11.1 **Remeshing**

Hao Li

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Outline

- *What* is remeshing?
- *Why* remeshing?
- *How* to do remeshing?

Outline

- ***What* is remeshing?**
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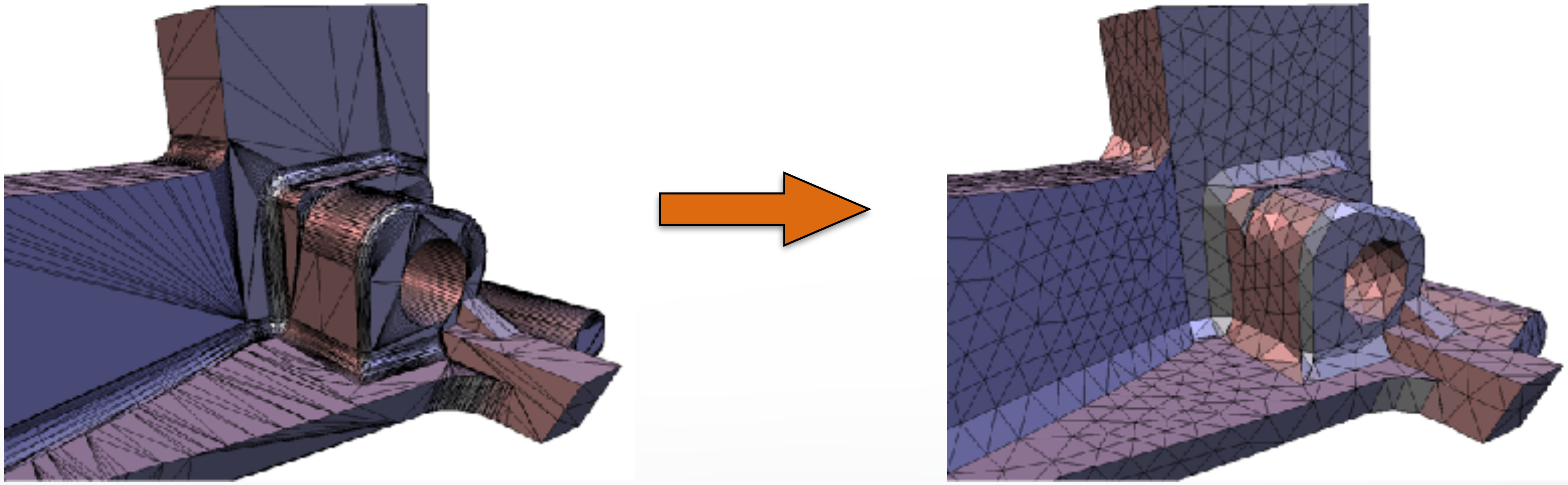
Definition

Given a 3D mesh

- Already a manifold mesh

Compute another mesh

- Satisfy some quality requirements
- Approximate well the input mesh



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- *What* is remeshing?
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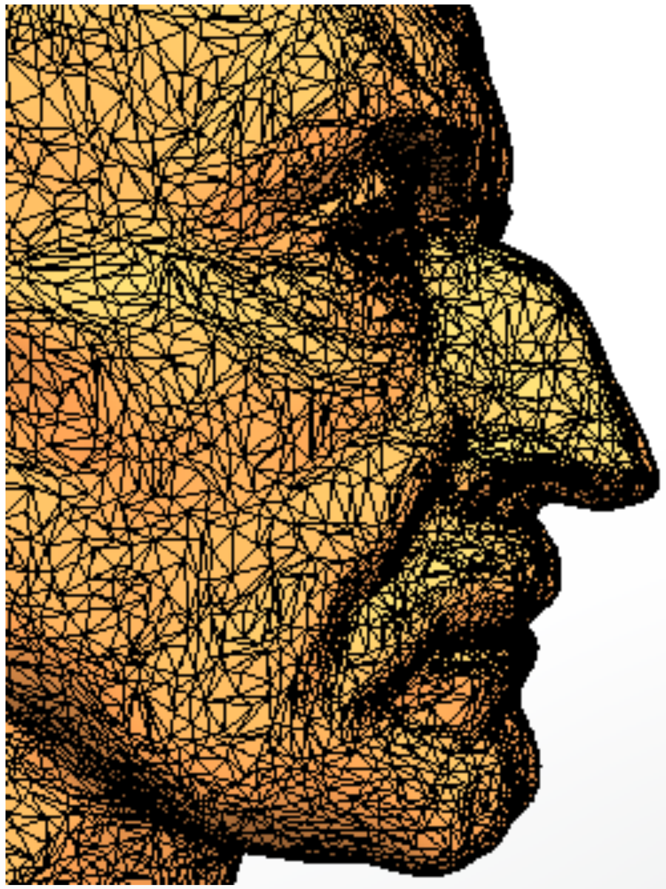
Motivation

Unsatisfactory “raw” mesh

- By scanning or implicit representations



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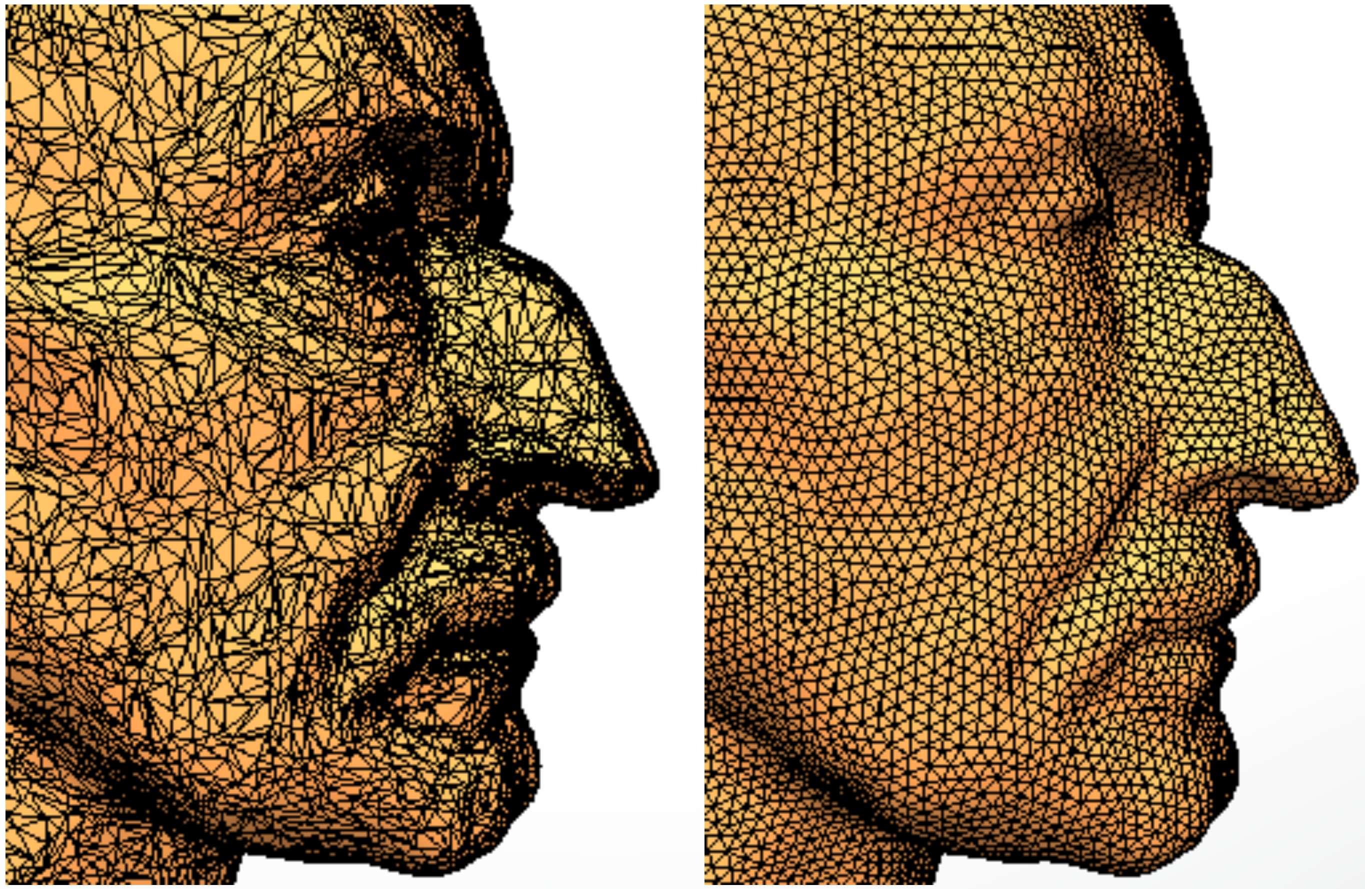
Unsatisfactory “raw” mesh

- By scanning or implicit representations

Improve mesh quality for further use



Motivation

Unsatisfactory “raw” mesh

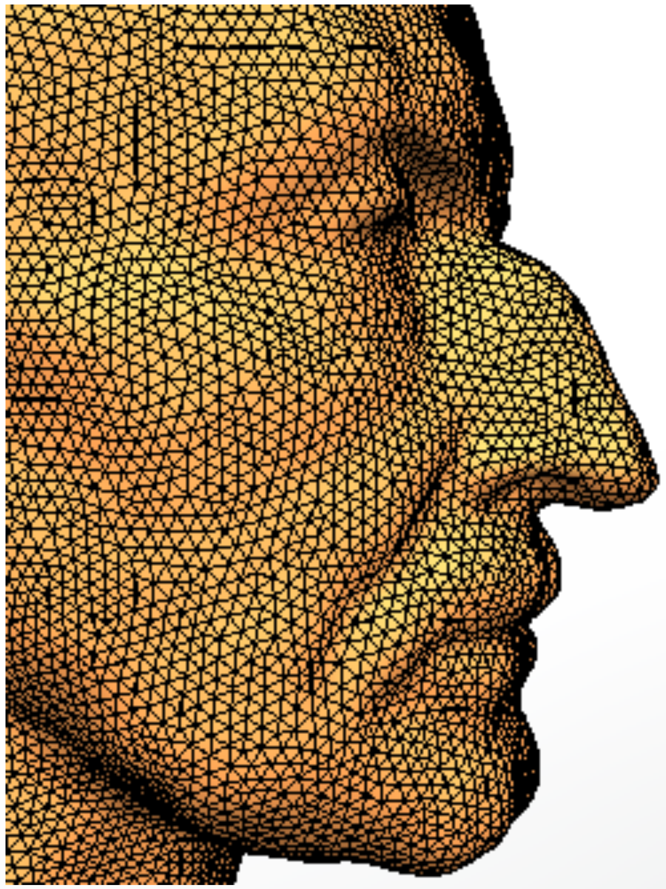
- By scanning or implicit representations

Improve mesh quality for further use

- Modeling: easy processing
- Simulation: numerical robustness
- ……

Quality requirements

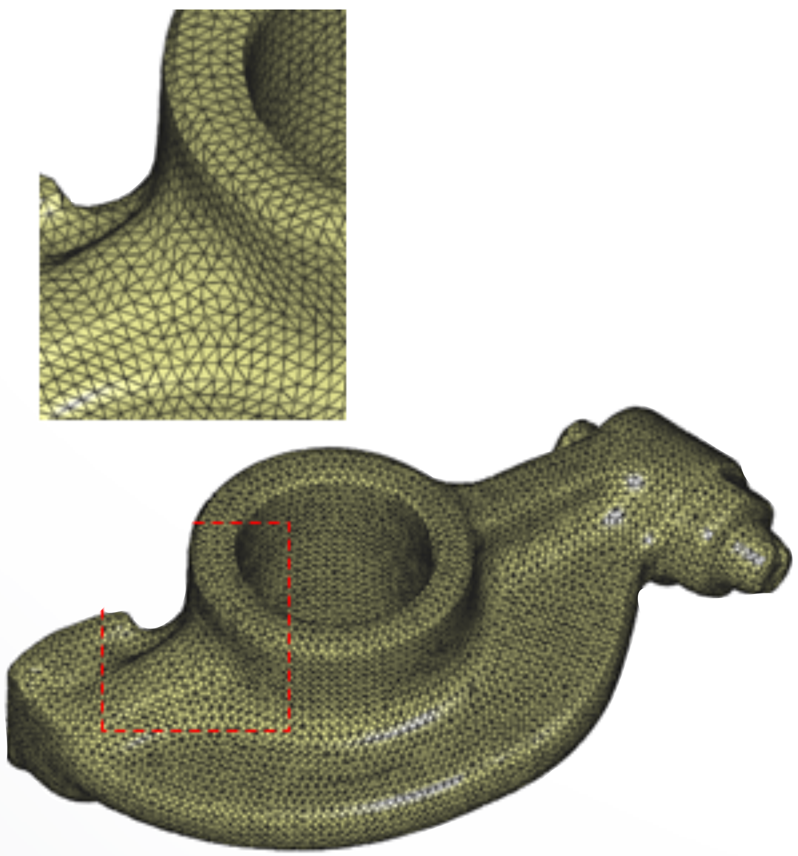
- Local structure
- Global structure



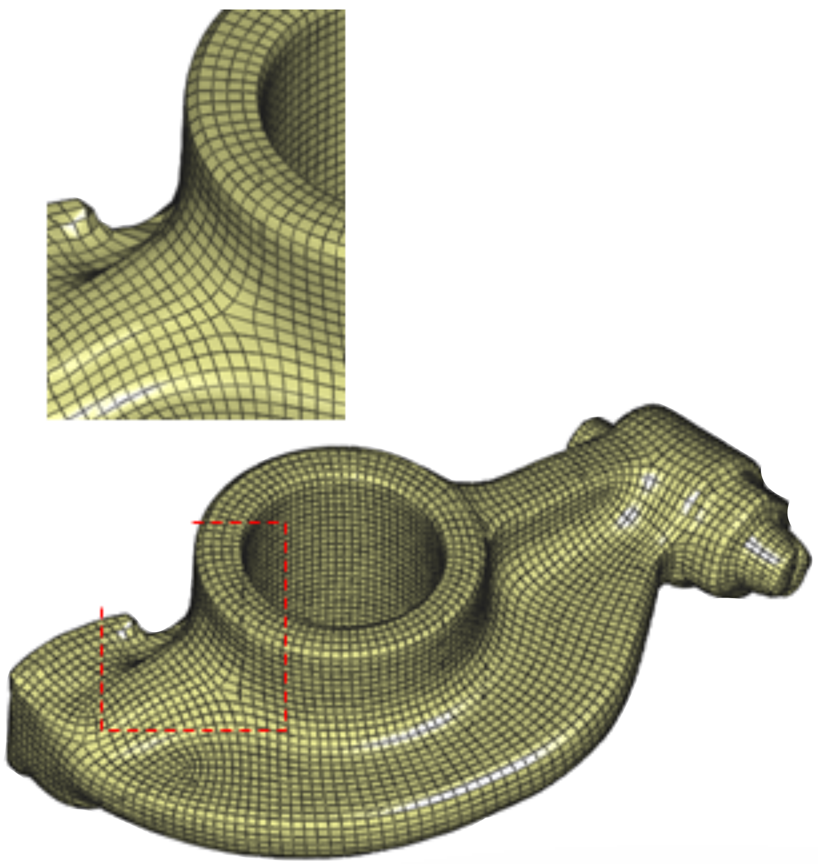
Local structure

Element type

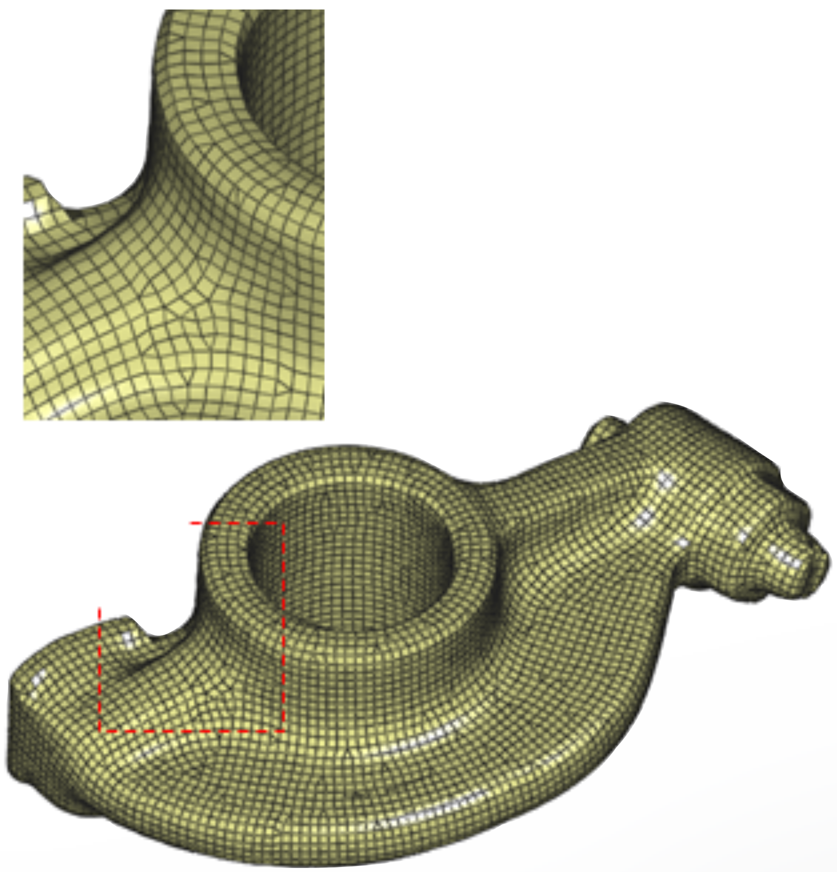
- Triangles vs. quadrangles



all-triangle mesh



all-quad mesh

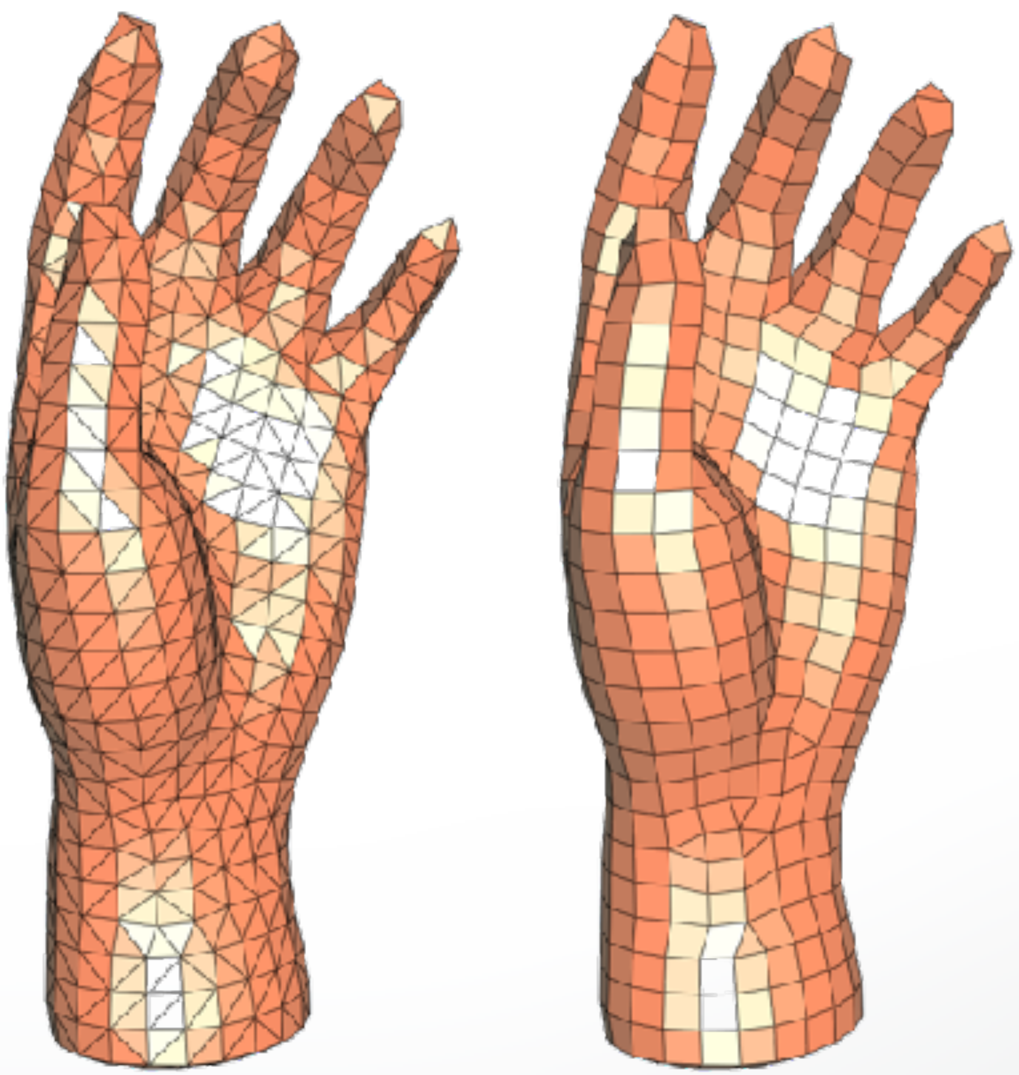


quad-dominant mesh

Local structure

Element type

- Triangles vs. quadrangles



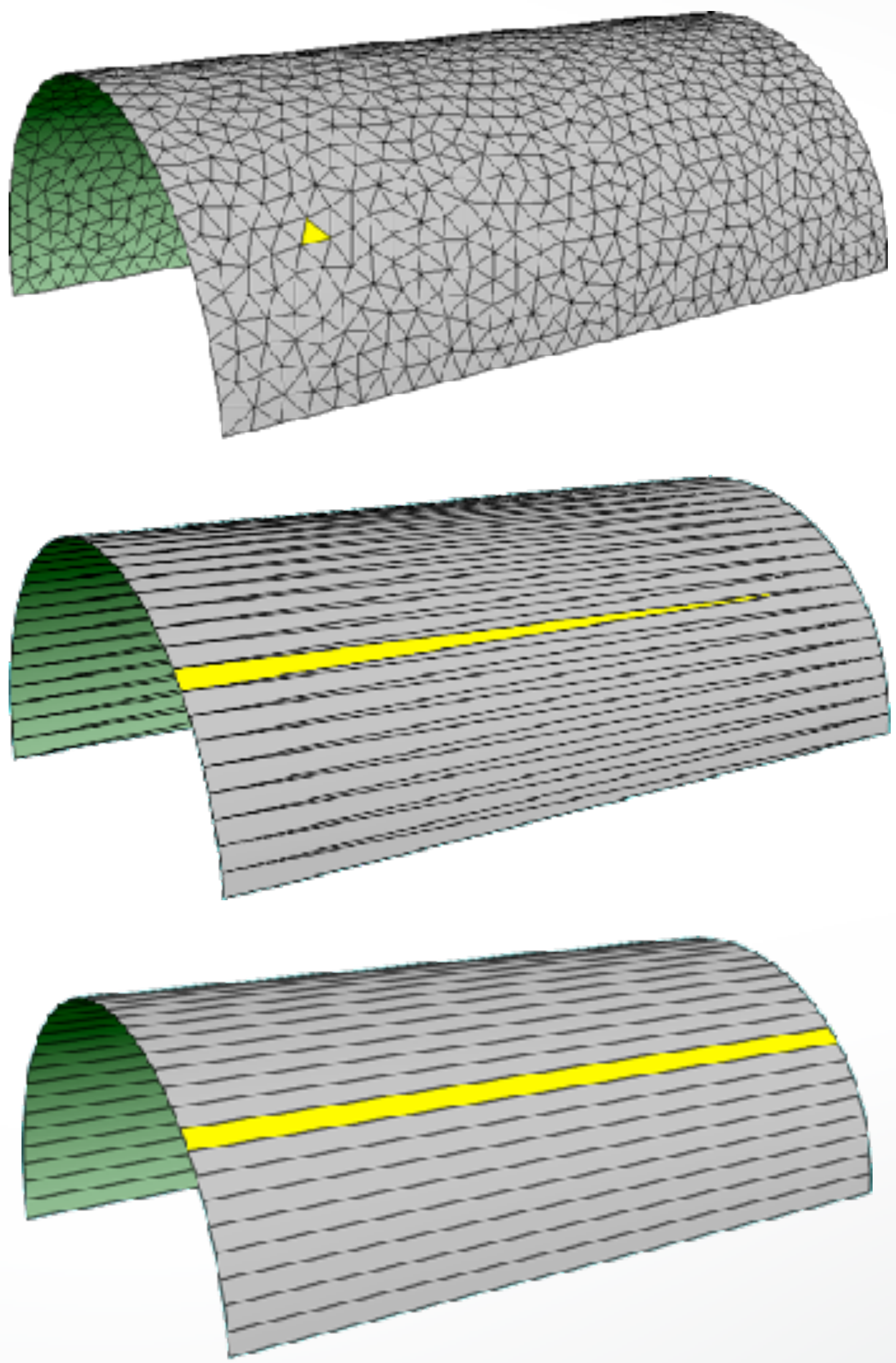
Local structure

Element type

- Triangles vs. quadrangles

Element shape

- Isotropic vs. anisotropic



Local structure

Element type

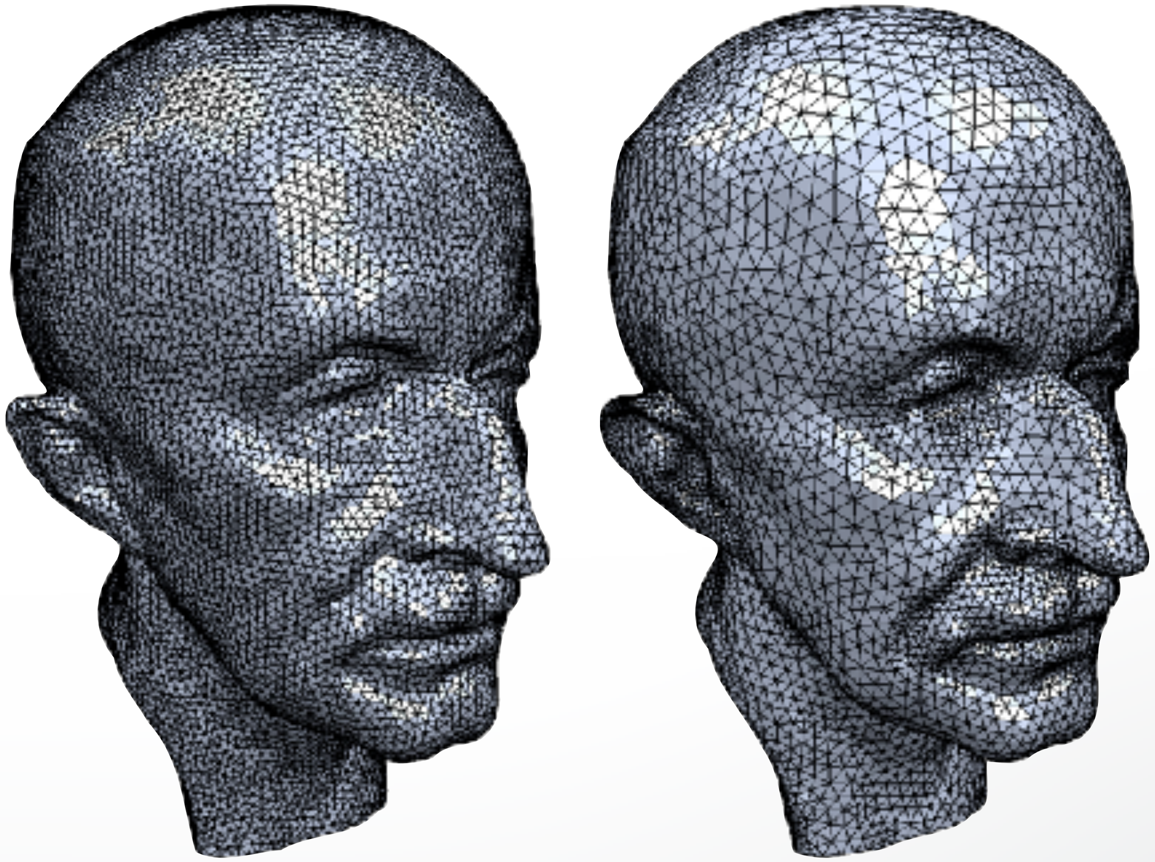
- Triangles vs. quadrangles

Element shape

- Isotropic vs. anisotropic

Element distribution

- Uniform vs. adaptive



Local structure

Element type

- Triangles vs. quadrangles

Element shape

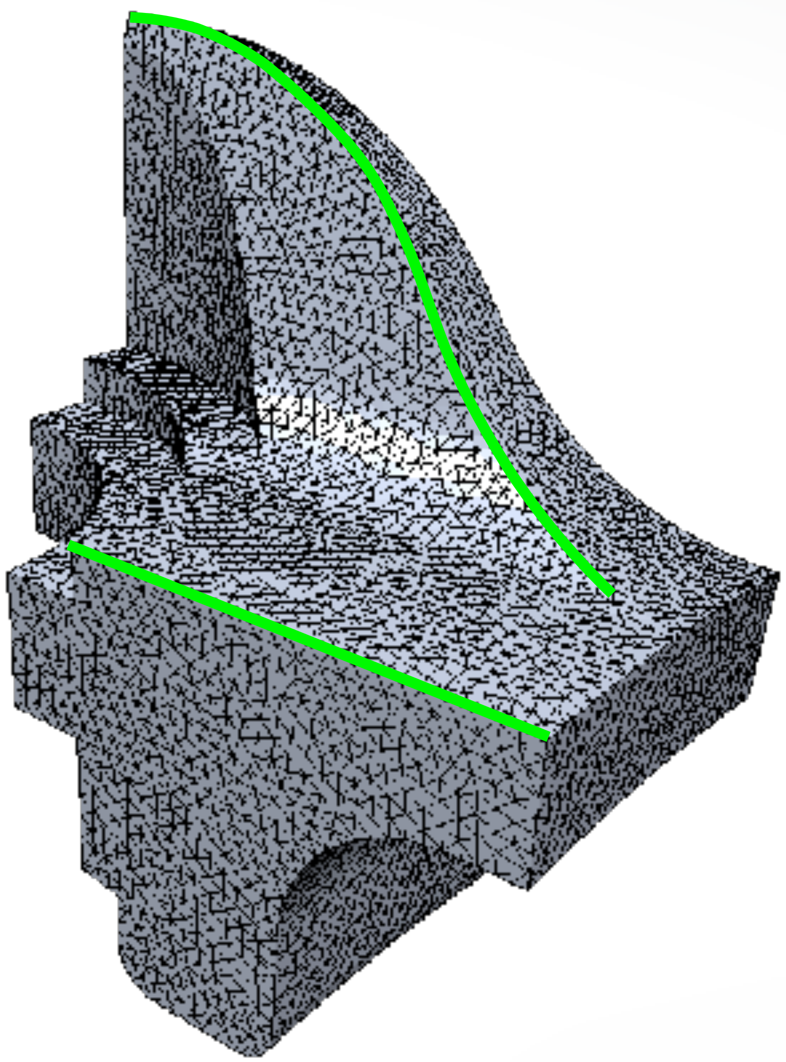
- Isotropic vs. anisotropic

Element distribution

- Uniform vs. adaptive

Element alignment

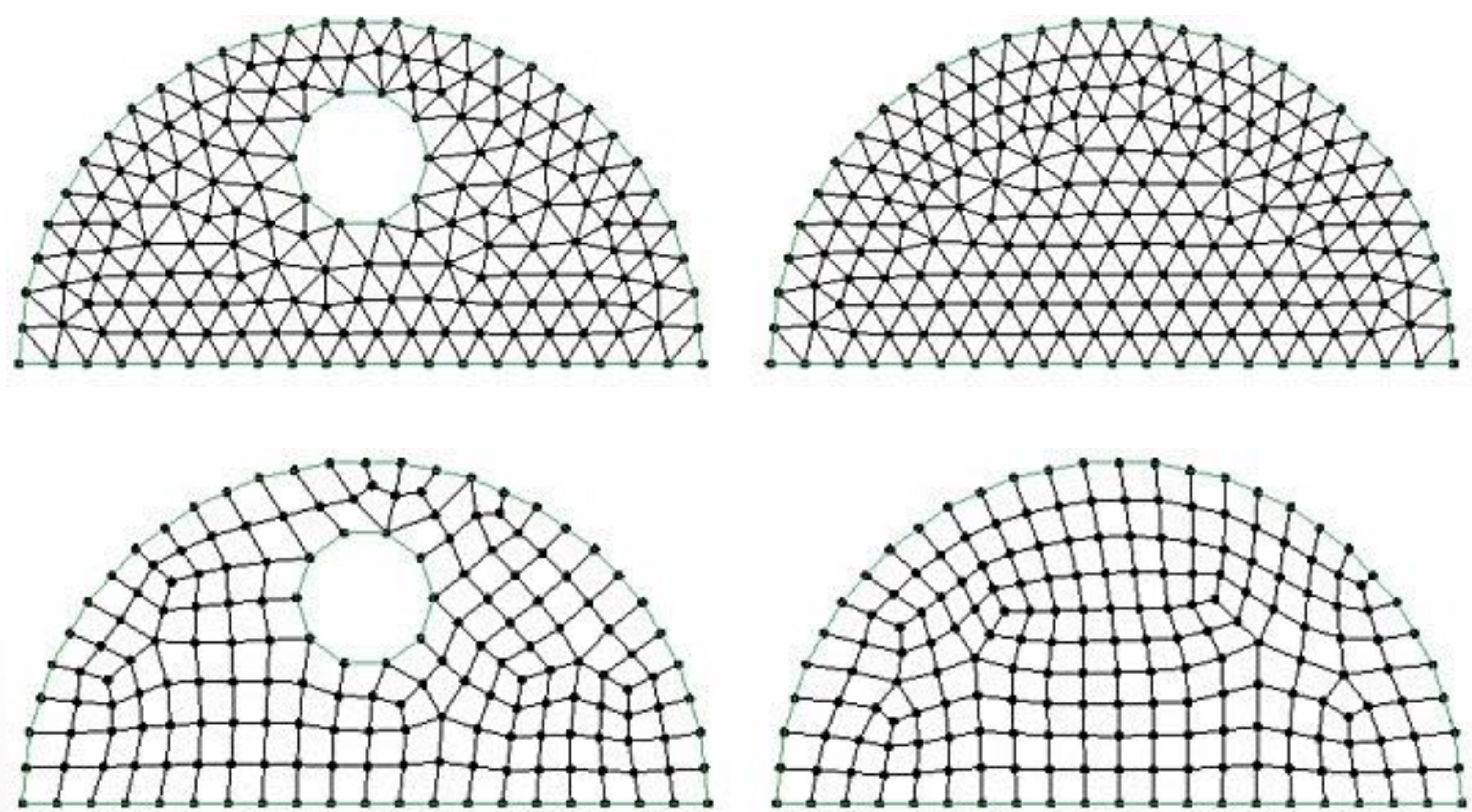
- Preserve sharp features and curvature lines



Global structure

Valence of a *regular* vertex

	Interior vertex	Boundary vertex
Triangle mesh	6	4
Quadrangle mesh	4	3



Global structure

Valence of a *regular* vertex

	Interior vertex	Boundary vertex
Triangle mesh	6	4
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Different types of mesh structure

- Irregular
- Semi-regular: multi-resolution analysis / modeling
- Highly regular: numerical simulation
- Regular: only possible for special models

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  - Isotropic remeshing
  - Anisotropic remeshing

Outline

- *What* is remeshing?
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  - **Isotropic remeshing**
  - Anisotropic remeshing

Isotropic remeshing

Incremental remeshing

- Simple to implement and robust
- Not need parameterization
- Efficient for high-resolution input

Variational remeshing

- Energy minimization
- Parameterization-based → expensive
- Works for coarse input mesh

Greedy remeshing

Isotropic remeshing

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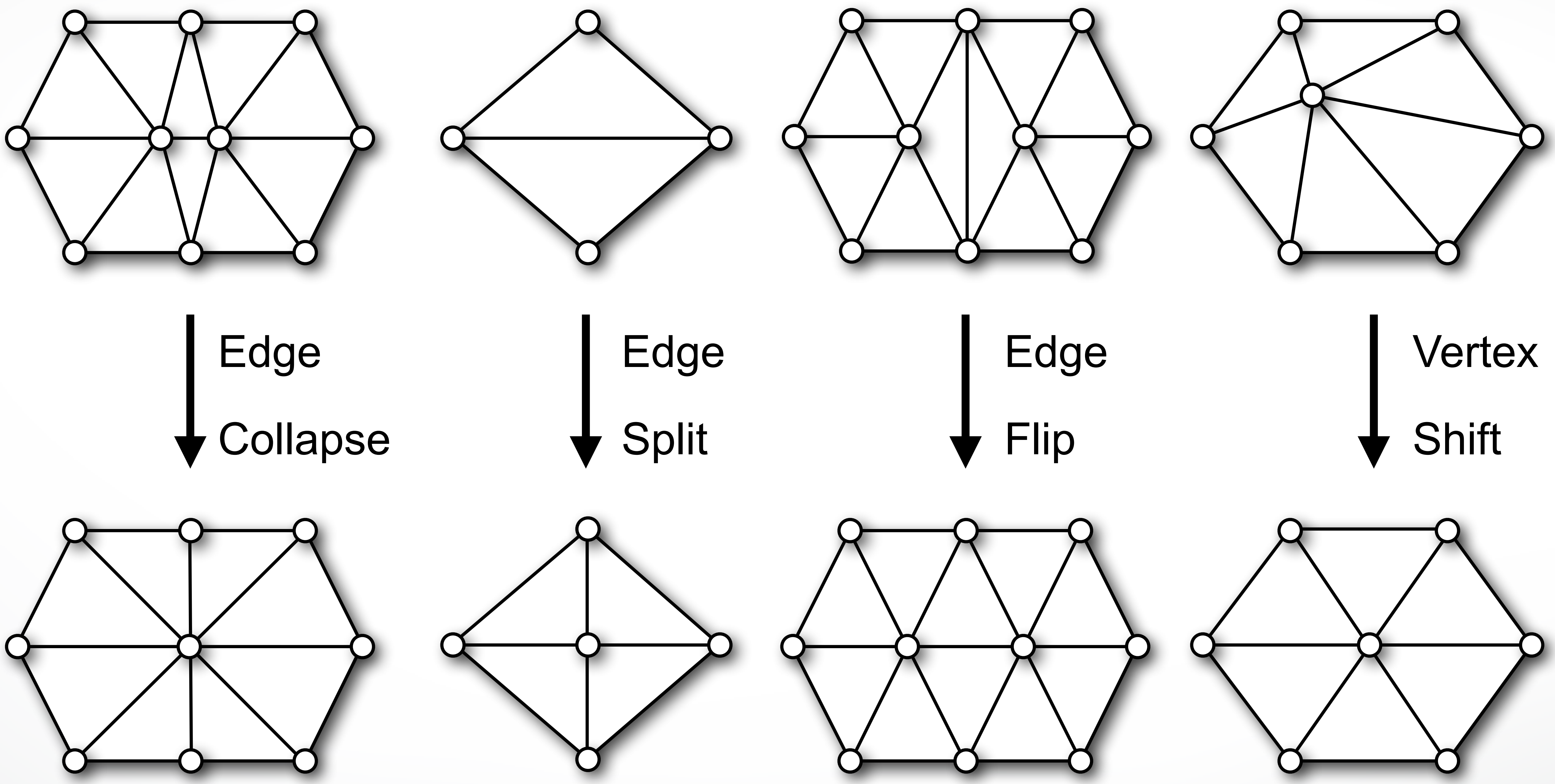
Local remeshing operators



Incremental remeshing

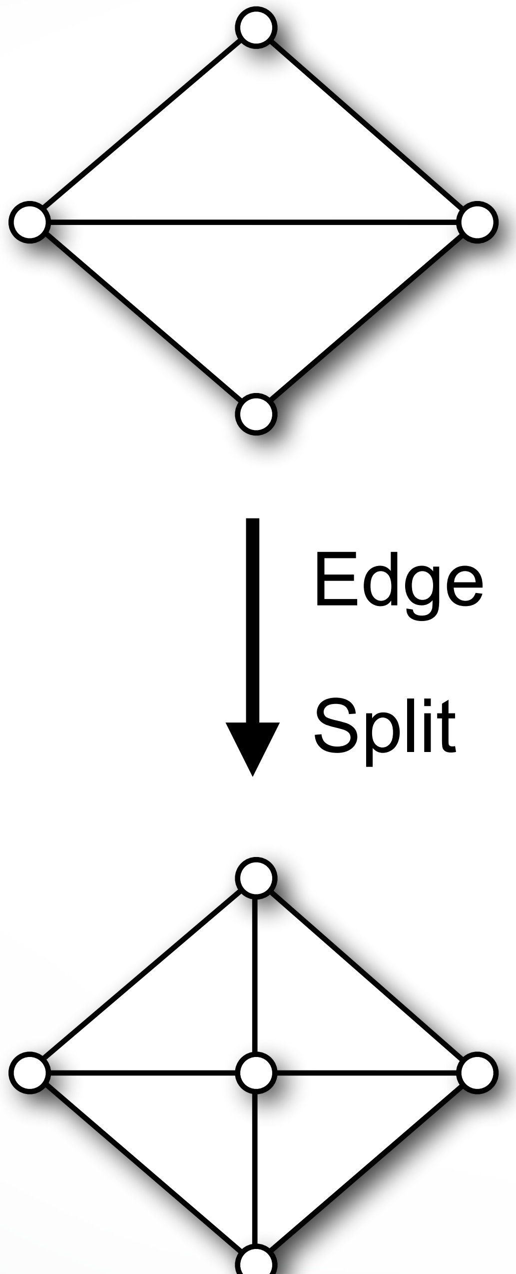
Specify target edge length L

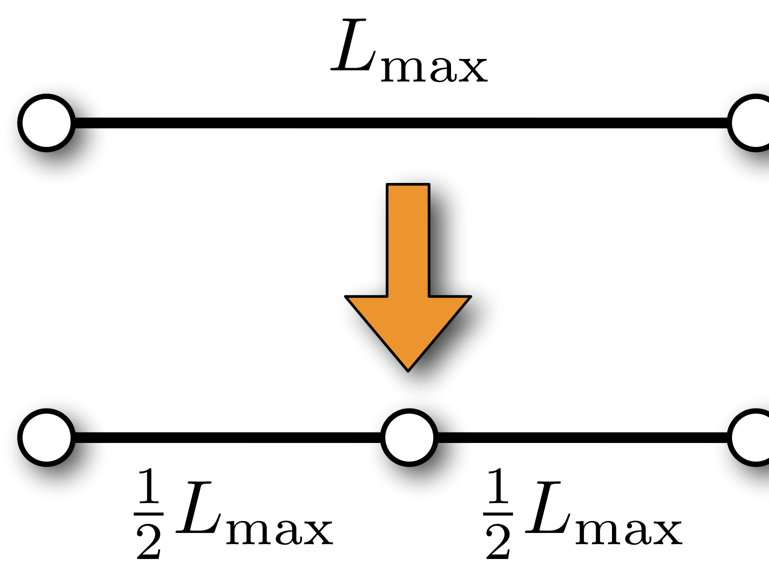
$$L_{max} = 4/3 * L; \; L_{min} = 4/5 * L;$$

Iterate:

1. Split edges longer than L_{max}
2. Collapse edges shorter than L_{min}
3. Flip edges to get closer to optimal valence
4. Vertex shift by tangential relaxation
5. Project vertices onto reference mesh

Edge split

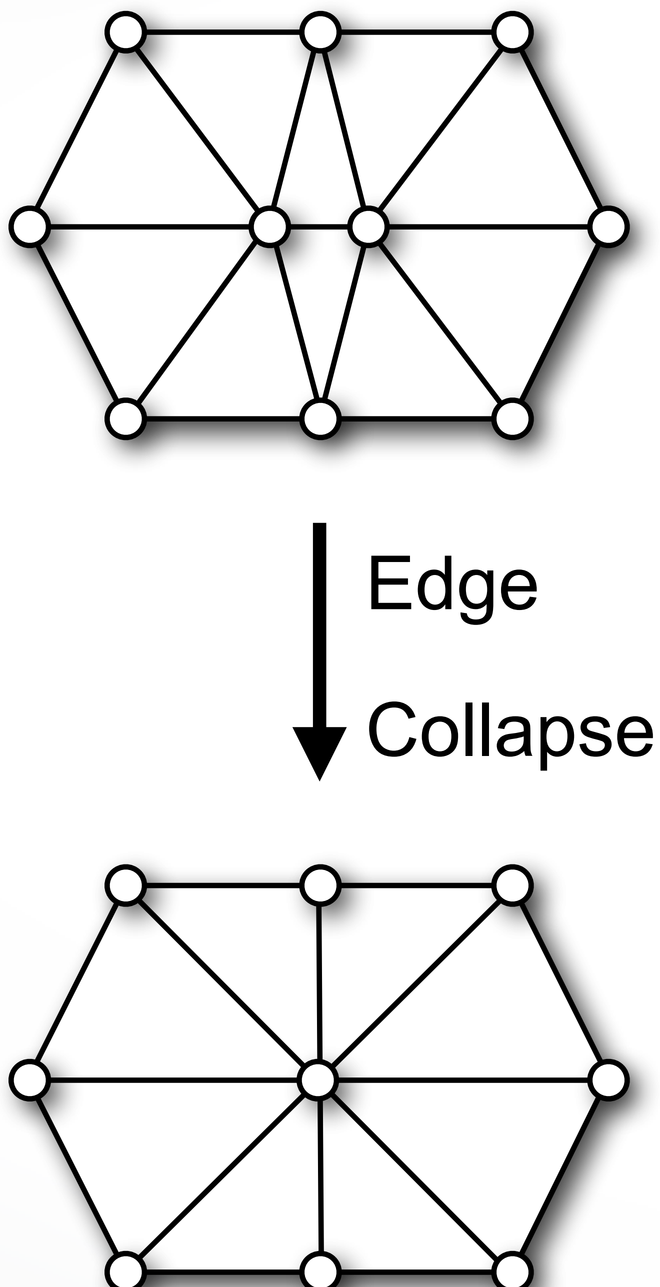


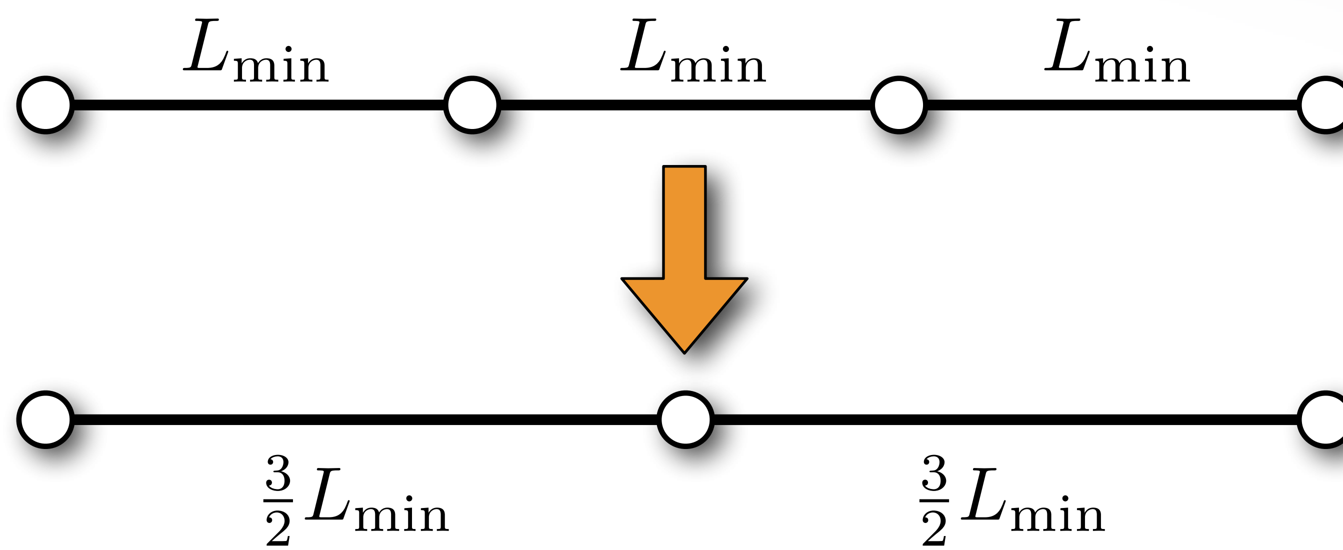


$$
\begin{aligned}
|L_{\max} - L| &= \left|\frac{1}{2}L_{\max} - L\right| \\
\Rightarrow L_{\max} &= \frac{4}{3}L
\end{aligned}
$$

Split edges longer than L_{max}

Edge collapse





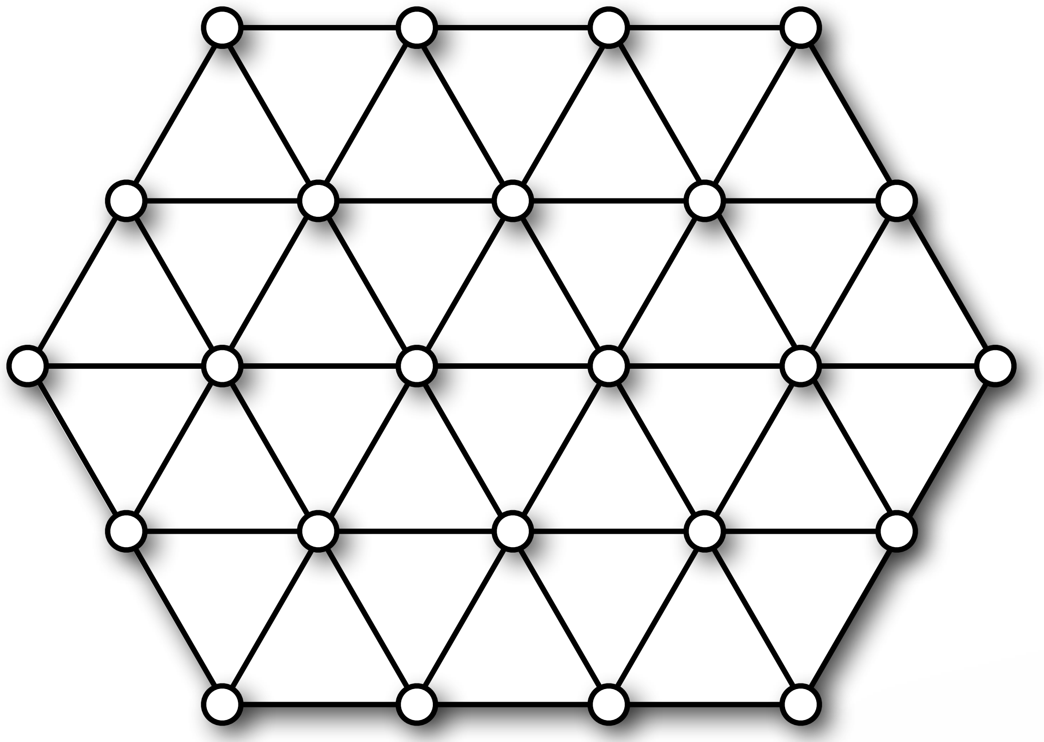
$$
\begin{aligned}
|L_{\min} - L| &= \left|\frac{3}{2}L_{\min} - L\right| \\
\Rightarrow L_{\min} &= \frac{4}{5}L
\end{aligned}
$$

Collapse edges shorter than $L_{\min}$

Edge flip

Optimal valence

- 6 for interior vertices
- 4 for boundary vertices



Edge flip

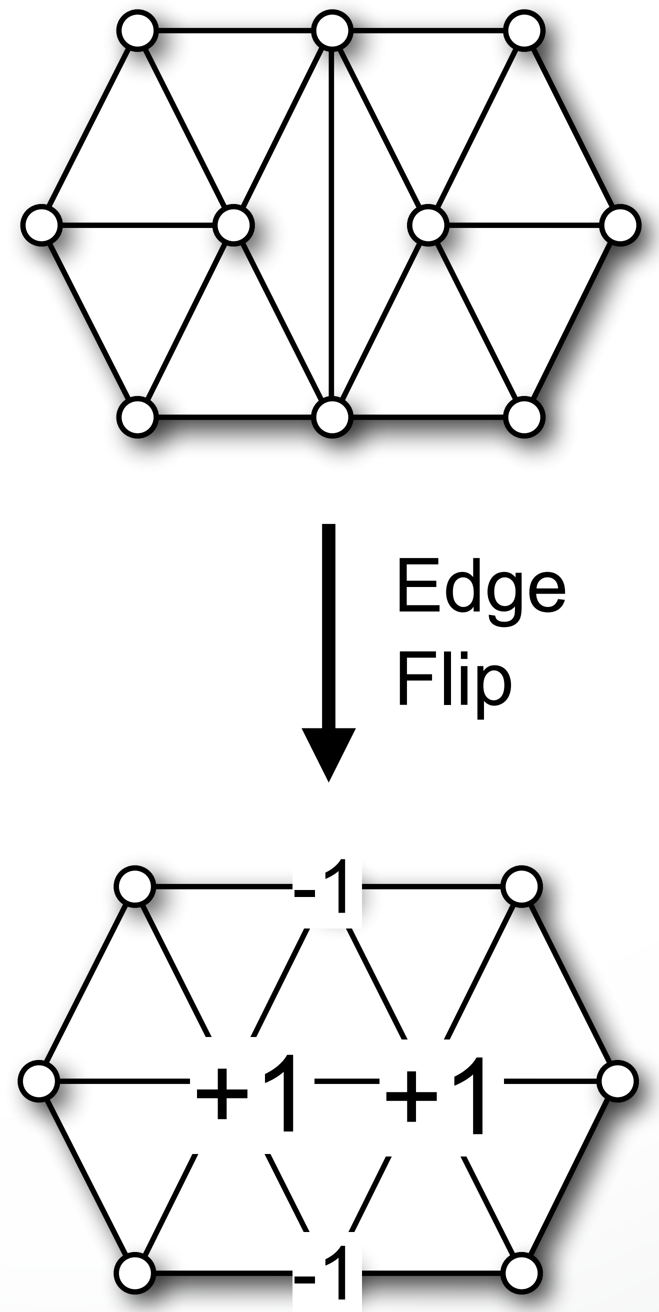
Optimal valence

- 6 for interior vertices
- 4 for boundary vertices

Improve valences

- Minimize valence excess

$$\sum_{i=1}^{4} (\text{valence}(v_i) - \text{opt\_valence}(v_i))^2$$

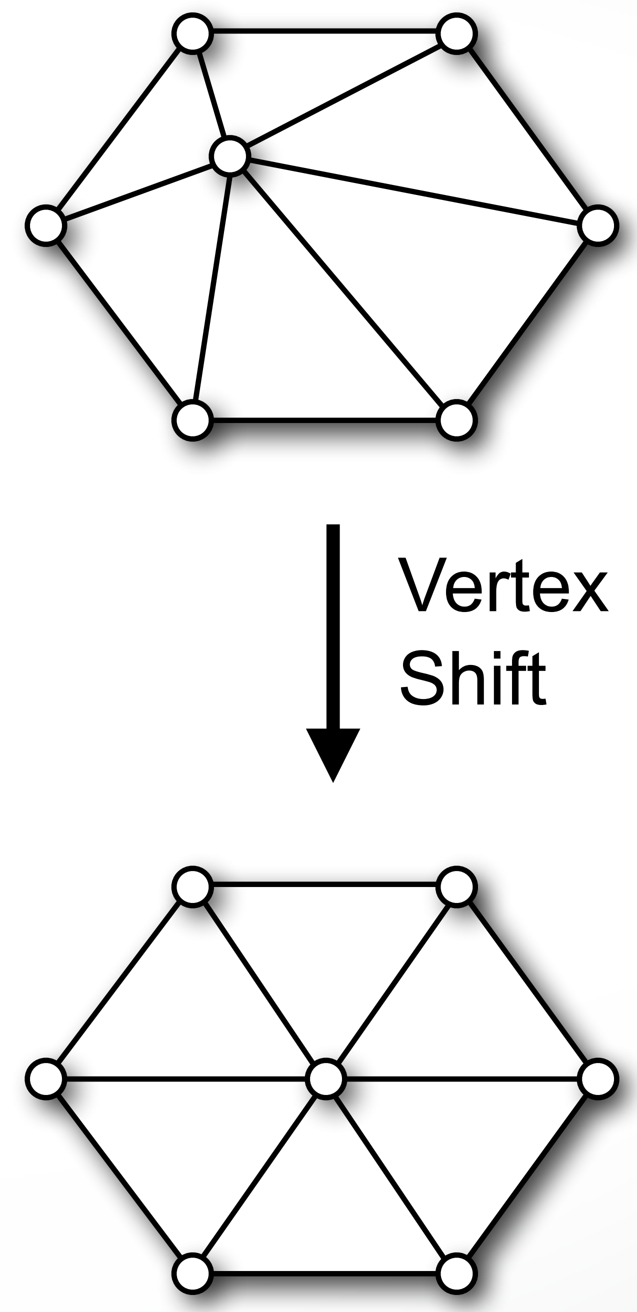


Vertex shift

Local “spring” relaxation

- Uniform Laplacian smoothing
- Barycenter of one-ring neighborhood

$$\mathbf{c}_i = \frac{1}{\text{valence}(v_i)} \sum_{j \in N(v_i)} \mathbf{p}_j$$



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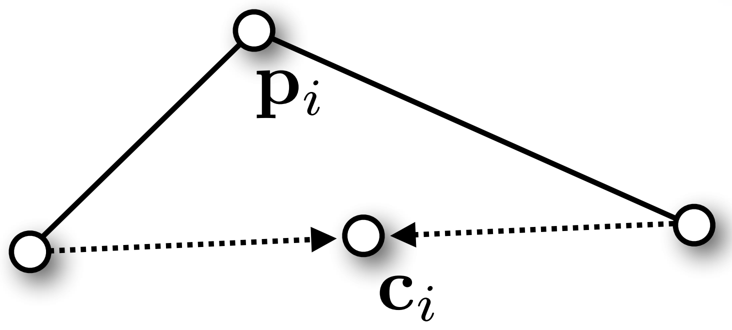

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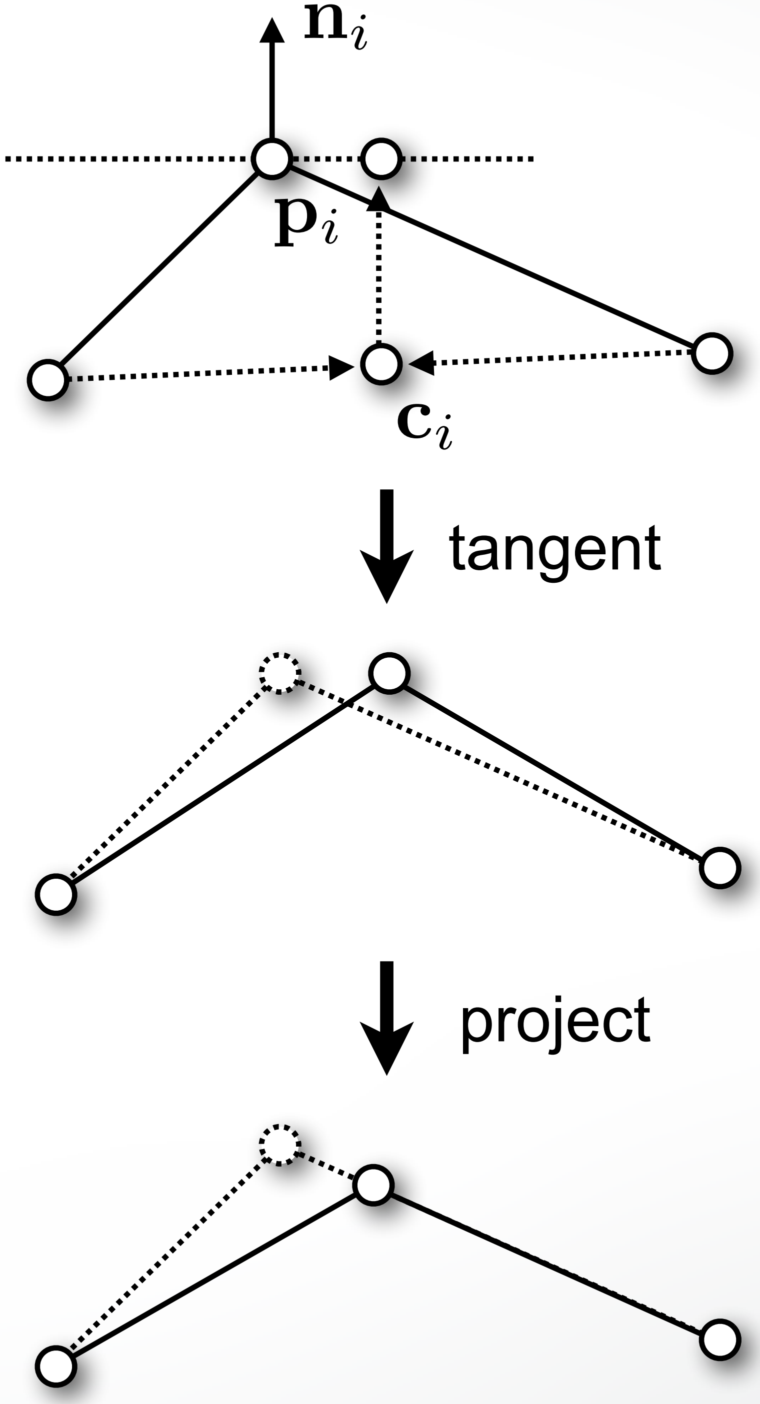
Keep vertex (approx.) on surface

- Restrict movement to tangent plane

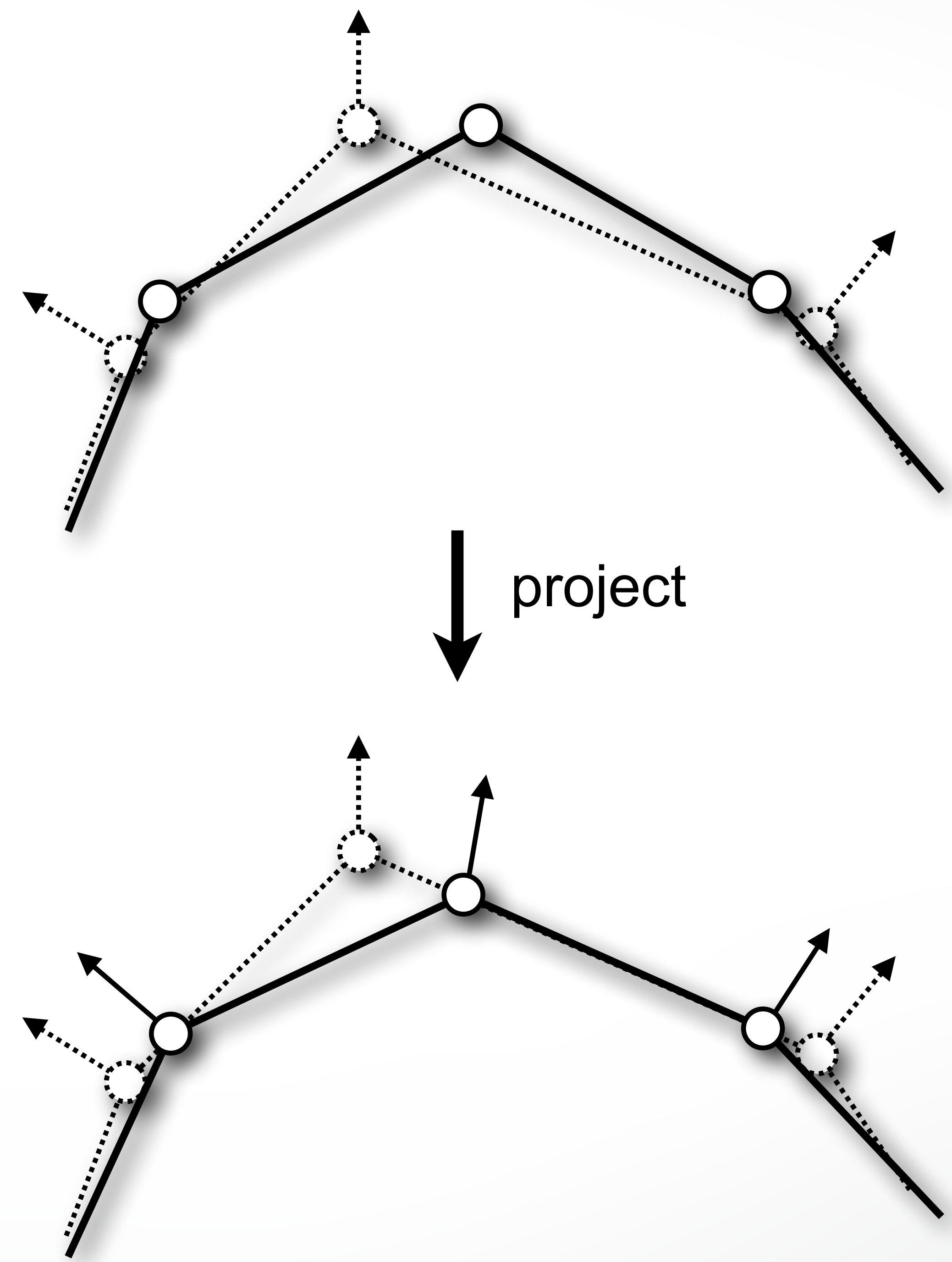
$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda \left(\mathbf{I} - \mathbf{n}_i \mathbf{n}_i^T \right) (\mathbf{c}_i - \mathbf{p}_i)$$



Vertex projection

Onto original reference mesh

- Find closet triangle
- Use BSP to accelerate → $O(\log n)$
- Barycentric interpolation to compute position & normal



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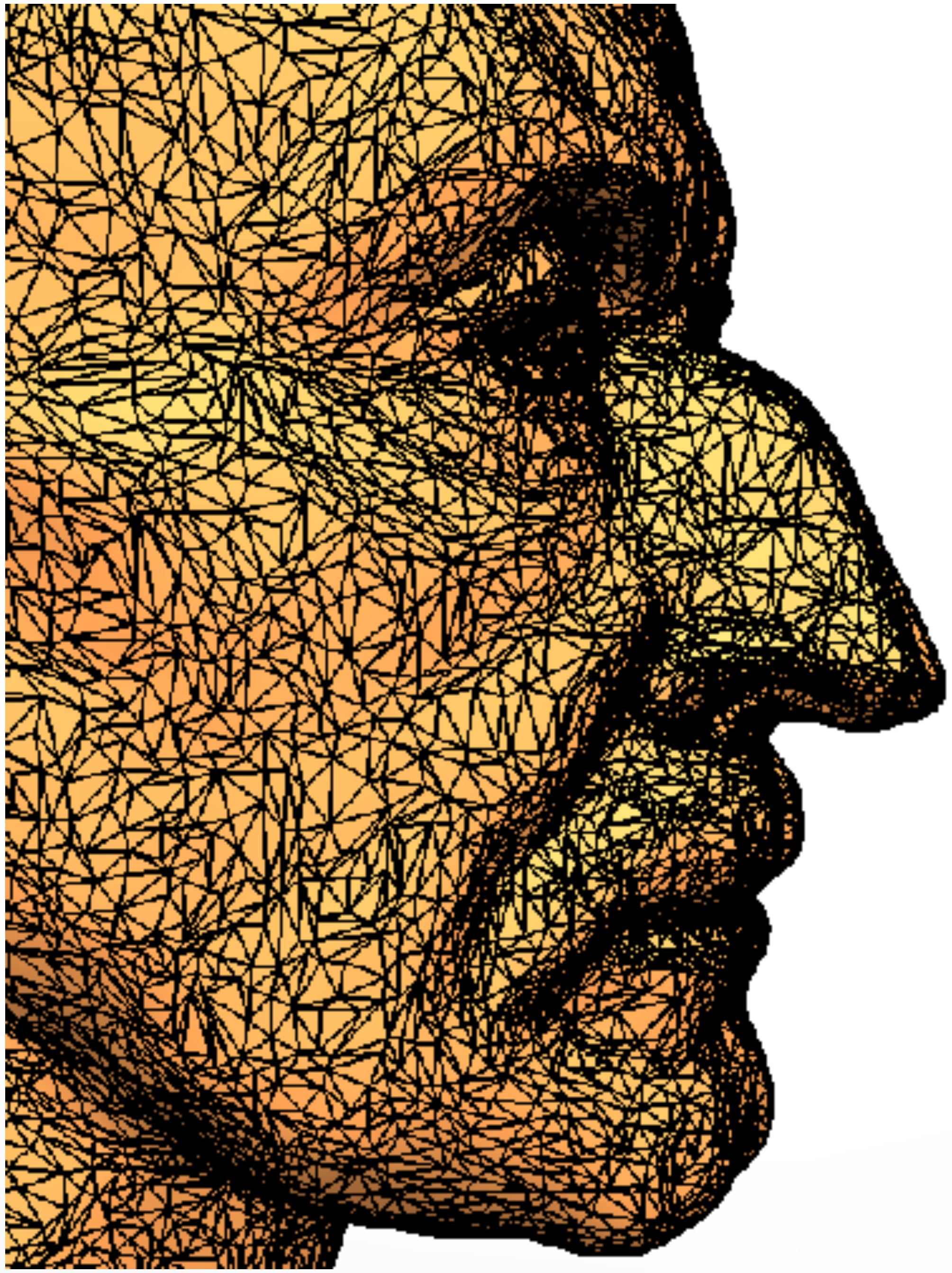
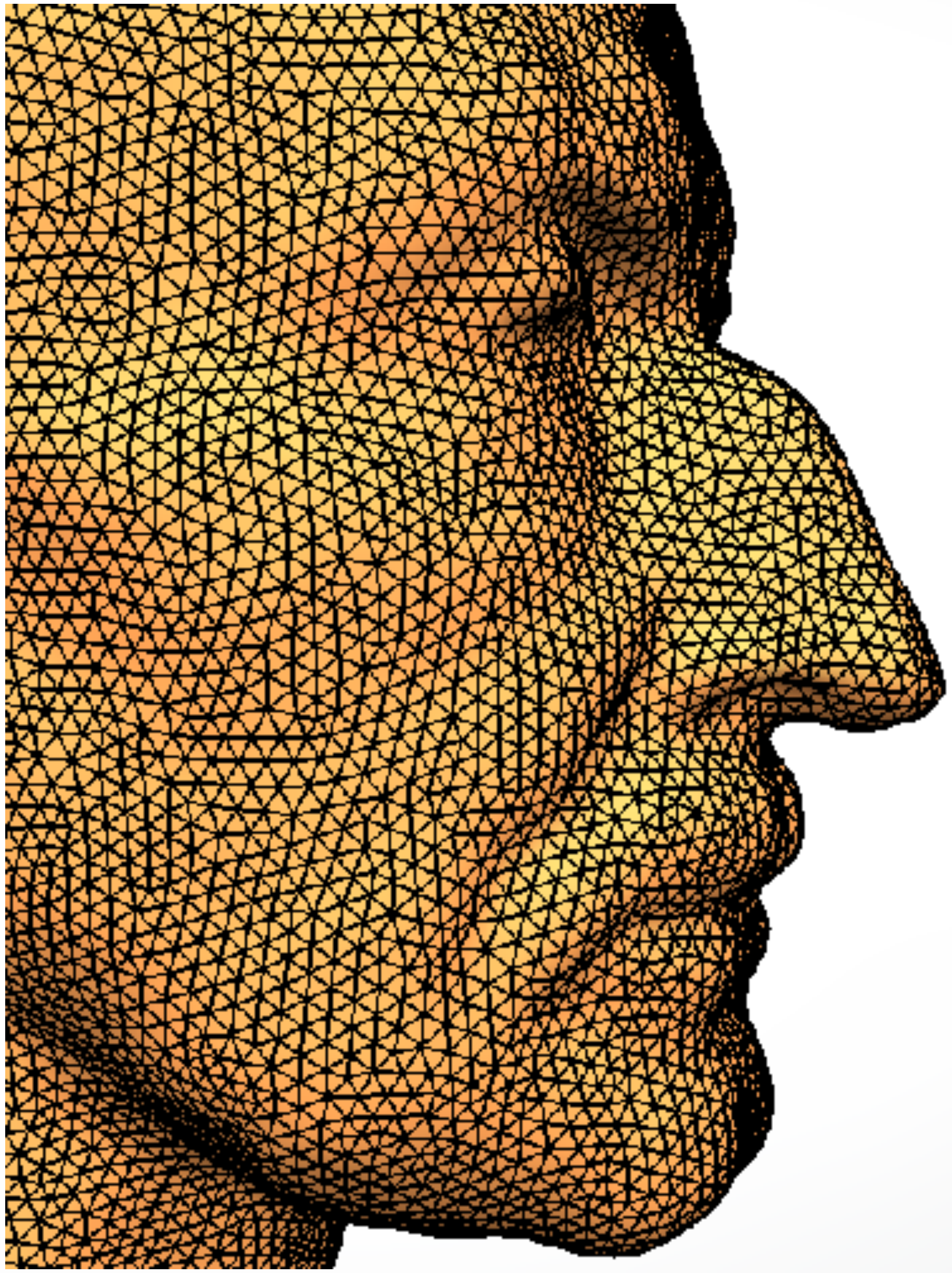
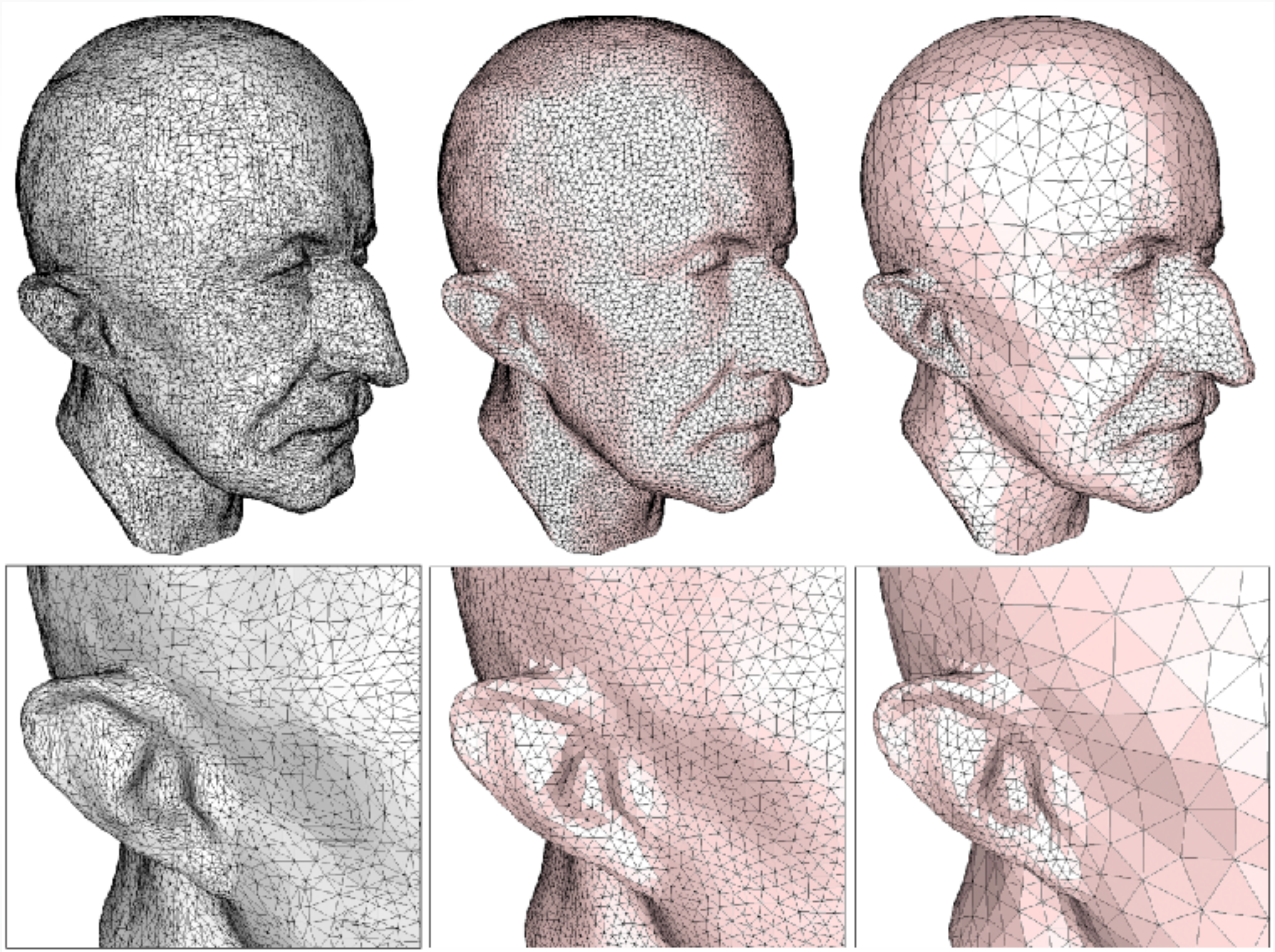
Remeshing result



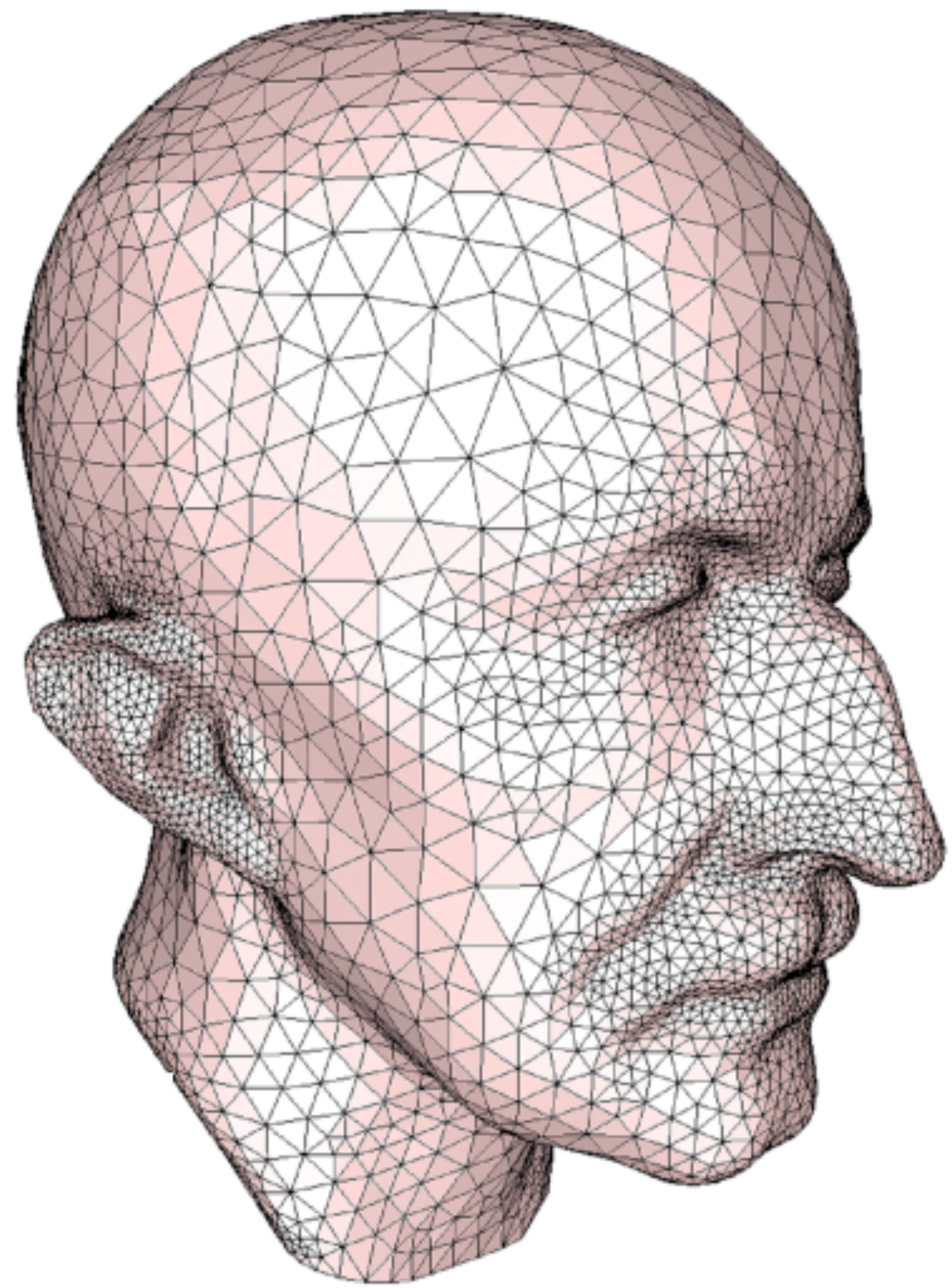


Adaptive remeshing

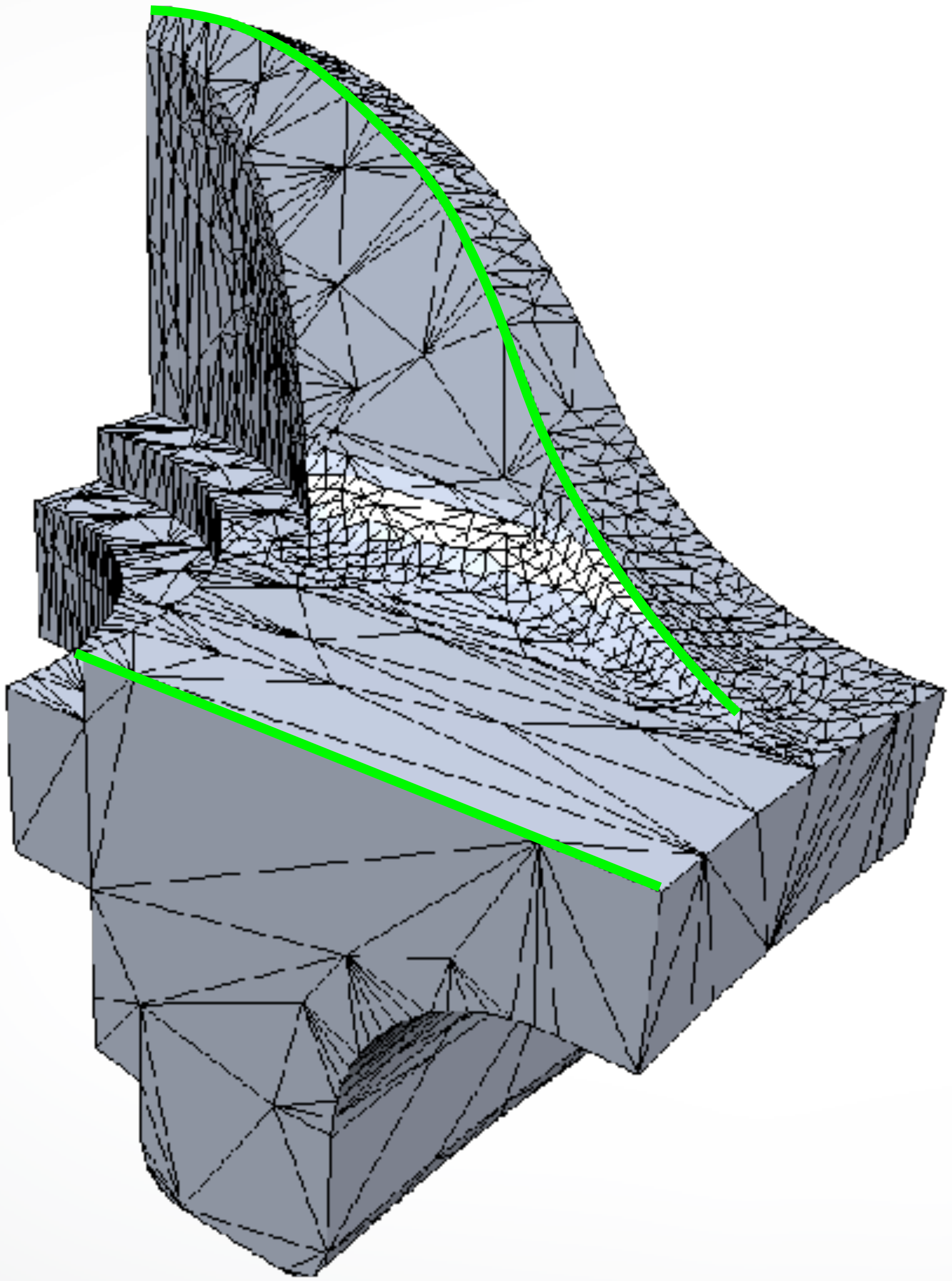


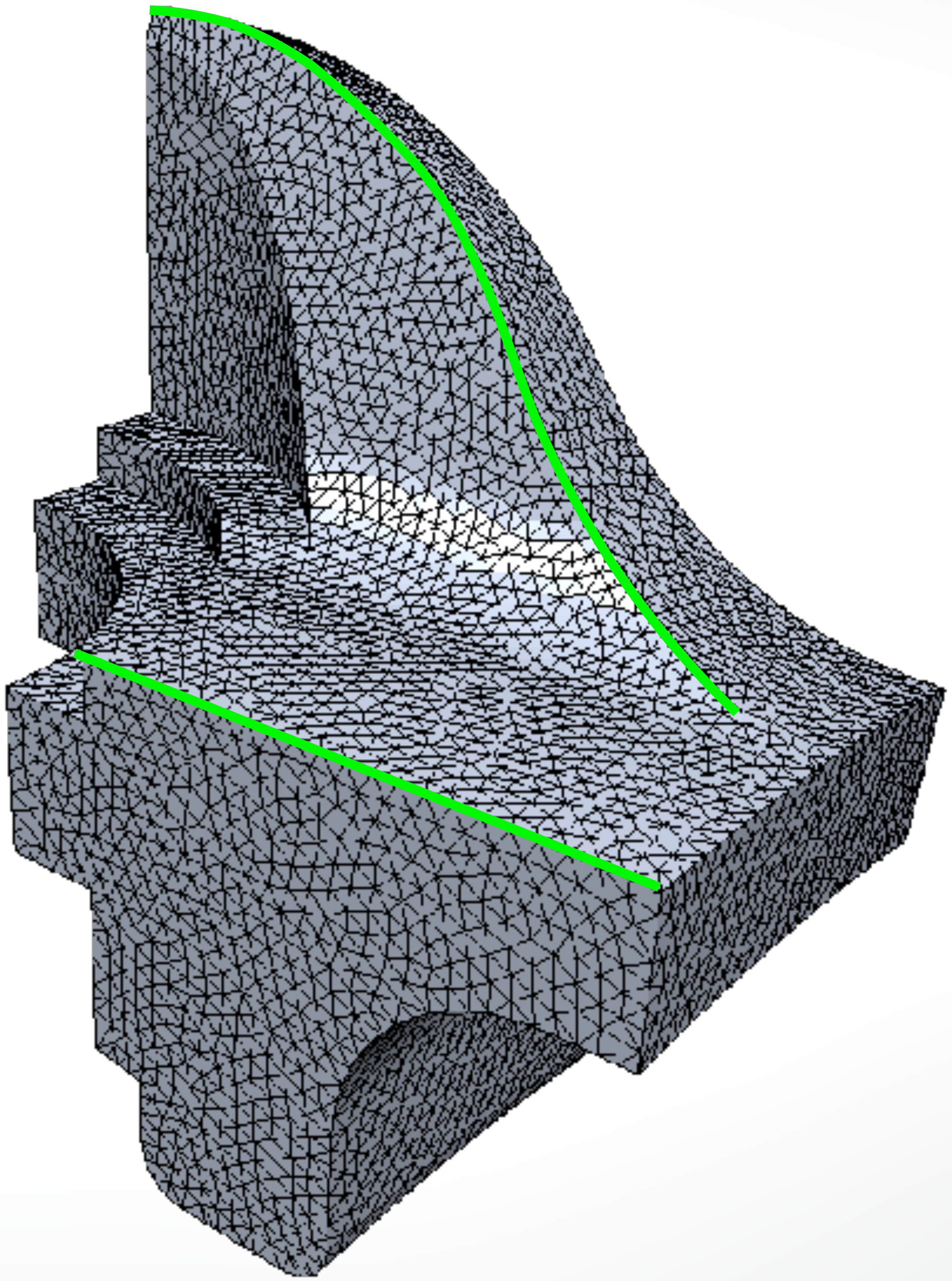
Adaptive remeshing

- **Compute maximum principle curvature on reference mesh**
- **Determine local target edge length from max-curvature**
- **Adjust edge split / collapse criteria accordingly**



Feature preservation





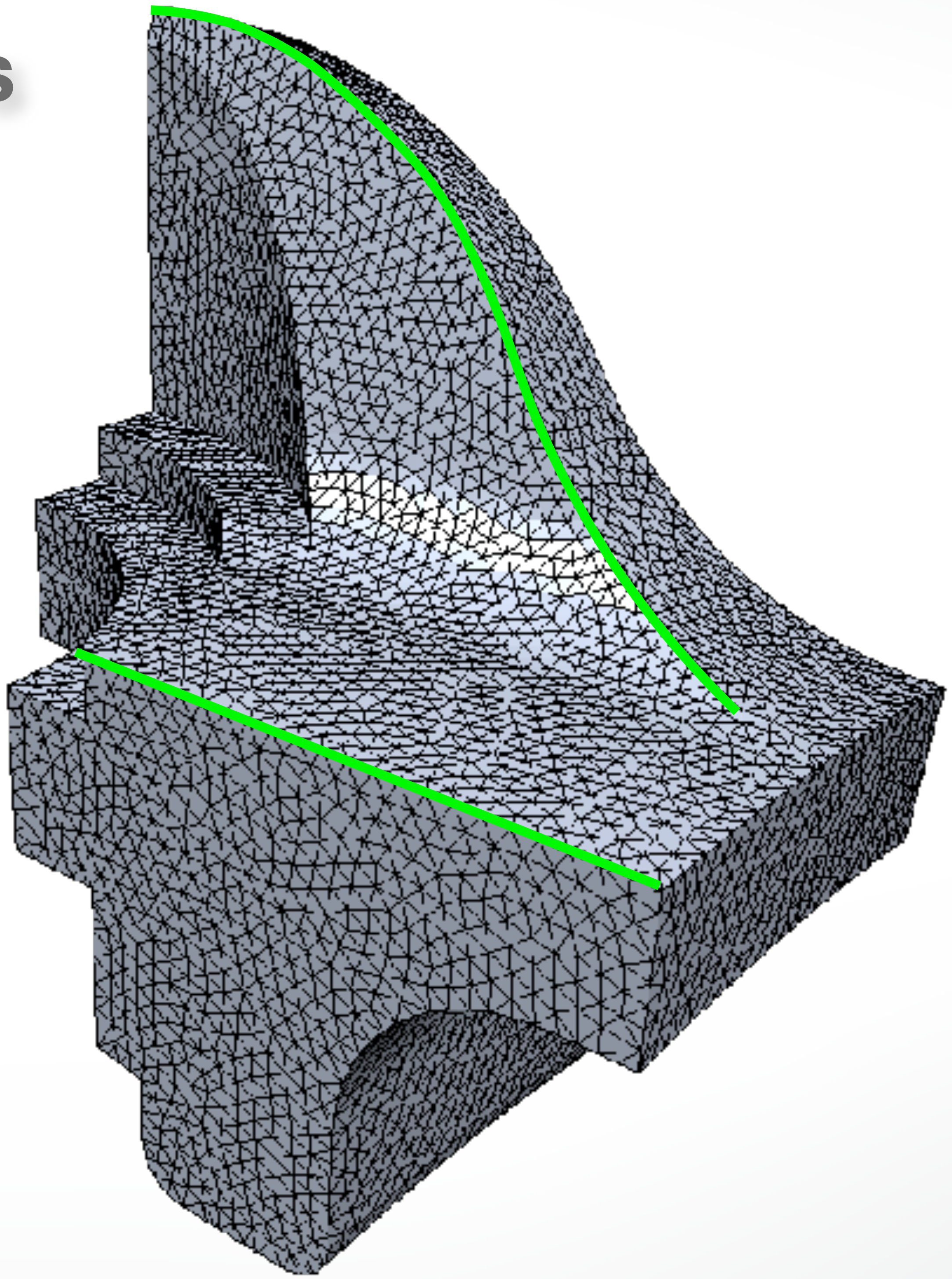
Feature preservation

Define feature edges / vertices

- Large dihedral angles
- Material boundaries

Adjust local operators

- Do not touch corner vertices
- Do not flip feature edges
- Collapse along features
- Univariate smoothing
- Project to feature curves



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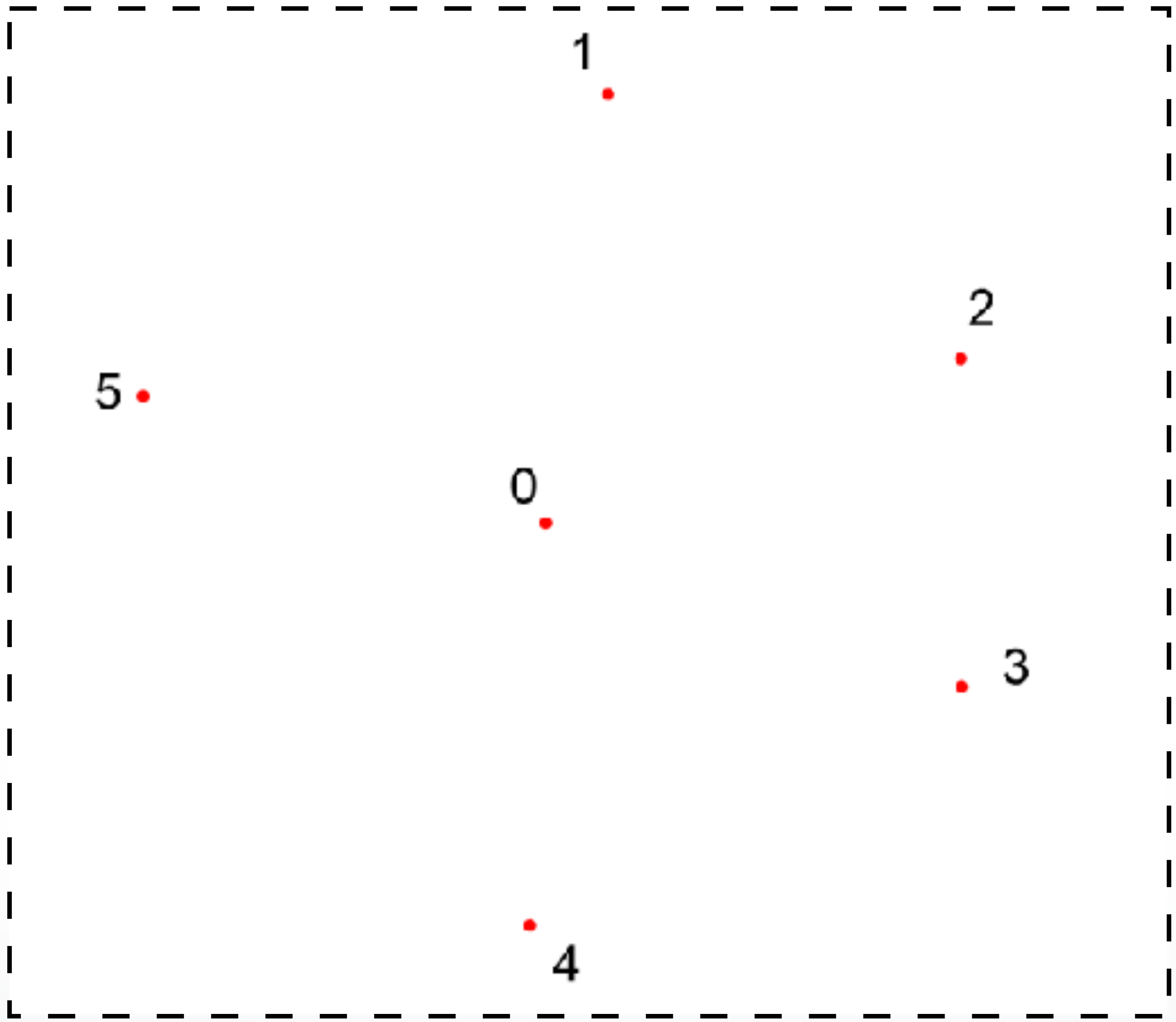
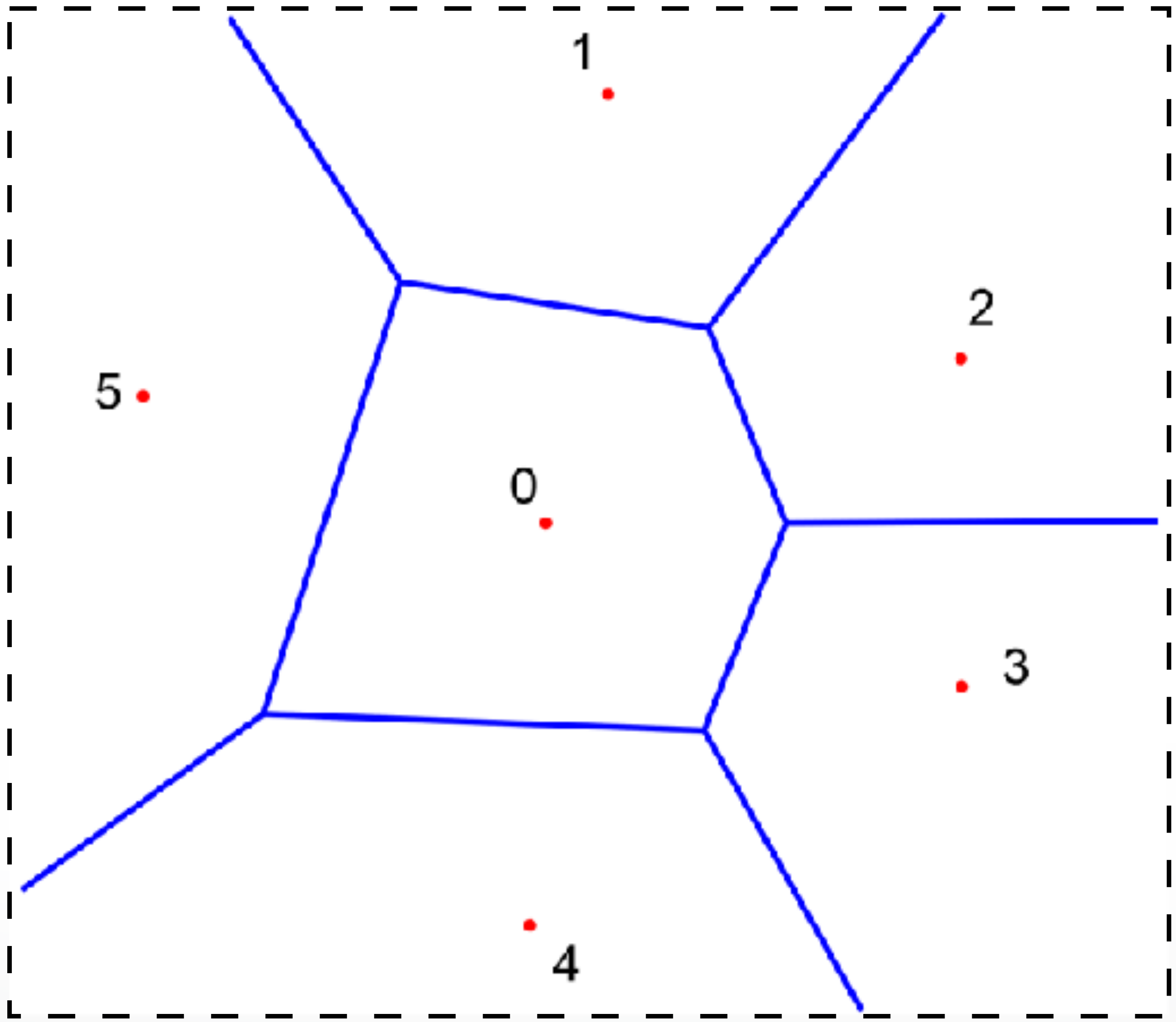
Voronoi Diagram



Voronoi Diagram

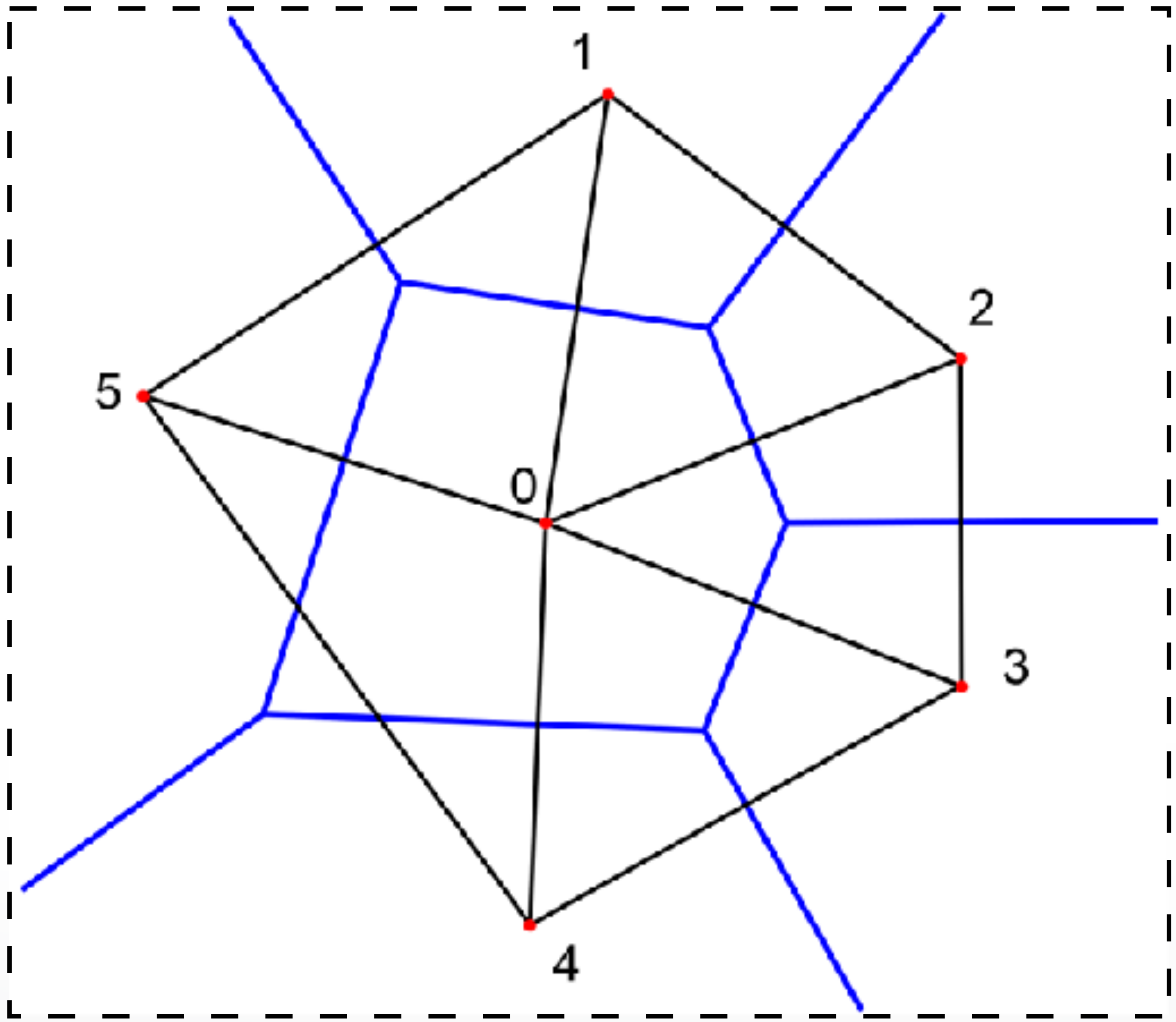
Divide space into a number of cells



Voronoi Diagram

Divide space into a number of cells

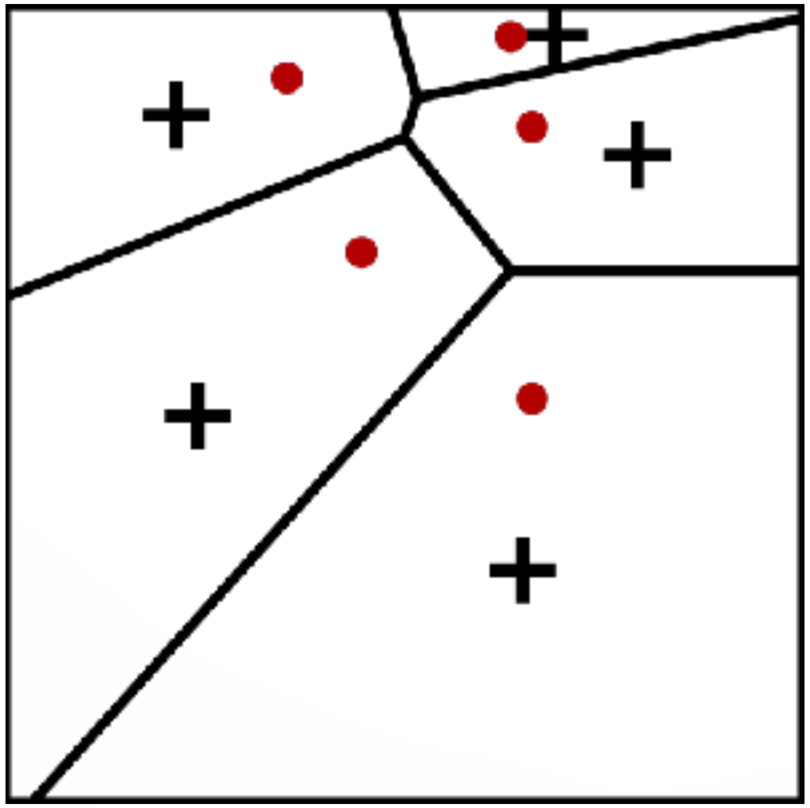
Dual graph: Delaunay triangulation



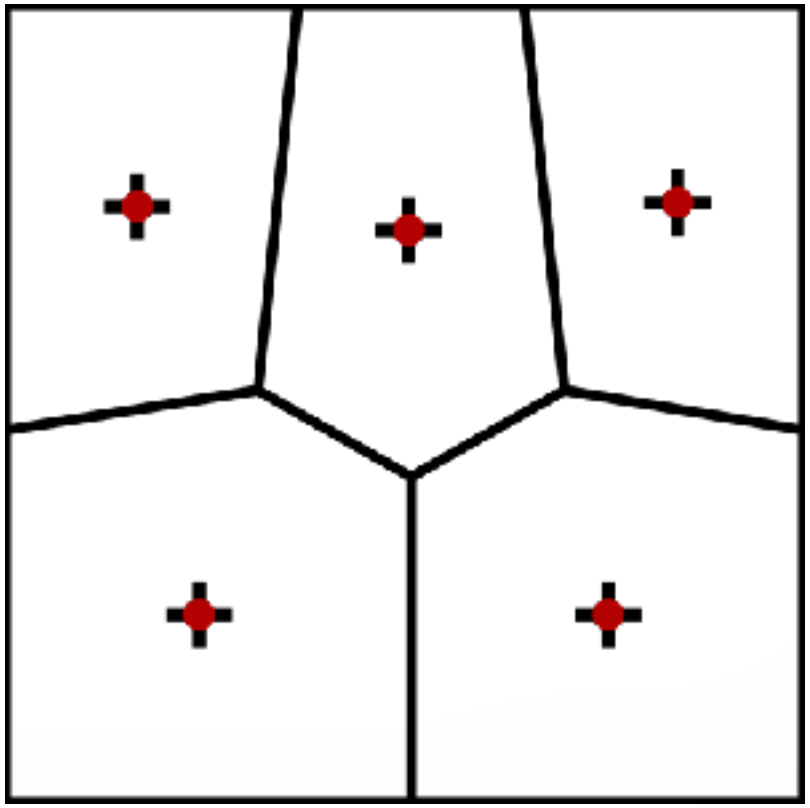
Centroidal Voronoi Diagram

For each cell

The generating point ● = mass of center ✚



non CVD



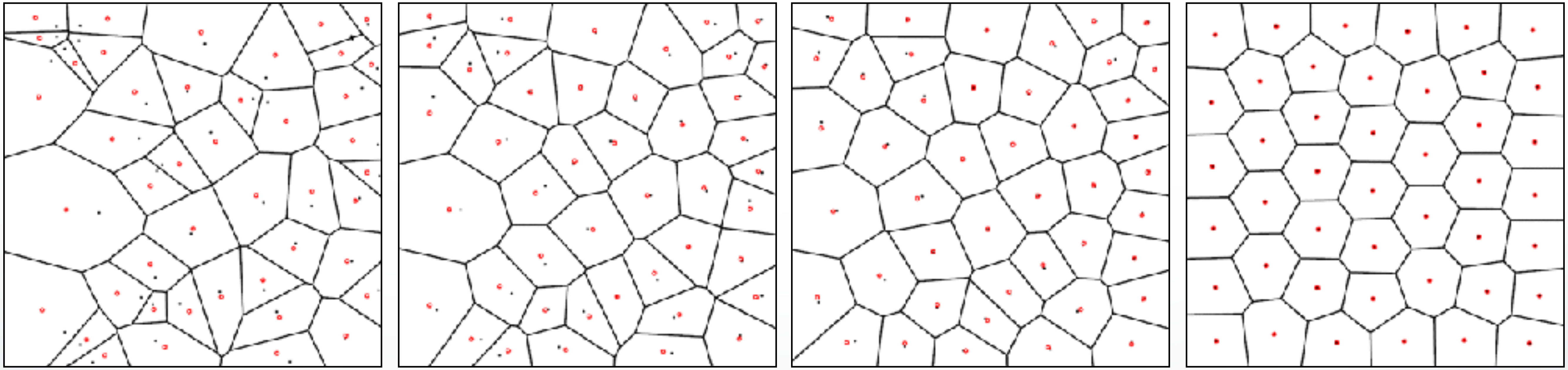
CVD

Centroidal Voronoi Diagram

Compute CVD by Lloyd relaxation

1. Compute Voronoi diagram of given points $\mathbf{p}_i$
2. Move points $\mathbf{p}_i$ to centroids $\mathbf{c}_i$ of their Voronoi cells V_i
3. Repeat steps 1 and 2 until satisfactory convergence

$$\mathbf{p}_i \;\leftarrow\; \mathbf{c}_i = \frac{\int_{V_i} \mathbf{x} \cdot \rho(\mathbf{x})\ \mathrm{d}\mathbf{x}}{\int_{V_i} \rho(\mathbf{x})\ \mathrm{d}\mathbf{x}}$$



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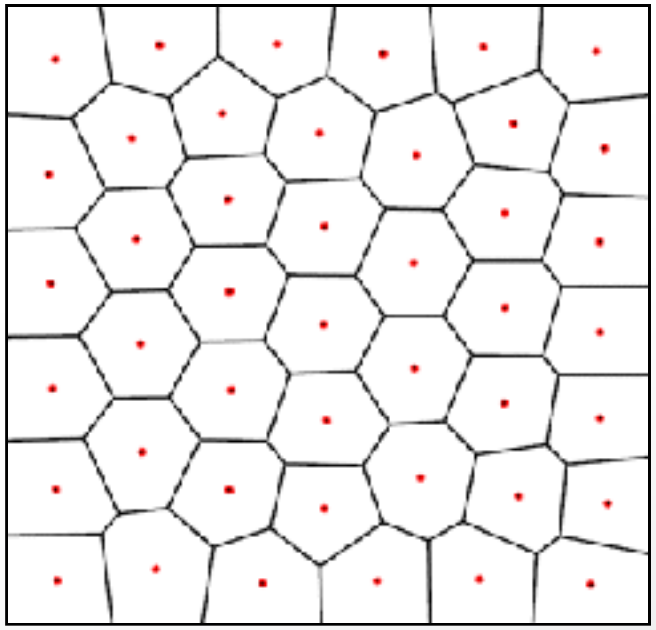
CVD maximizes compactness

- Minimize the energy:

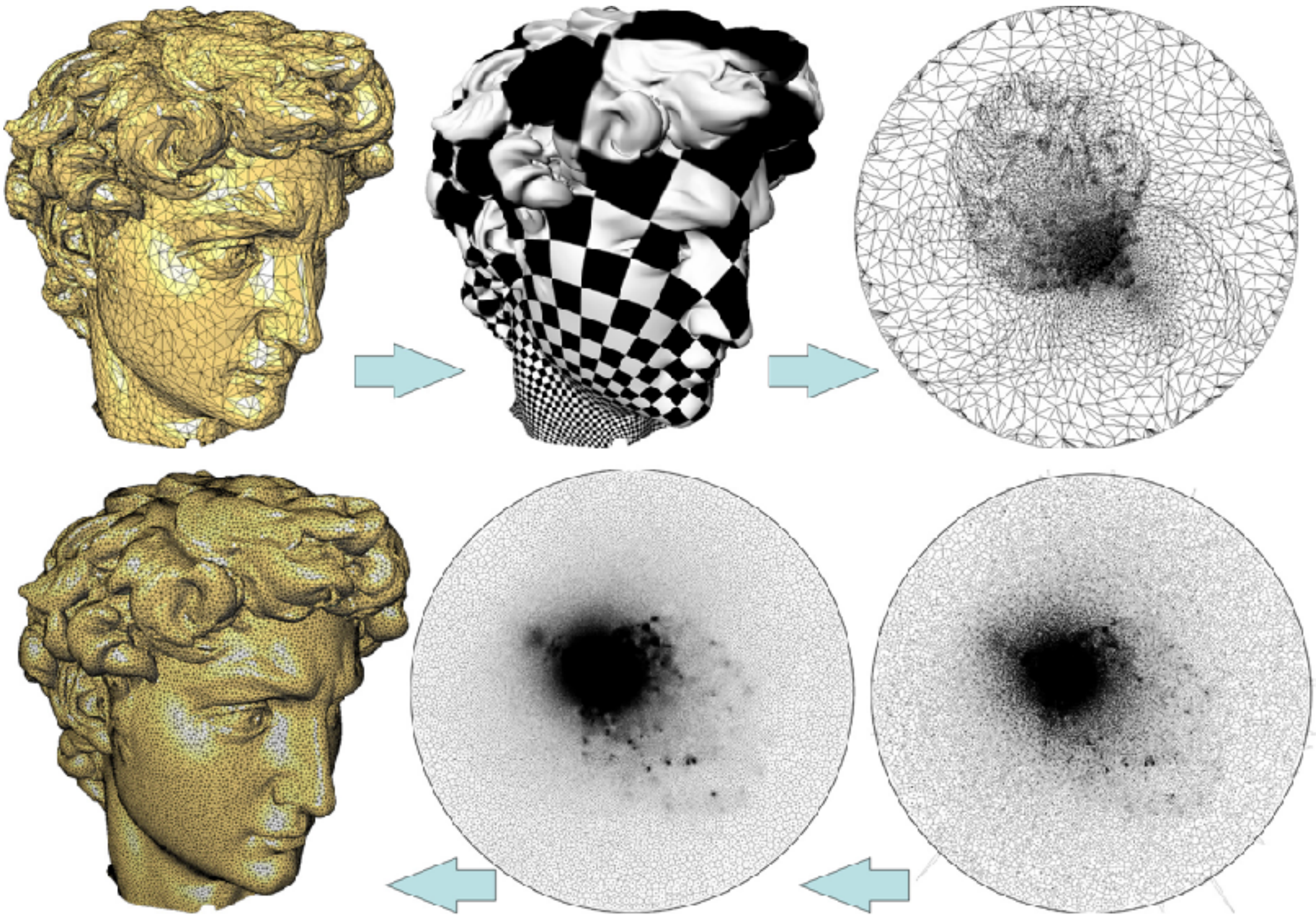
$$\sum_i \int_{V_i} \rho(\mathbf{x}) \, \|\mathbf{x} - \mathbf{p}_i\|^2 \; \mathrm{d}\mathbf{x} \;\rightarrow\; \min$$



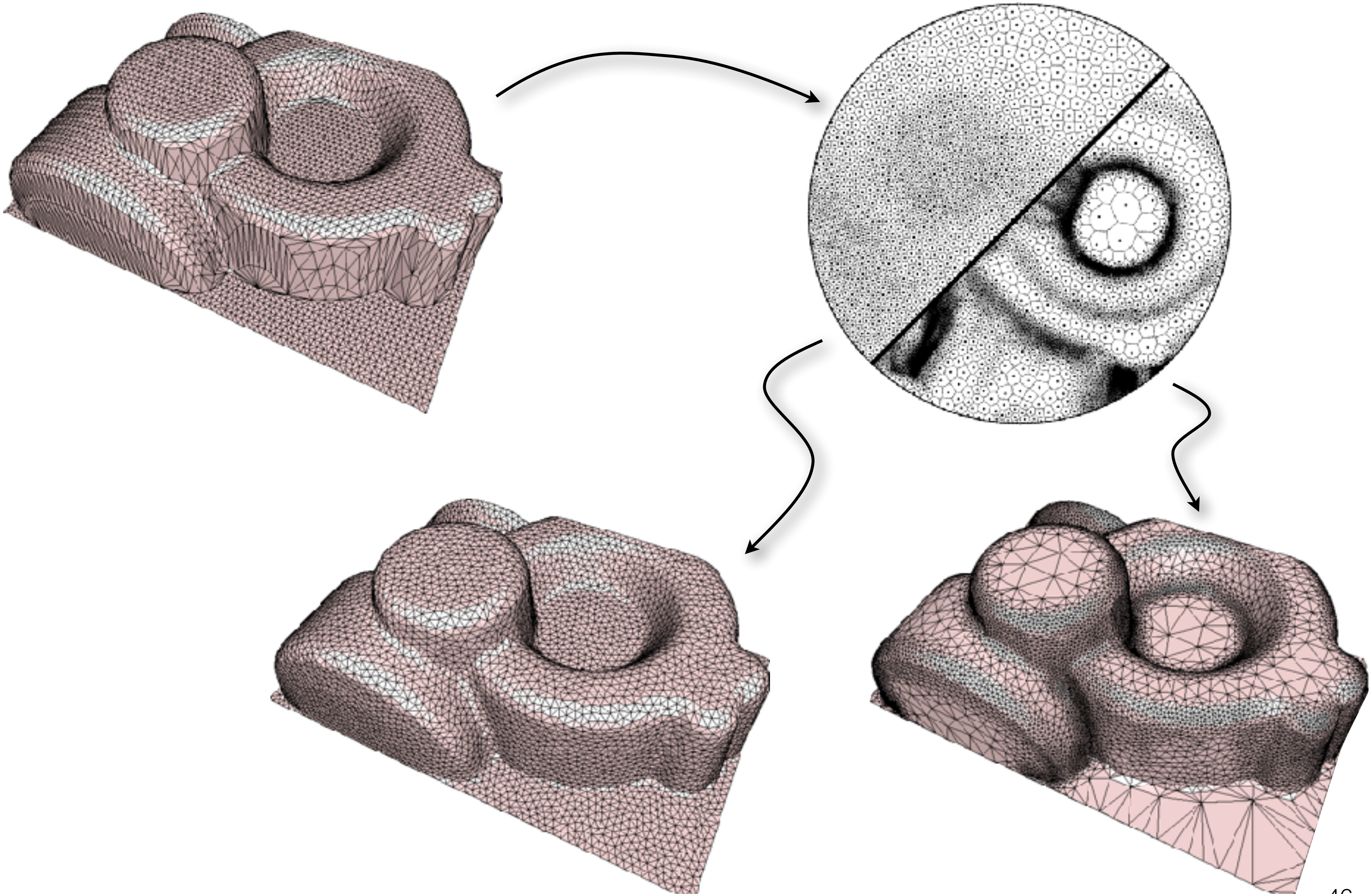
Variational remeshing

1. **Conformal parameterization of input mesh**
2. **Compute local density**
3. **Perform in 2D parameter space**
   A. Randomly sample according to local density
   B. Compute CVD by Lloyd relaxation
4. **Lift 2D Delaunay triangulation to 3D**

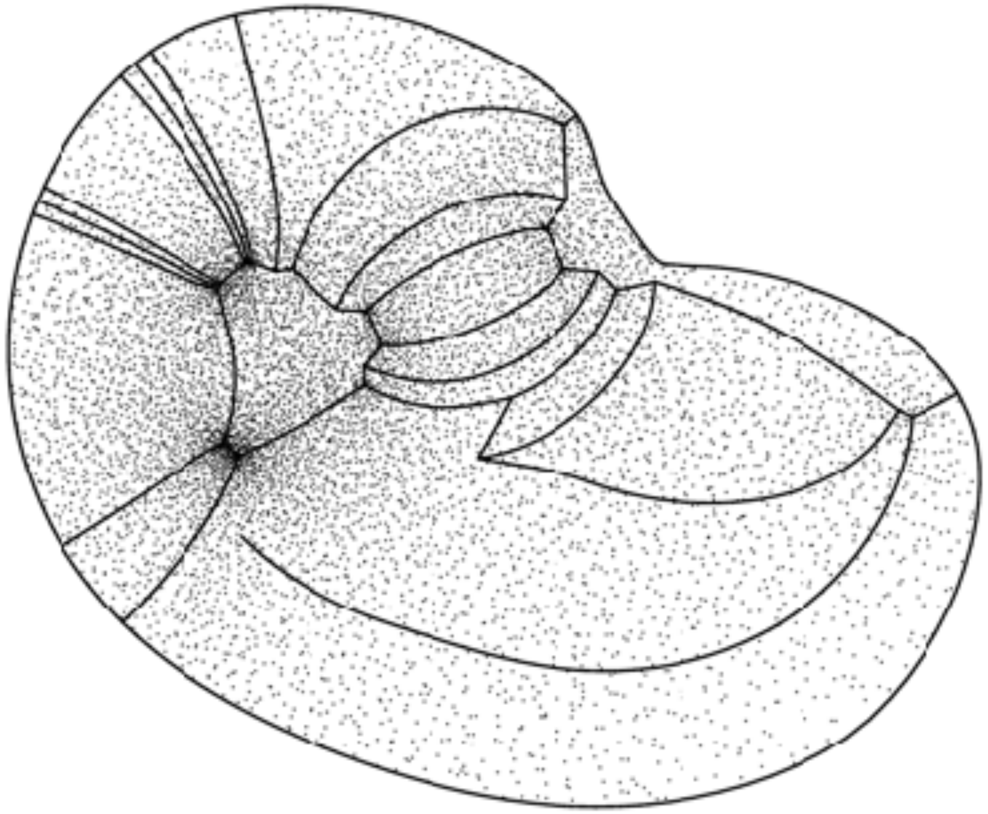
Variational remeshing

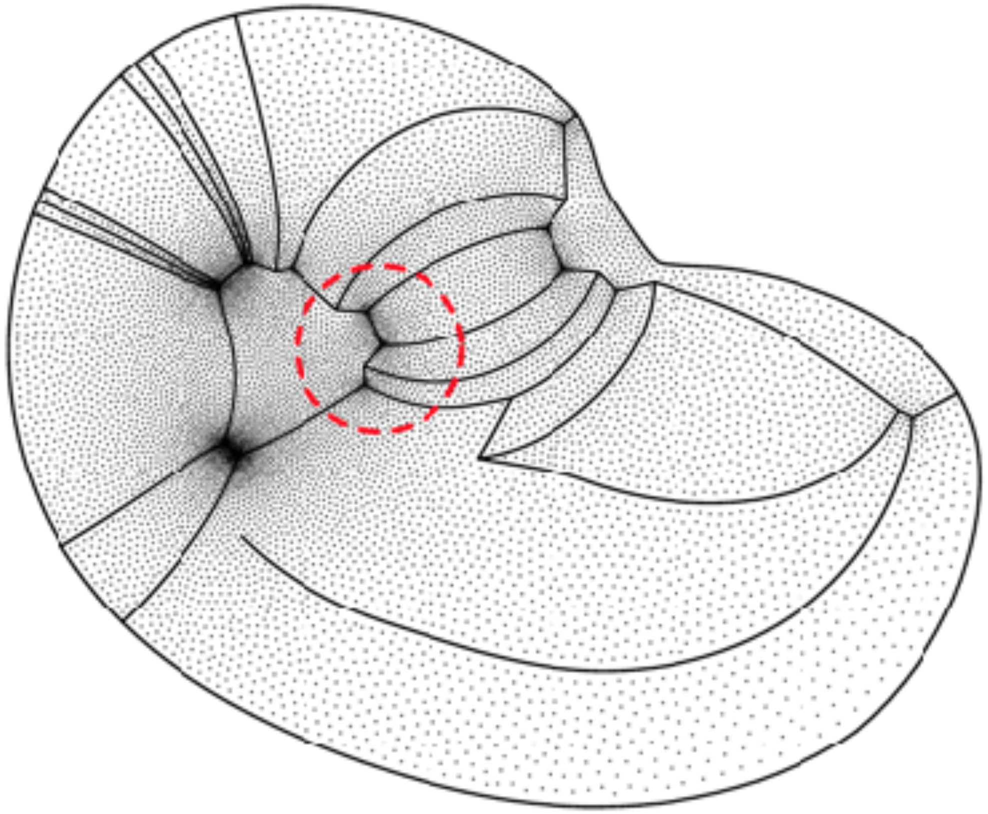


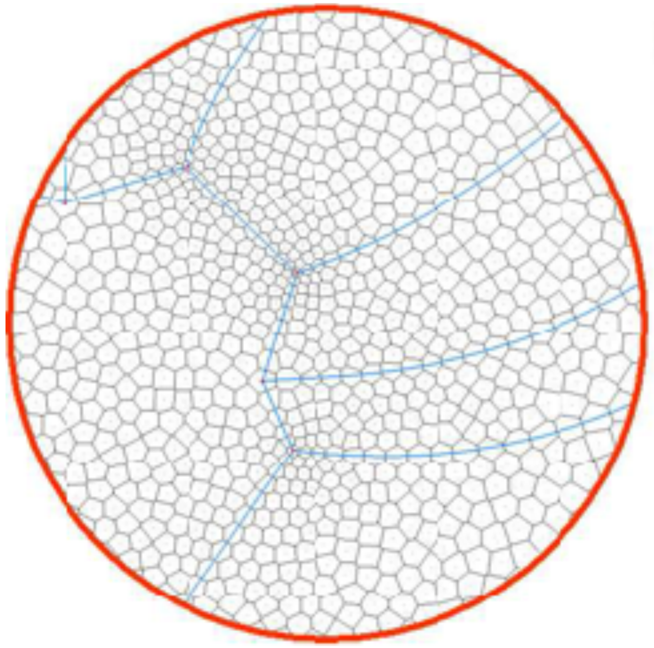
Adaptive remeshing

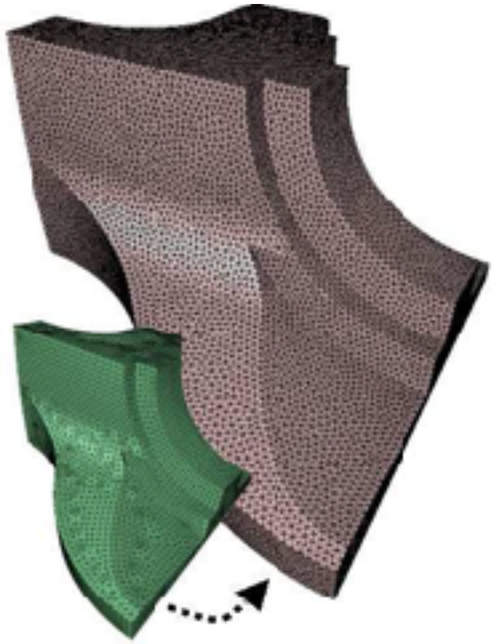


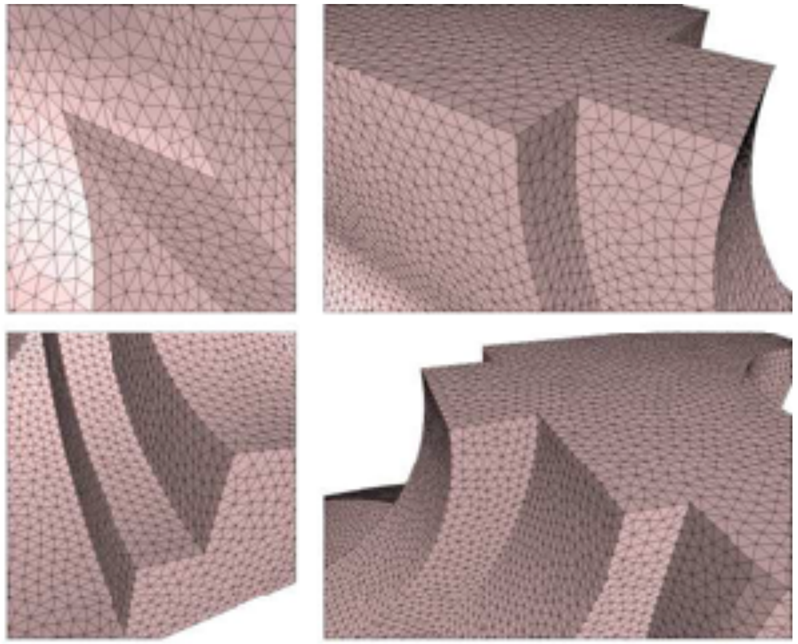
Feature preservation











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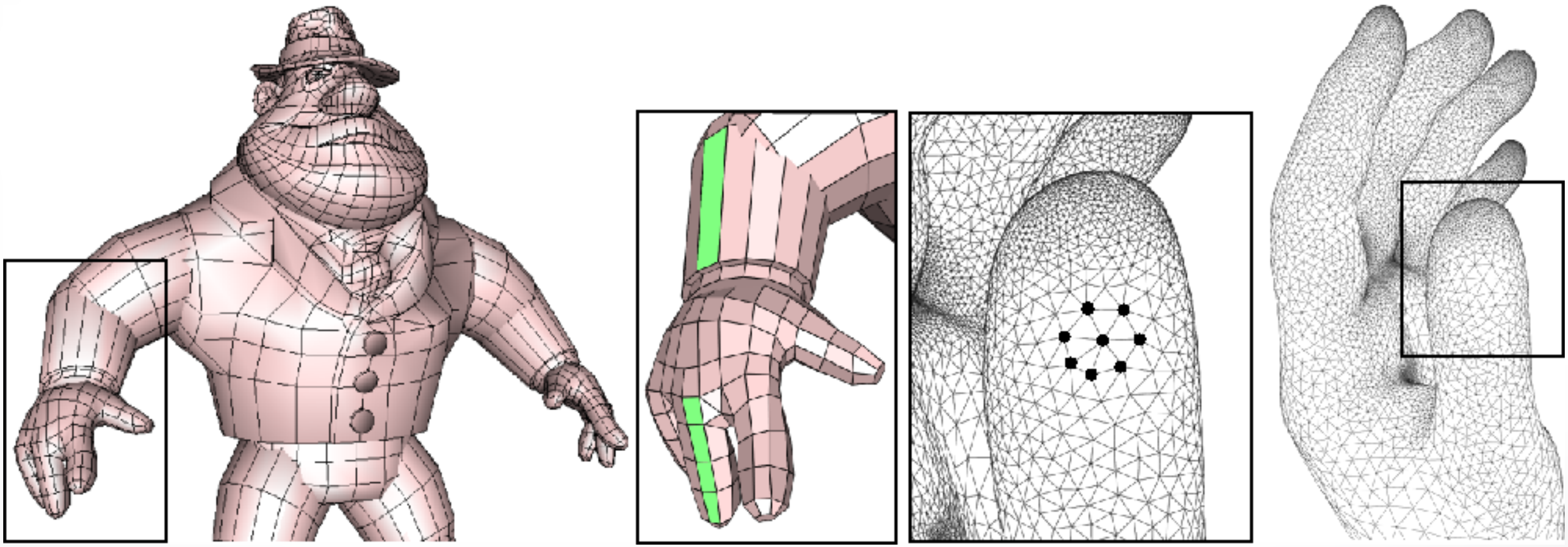
  - Isotropic remeshing
  - **Anisotropic remeshing**

Anisotropic remeshing

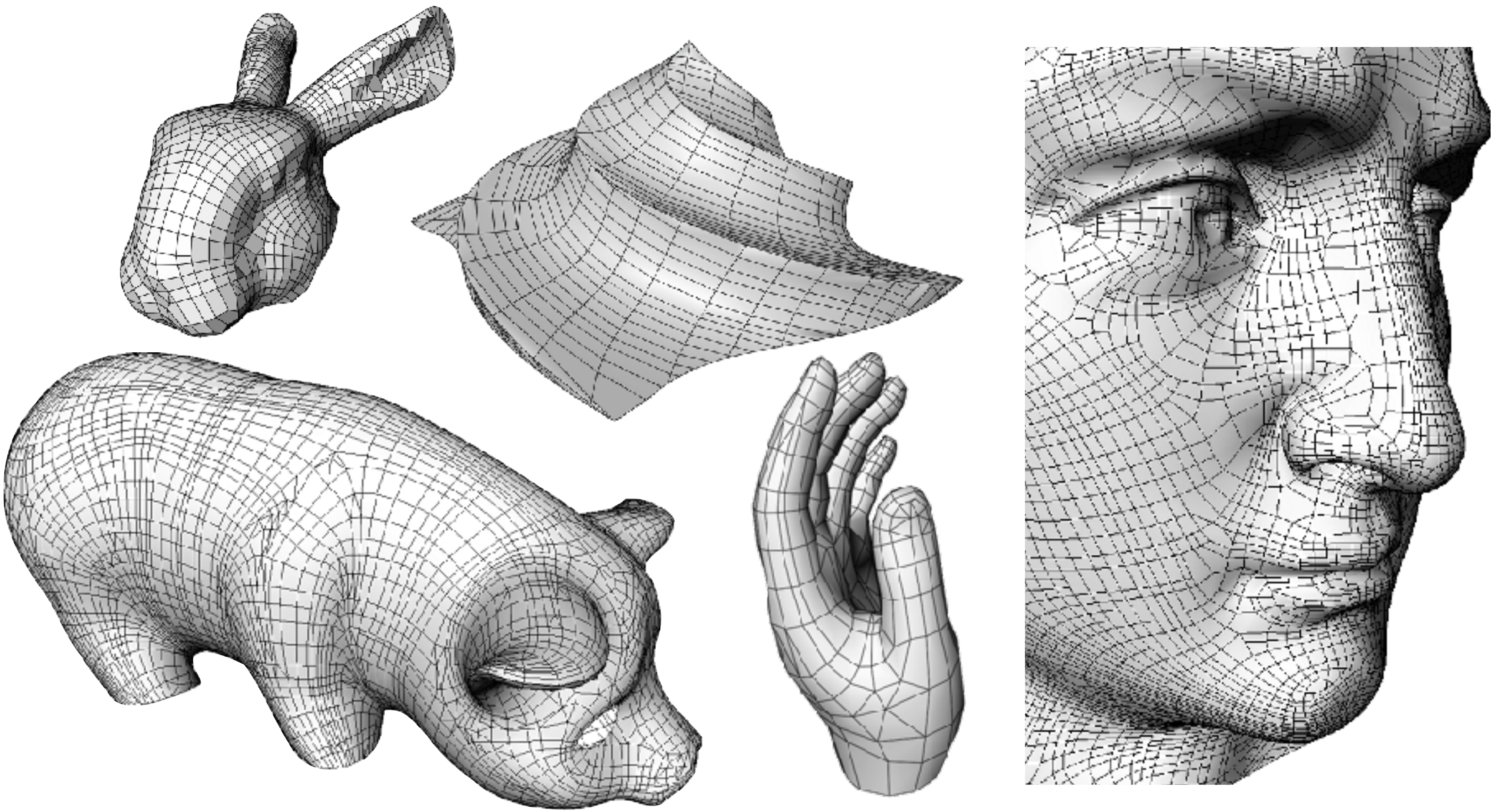
Artist-designed models

- Conform to the anisotropy of a surface



Anisotropic remeshing

[Alliez et al. 2003] *Anisotropic Polygonal Remeshing.*



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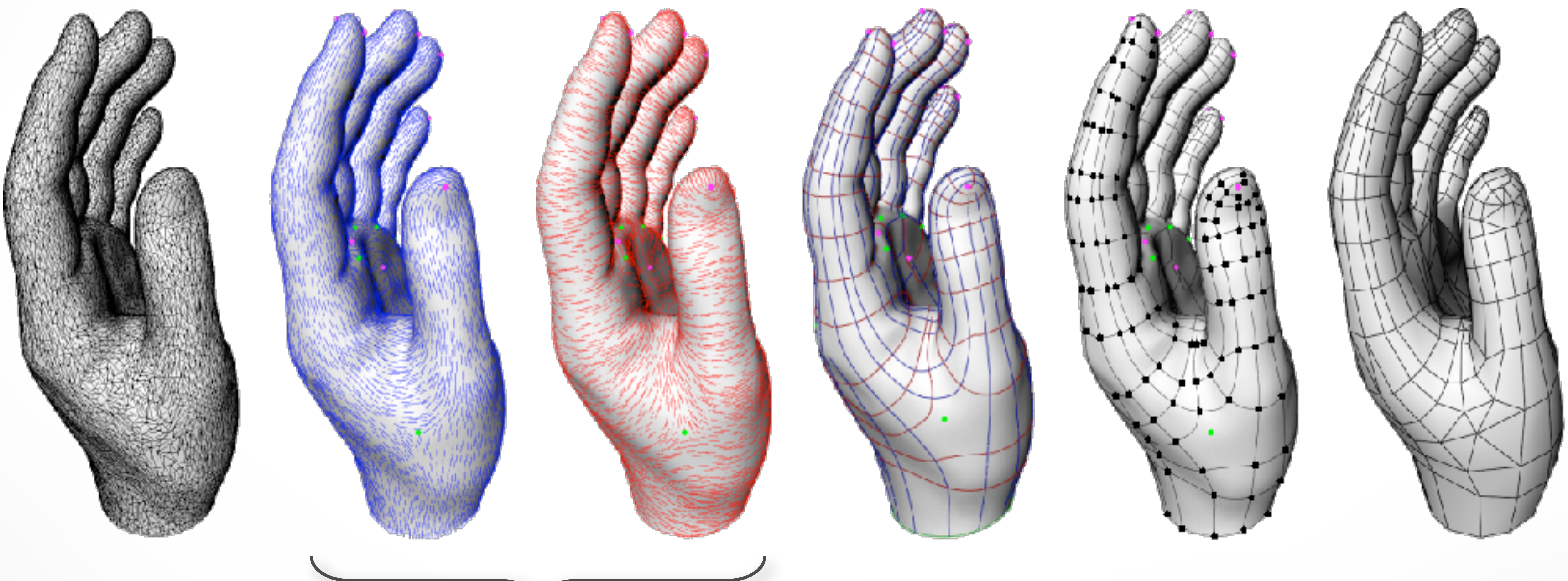
[Alliez et al. 2003] ***Anisotropic Polygonal Remeshing.***



input mesh

principal direction fields

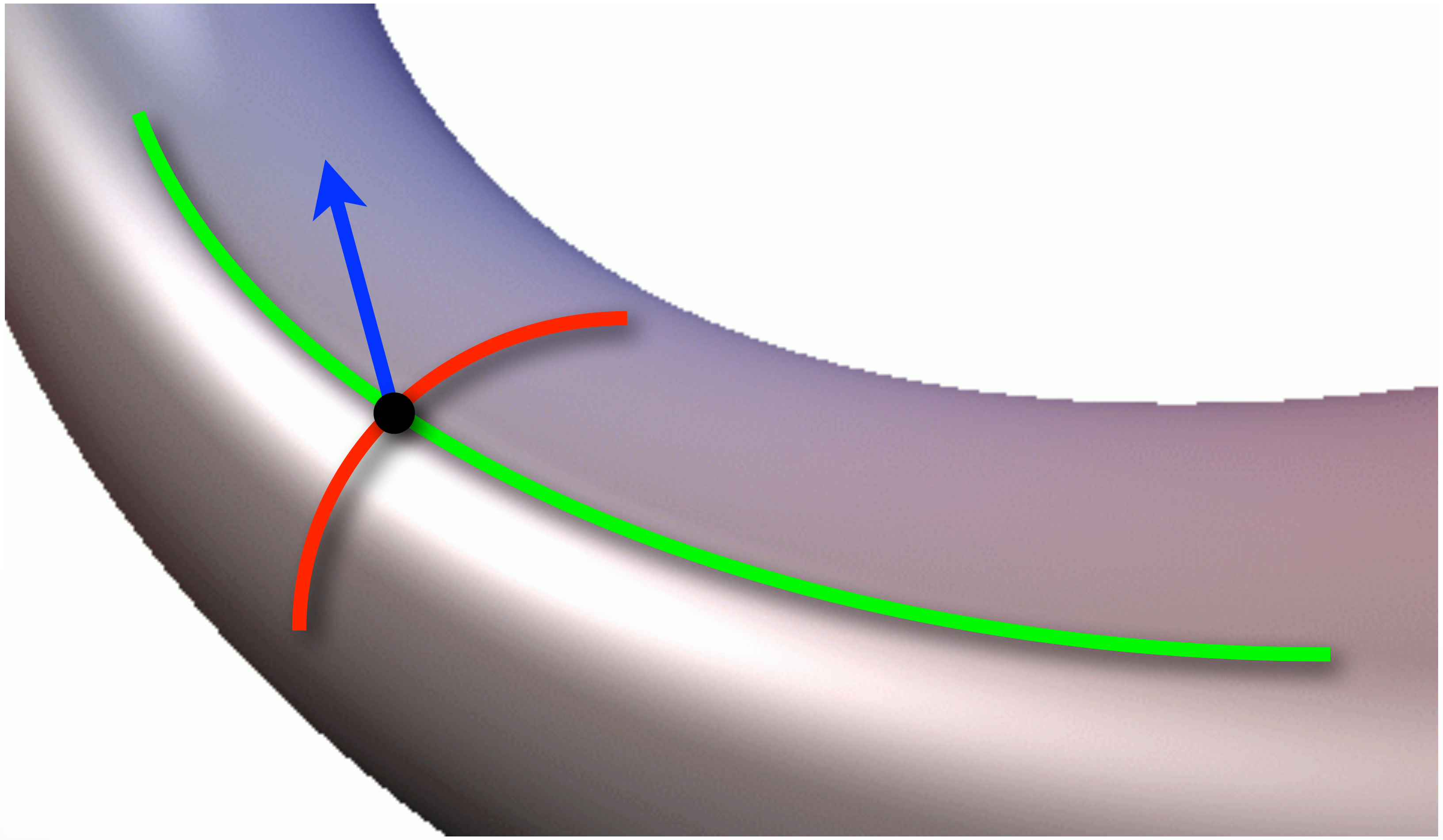
sampling

meshing

output mesh

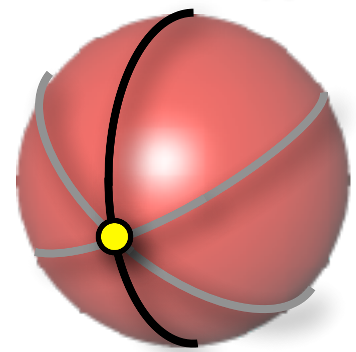
Differential geometry

- A local *orthogonal* frame: min/max curvature directions and normal

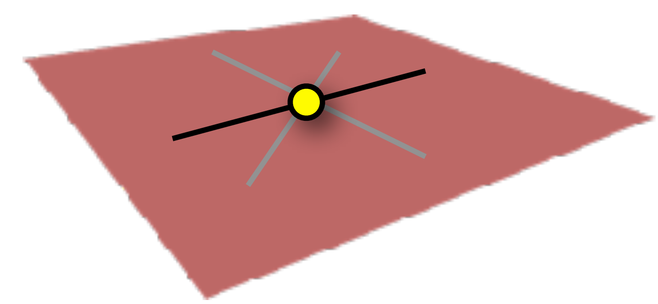


3D curvature tensor

Isotropic



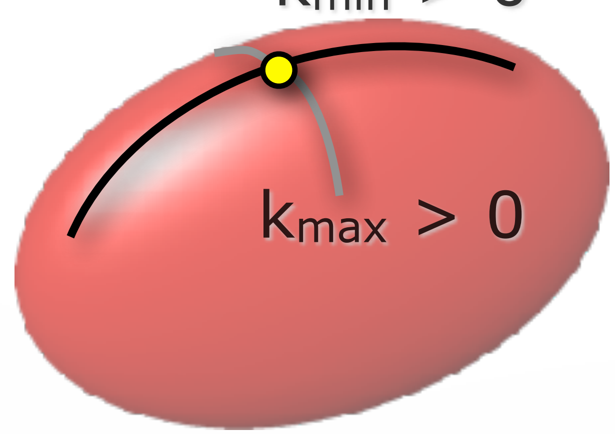
spherical



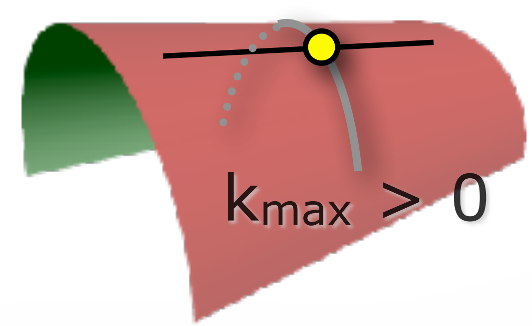
planar

Anisotropic

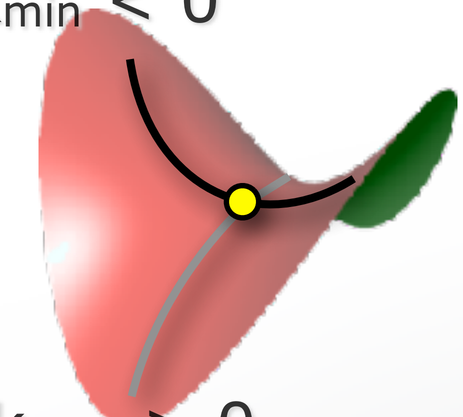
2 principal directions



elliptic

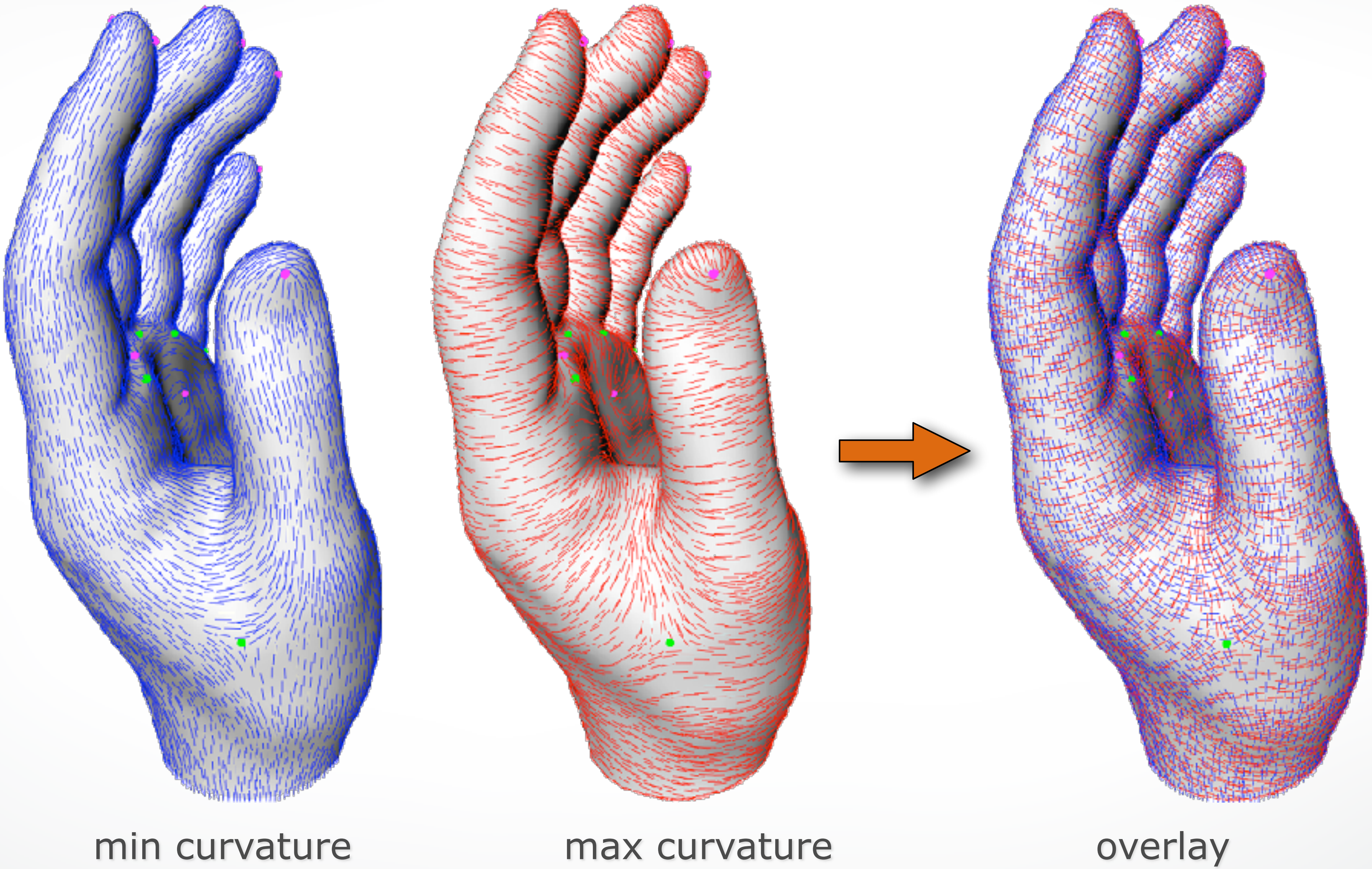


parabolic

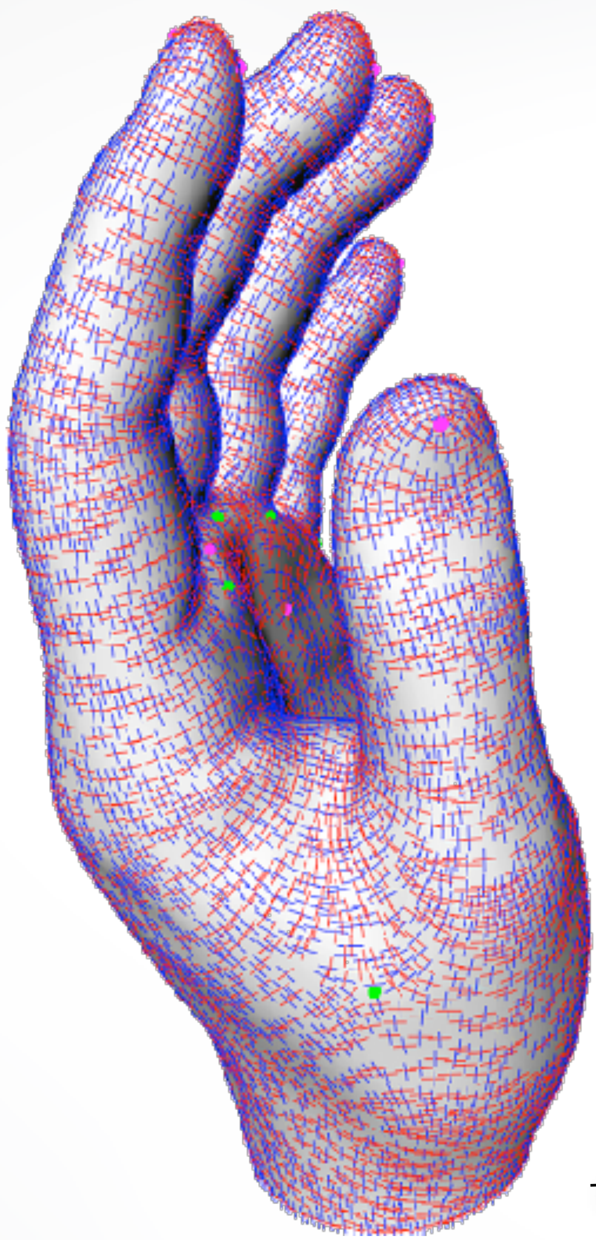


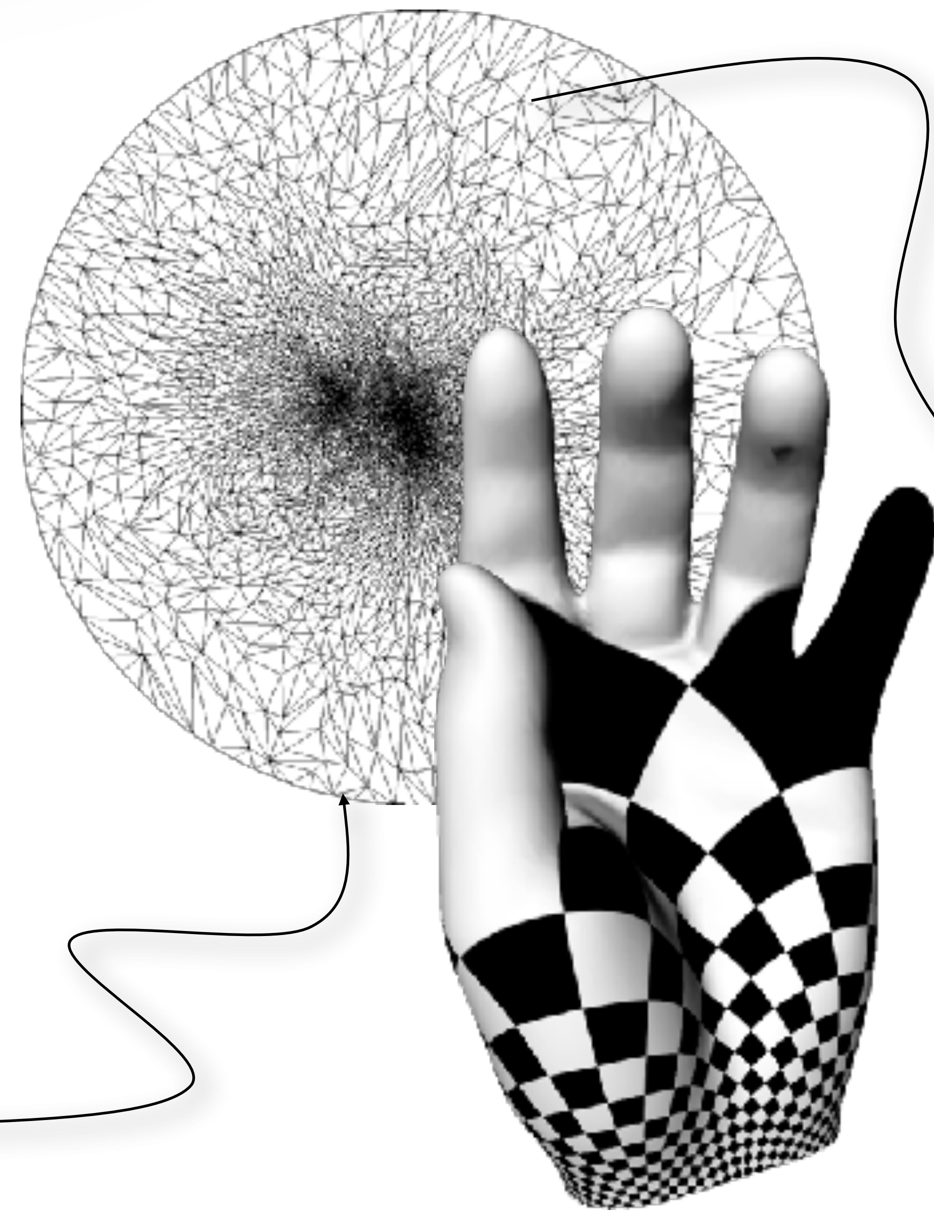
hyperbolic

Principal direction fields

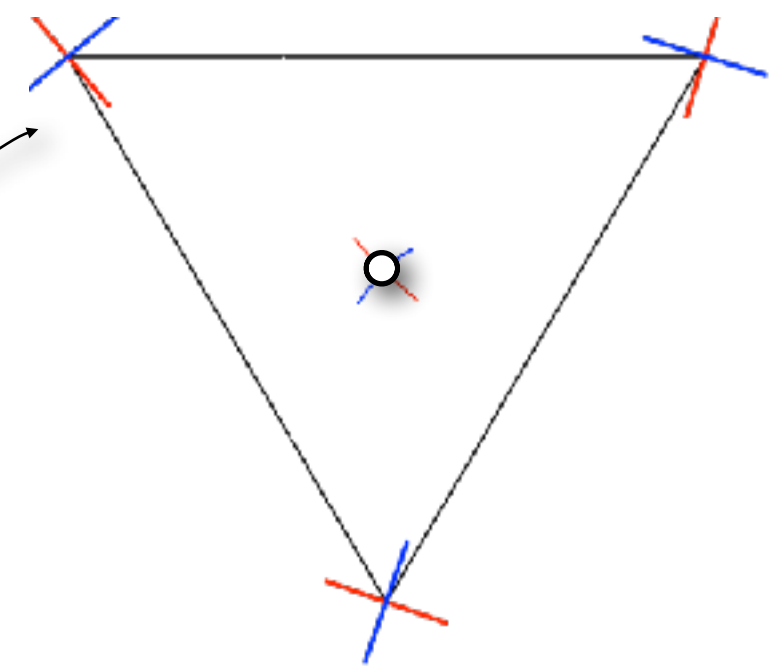


Flattening to 2D





piecewise linear
interpolation of
2D tensors



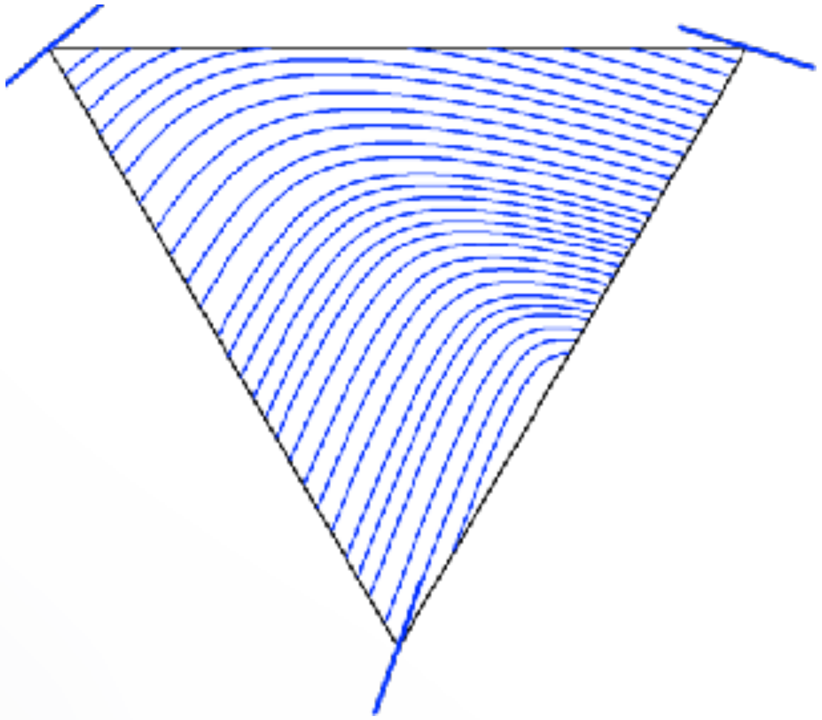
one 3D tensor
per vertex

discrete conformal
parameterization

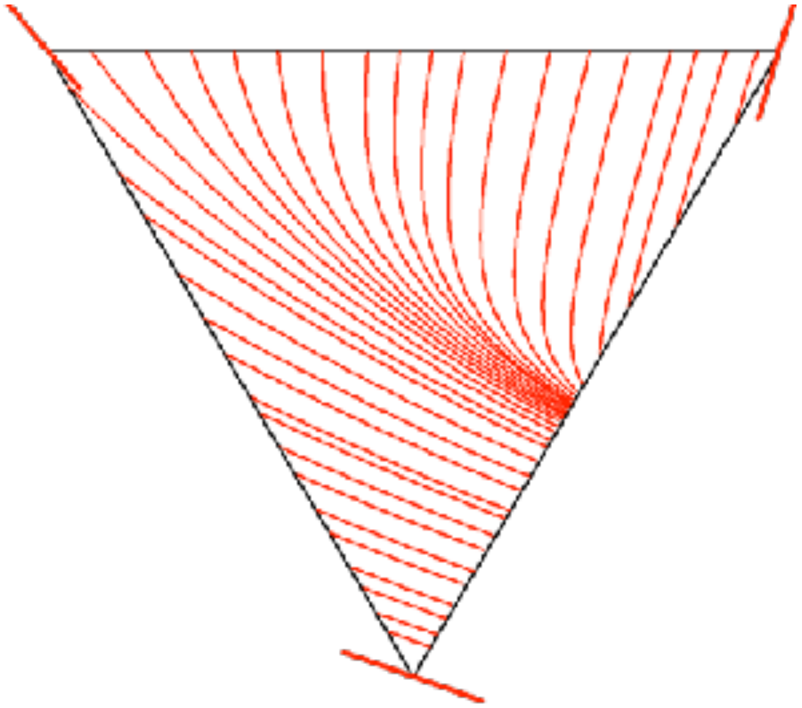
2D tensor **field**
using barycentric
coordinates

2D direction fields

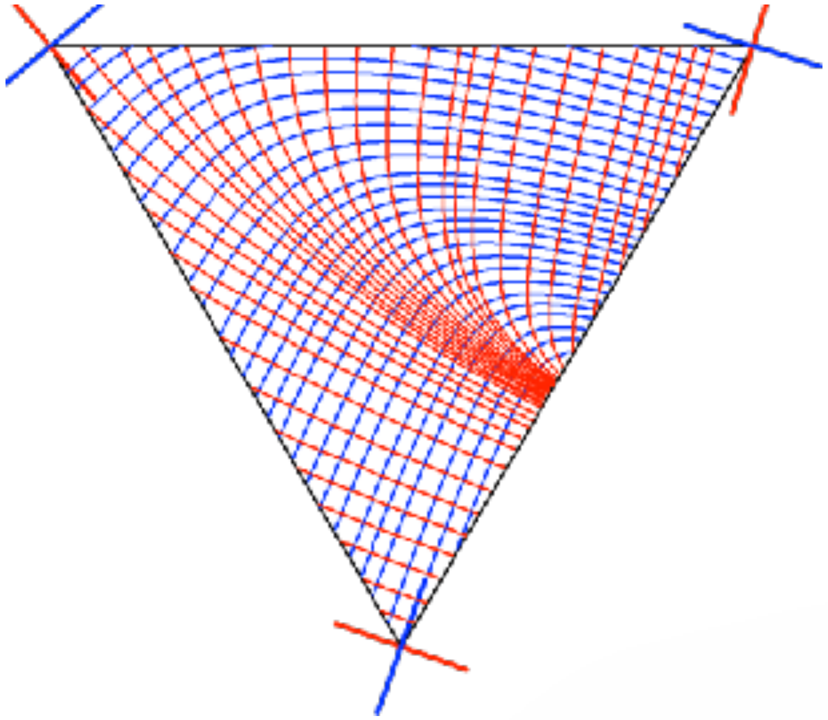
- **Regular case**



minor foliation



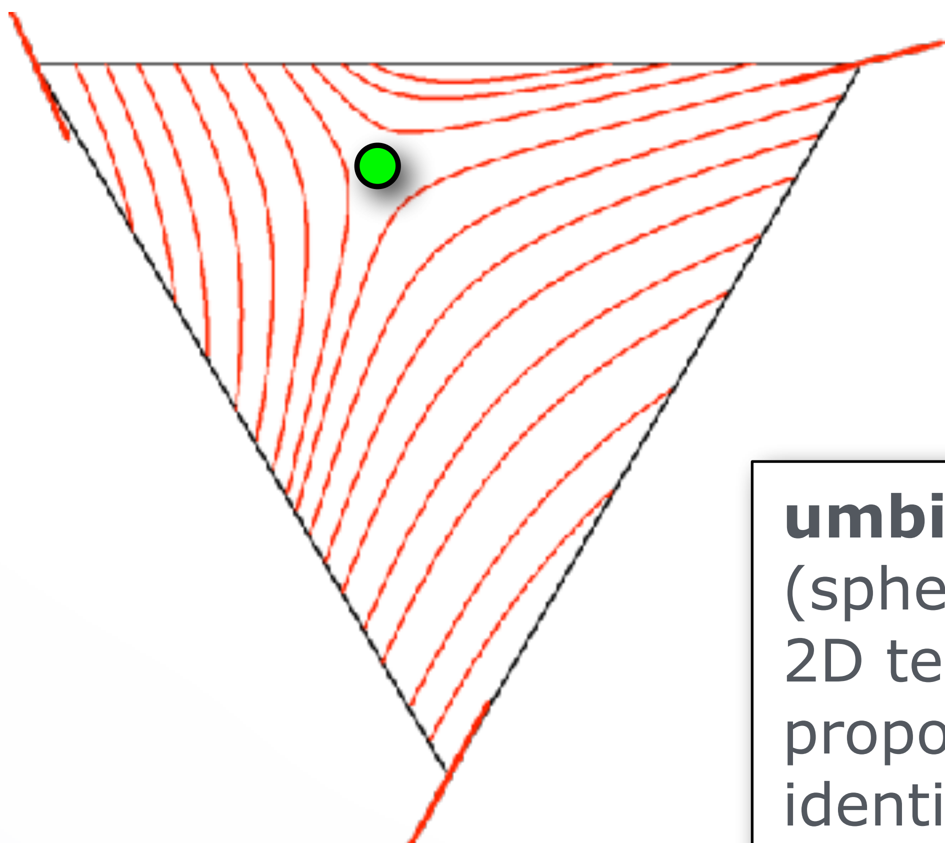
major foliation



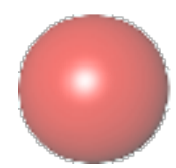
principal foliations

2D direction fields

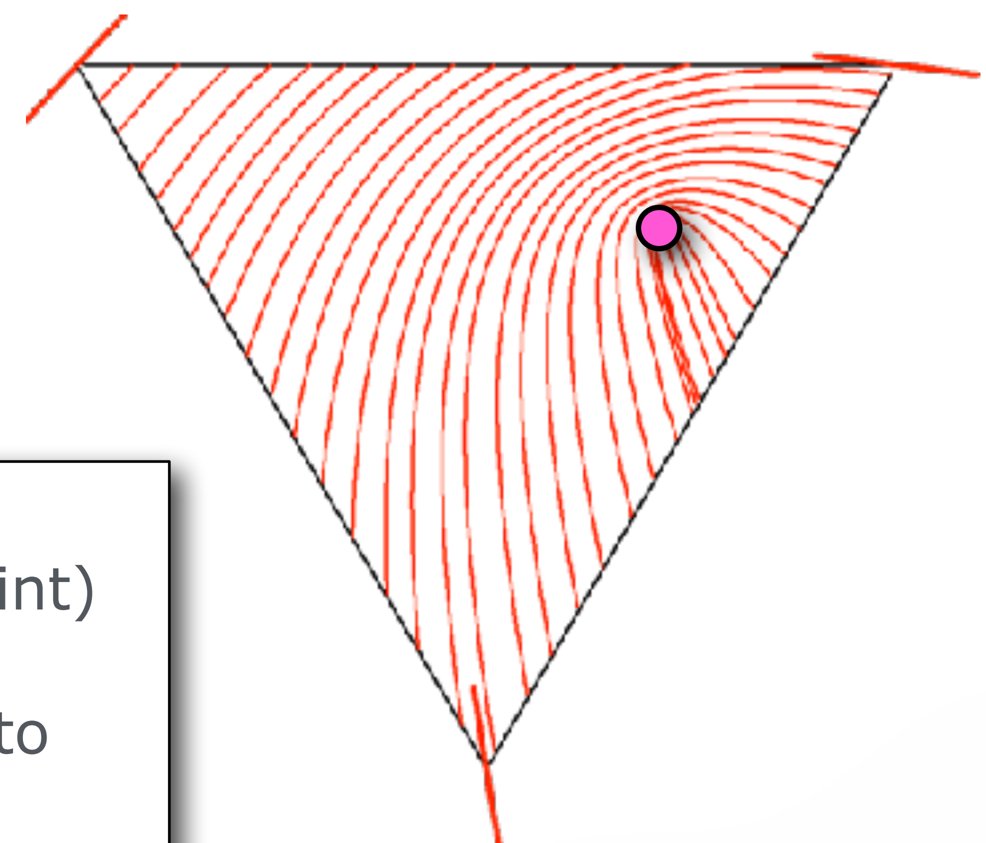
- **Singularities**



trisector

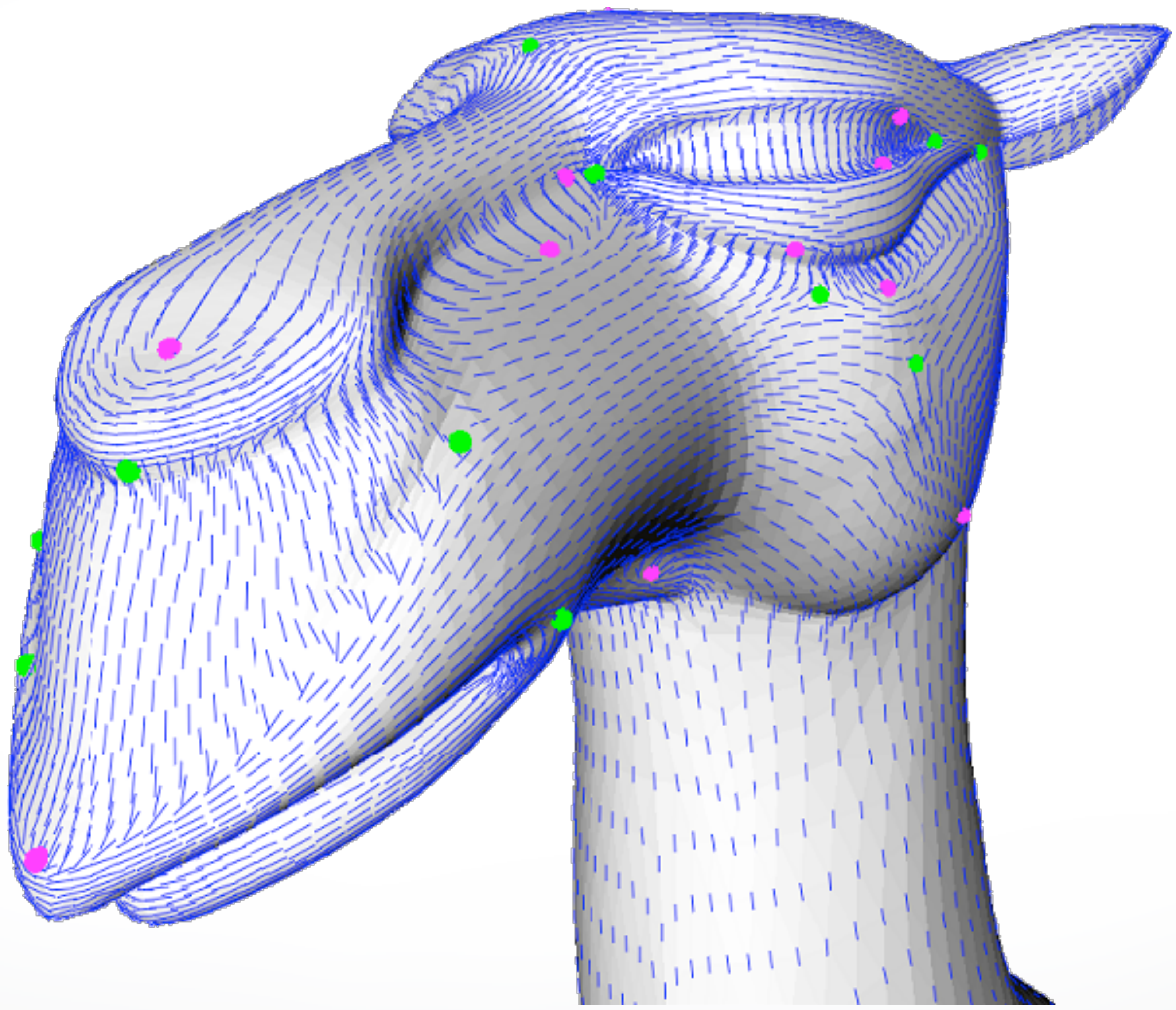


umbilic
(spherical point)
2D tensor
proportional to
identity

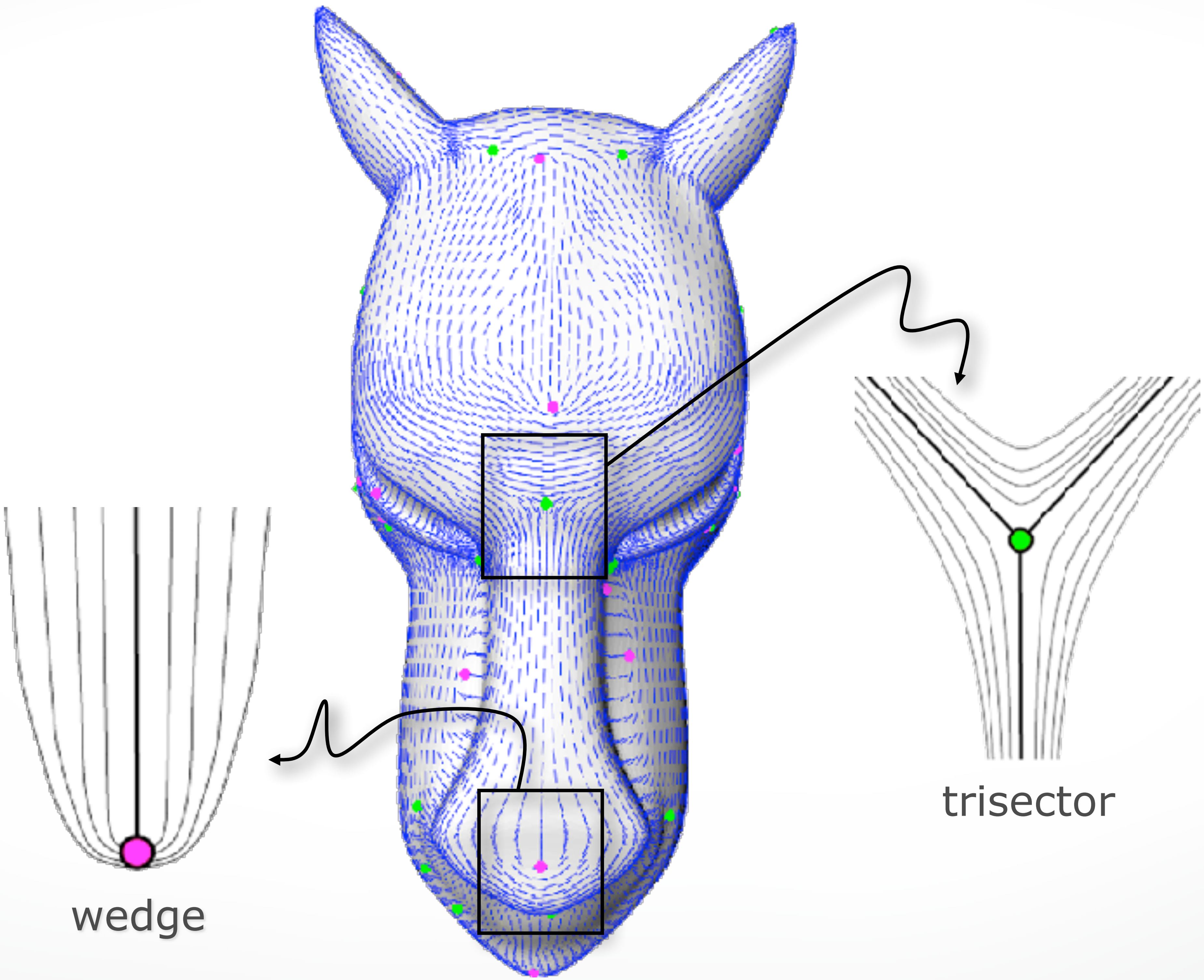


wedge

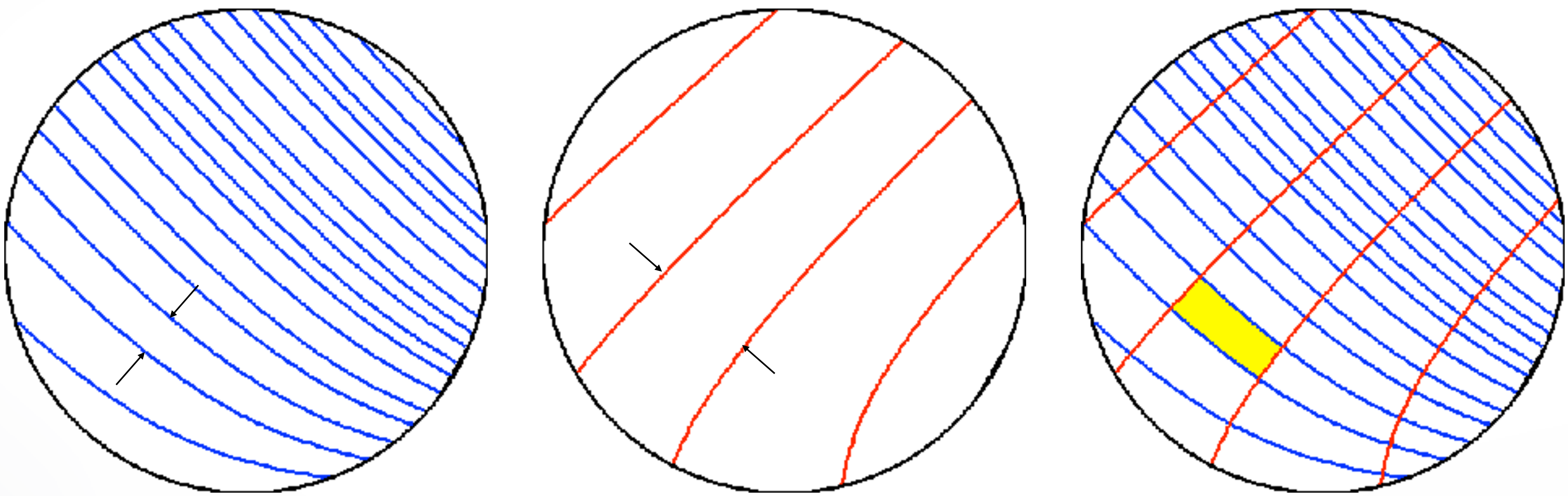
Umbilics



Umbilics

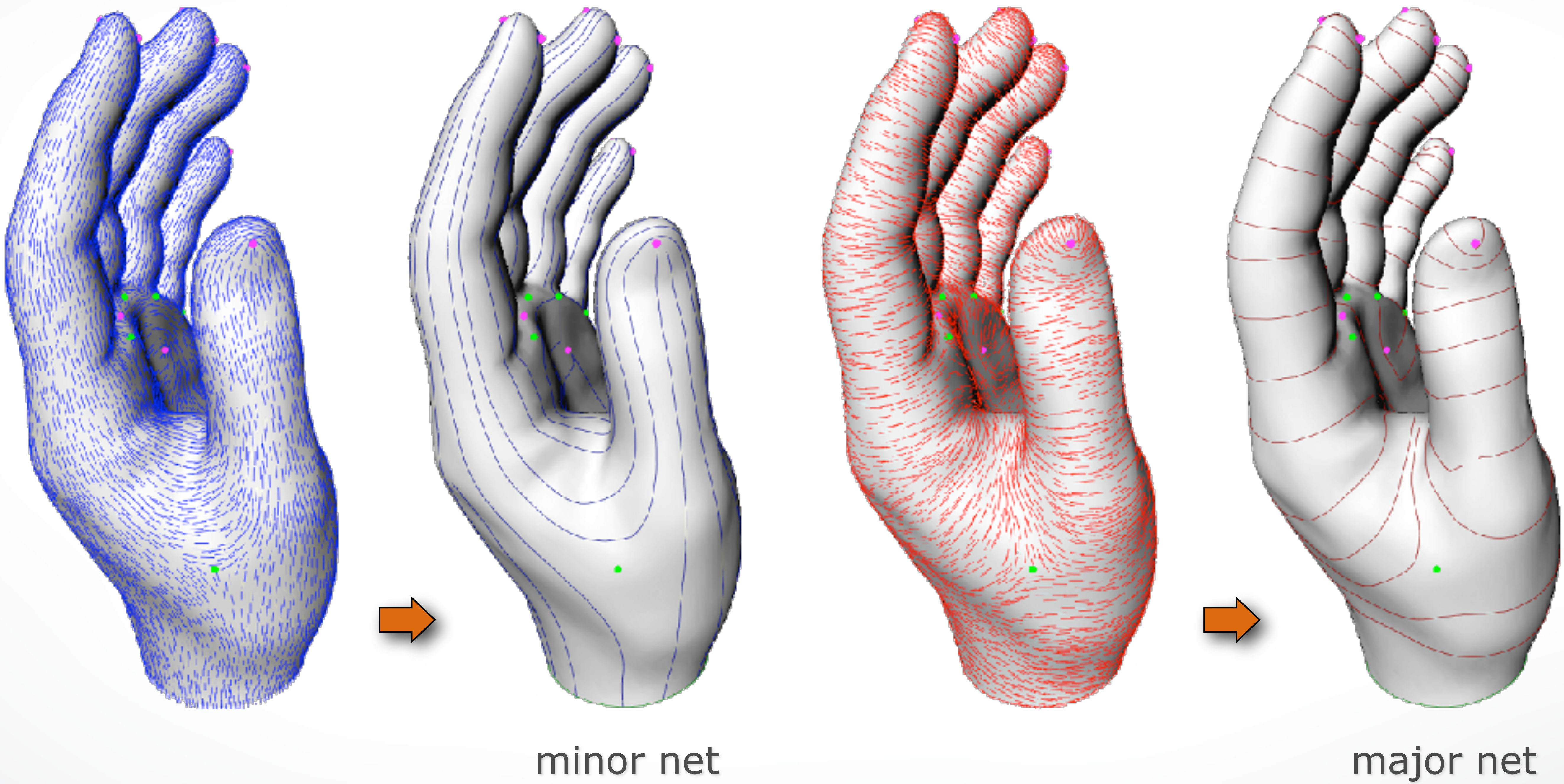


Lines of curvature



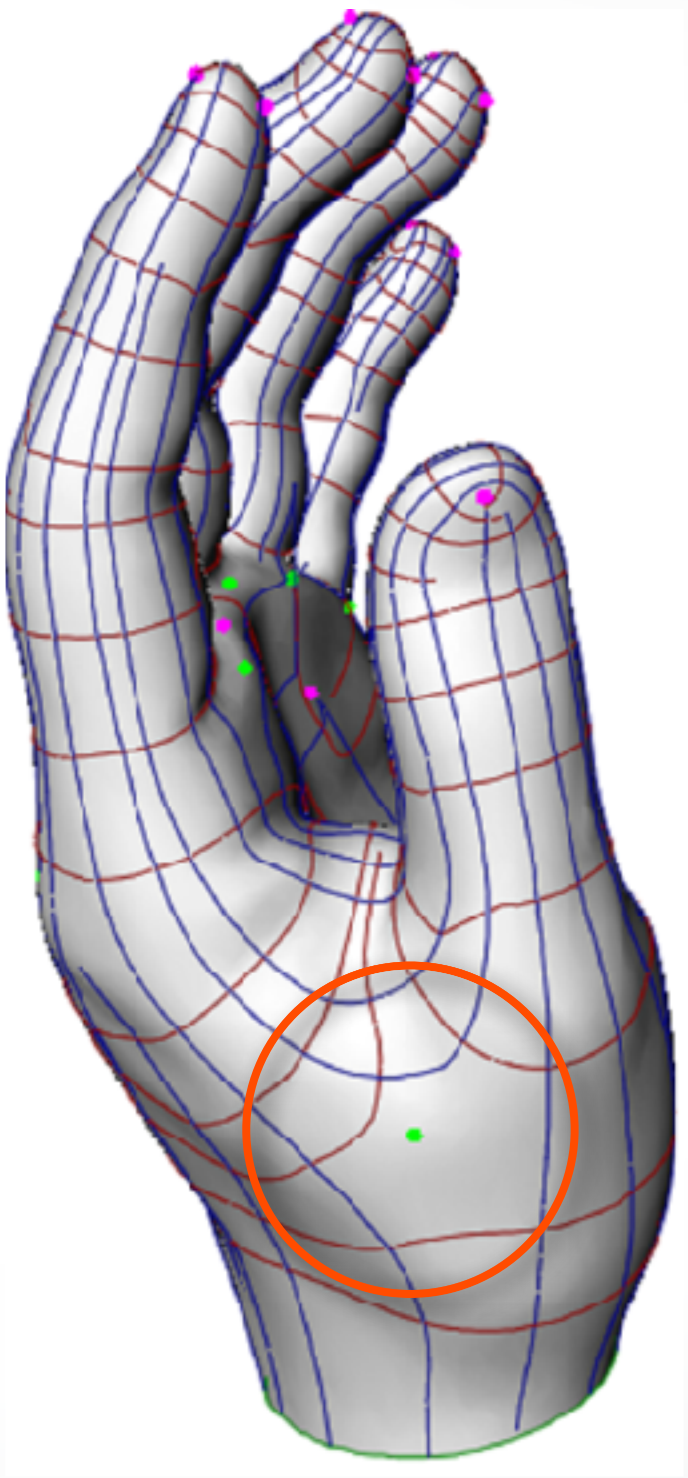
minor net | major net | overlay

Lines of curvature

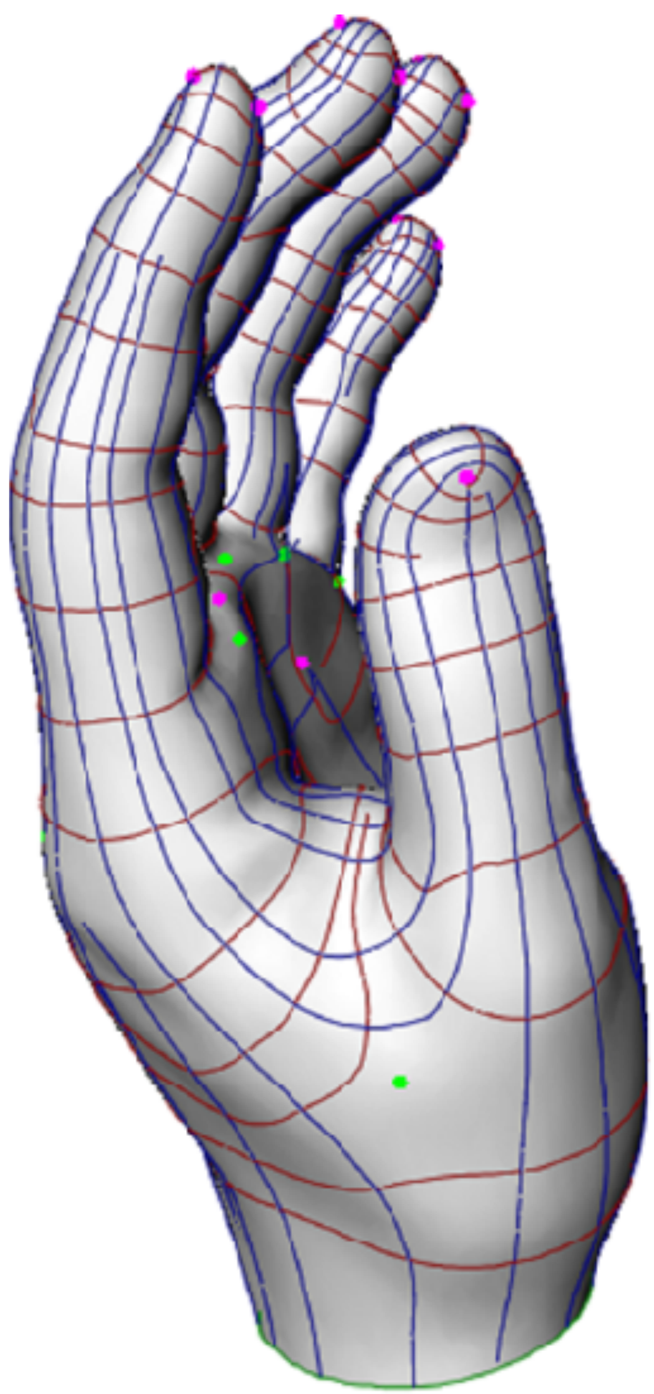


Overlay

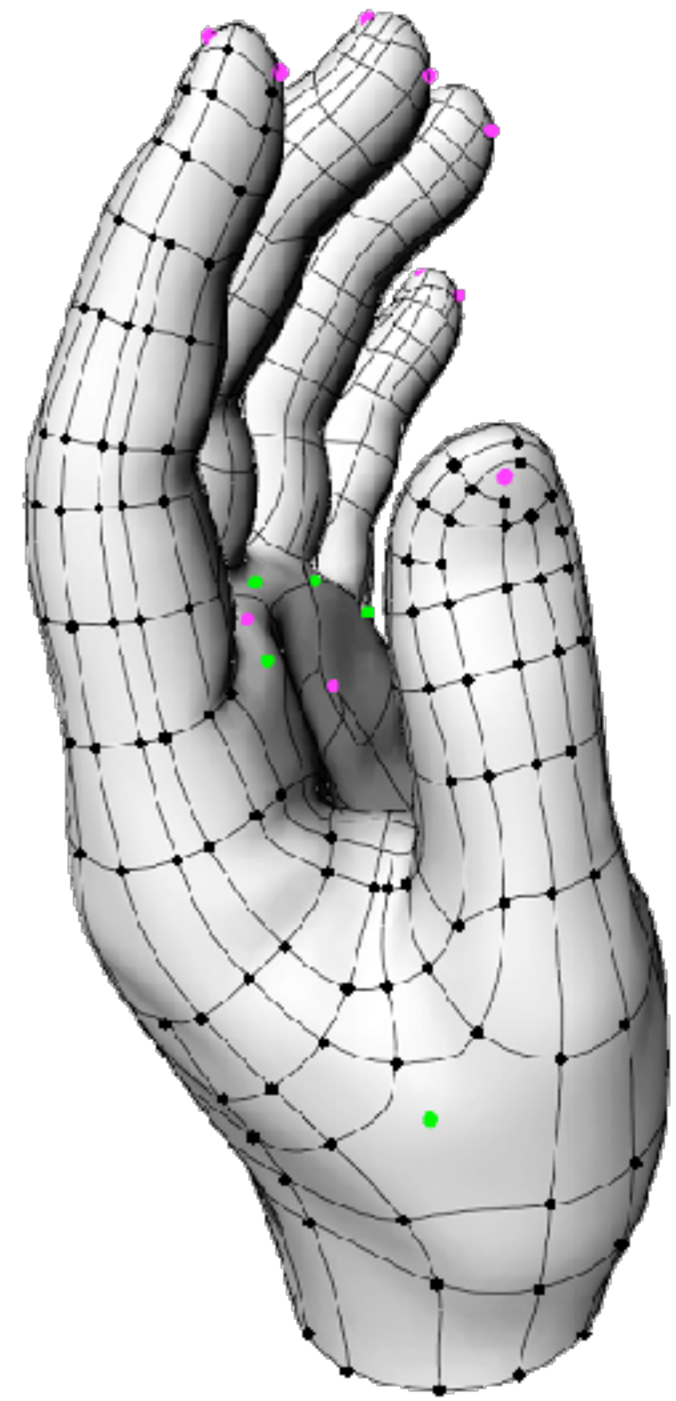
- **Overlay curvature lines in anisotropic regions**
- **Add umbilical points in isotropic regions**



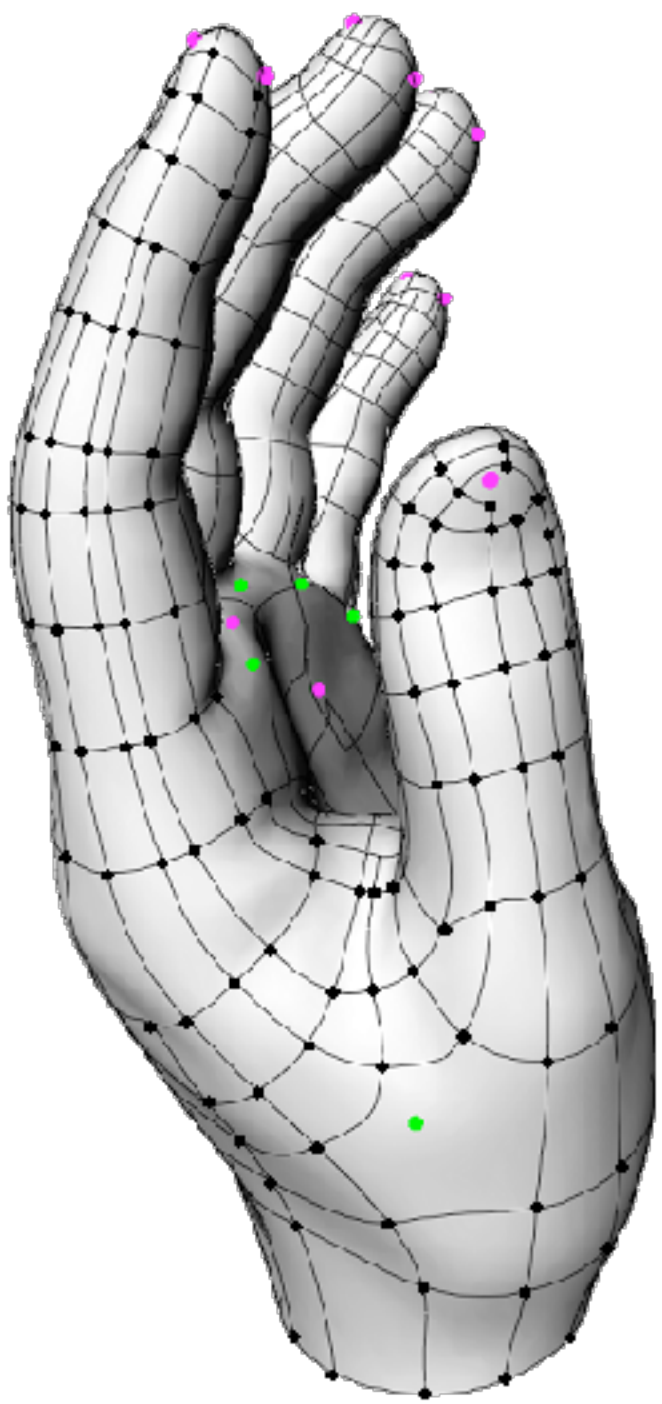
Vertices

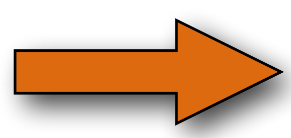


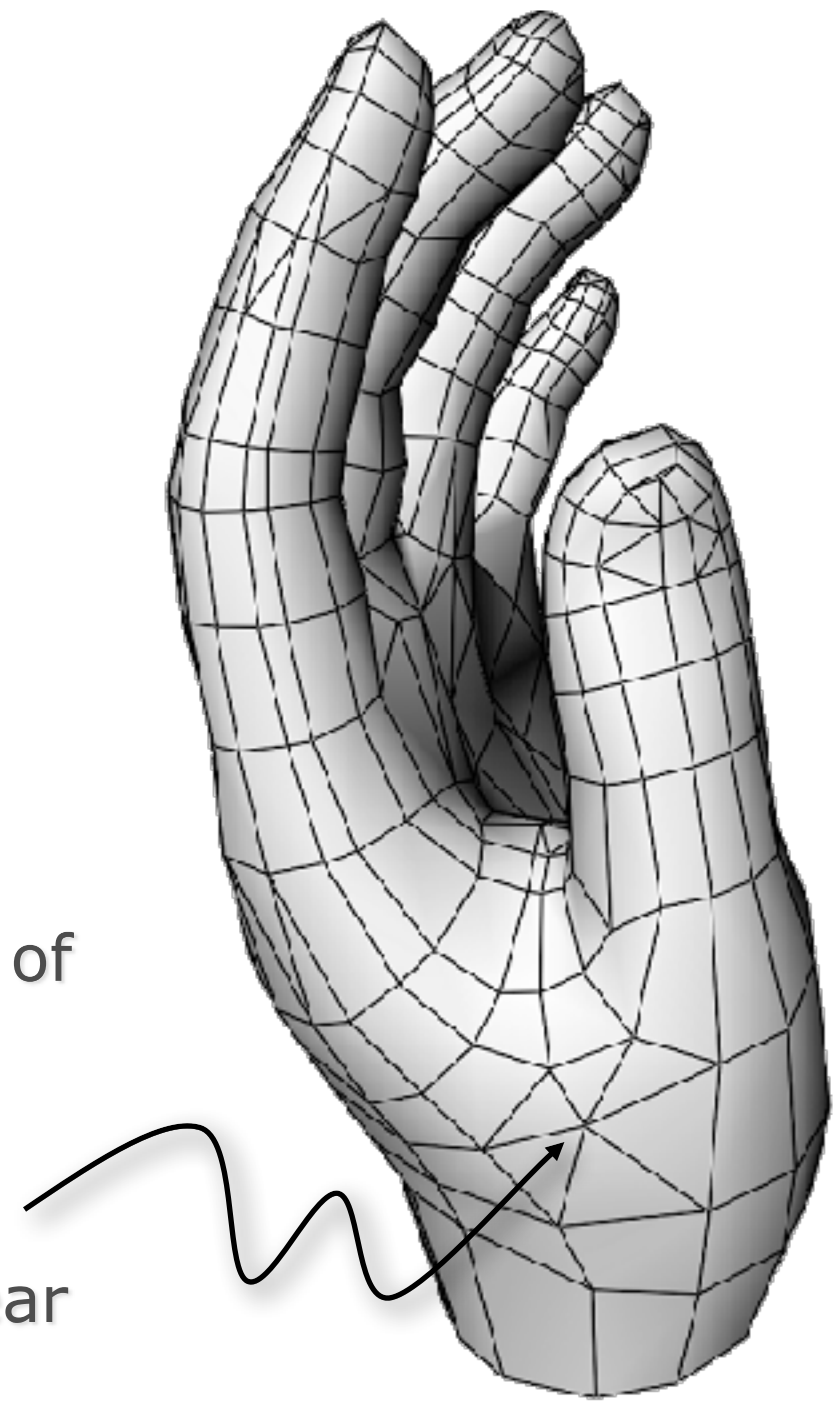
intersect lines of
curvatures



Edges

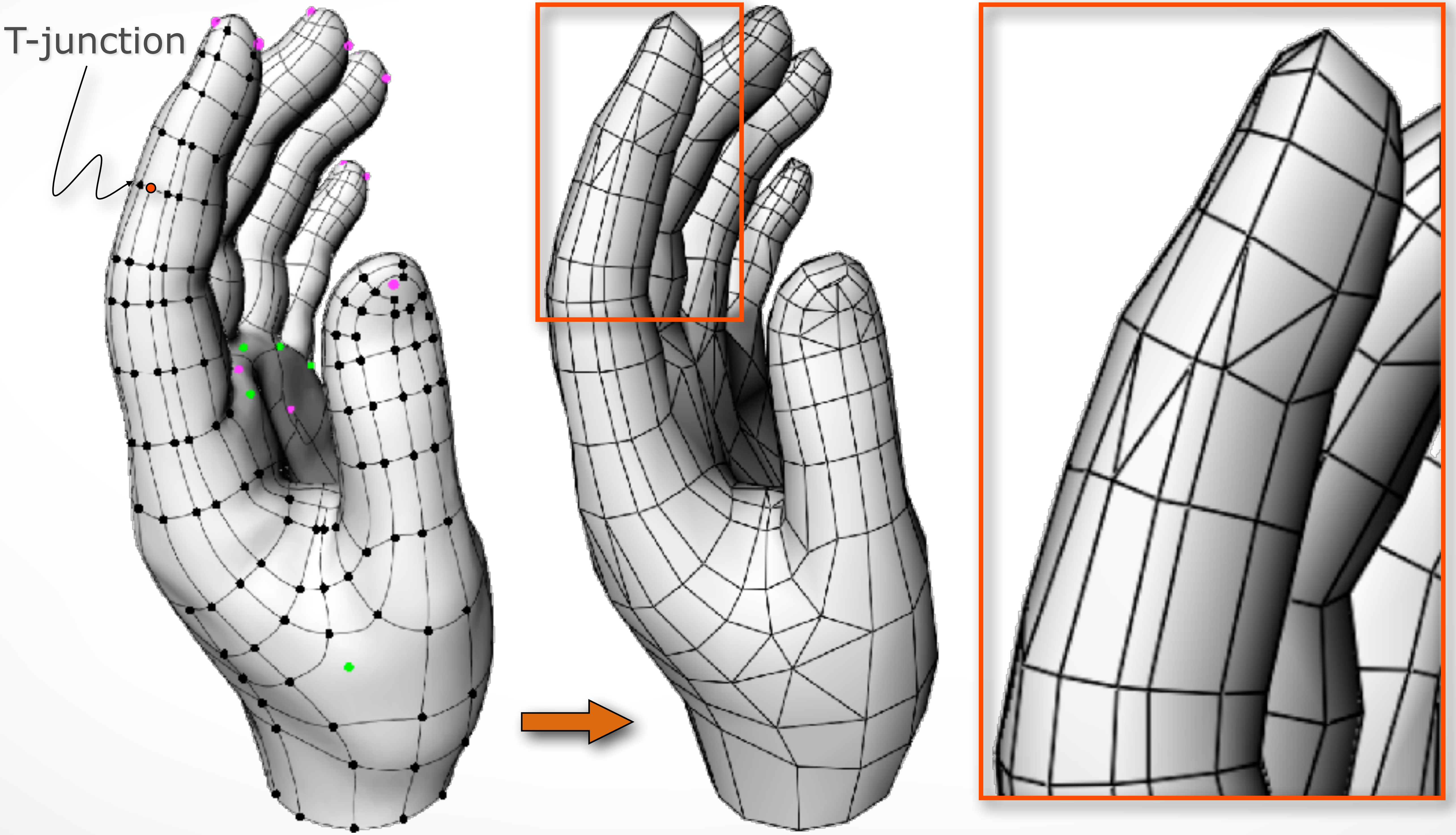




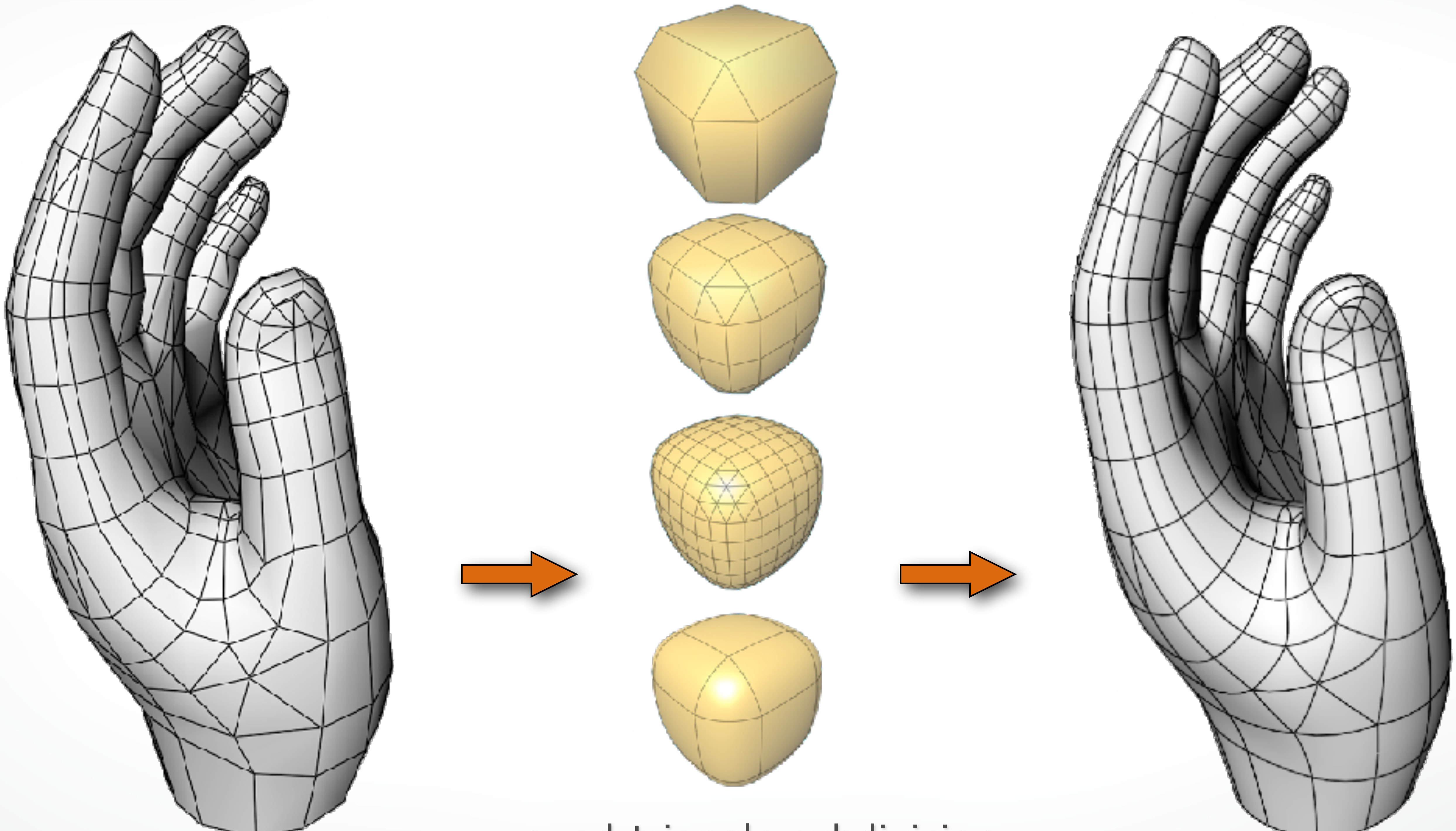


straighten lines of curvatures
+
Delaunay triangulation near umbilics

Resolve T-junctions



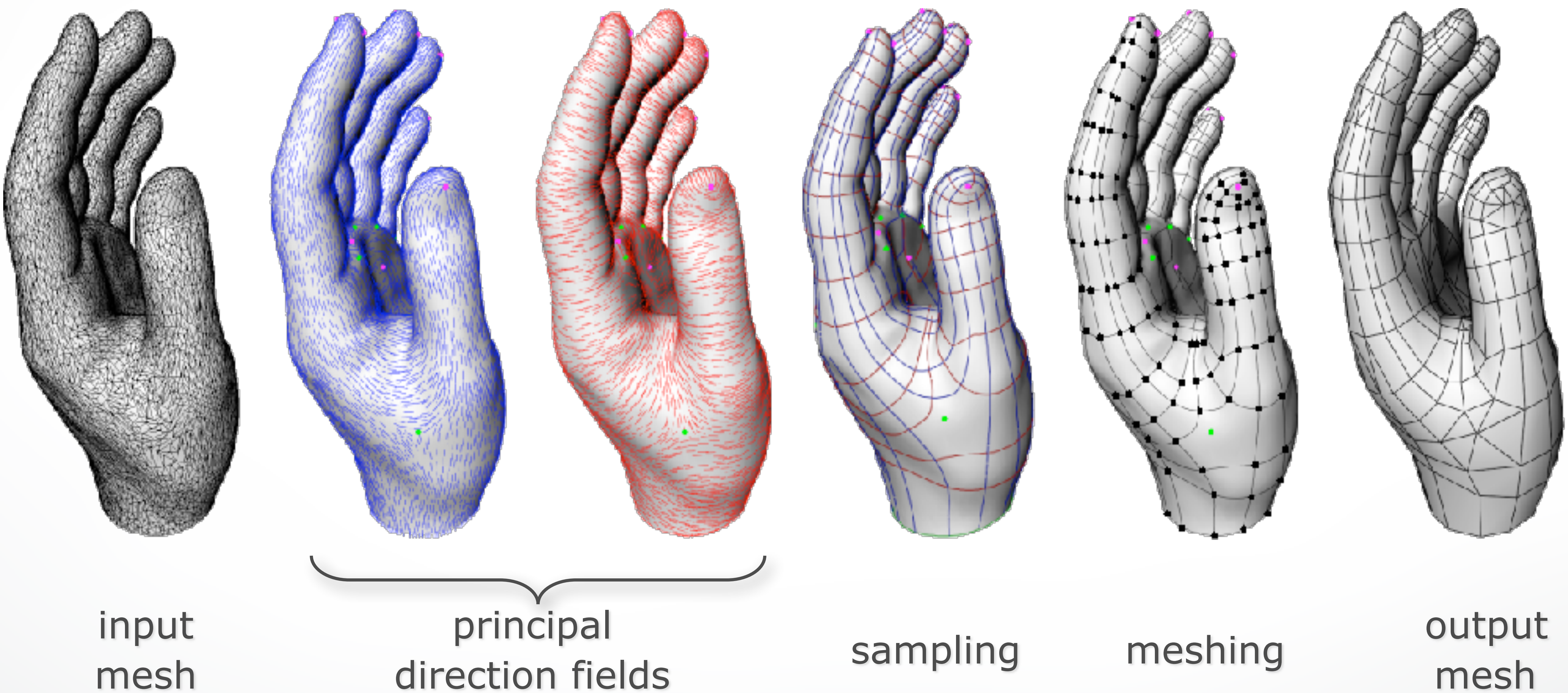
Smoothing



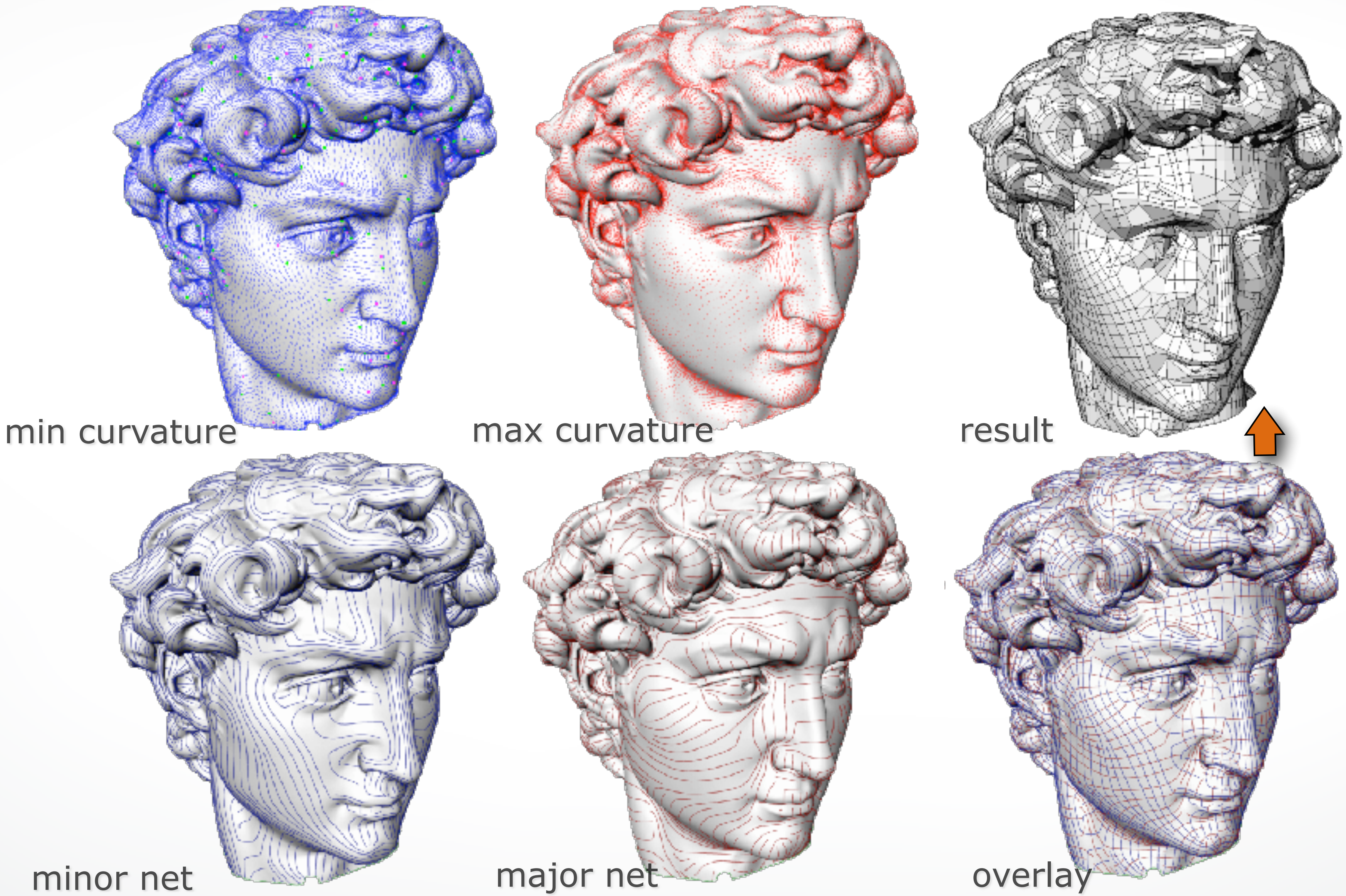
quad-triangle subdivision

Anisotropic remeshing

[Alliez et al. 2003] *Anisotropic Polygonal Remeshing.*

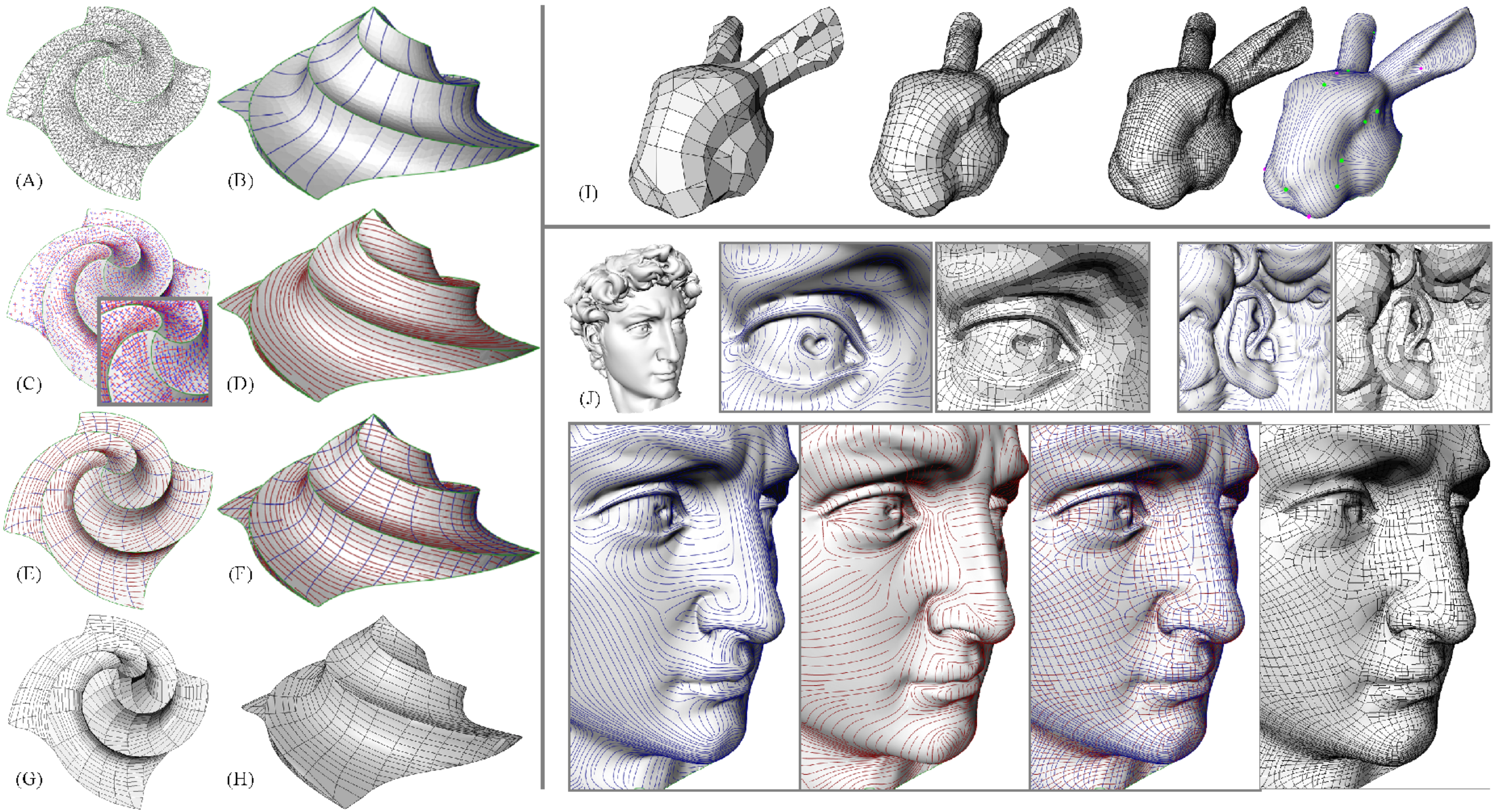


Remeshing results



Remeshing results

[Alliez et al. 2003] *Anisotropic Polygonal Remeshing.*



Tools

MeshLab

- meshlab.sourceforge.net
- open source
- available for Windows, MacOSX, and Linux

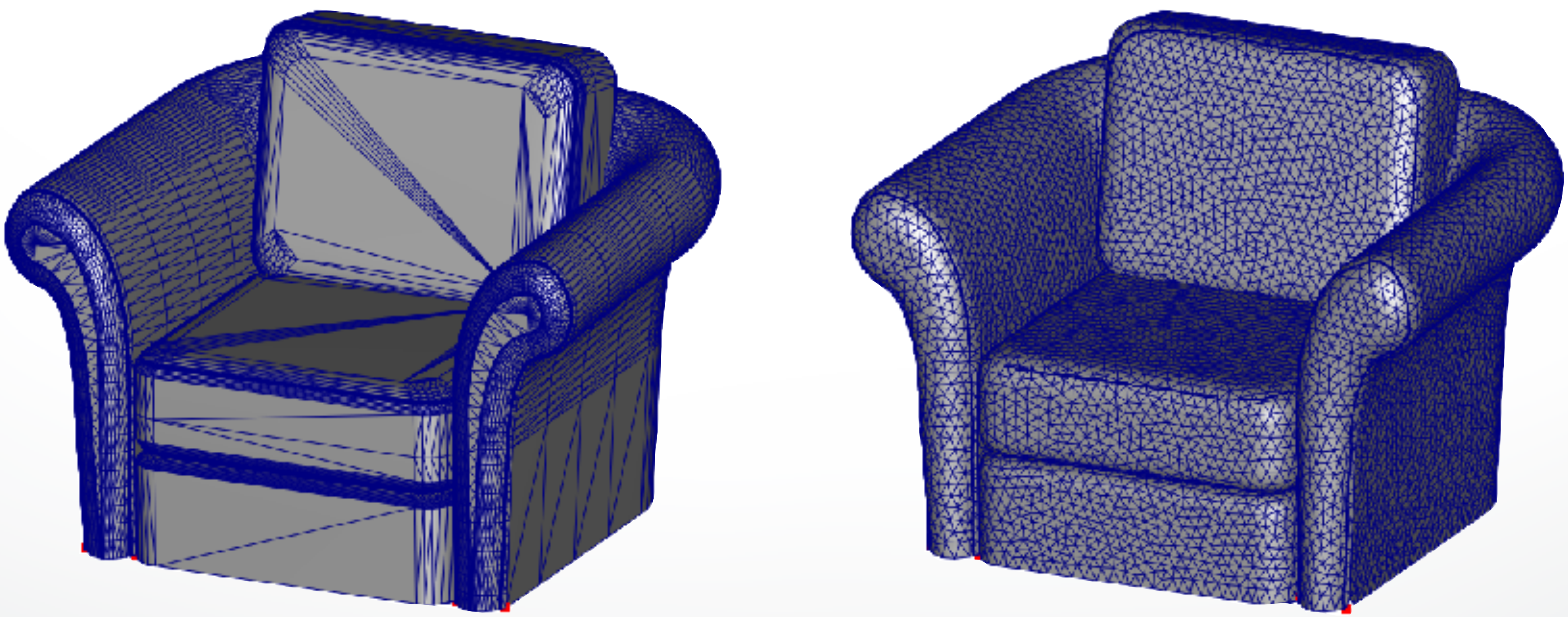
Graphite

- http://alice.loria.fr/index.php/software/3-platform/22-graphite.html
- available for Windows
- MacOSX or Linux?

Remeshing via Graphite

“Mesh” → “remesh” → “pliant” →

- [Optional] flag border as feature
- [Optional] flag sharp edges as feature (dihedral angle)
- [Optional] estimate edge size (bounding box divisions)
- remesh (target edge length)



Literature

- Textbook: Chapter 6
- Alliez et al, *"Interactive geometry remeshing"*, SIGGRAPH 2002
- Alliez et al, *"Isotropic surface remeshing"*, SMI 2003
- Alliez et al, *"Anisotropic polygonal remeshing"*, SIGGRAPH 2003
- Vorsatz et al, *"Dynamic remeshing and applications"*, Solid Modeling 2003
- Botsch & Kobbelt, *"A remeshing approach to multiresolution modeling"*, Symp. on Geometry Processing 2004
- Marinov et al, *"Direct anisotropic quad-dominant remeshing"*, Pacific Graphics 2004
- Alliez et al, *"Recent advances in remeshing of surfaces"*, AIM@Shape state of the art report, 2006

http://cs621.hao-li.com

Thanks!