

*Spring 2019*

CSCI 621: **Digital Geometry Processing**

# **Exercise 4. Surface Quality and Smoothing**



Zeng Huang

<http://cs621.hao-li.com>

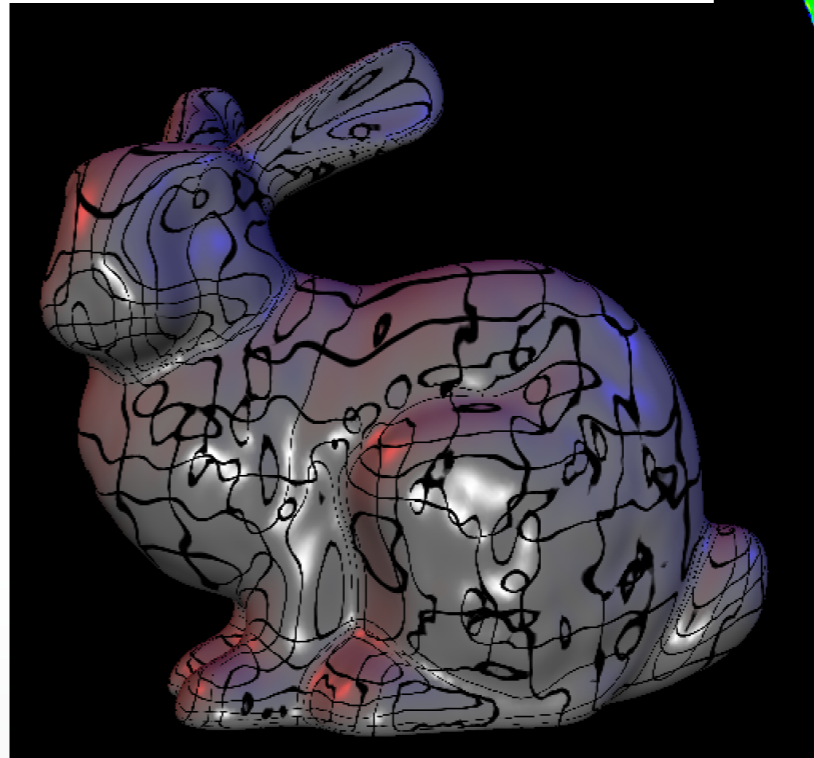
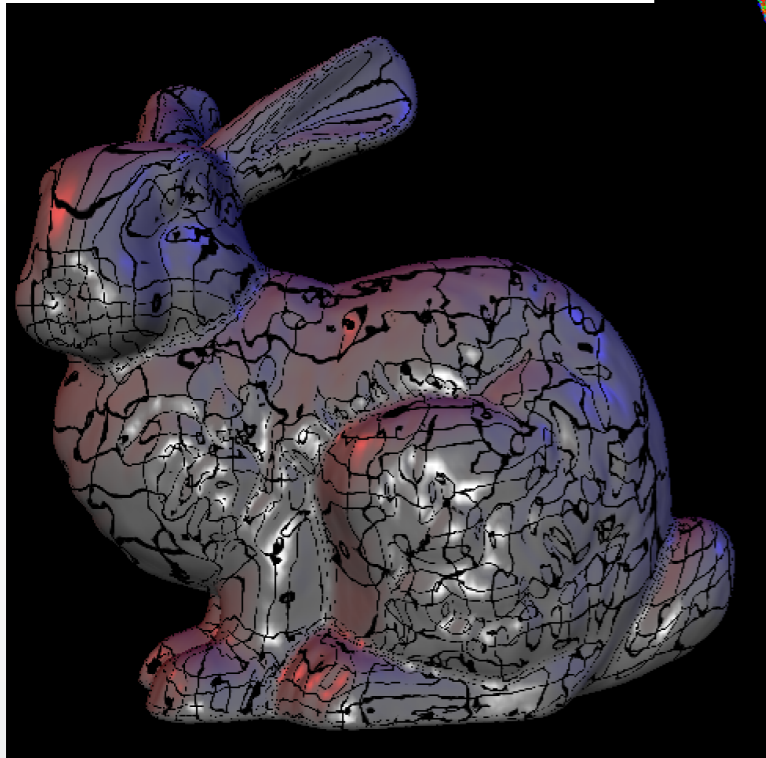
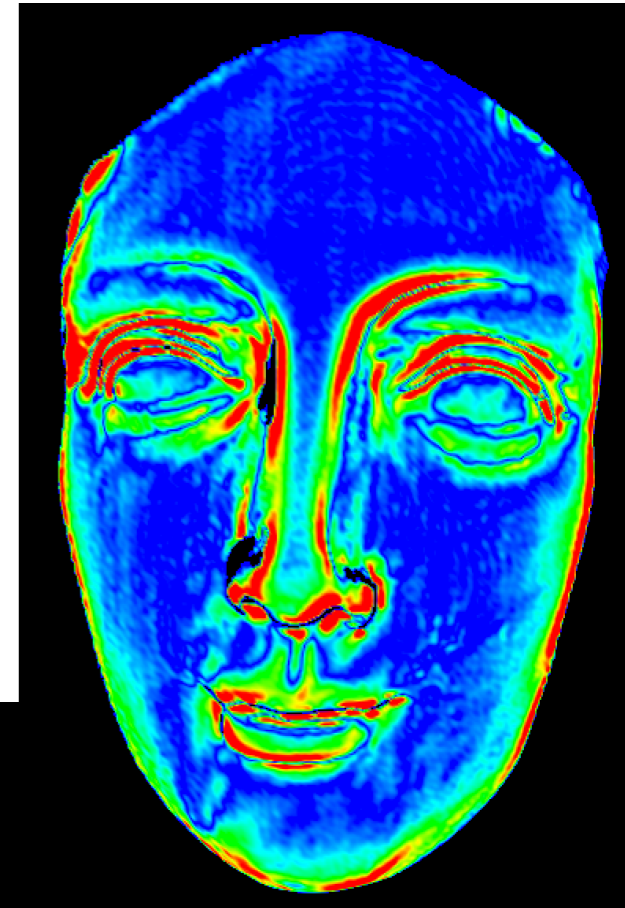
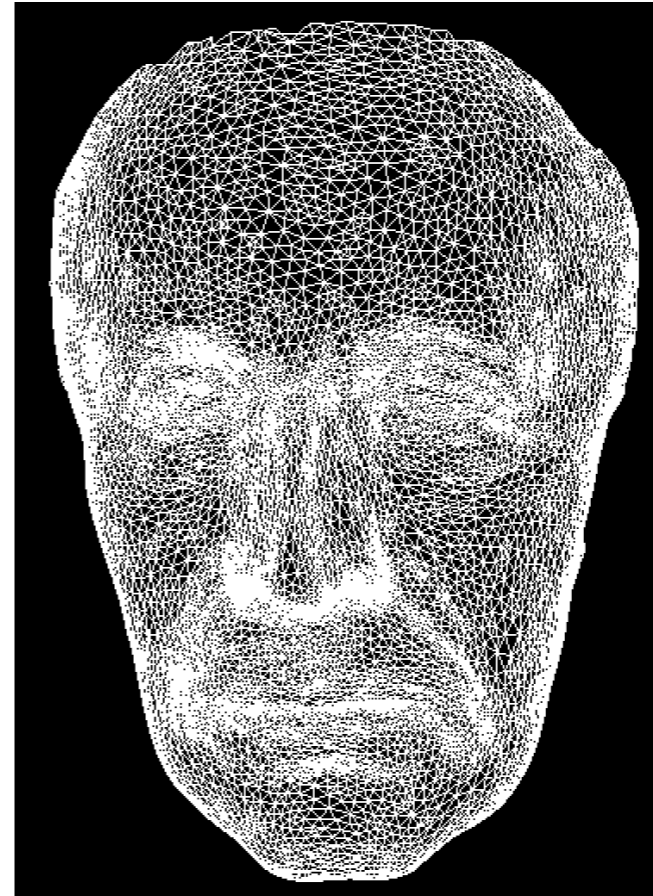
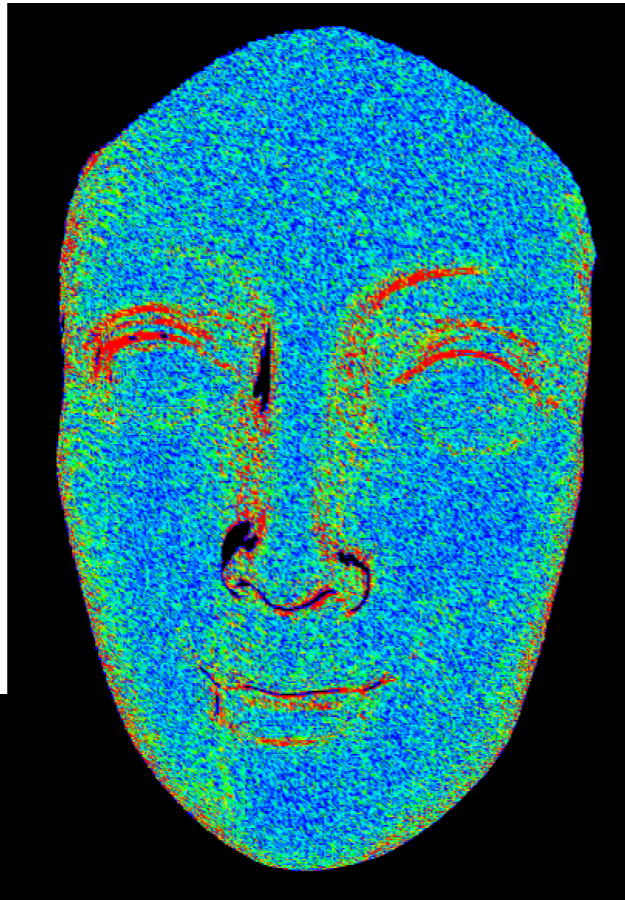
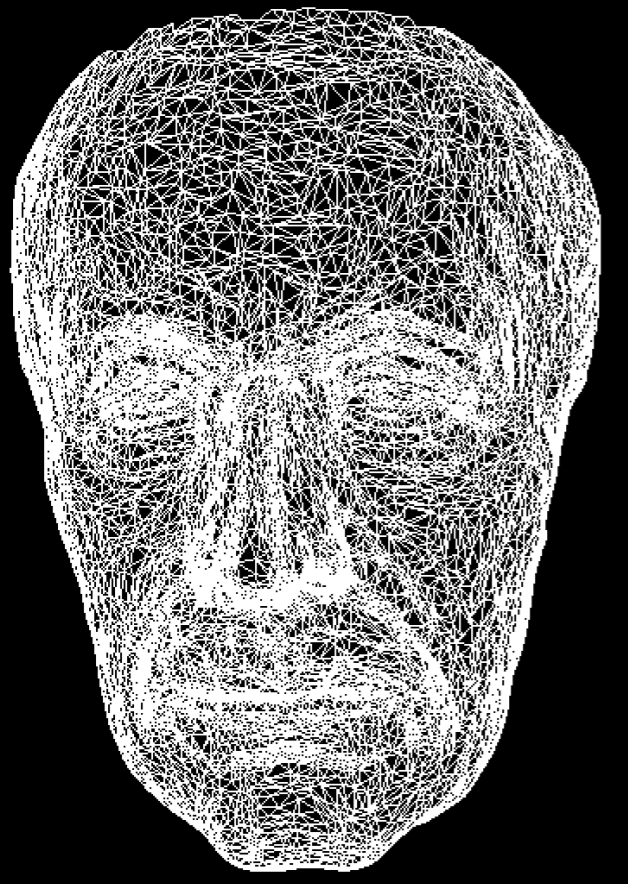
# Surface Smoothing

- Spectral analysis
- **Diffusion flow**
  - **Uniform Laplace operator**
  - **Laplacian-Beltrami operator**
- Energy minimization

# Uniform Laplacian Surface Smoothing

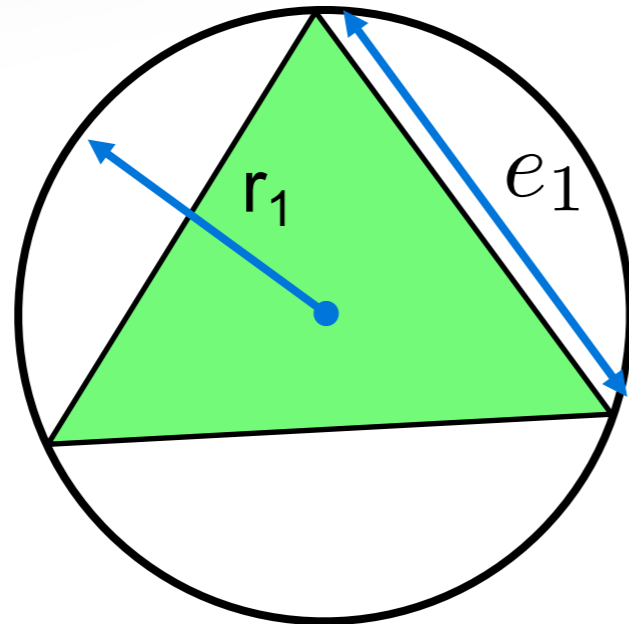
- Uniform Laplace operator  $L_U(v) = \left(\frac{1}{n} \sum_i v_i\right) - v$
- Mesh smoothing  $v' = v + \frac{1}{2} \cdot L_U(v)$
- Implement uniform Laplace operator in `QualityViewer::calc_uniform_mean_curvature()` in `QualityViewer.cc`
- Implement uniform Laplacian smoothing `SmoothViewer::uniform_smooth()` in `SmoothViewer.cc`

# Uniform Laplacian Surface Smoothing

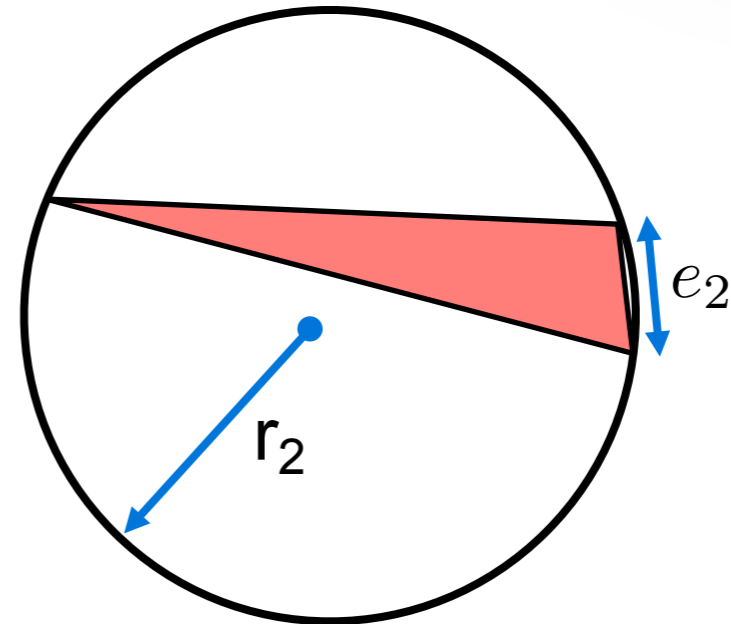


# Triangle Quality

good triangle



bad triangle



$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$

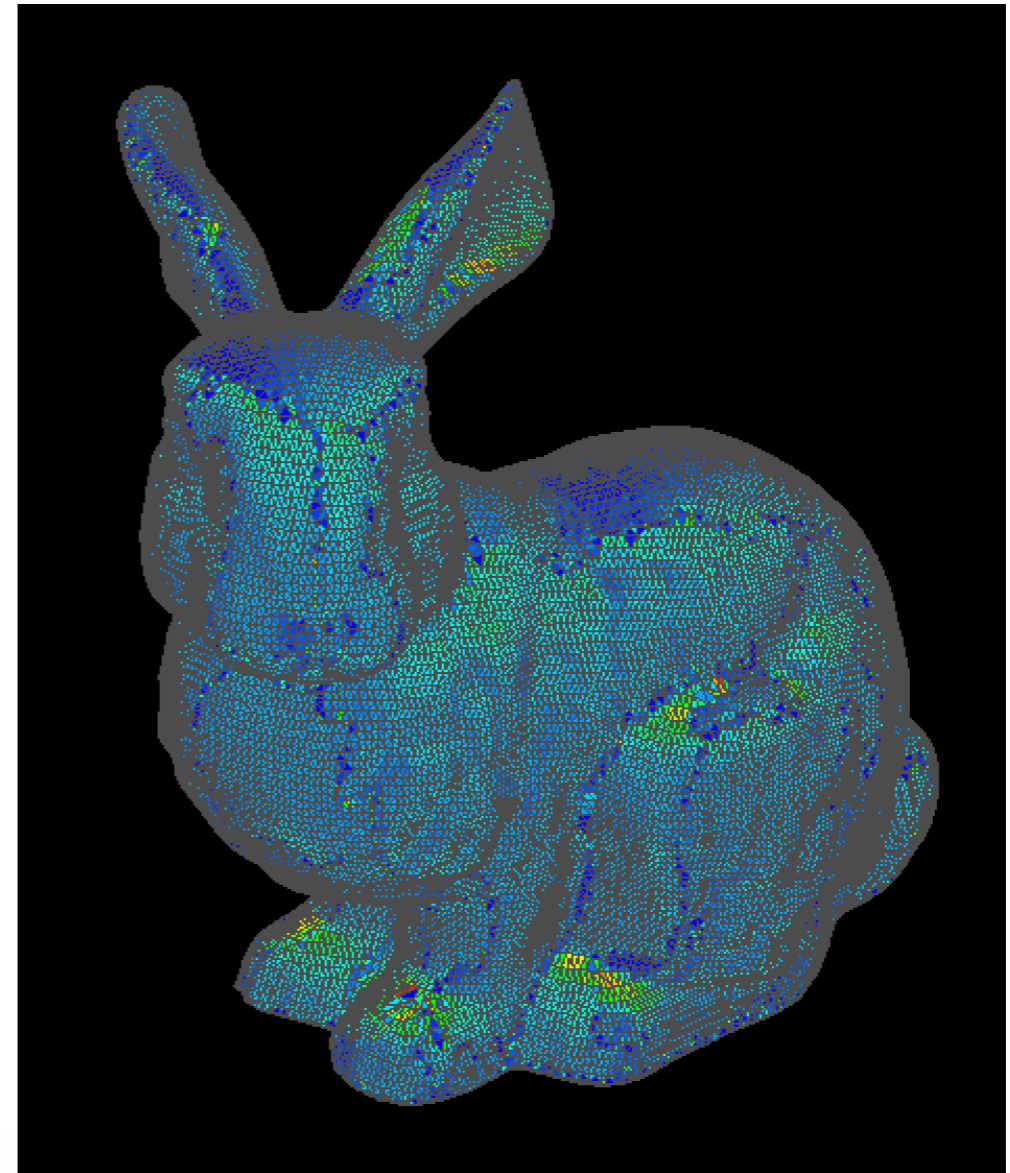
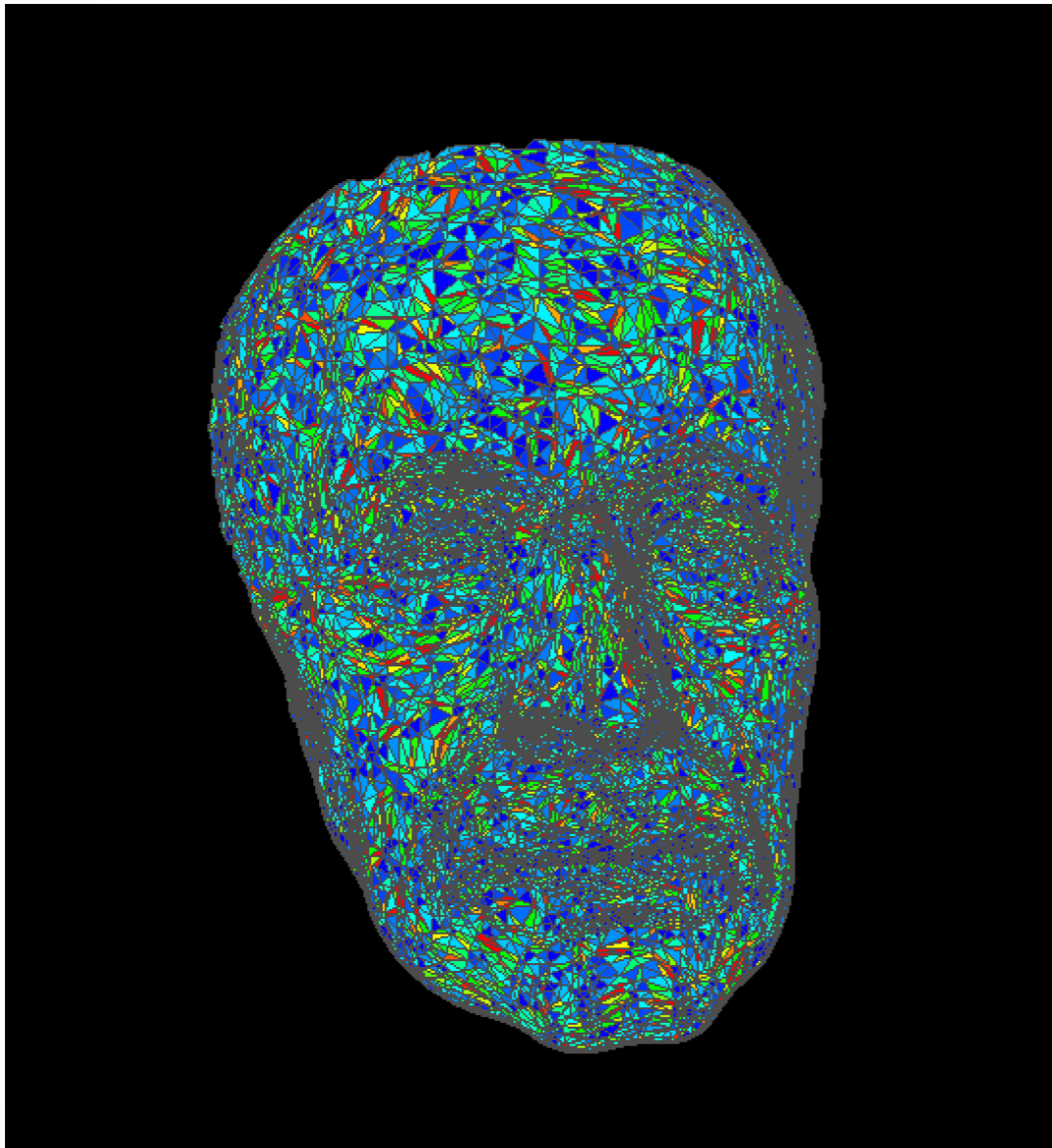
- Assess triangle quality by the circumradius to the minimum edge length ratio

- Circumradius is computed by  $A = \frac{|a| \cdot |b| \cdot |c|}{4 \cdot r} = \frac{|a \times b|}{2}$

- Implement in `QualityViewer::`

`calc_triangle_quality()` in `QualityViewer.cc`

# Triangle Quality



# Laplace-Beltrami curvature and smoothing

- Laplace-Beltrami Operator

$$L_B(v) = \frac{1}{2A} \sum_i ((\cot \alpha_i + \cot \beta_i)(v_i - v))$$

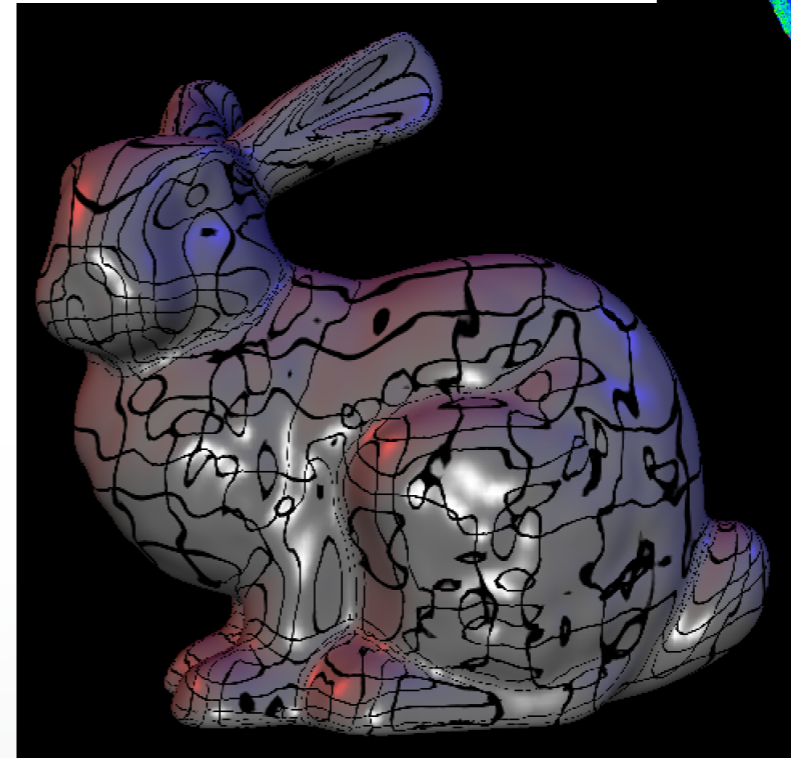
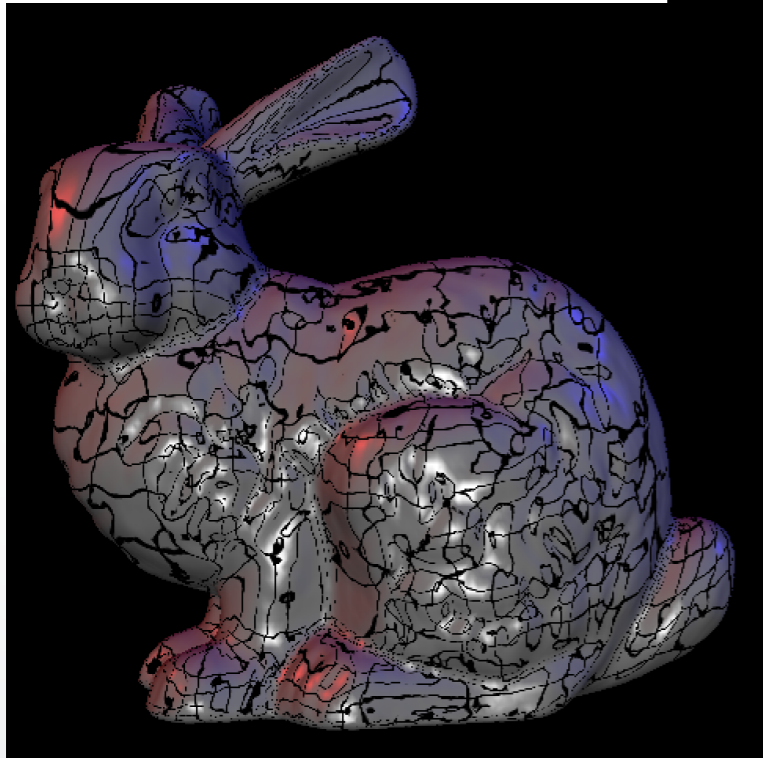
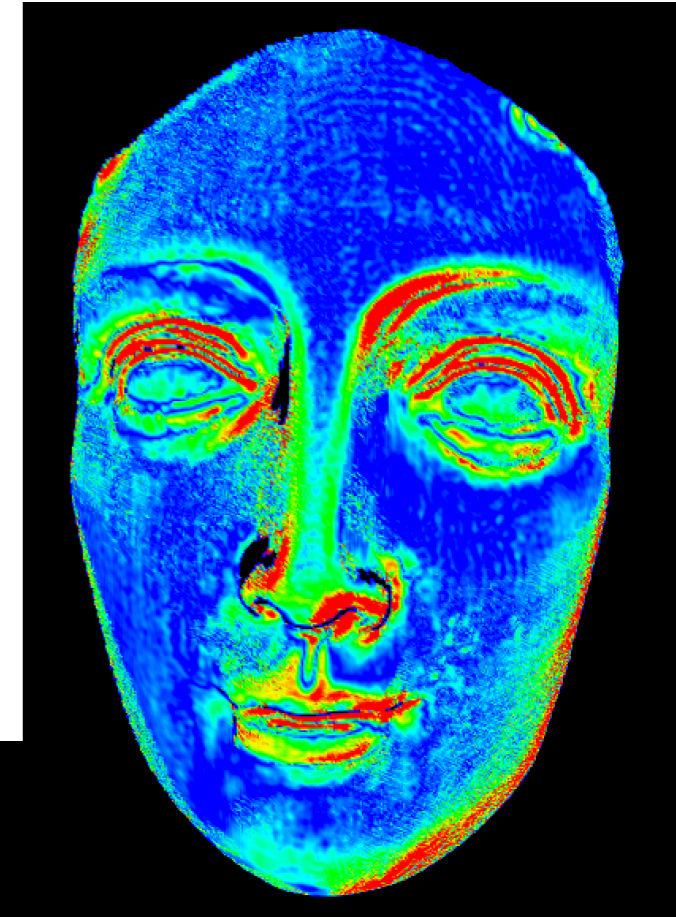
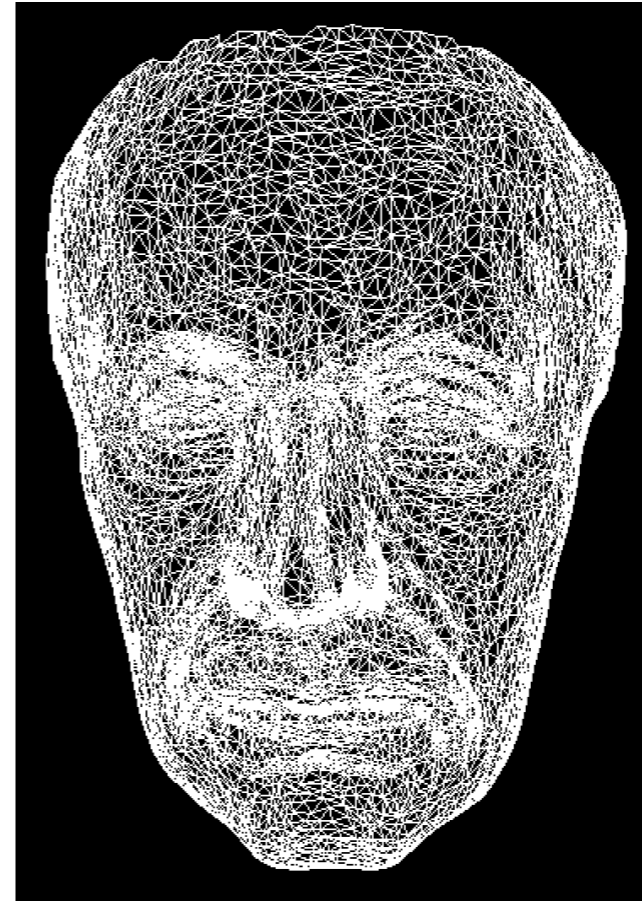
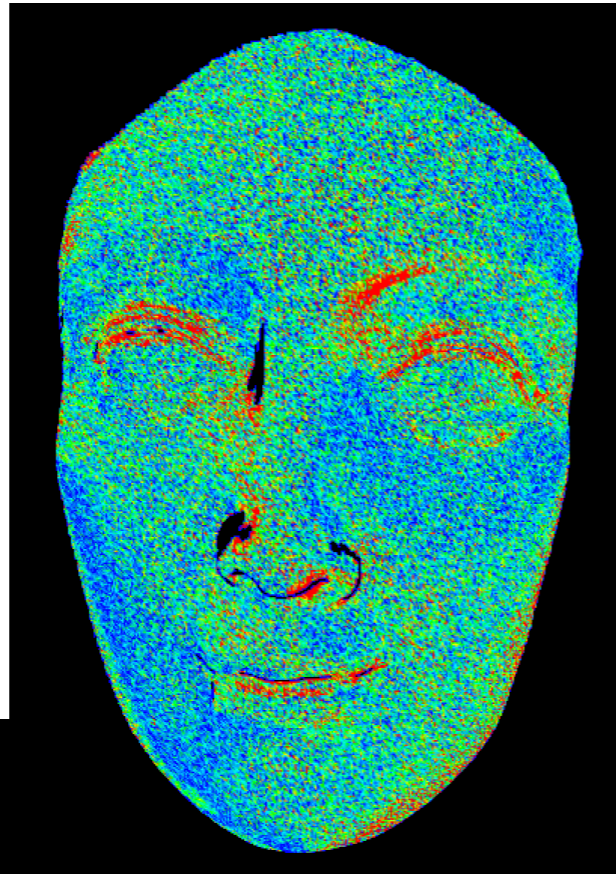
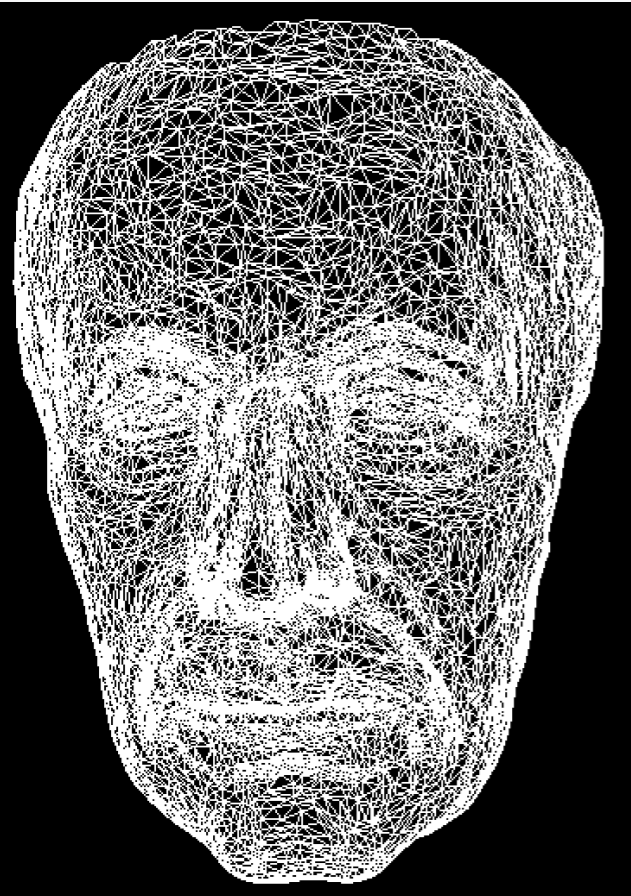
- Compute mean curvature using Laplace-Beltrami weights in `QualityViewer::`

`calc_mean_curvature()` in `QualityViewer.cc`

- Implement smoothing in `SmoothViewer::`

`smooth()` in `SmoothViewer.cc`

# Laplace-Beltrami curvature and smoothing

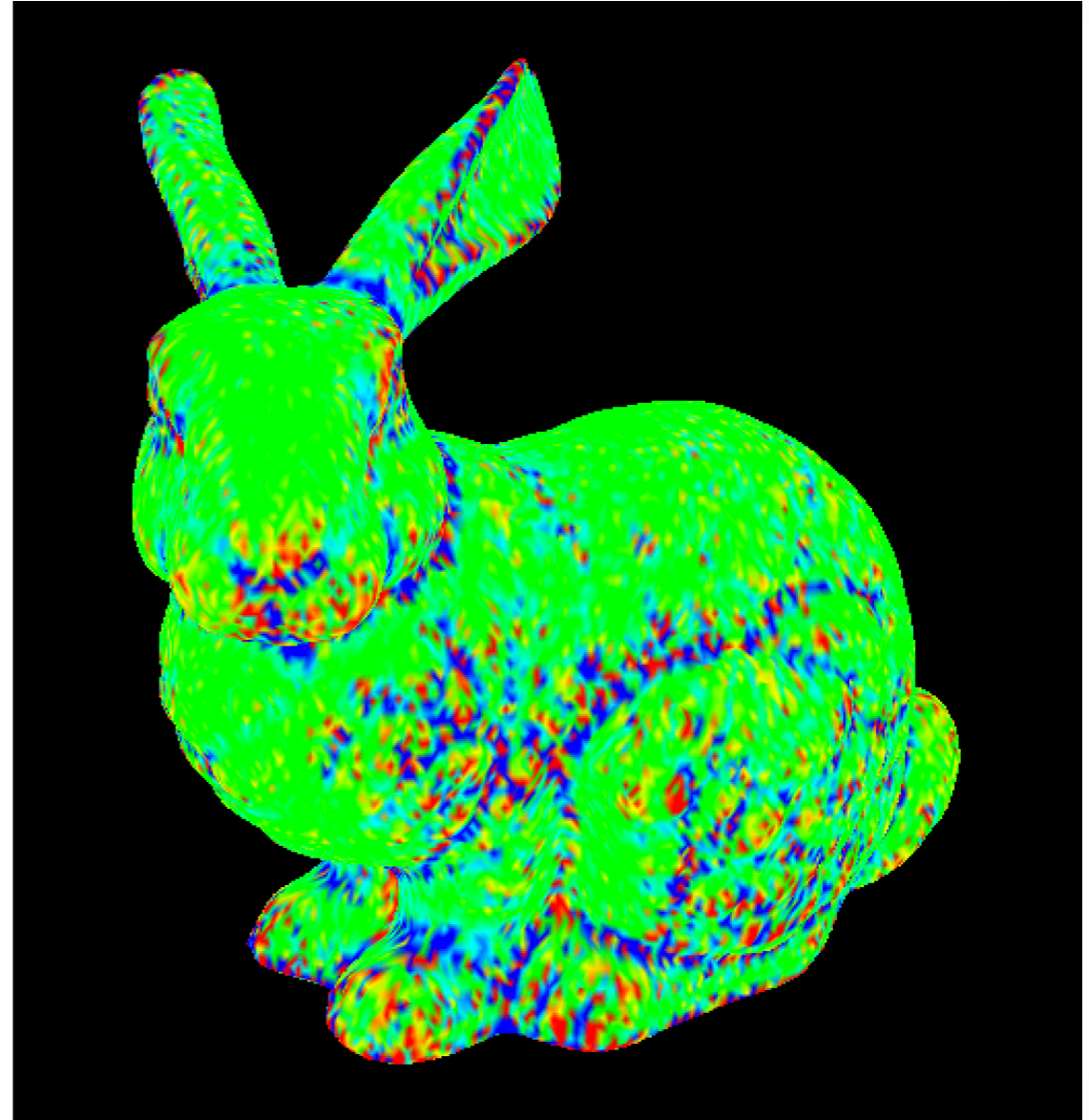
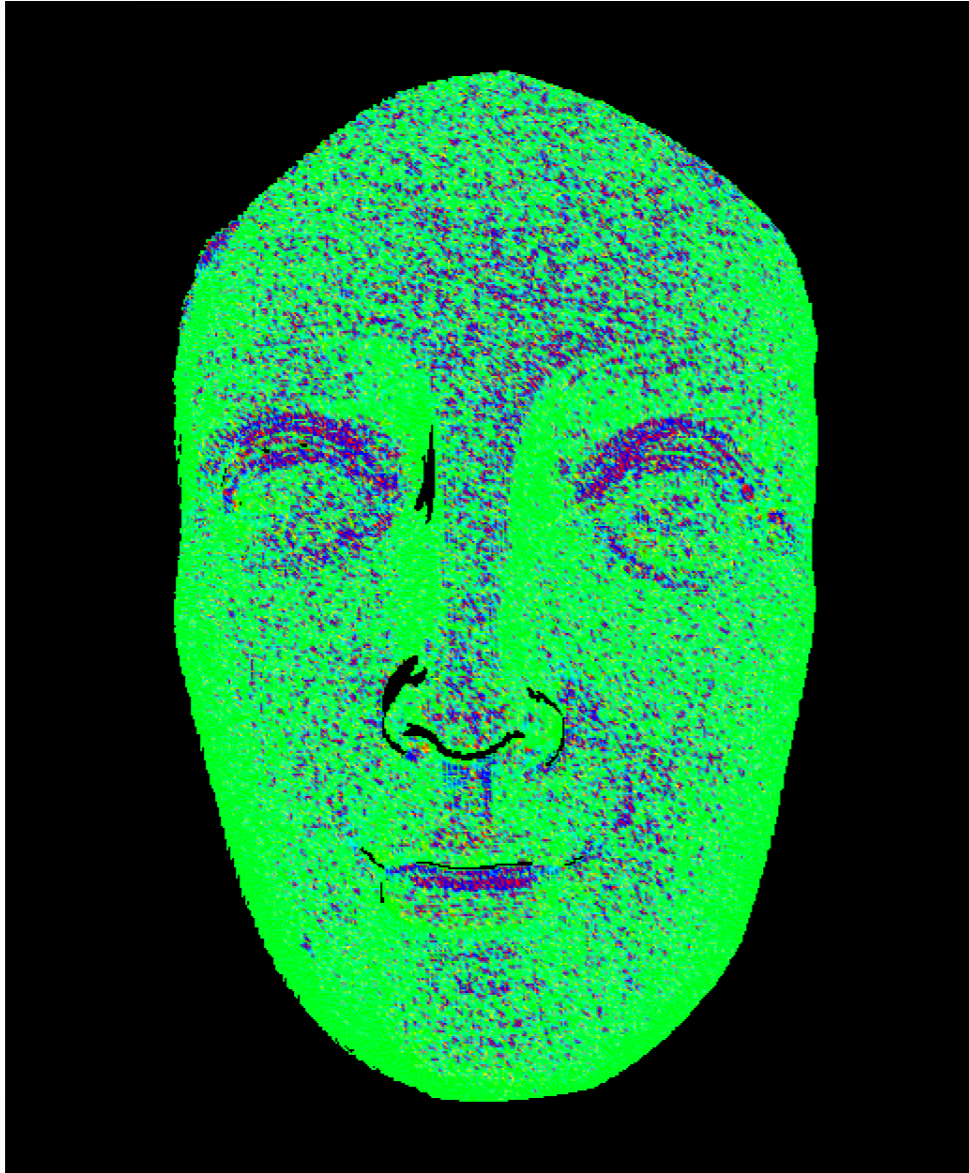




# Gaussian Curvature

- Gaussian curvature  $G = (2\pi - \sum_j \theta_j) / A$
- `QualityViewer::calc_gauss_curvature()` in `QualityViewer.cc`

# Gaussian Curvature



# Submission

- Deadline: **Mar 19, 2019 12:00 pm**
- Upload a .zip compressed file named “Exercise4-YourName.zip” to Blackboard.
- Include a “read.txt” file describing how you solve each exercise and the encountered problems

<http://cs621.hao-li.com>

# Thanks!

