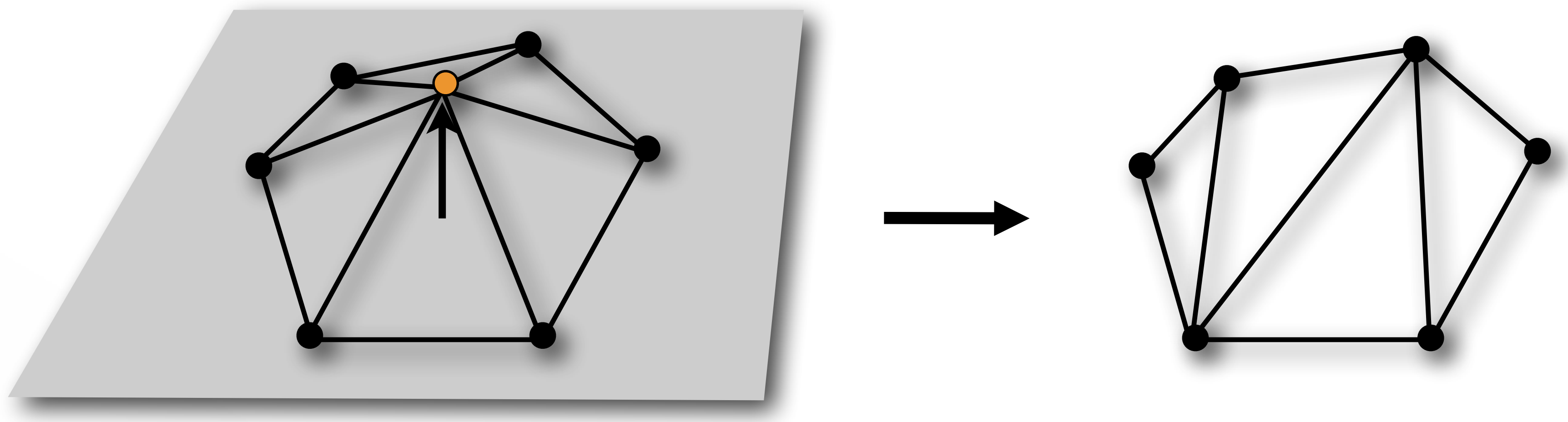
Incremental Decimation

- General Setup
- Decimation operators
- **Error metrics**
- Fairness criteria
- Topology changes

Local Error Metrics

Local distance to mesh [Schröder et al. '92]

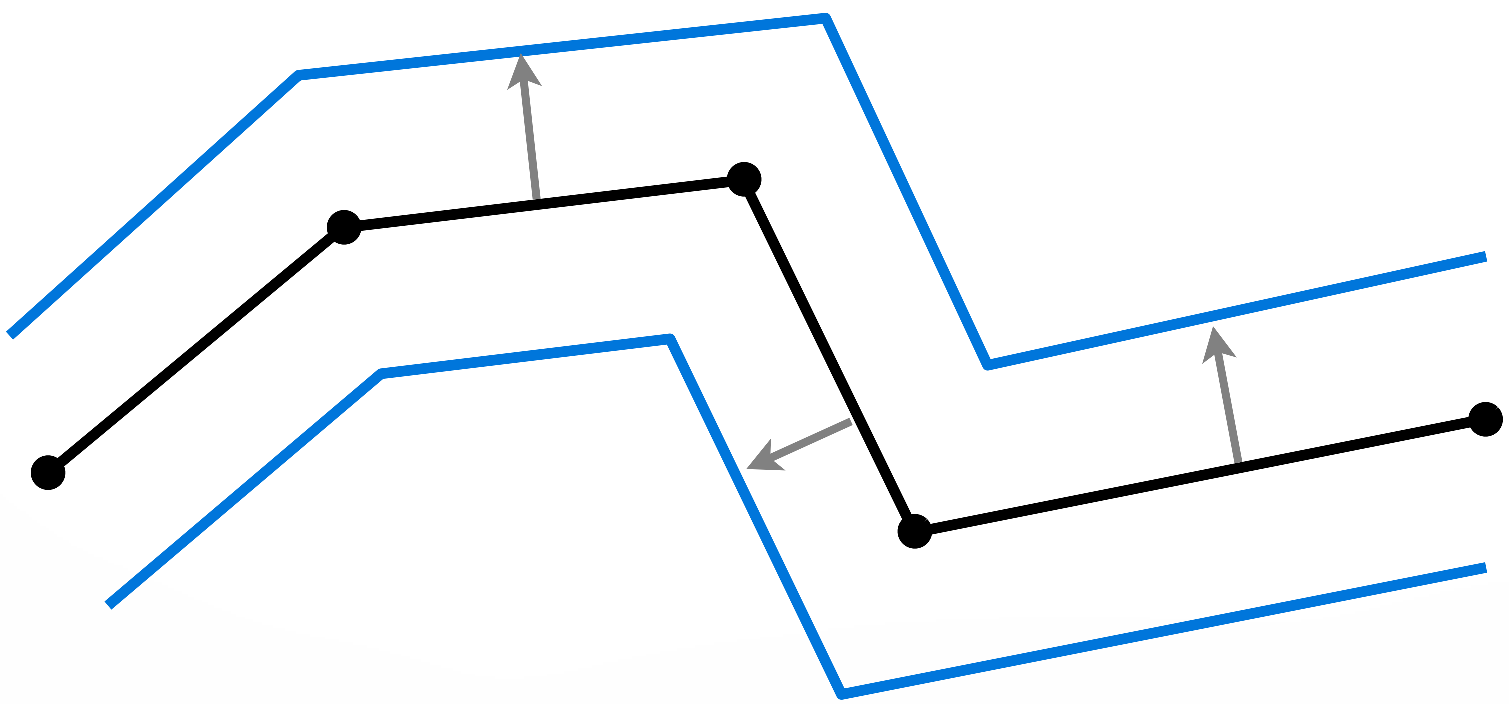
- Compute average plane
- No comparison to original geometry



Global Error Metrics

Simplification envelopes [Cohen al. ‘96]

- Compute (non-intersecting) offset surfaces
- Simplification guarantees to stay within bounds

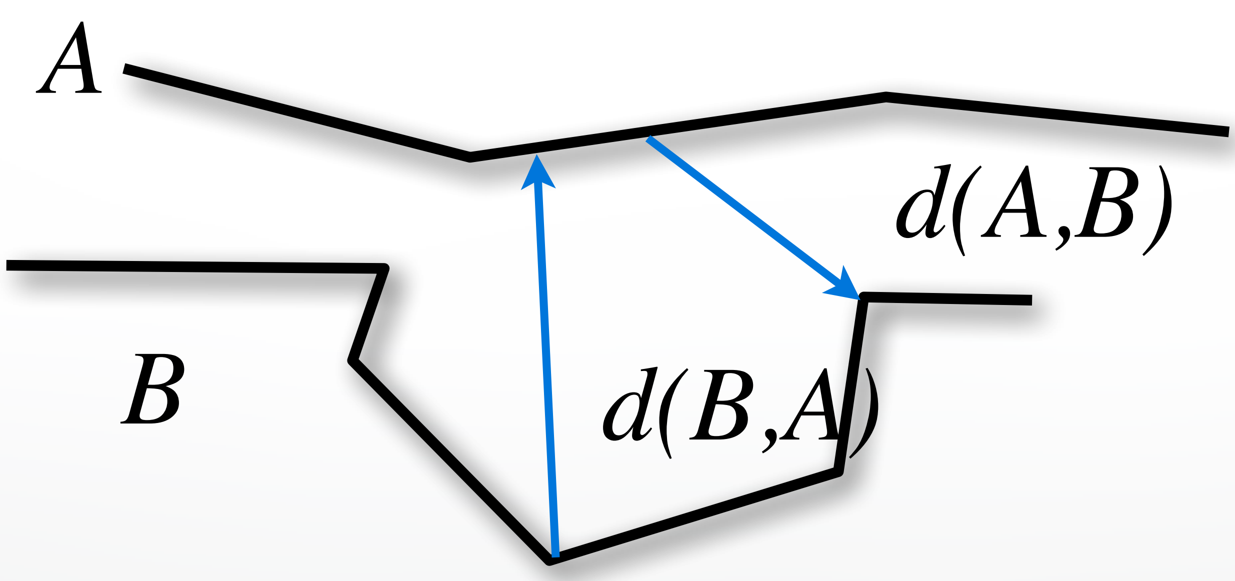


Global Error Metrics

(Two-sided) Hausdorff distance: Maximum distance between two shapes

$$d(A,B) := \max_{\mathbf{a}\in A} \min_{\mathbf{b}\in B} \|\mathbf{a}-\mathbf{b}\|$$

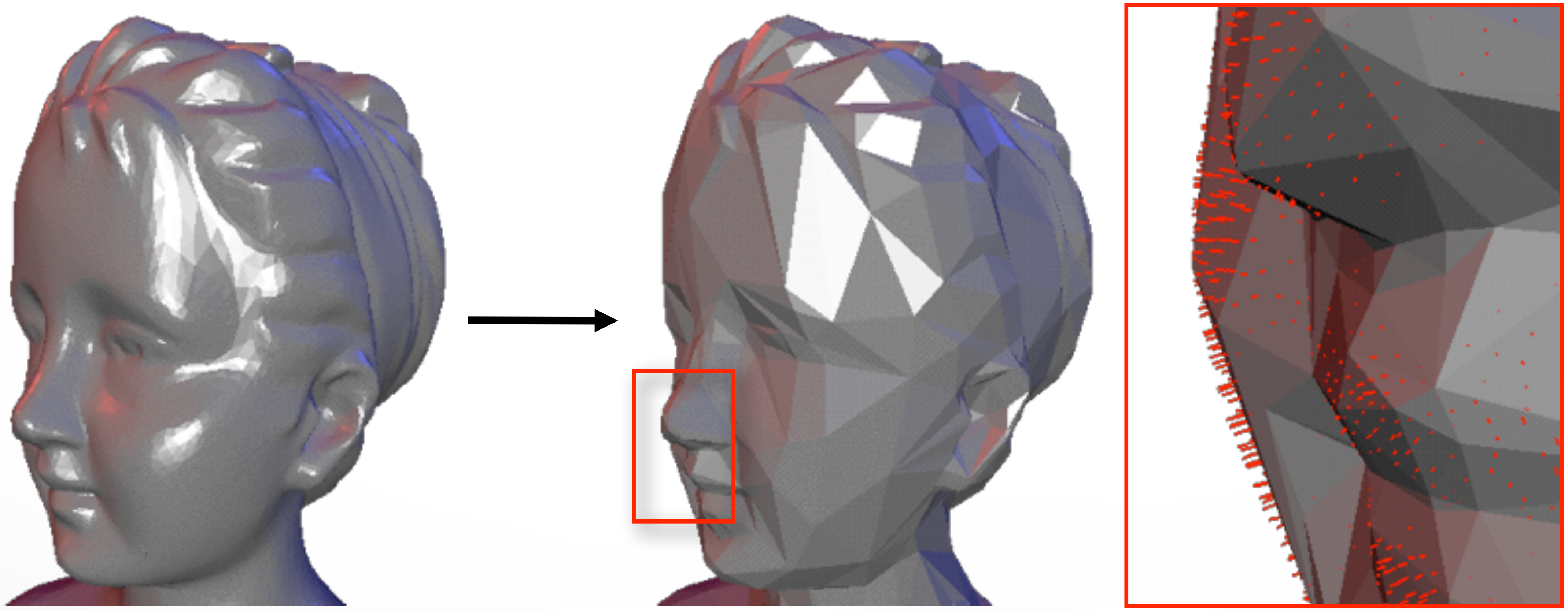
- In general $d(A,B) \neq d(B,A)$
- Computationally involved



Global Error Metrics

Scan data: One-sided Hausdorff distance sufficient

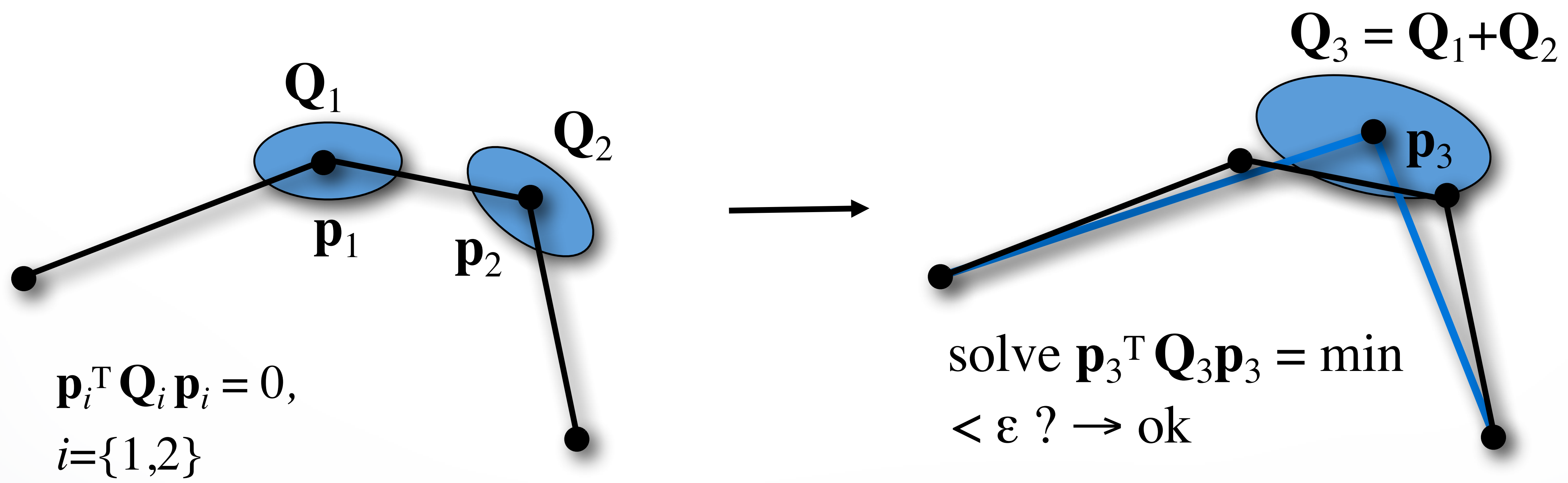
- From original vertices to current surface



Global Error Metrics

Error quadrics [Garland, Heckbert 97]

- Squared distance to planes at vertex
- No bound on true error



Global Error Metrics

Initialization:

- Assign each vertex the quadric built from all its incident triangles' planes

Decimation:

- After collapsing edge $(\mathbf{p}_1, \mathbf{p}_2) \to \mathbf{p}_3$, simply add the corresponding quadrics: $\mathbf{Q}_3 = \mathbf{Q}_1 + \mathbf{Q}_2$

Memory consumption

- Quasi-global error metric with 10 floats per vertex

Complexity

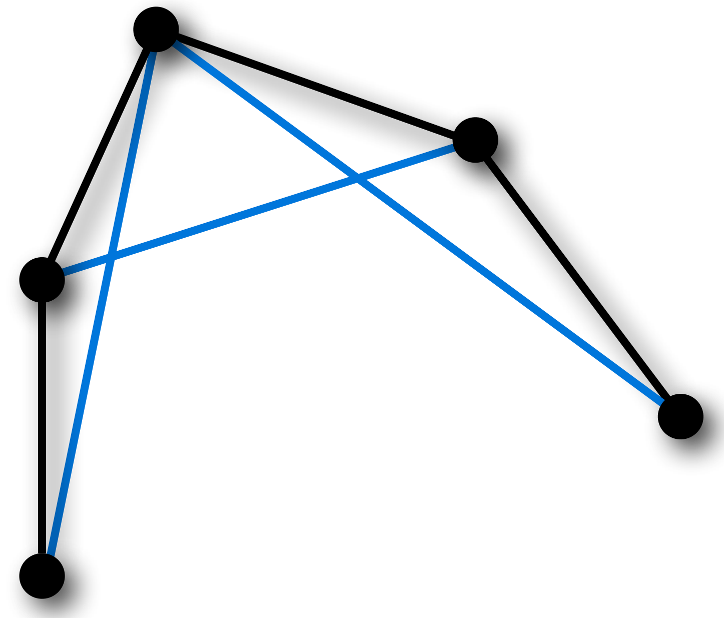
- $N =$ number of vertices
- Priority Queue for half edges
  - $6N \log(6N)$
- Error control
  - Local $O(1) \Rightarrow$ global $O(N)$
  - Local $O(N) \Rightarrow$ global $O(N^2)$

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- **Fairness criteria**
- Topology changes

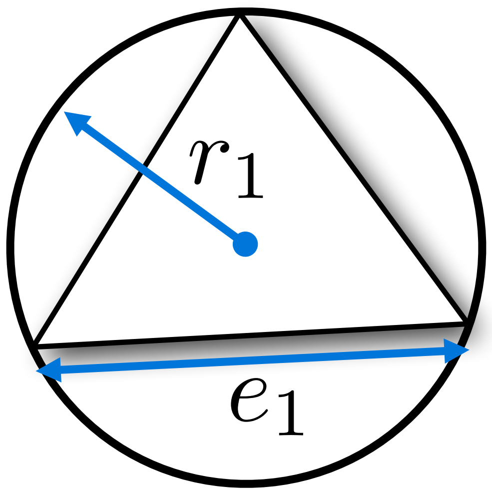
Fairness Criteria

- Rate quality after decimation
  - Approximation error

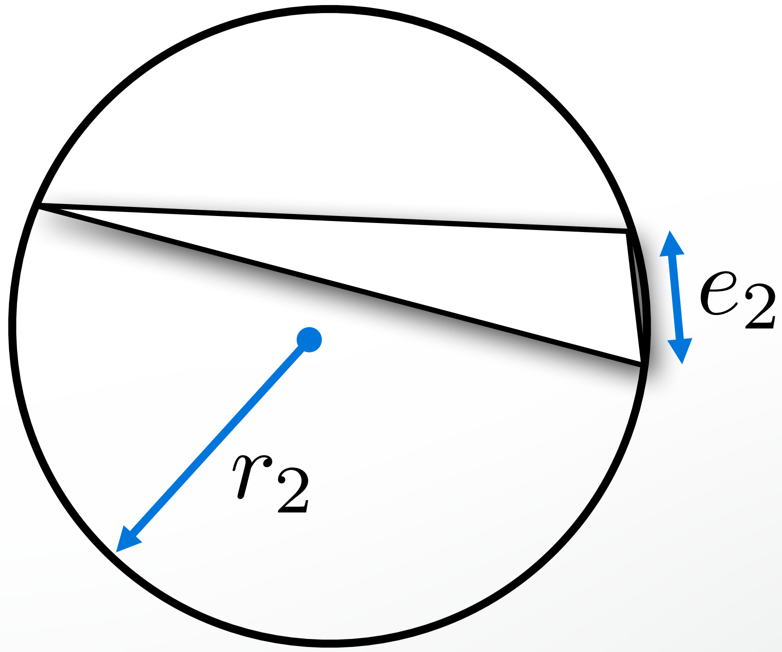


Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape

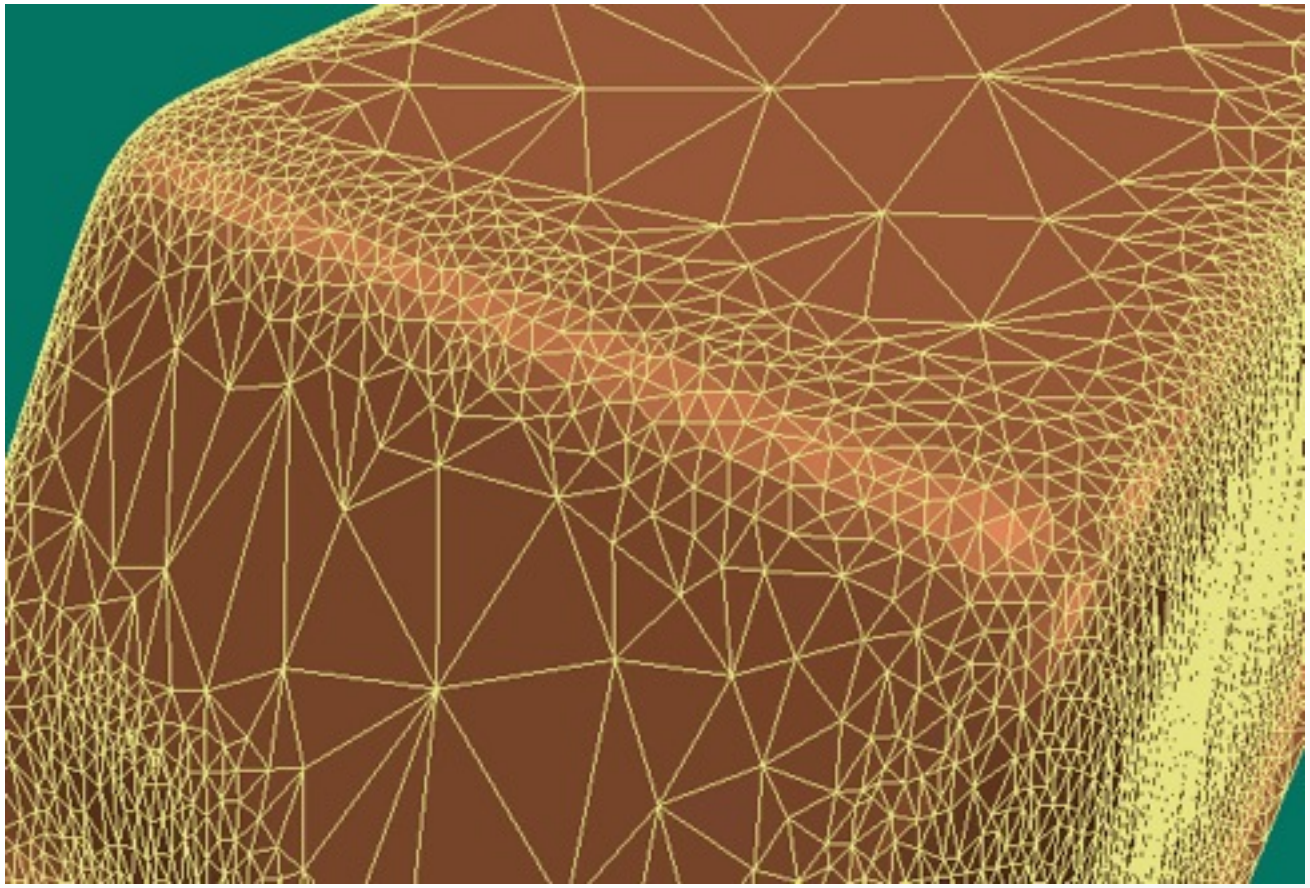


$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$



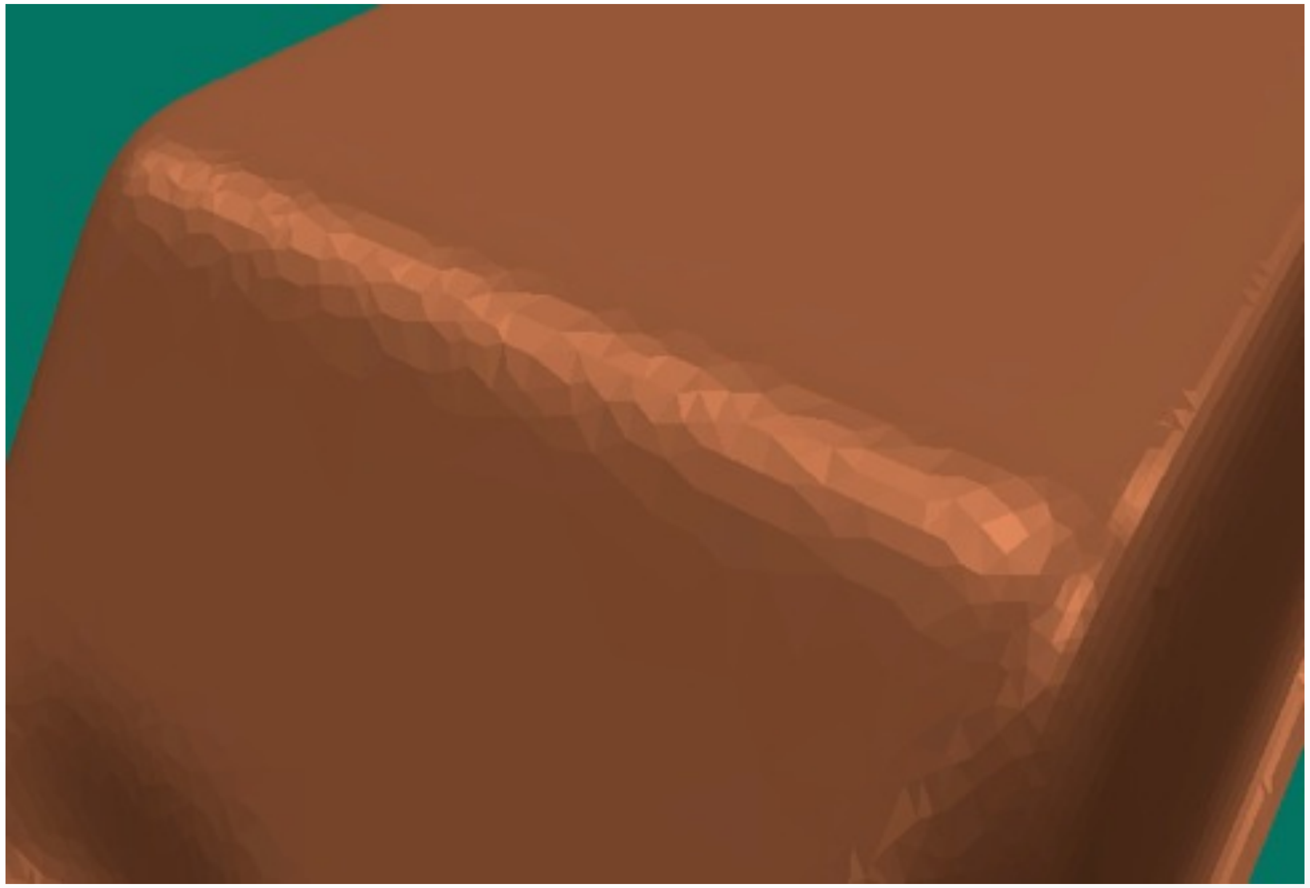
Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape



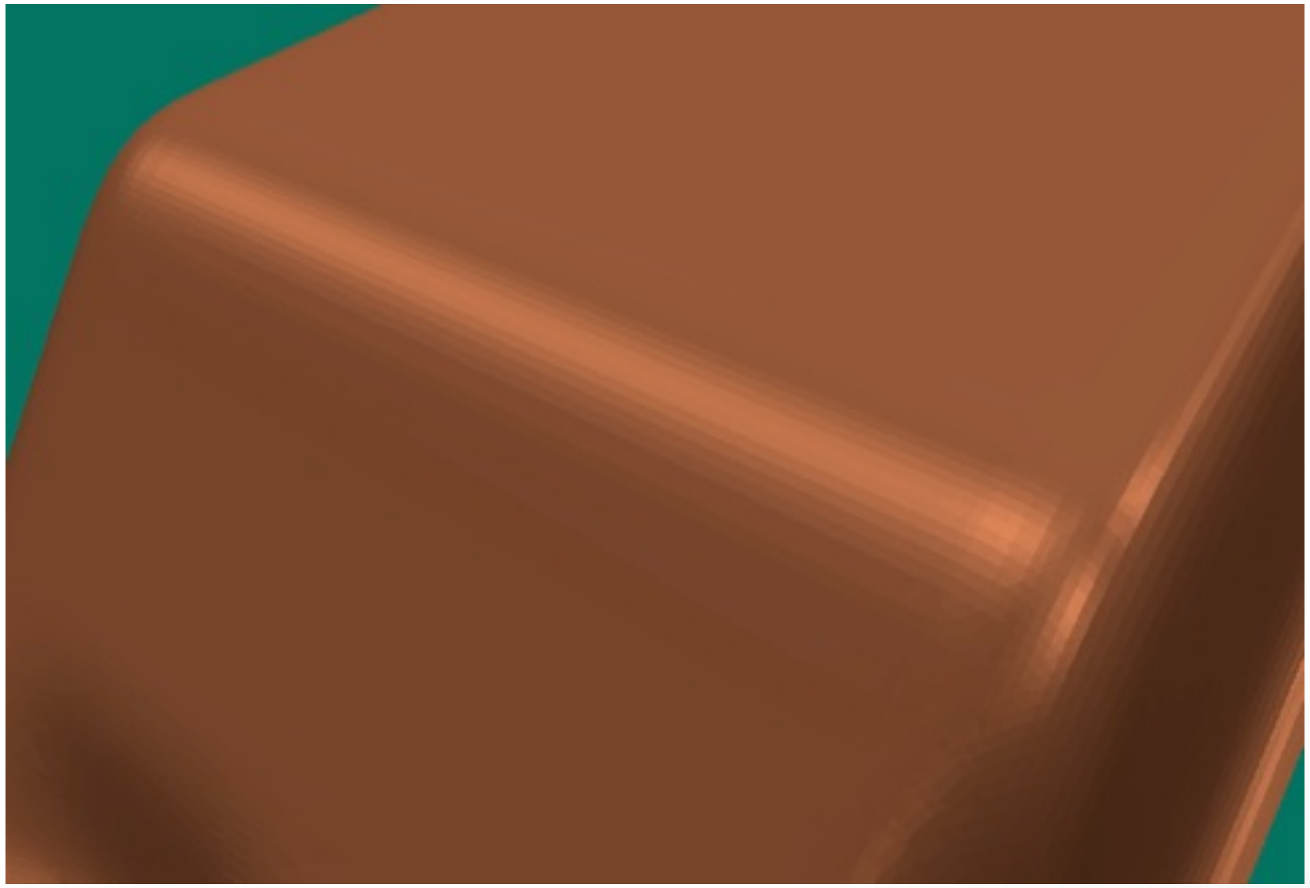
Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape



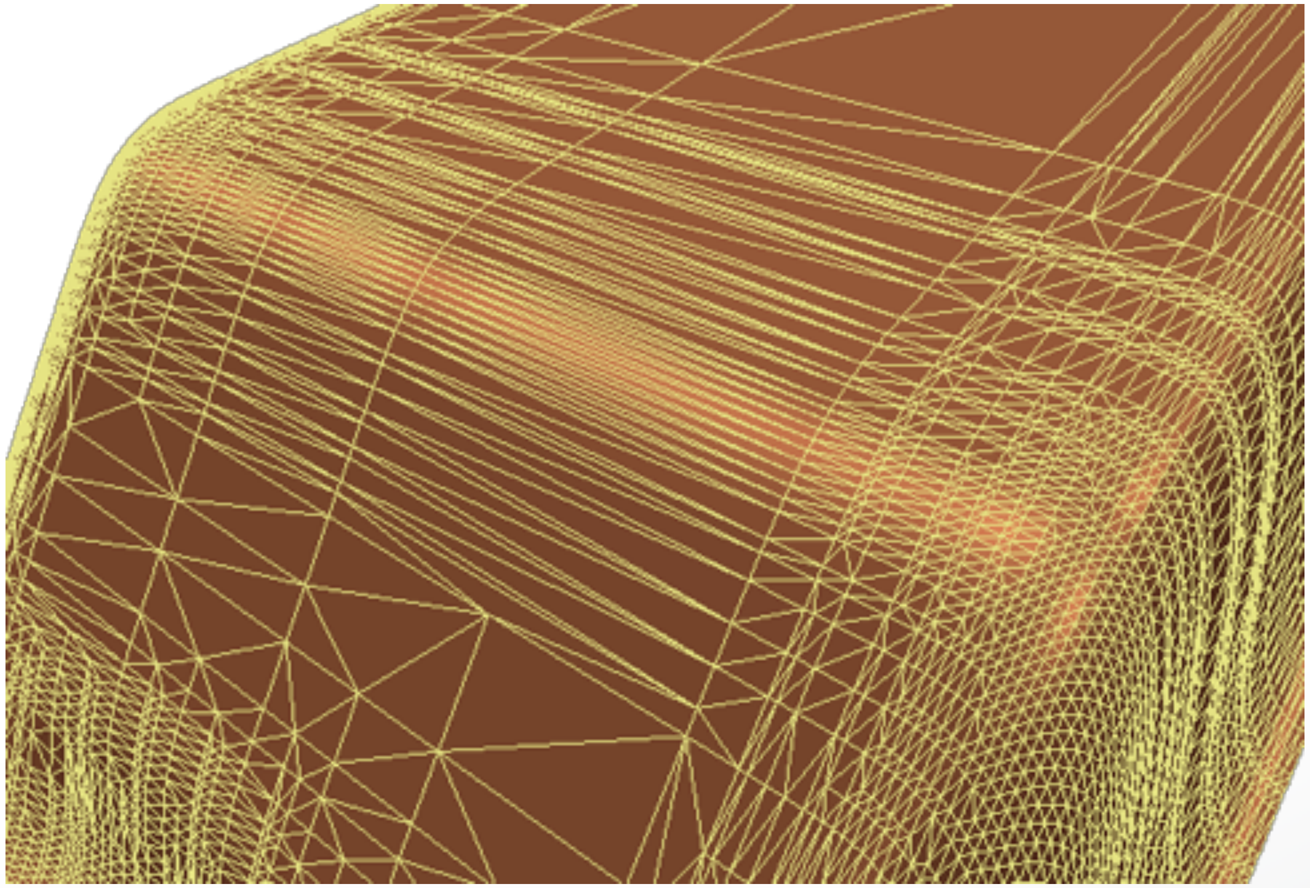
Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles



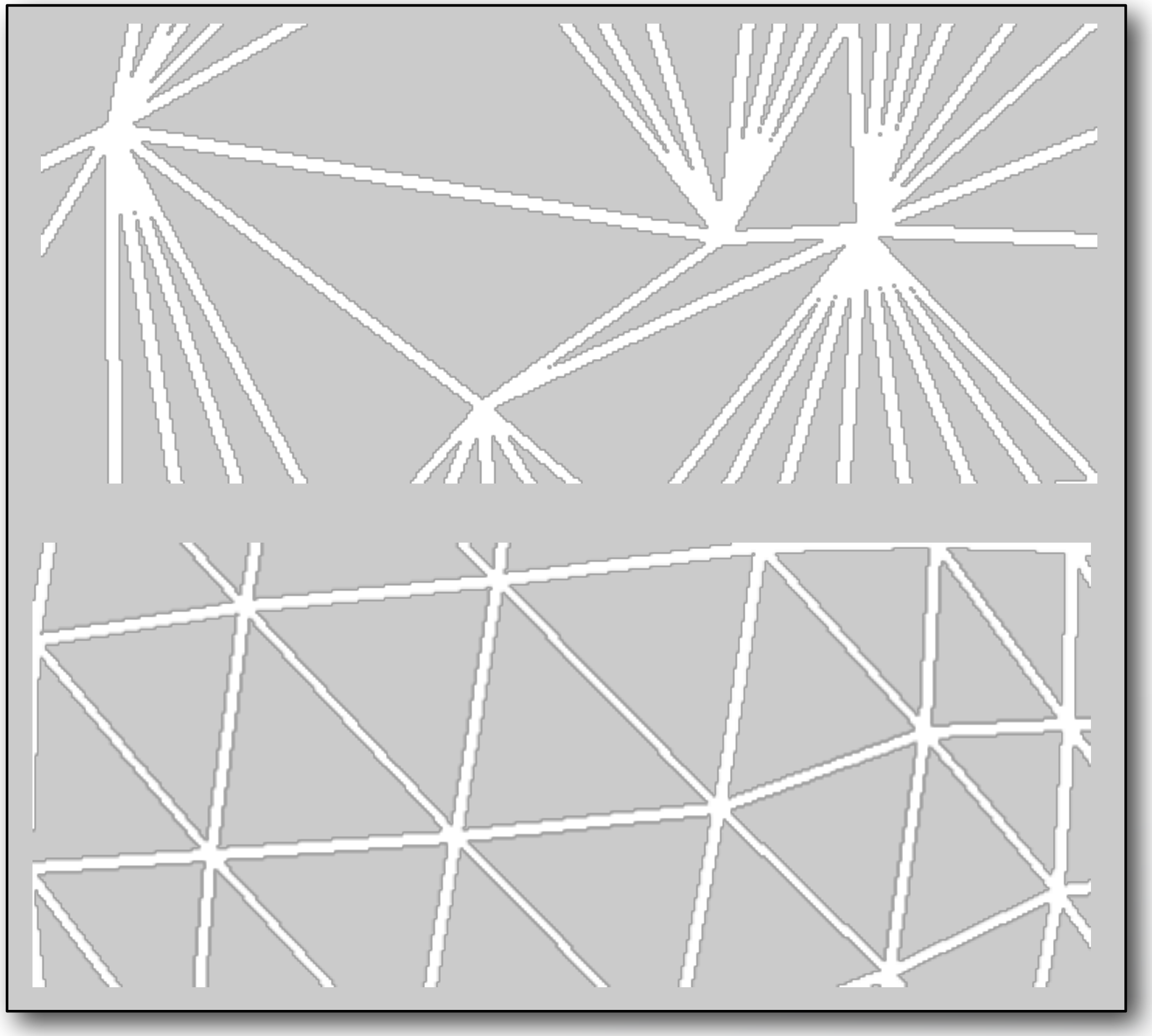
Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles



Fairness Criteria

- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance



Fairness Criteria

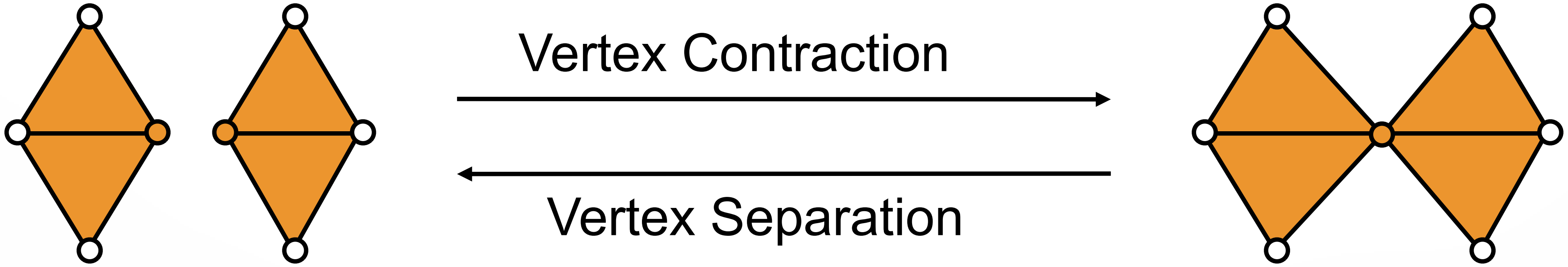
- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - Color differences
  - …

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- **Topology changes**

Fairness Criteria

- Merge vertices across non-edges
  - Changes mesh topology
  - Need spatial *neighborhood* information
  - Generates *non-manifold* meshes



Comparison

- **Vertex clustering**
  - fast but difficult to control simplified mesh
  - topology changes, non-manifold meshes
  - global error bound, but often not close to optimum
- **Iterative decimation with quadric error metrics**
  - good trade-off between mesh-quality and speed
  - explicit control over mesh topology
  - restricting normal deviation improves mesh quality

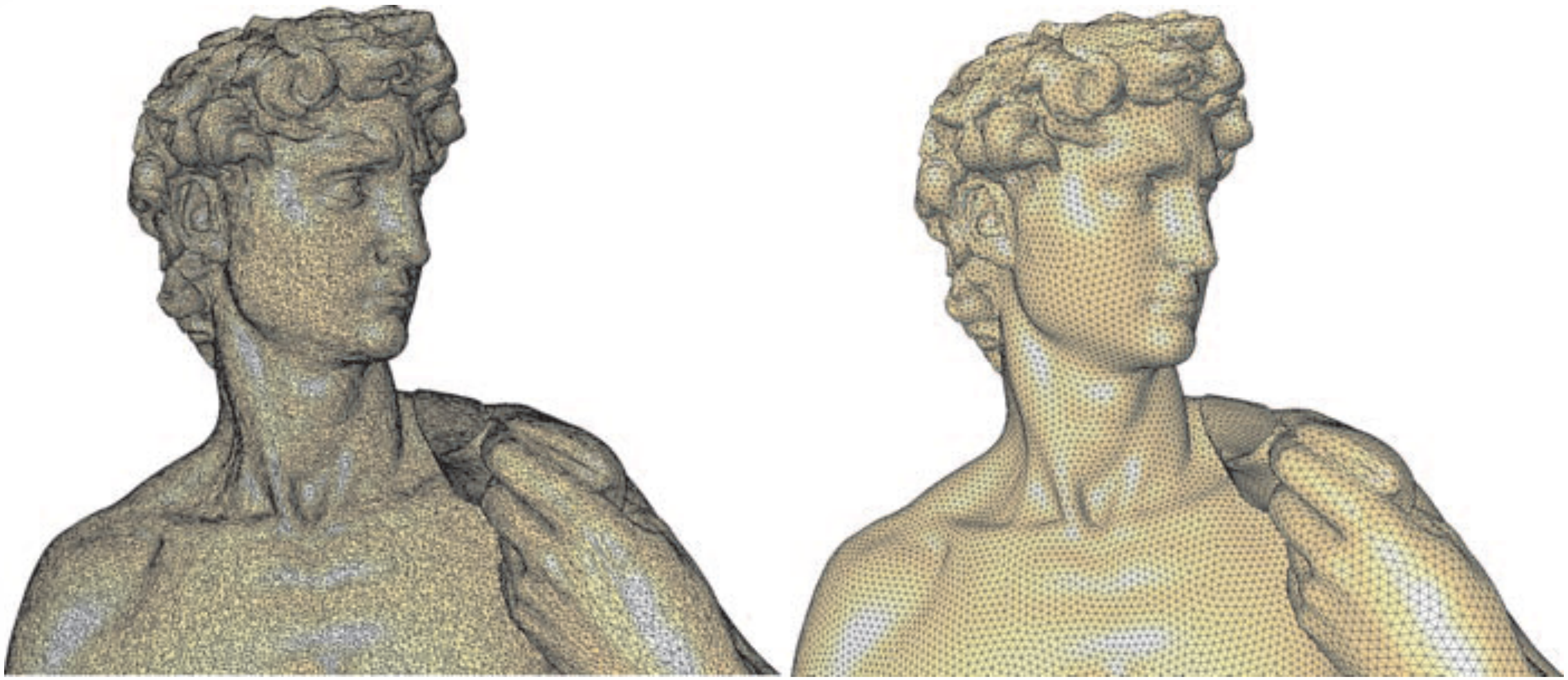
Literature

- Quadric-based simplification
  - http://graphics.cs.uiuc.edu/~garland/software/qslim.html
  - http://www.openmesh.org

- Garland, Heckbert: Surface simplification using quadric error metrics, SIGGRAPH 1997.

- Kobbelt et al., A general framework for mesh decimation, Graphics Interface 1998.

Next Time



Remeshing

http://cs621.hao-li.com

Thanks!