6.1 **Shape Matching**

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Acknowledgement

Images and Slides are courtesy of

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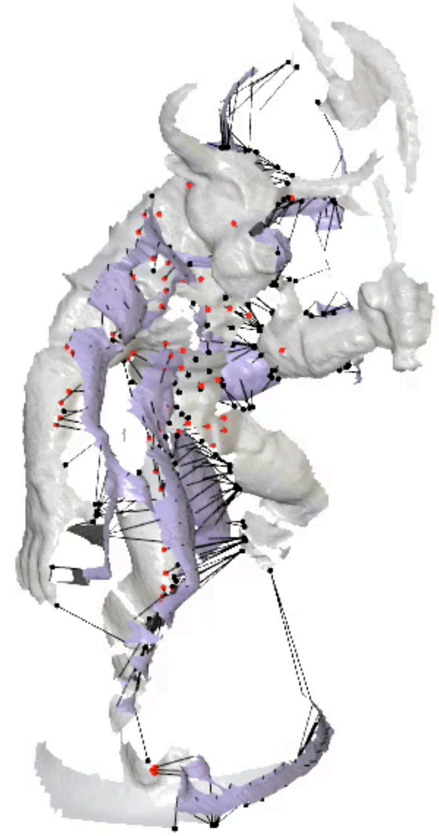
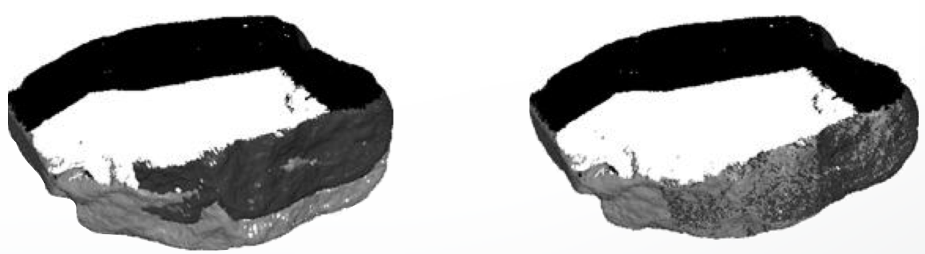
- ICCV Course 2005: http://gfx.cs.princeton.edu/proj/iccv05_course/

Last Time

Surface Registration

- Pairwise ICP & Variants
- Point-to-point/plane metric
- BSP closes point search
- Stability Analysis
- Global Registration

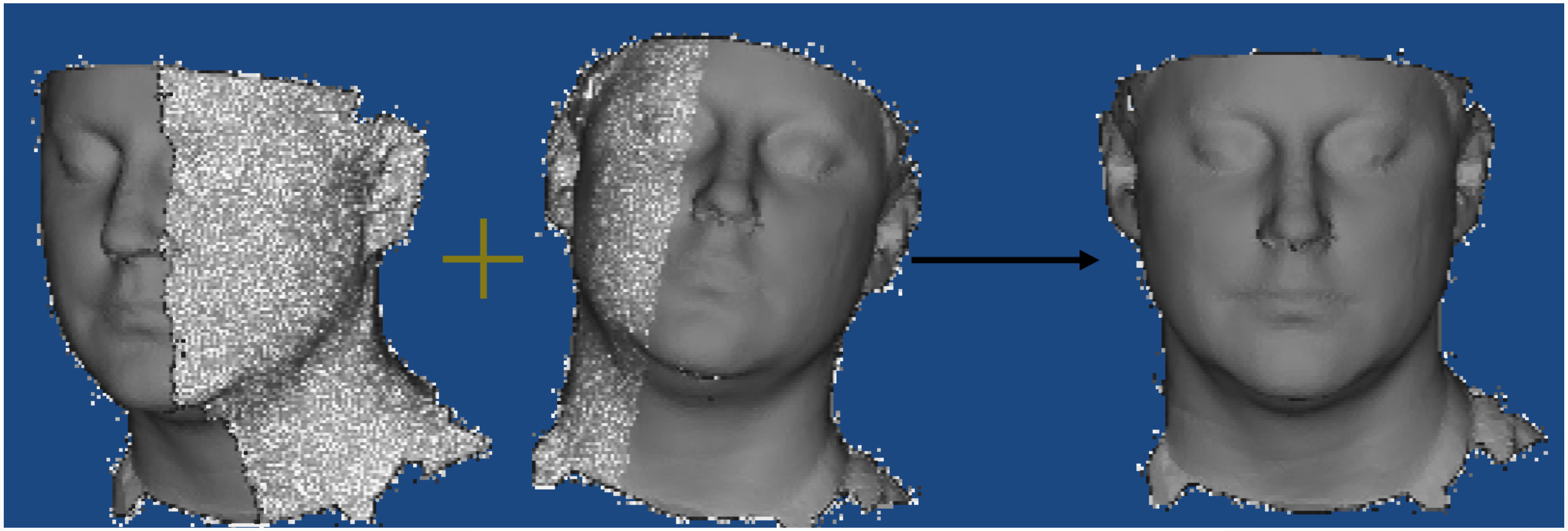




Shape Matching for Model Alignment

Goal

- Given two partially overlapping scans, compute transformation that aligns the two.
- **No assumption** about rough initial alignment

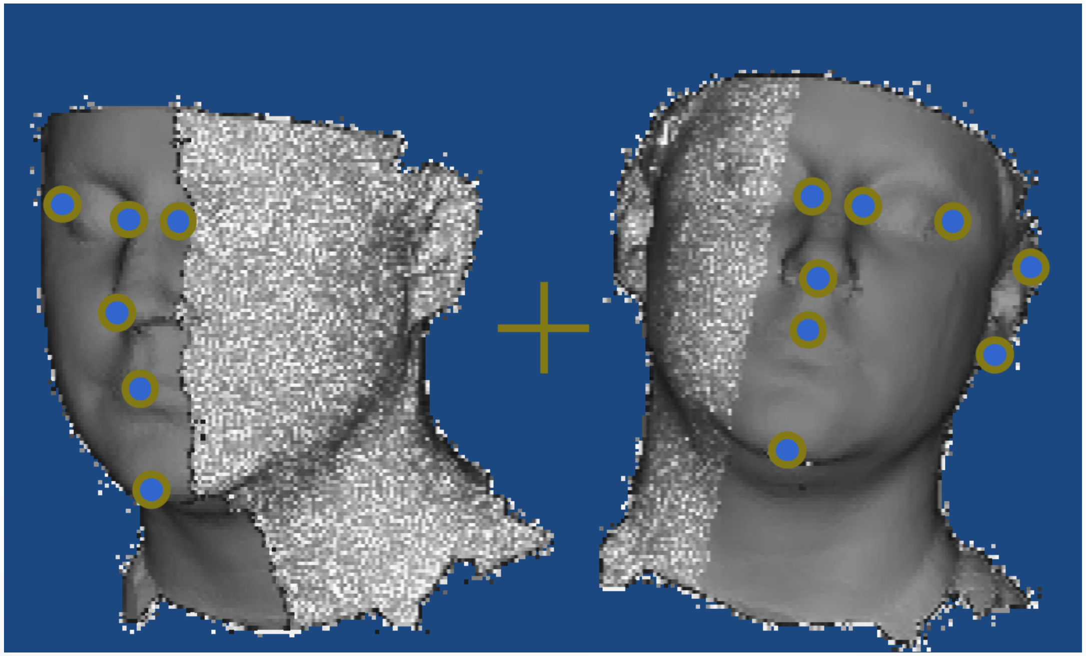


Partially Overlapping Scans **Aligned Scans**

Shape Matching for Model Alignment

Approach

- Find feature points on the two scans

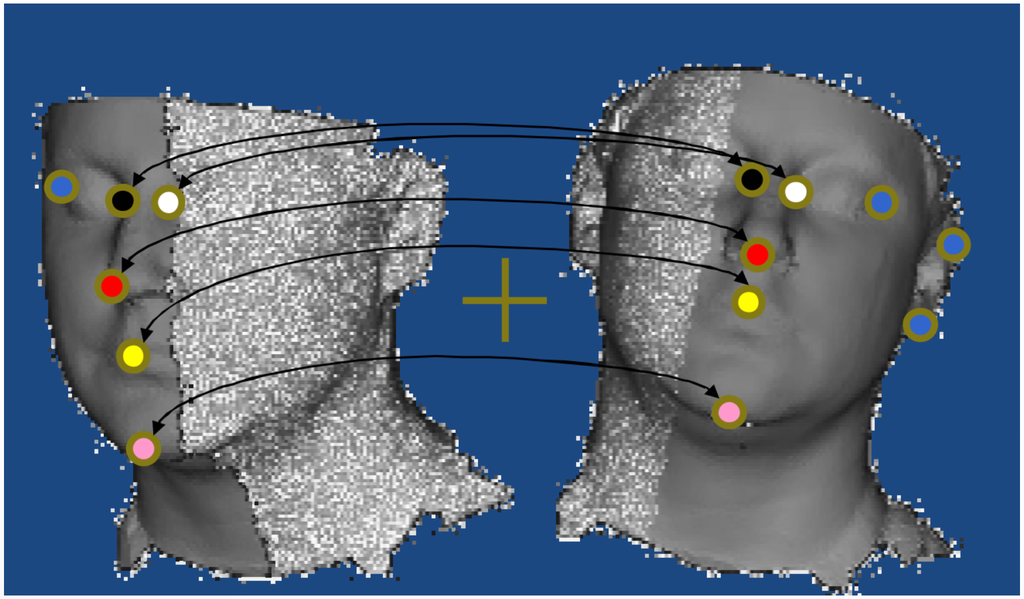


Partially Overlapping Scans

Shape Matching for Model Alignment

Approach

- Find feature points on the two scans
- Establish correspondences

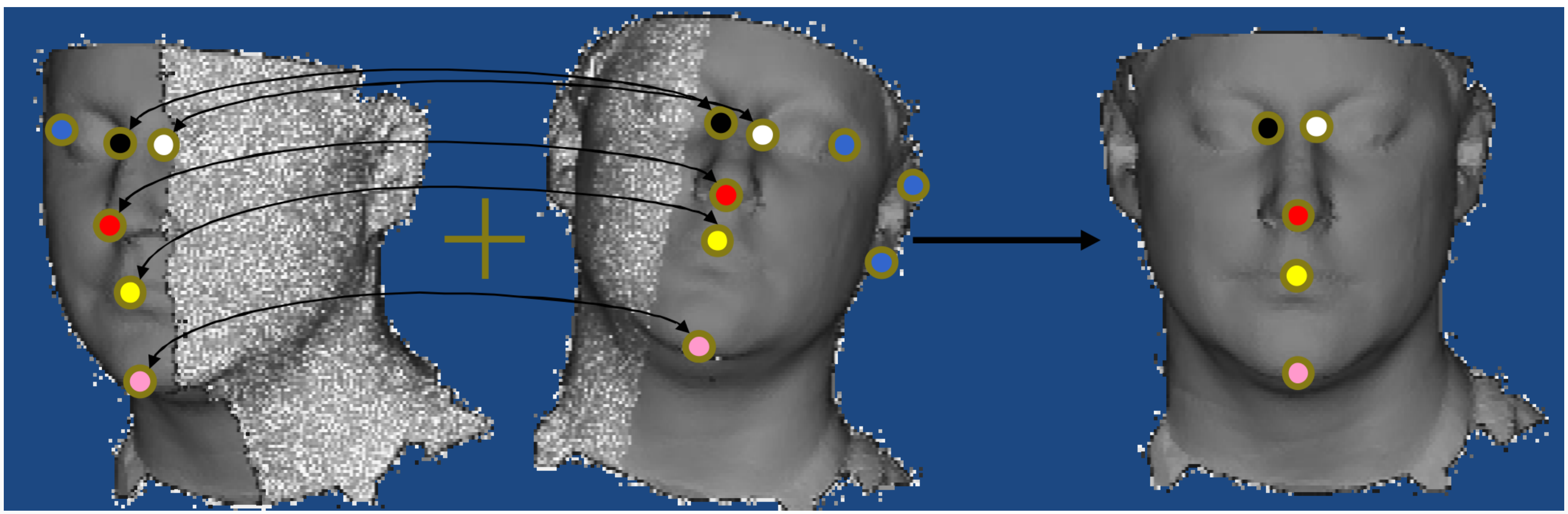


Partially Overlapping Scans

Shape Matching for Model Alignment

Approach

- Find feature points on the two scans
- Establish correspondences
- Compute the alignment



Partially Overlapping Scans **Aligned Scans**

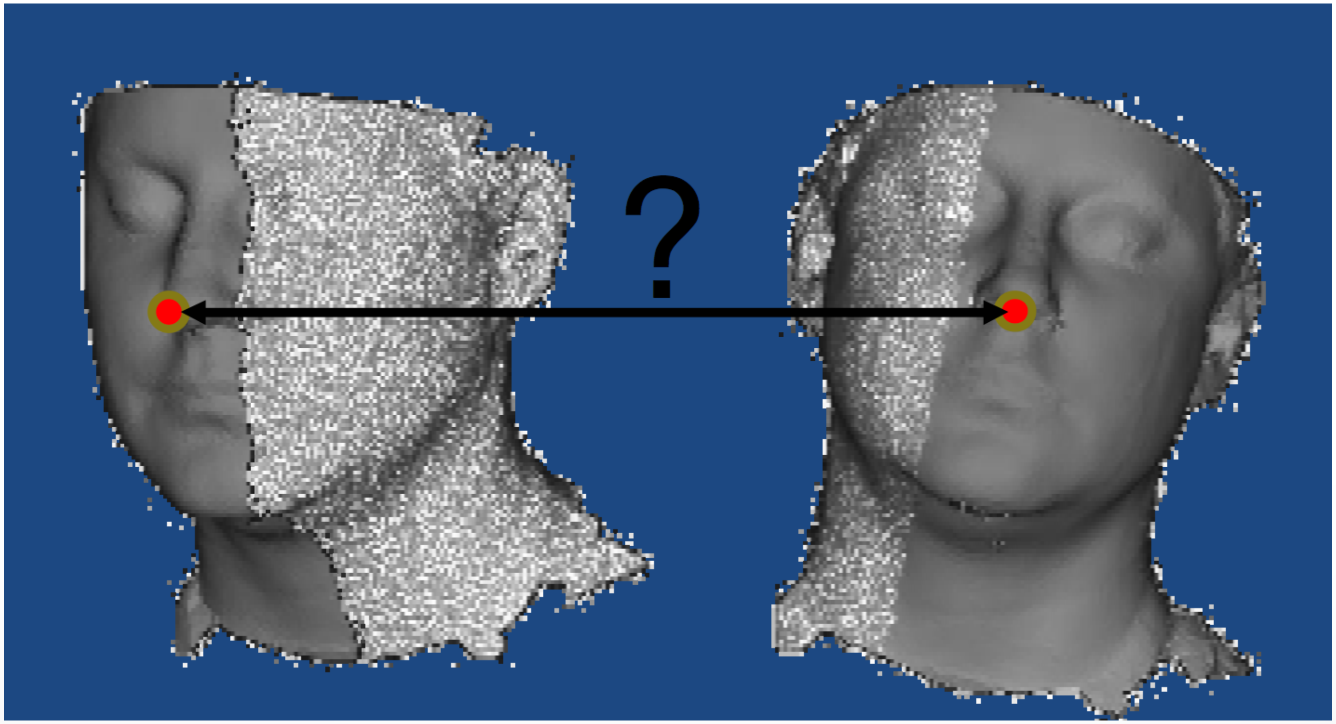
Outline

- **Global Shape Correspondence**
  - Shape Descriptors
  - Alignment
- **Partial Shape Correspondence**
  - From Global to Local
  - Pose Normalization
  - Partial Shape Descriptors
- **Registration**
  - Closed Form Solutions
  - Branch & Bound
  - Random Sample Consensus (RANSAC)

Correspondence

Goal

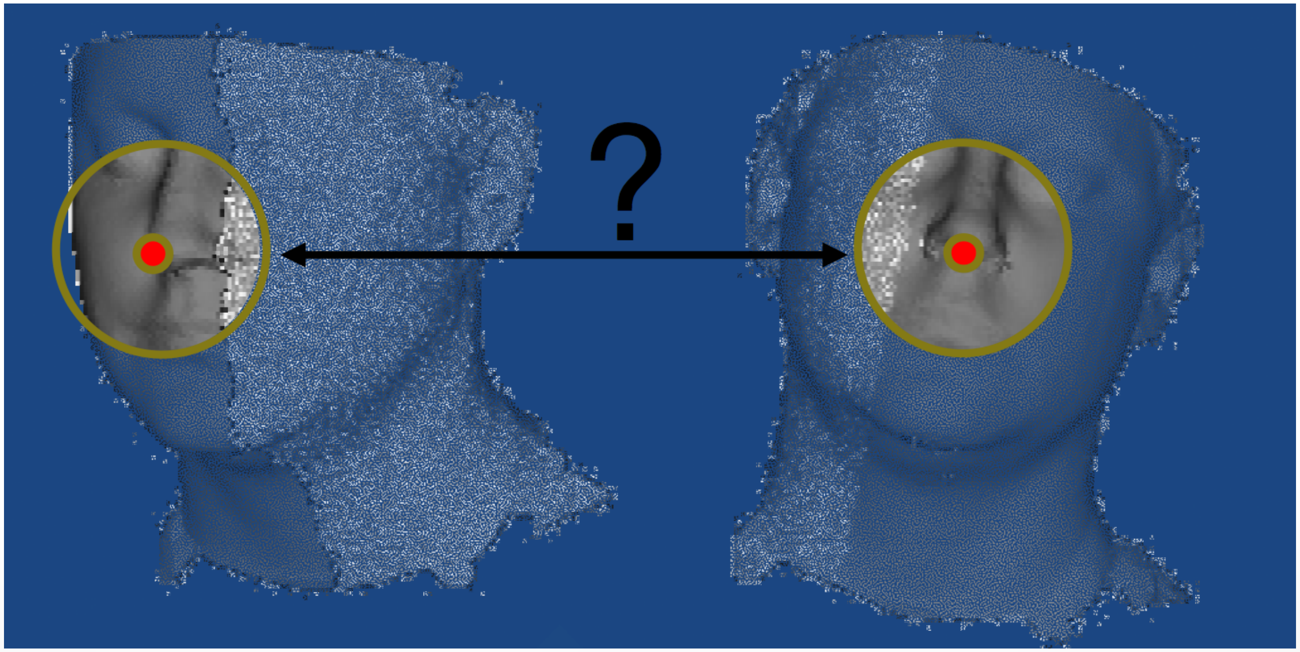
- Identify when two points on different scans represent the same feature



Local **Correspondence**

Goal

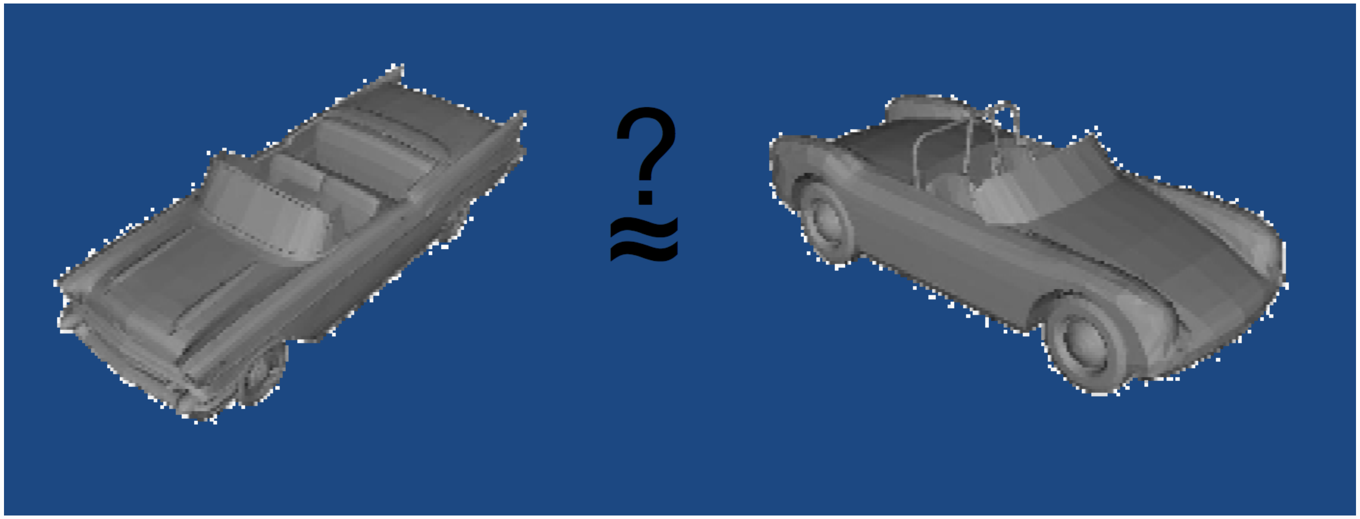
- Identify when two points on different scans represent the same feature
  - Are the surrounding regions similar?



Global **Correspondence**

More Generally:

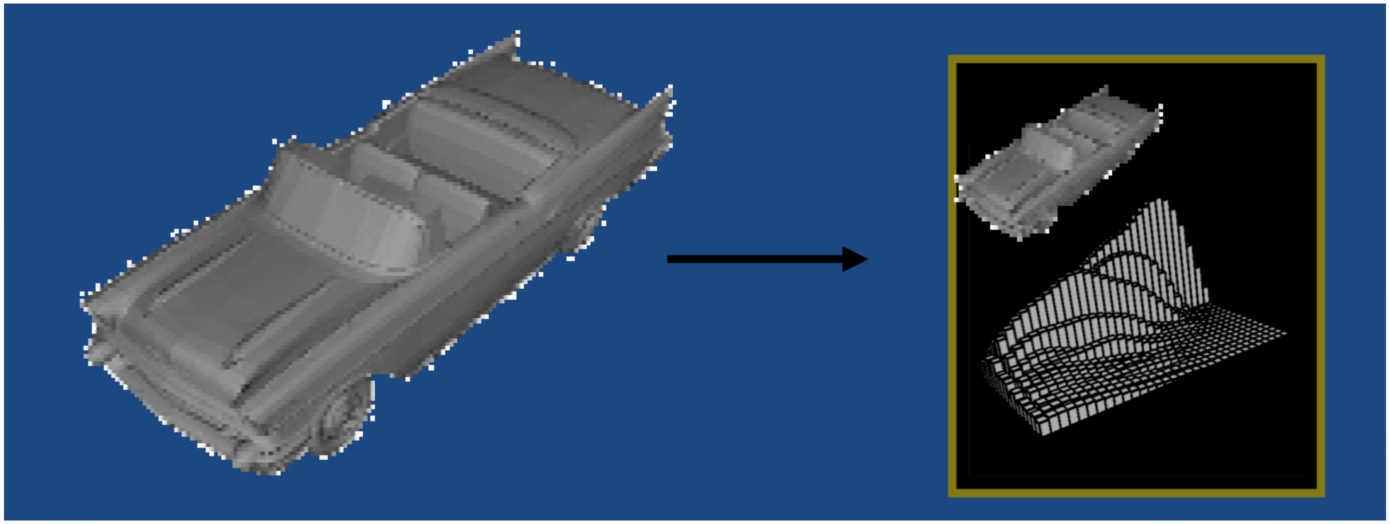
- Given two models, determine if they represent the same/ similar shapes
  - models can have different representations, tesselations, topologies, etc.



Global **Correspondence**

Approach:

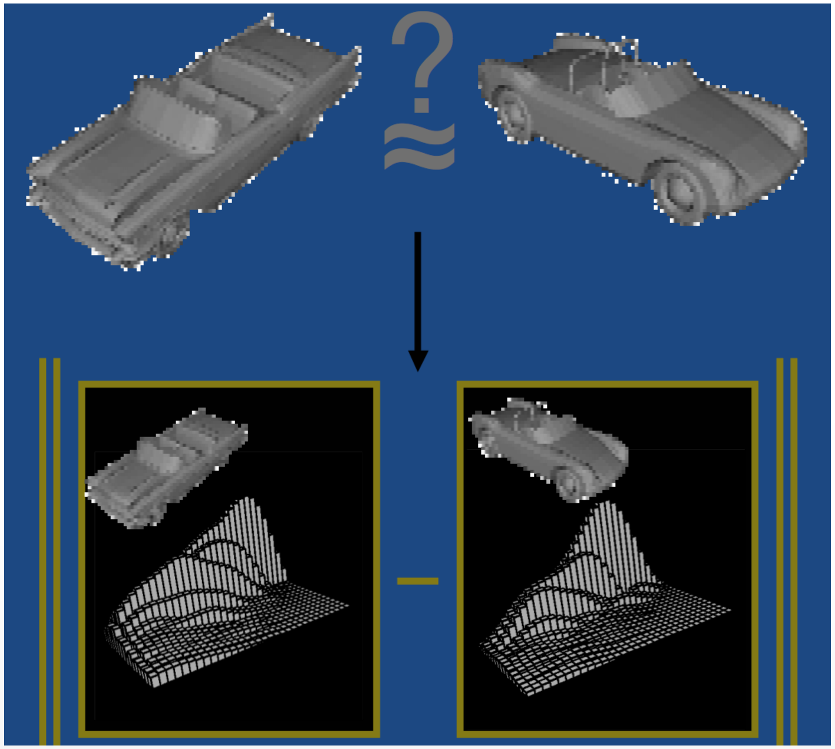
- Represent each model by a shape descriptor:
  - A **structured** abstraction of a 3D model
  - that captures **salient** shape information



Global **Correspondence**

Approach:

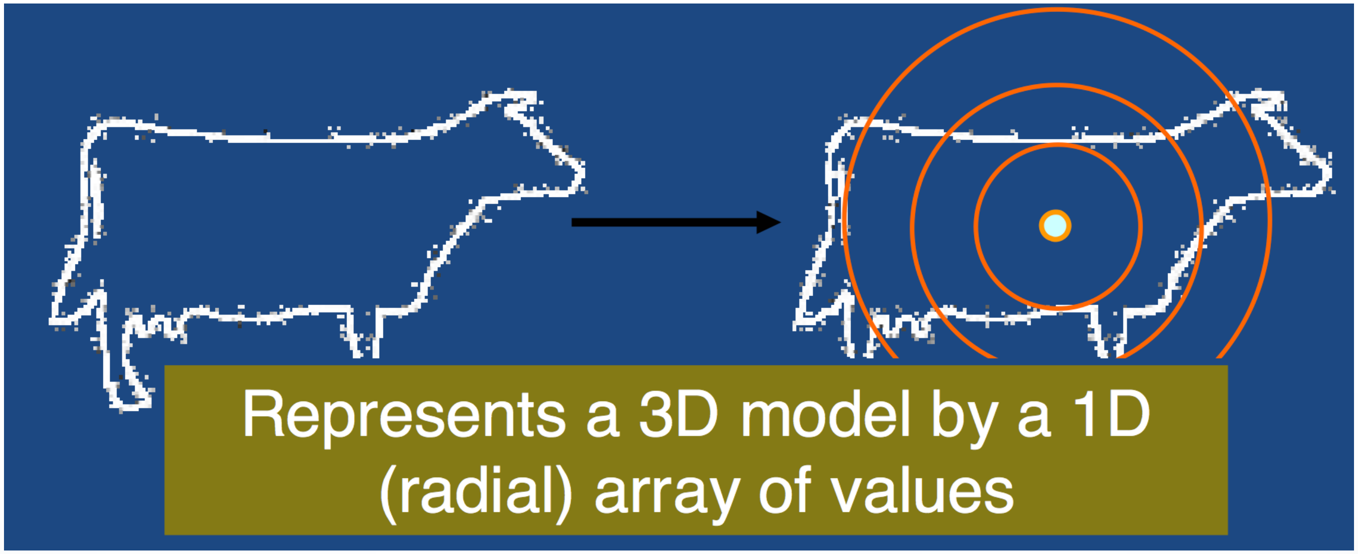
- Represent each model by a shape descriptor:
- Compare shapes by comparing their shape descriptors



Shape Descriptors: Examples

Shape Histograms

- Shape descriptor stores a histogram of how much surface **area** resides within different **concentric shells** in space

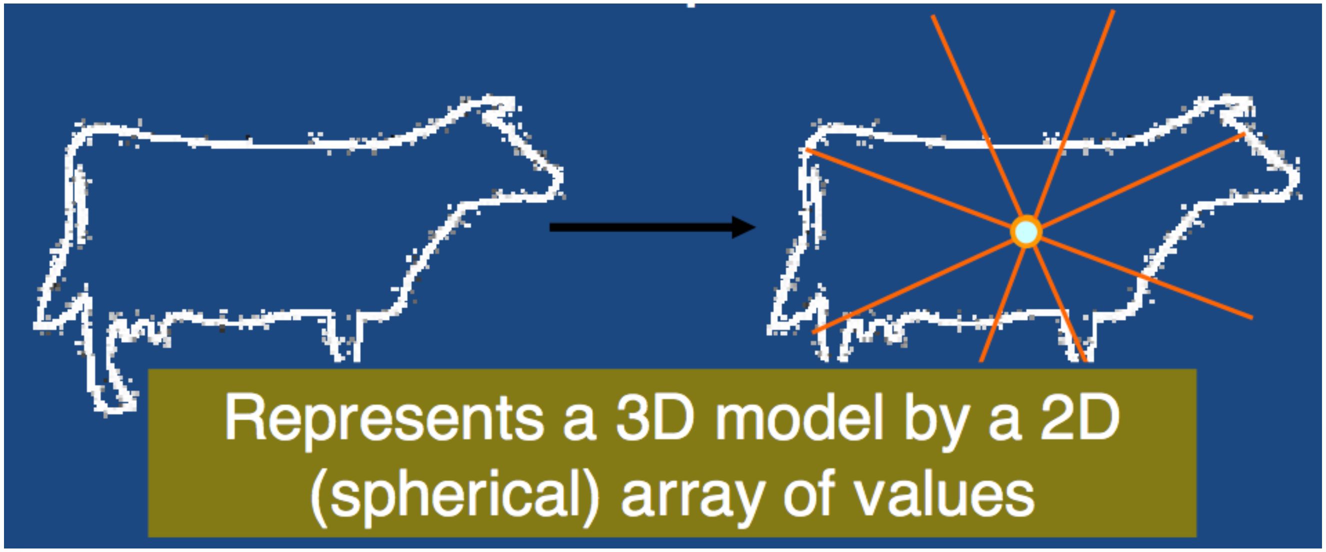


[Ankerst et al. 1999]

Shape Descriptors: Examples

Shape Histograms

- Shape descriptor stores a histogram of how much surface **area** resides within different **sectors** in space

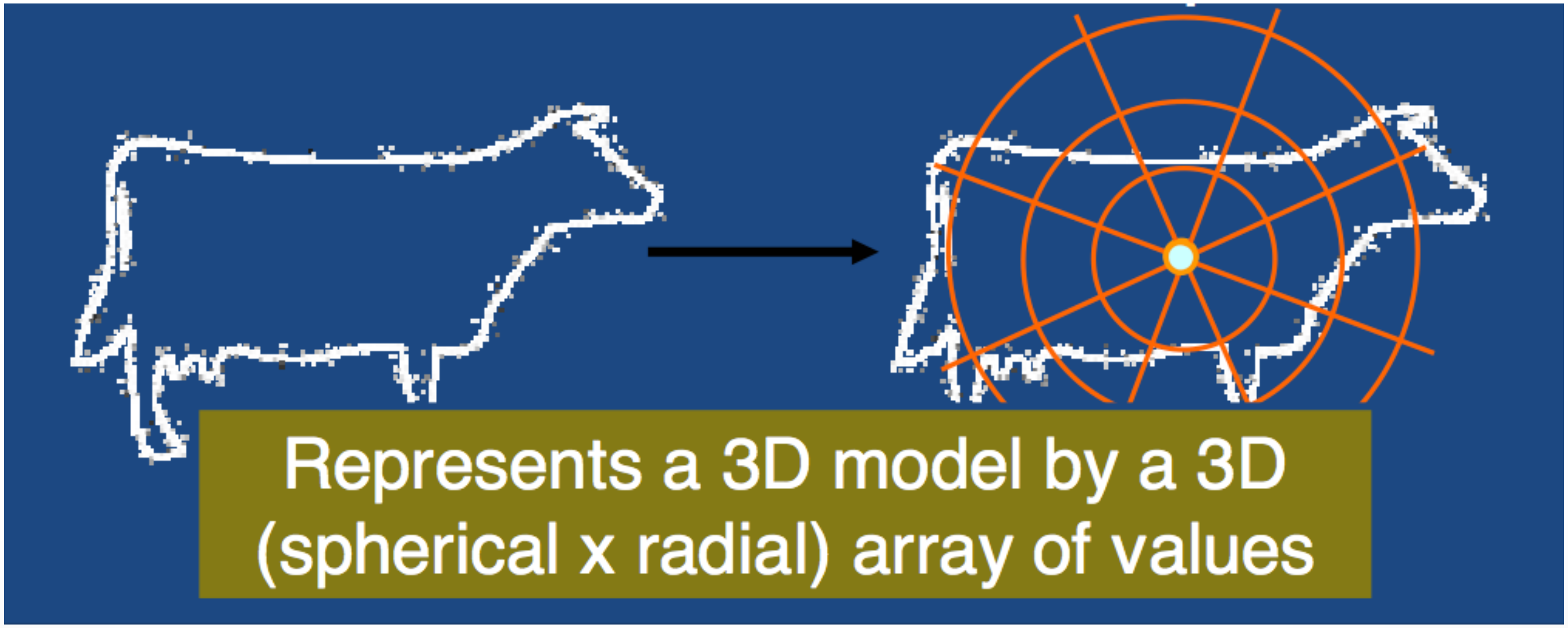


[Ankerst et al. 1999]

Shape Descriptors: Examples

Shape Histograms

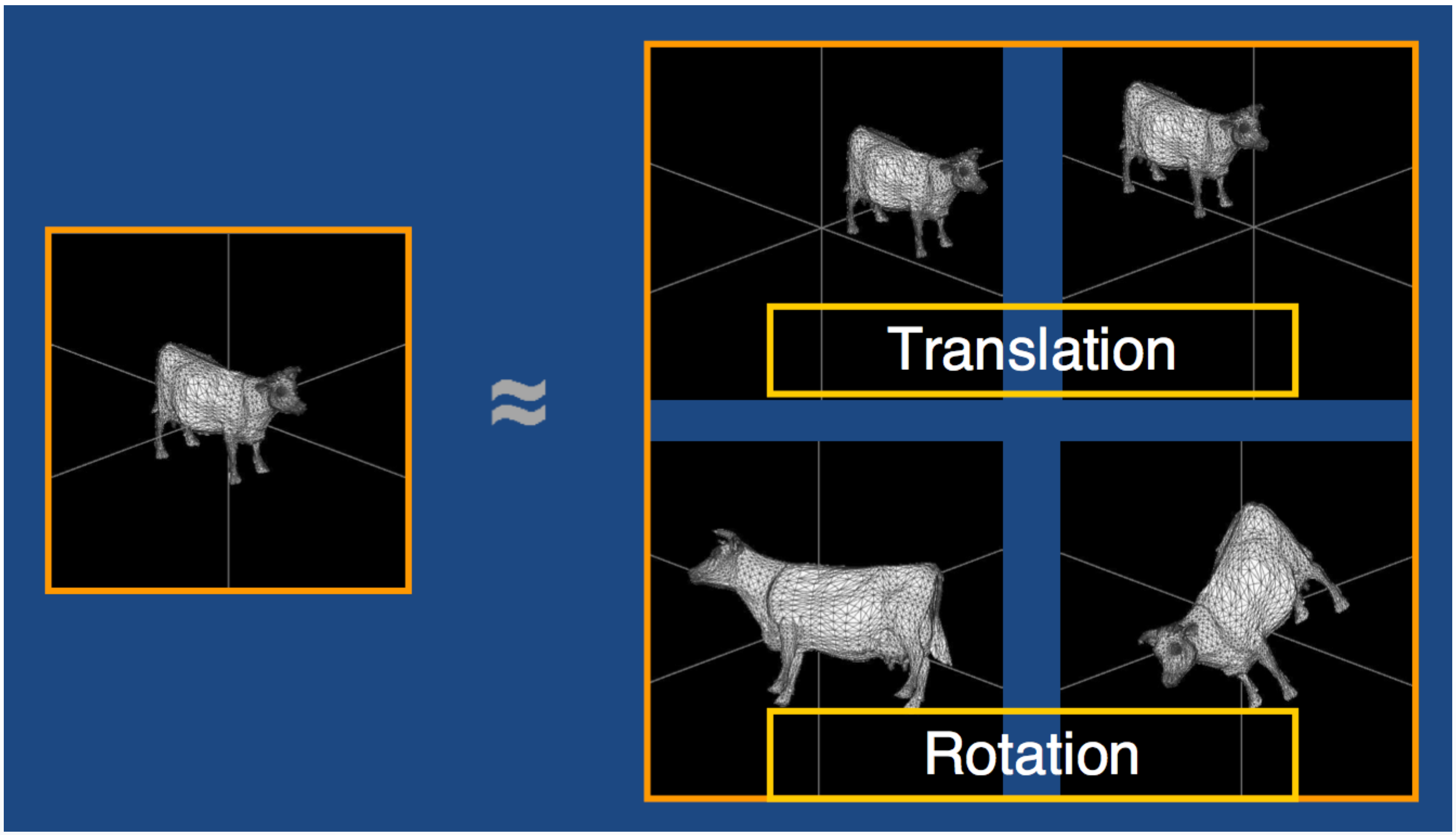
- Shape descriptor stores a histogram of how much surface **area** resides within different **shells** and **sectors** in space



[Ankerst et al. 1999]

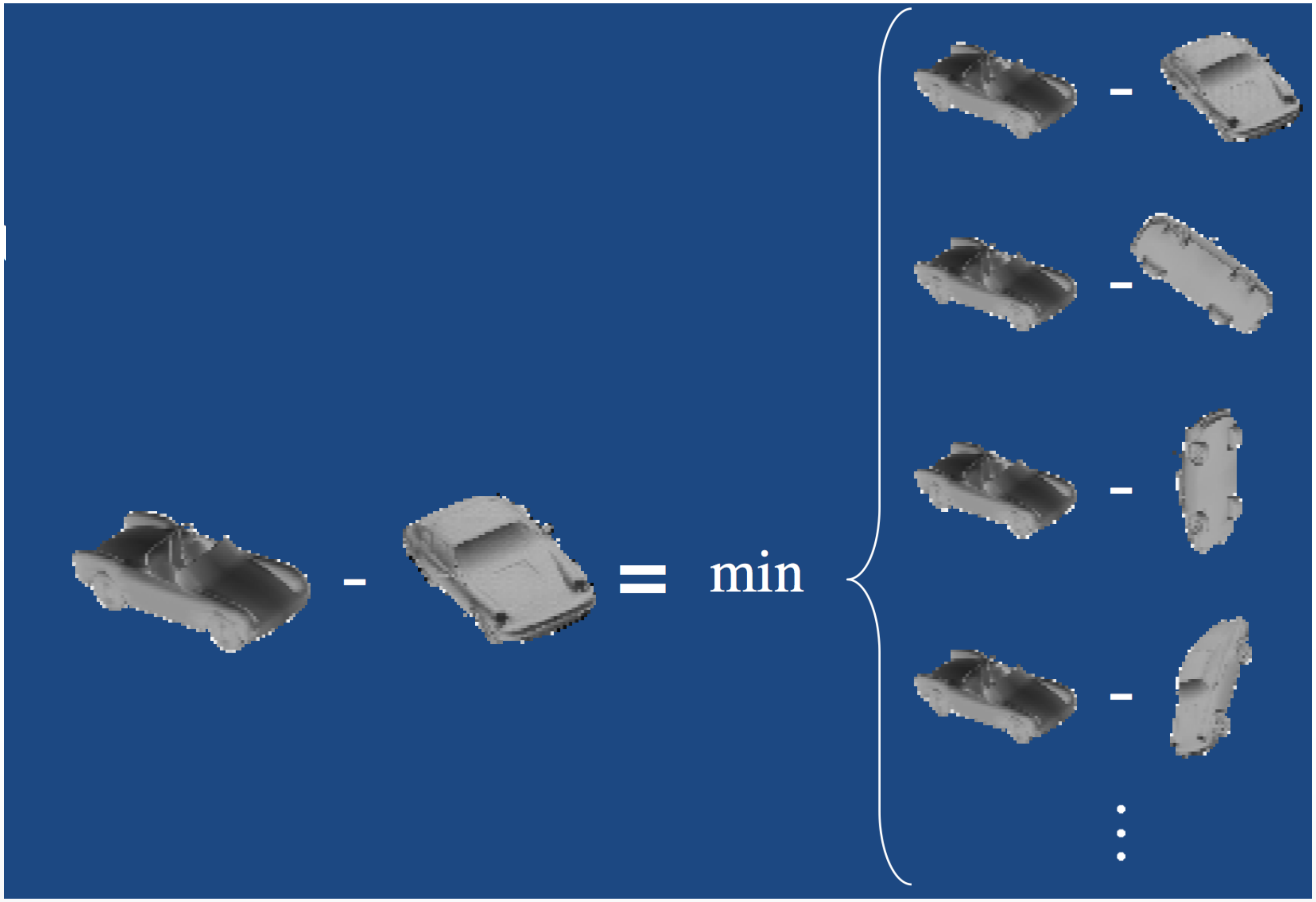
Shape Descriptors: Challenge

- The **shape** of a model does not change when a rigid body transformation is applied to the model.



Shape Descriptors: Challenge

- To compare two models, we need them at their optimal alignment



Shape Descriptors: Alignment

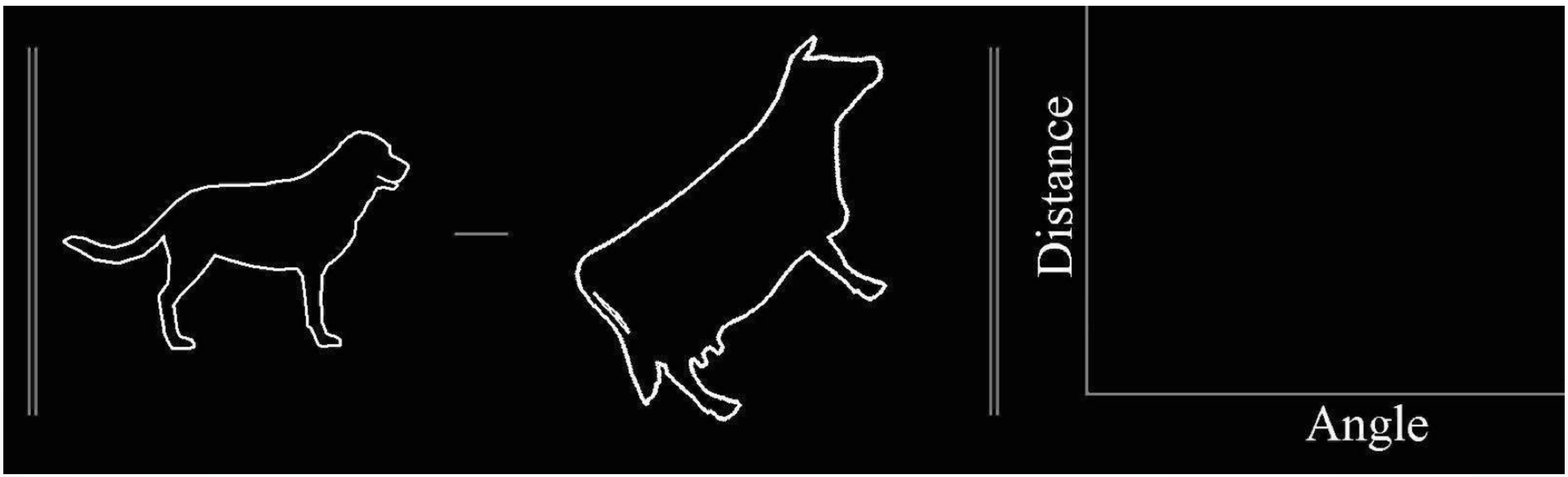
Three general methods:

- Exhaustive Search
- Normalization
- Invariance

Shape Descriptors: Alignment

Exhaustive Search:

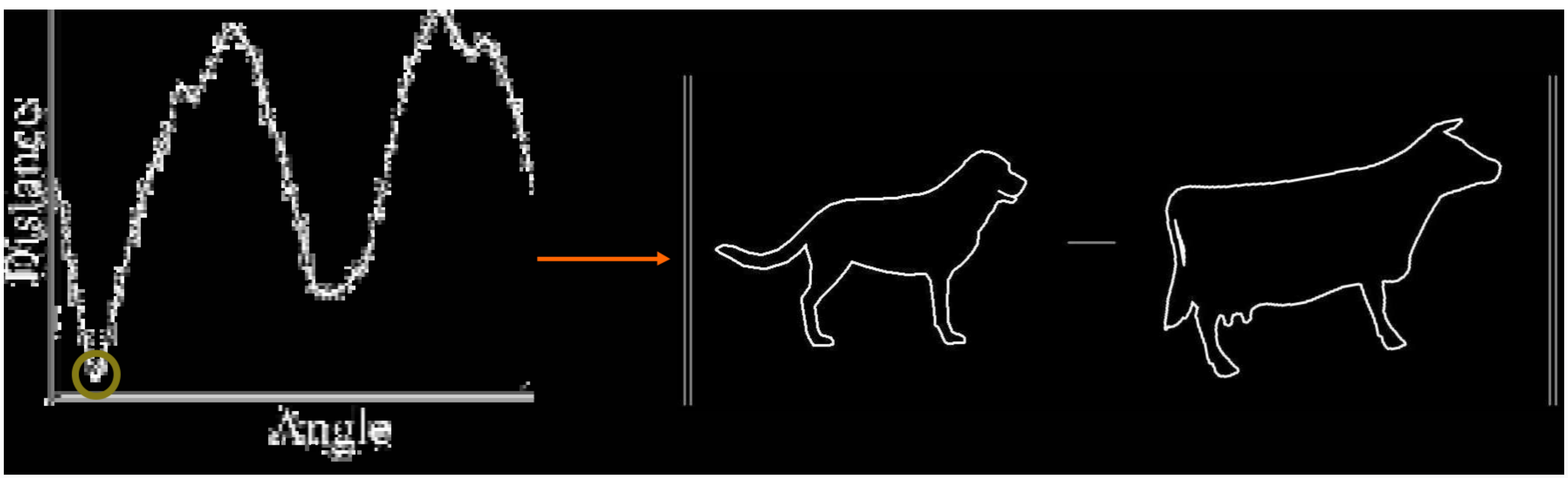
- Compare at all alignments



Exhaustive search for optimal rotation

Exhaustive Search:

- Compare at all alignments
- Correspondence is determined by the alignment at which the models are closest



Exhaustive search for optimal rotation

Shape Descriptors: Alignment

Exhaustive Search:

- Compare at all alignments
- Correspondence is determined by the alignment at which the models are closest

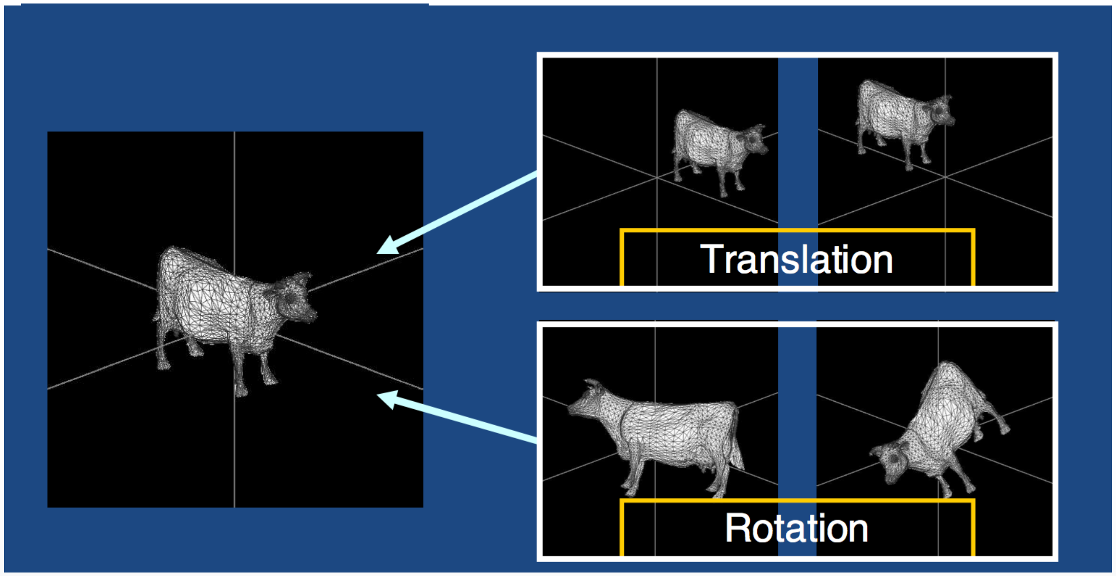
Properties:

- Gives the correct answer (w.r.t. the metric)
- While slow on a single processor, it can be parallelized (Clusters? Multi-Threading? GPU?)

Shape Descriptors: Alignment

Normalization:

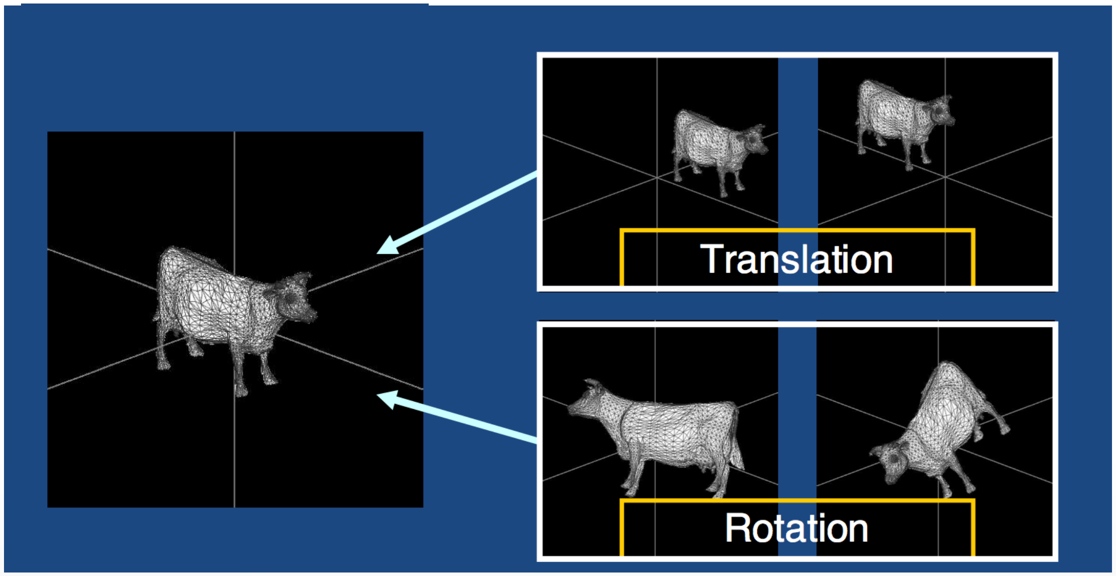
- Put each model into a canonical frame:
  - Translation
  - Rotation



Shape Descriptors: Alignment

Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
  - Rotation



Shape Descriptors: Alignment

Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
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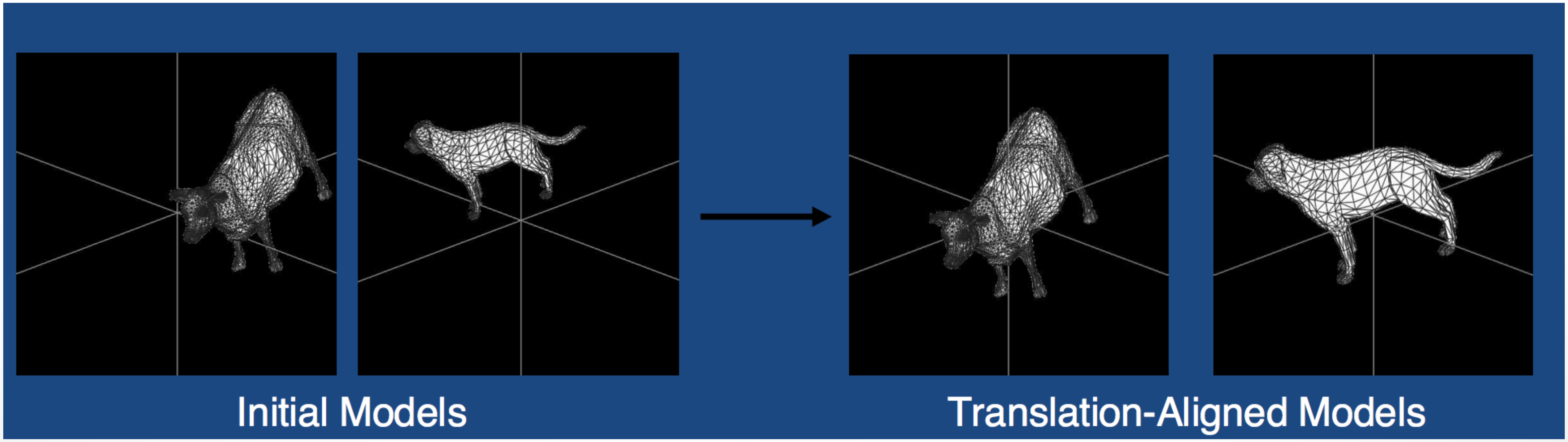
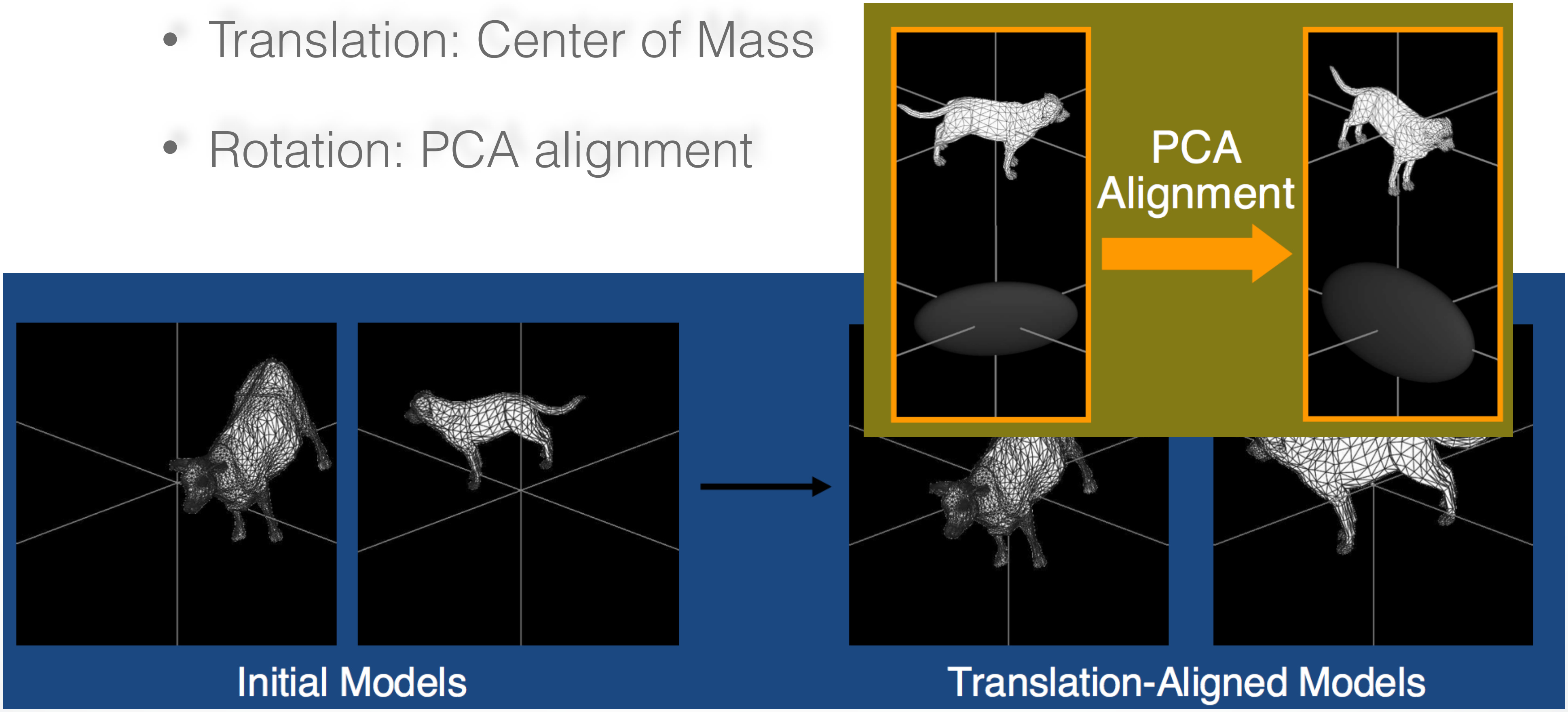


Shape Descriptors: Alignment

Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
  - Rotation: PCA alignment



Shape Descriptors: Alignment

Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
  - Rotation: PCA alignment

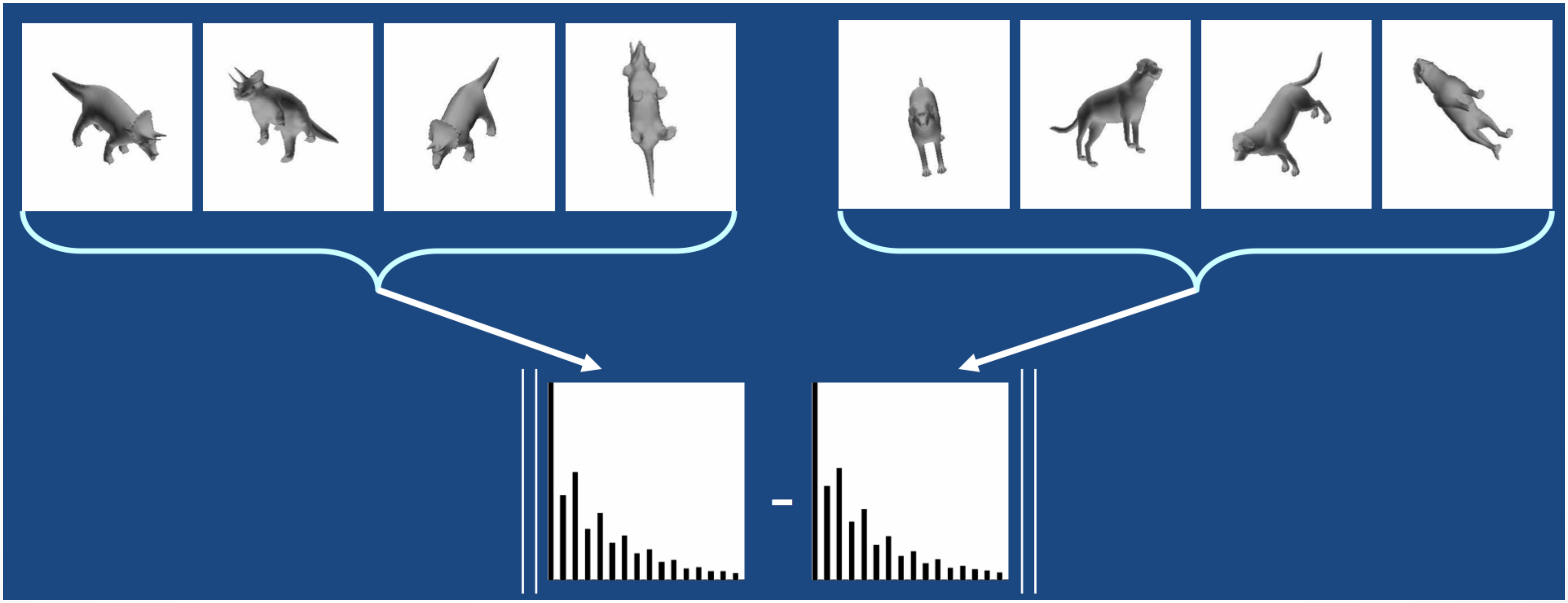
Properties:

- Efficient
- Not always robust
- Not suitable for local feature matching

Shape Descriptors: Alignment

Invariance:

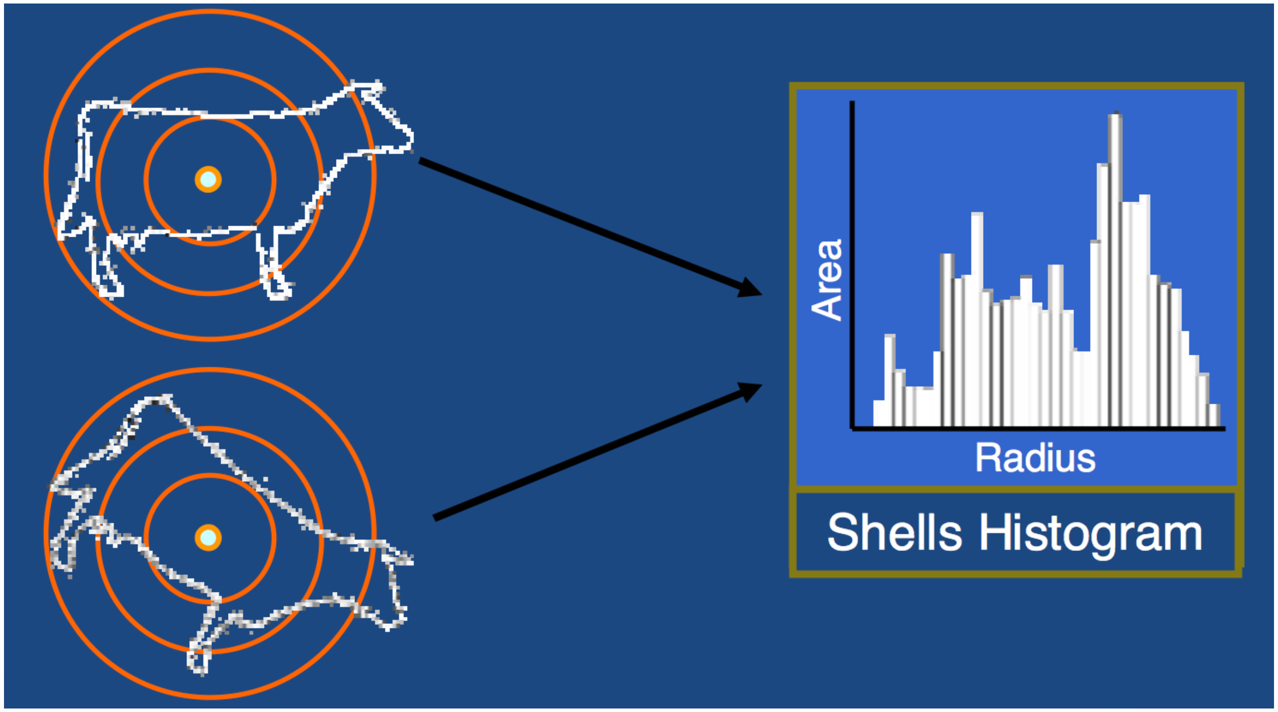
- Represent a model by a shape descriptor that is independent of the pose.



Shape Descriptors: Alignment

Example: Ankerst's Shells

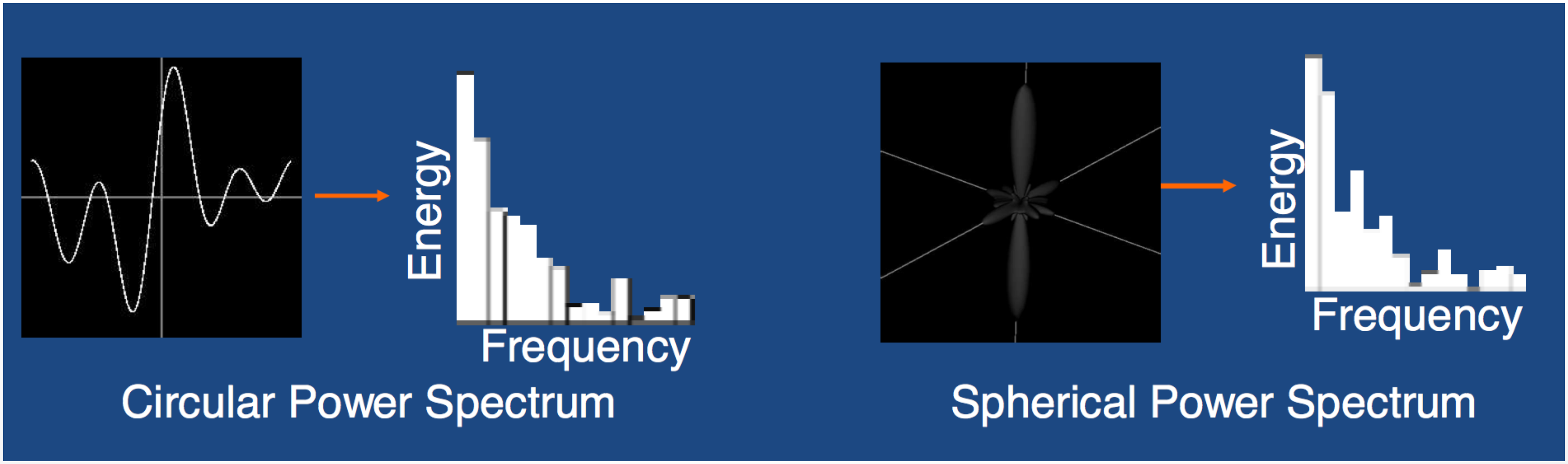
- A histogram of the radial distribution of surface area



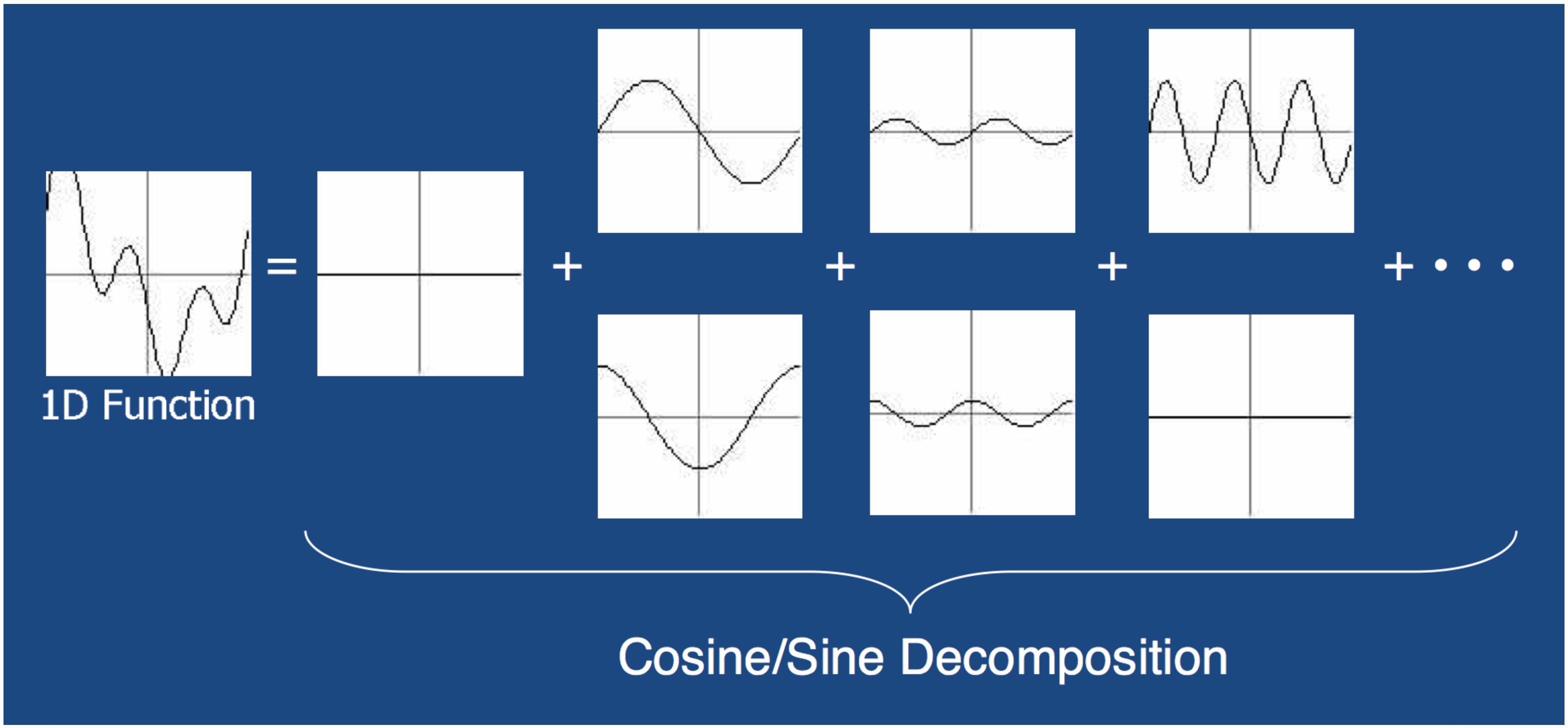
Shape Descriptors: Alignment

Invariance

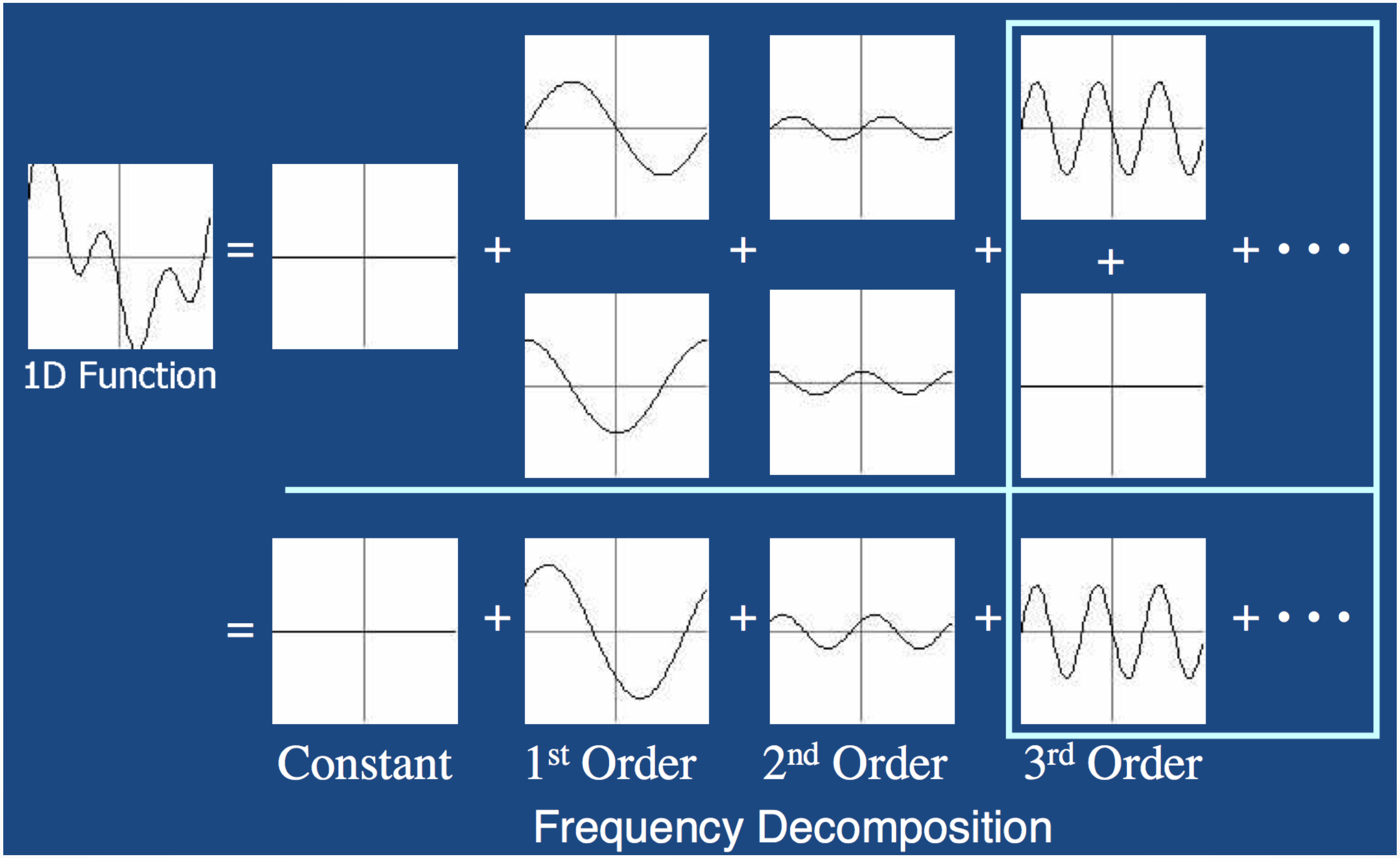
- Power spectrum representation
    - Fourier transform for translations
    - Spherical harmonic transform for rotations



Translation Invariance

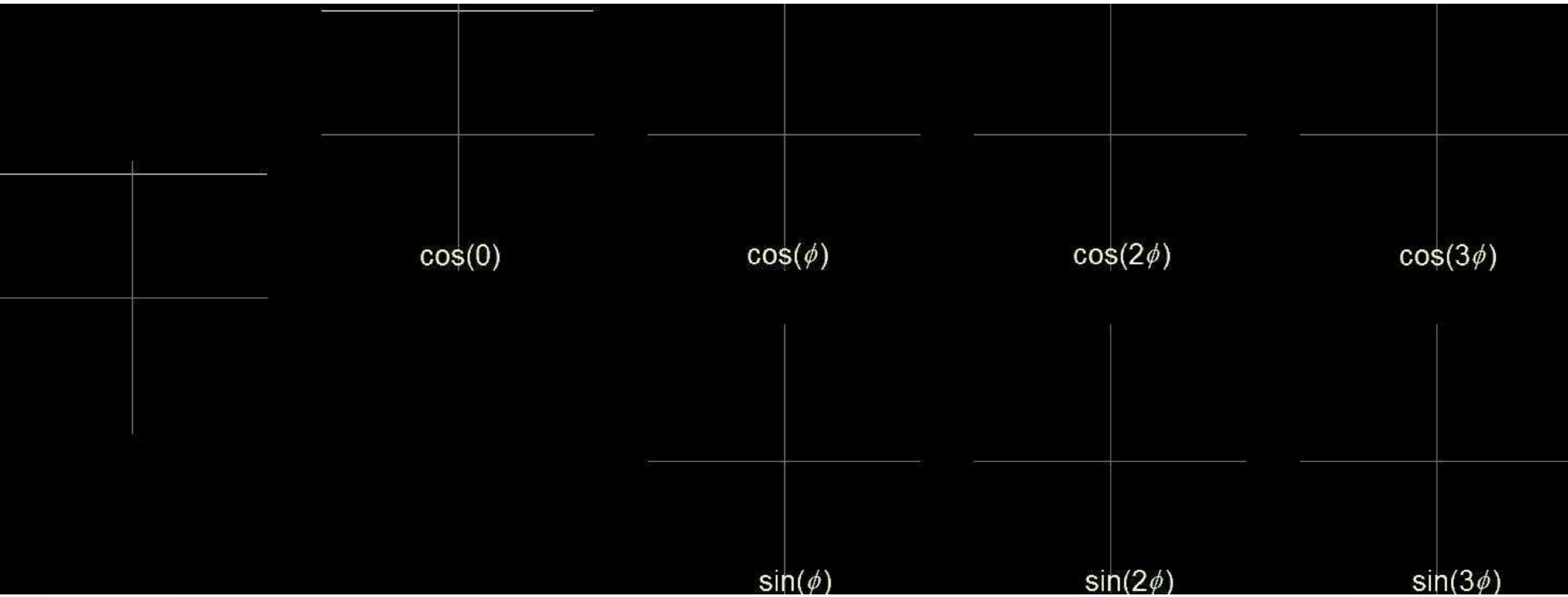


Translation Invariance



Translation Invariance

Frequency subspaces are fixed by rotations:



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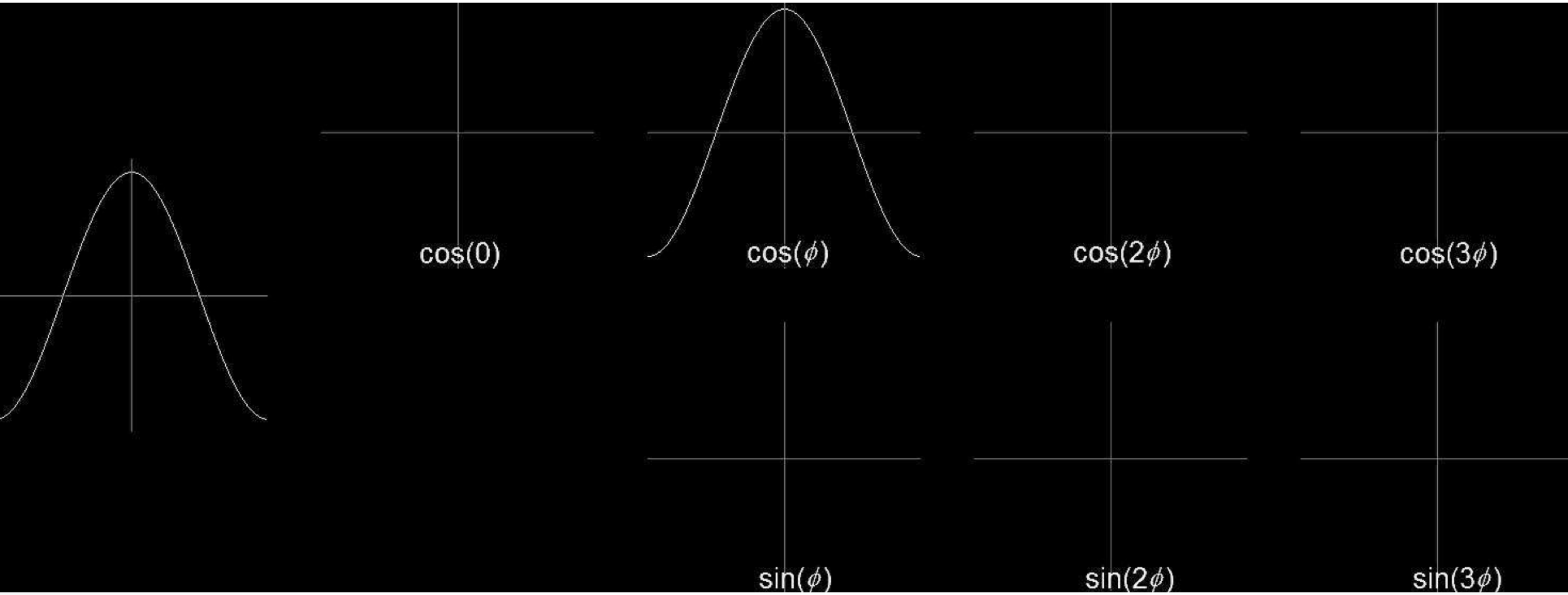

Translation Invariance

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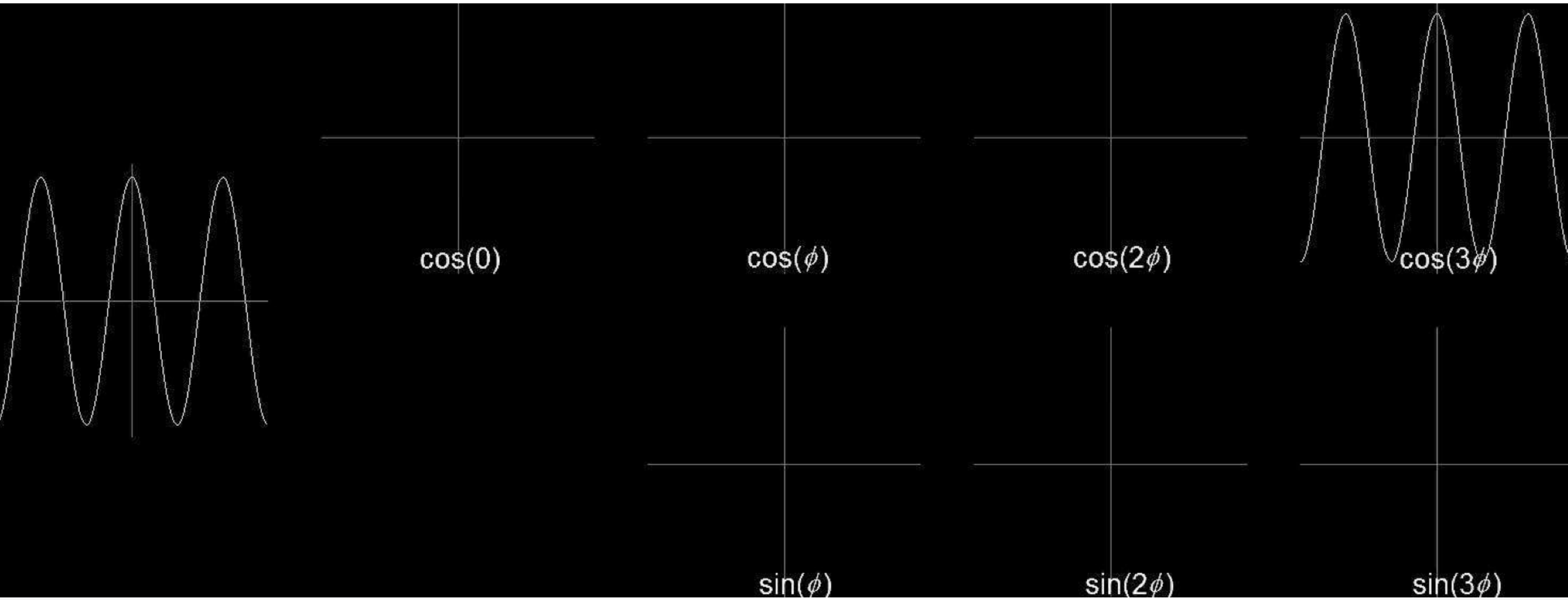
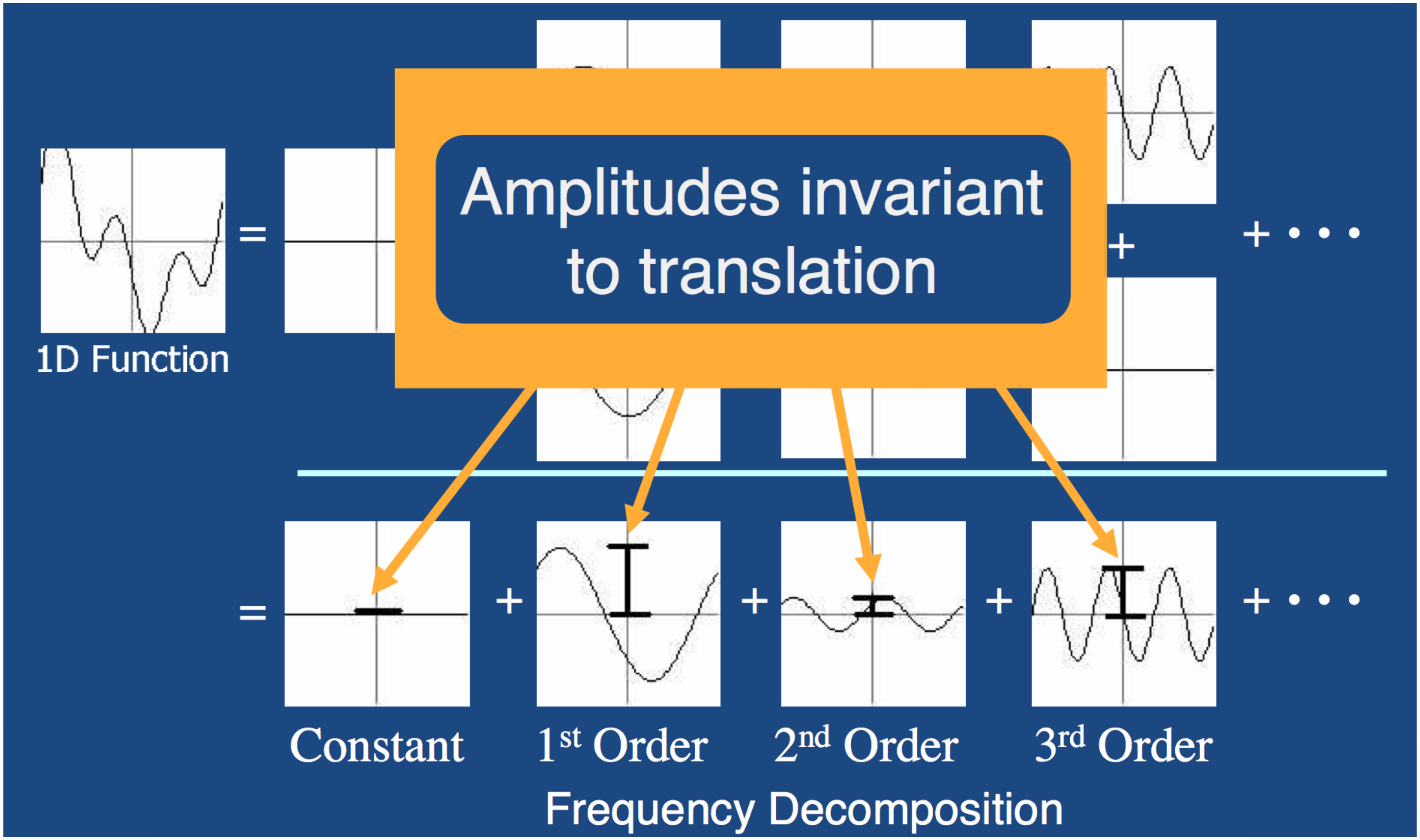
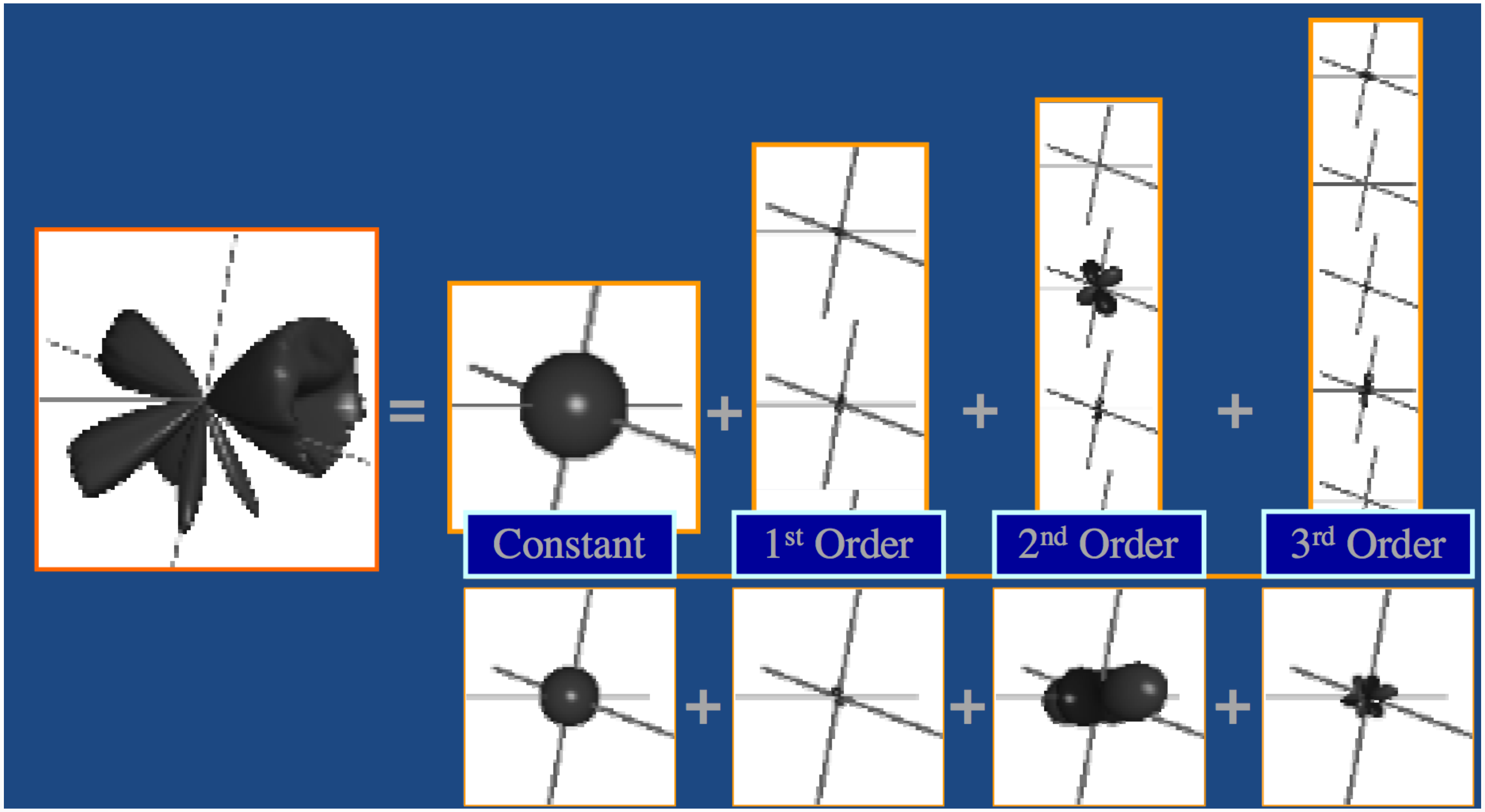


Translation Invariance



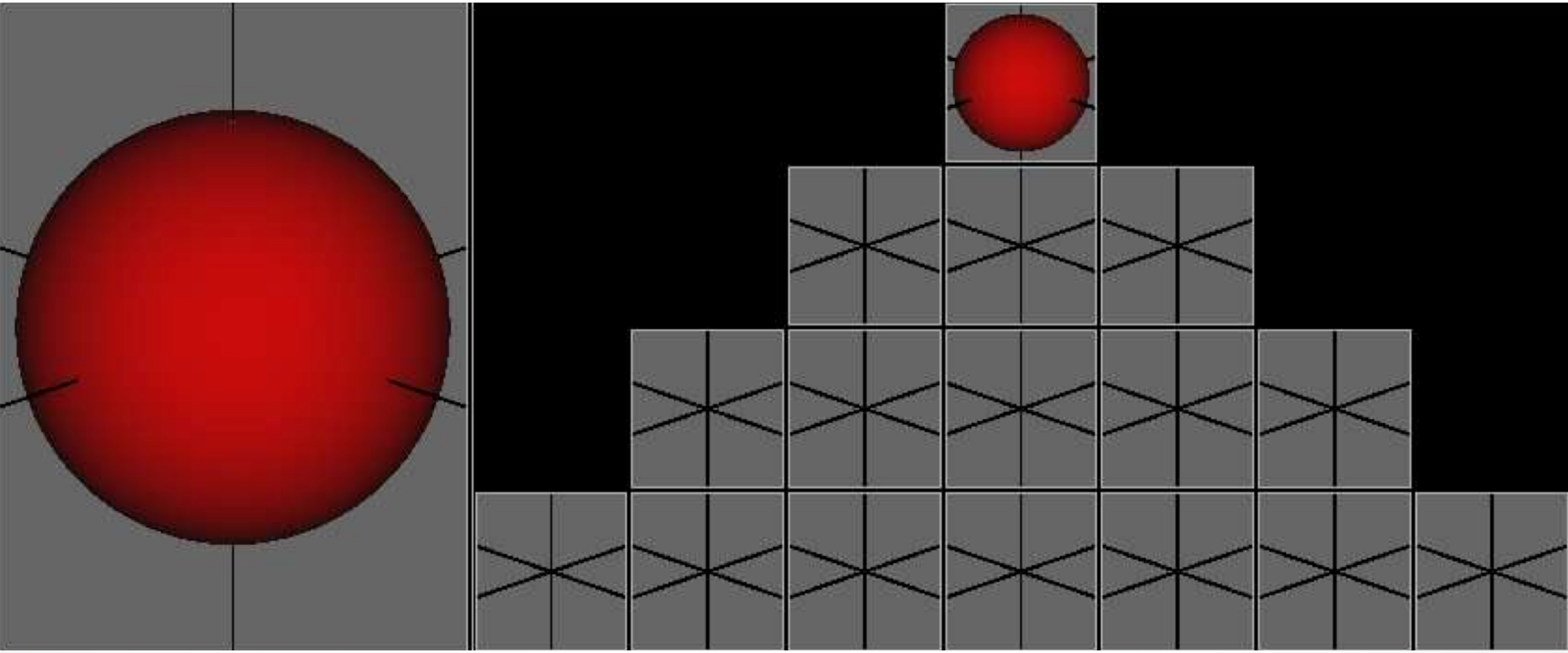
Rotation Invariance

Represent each spherical function as a sum of harmonic frequencies (orders)



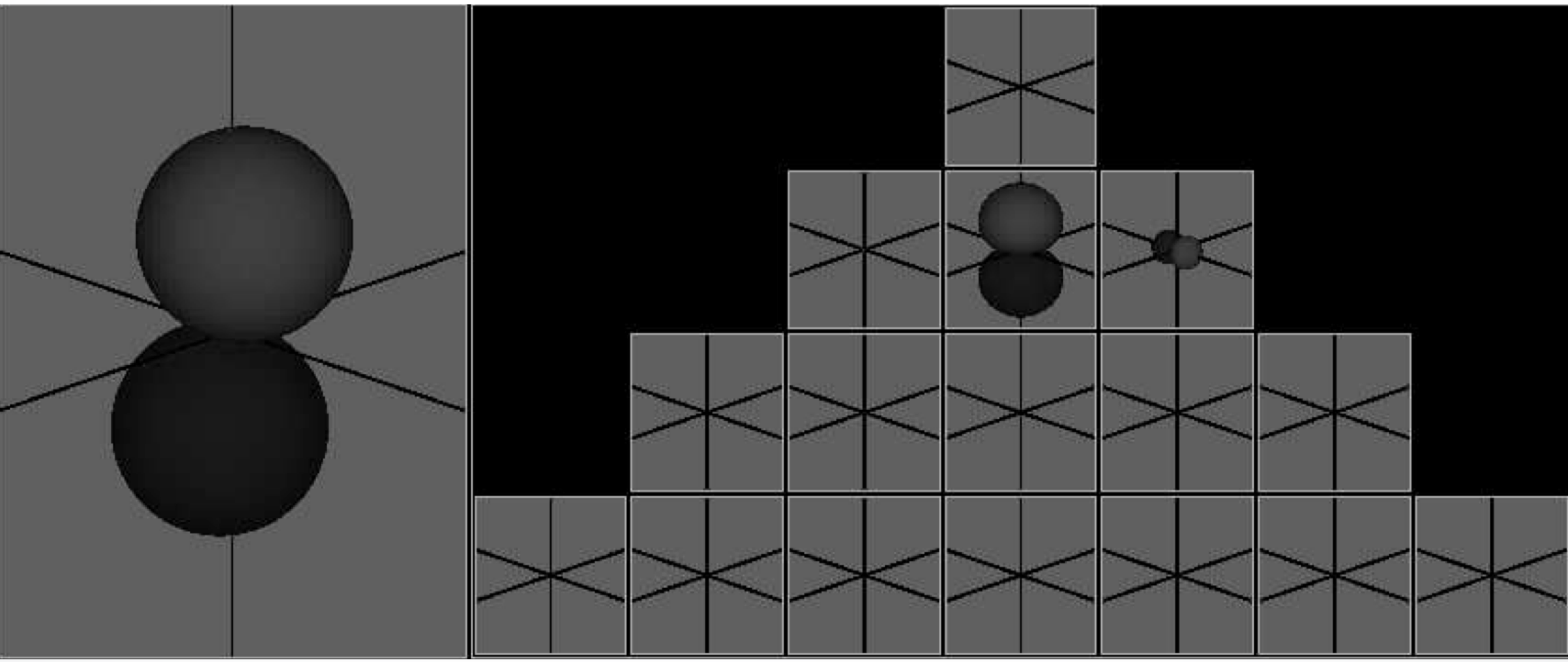
Rotation Invariance

Frequency subspaces are fixed by rotations



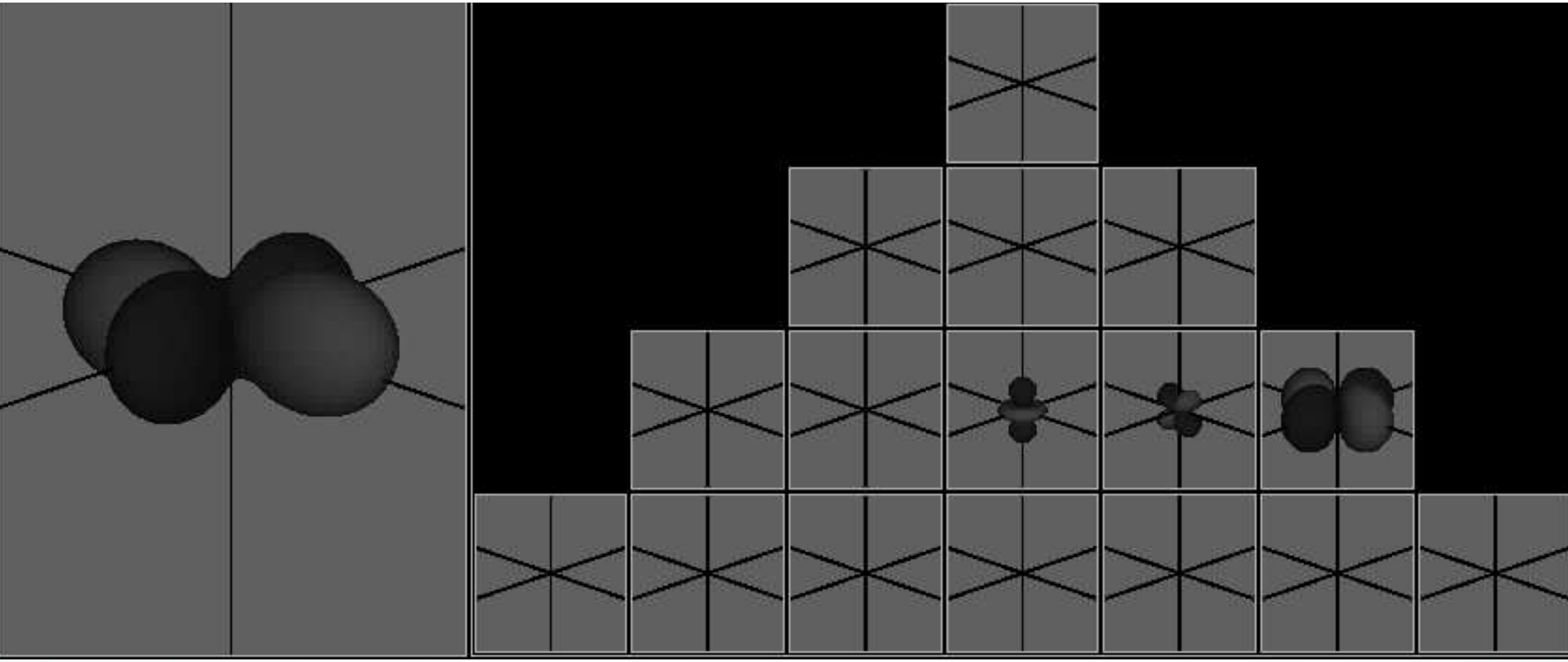
Rotation Invariance

Frequency subspaces are fixed by rotations



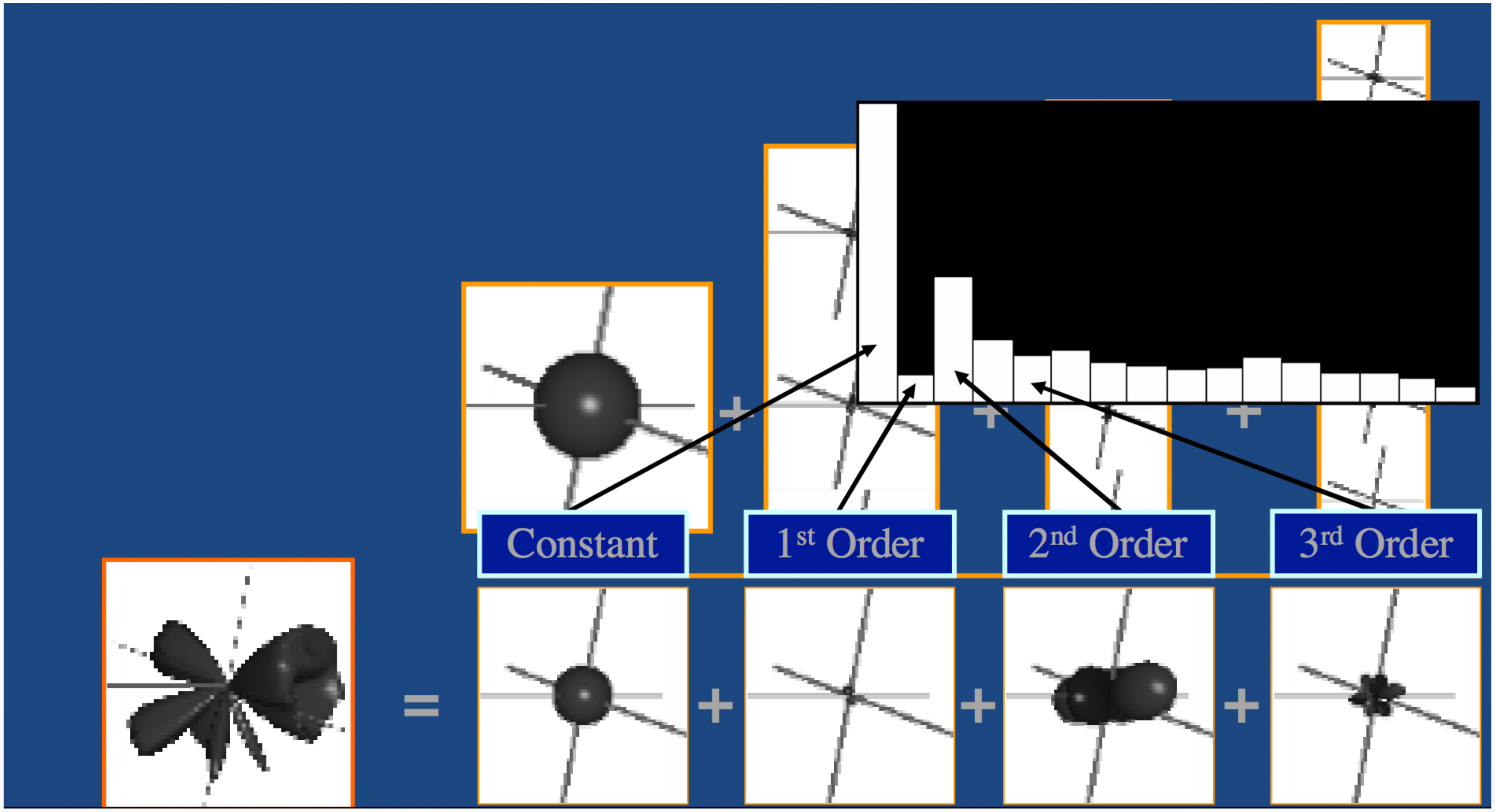
Rotation Invariance

Frequency subspaces are fixed by rotations



Rotation Invariance

Store "how much" (L2-norm) of the shape resides in each frequency to get a rotatin invariant representation



Shape Descriptors: Alignment

Invariance:

- Represent a model by a shape descriptor that is independent of the pose

Properties:

- Compact representation
- Not always discriminating

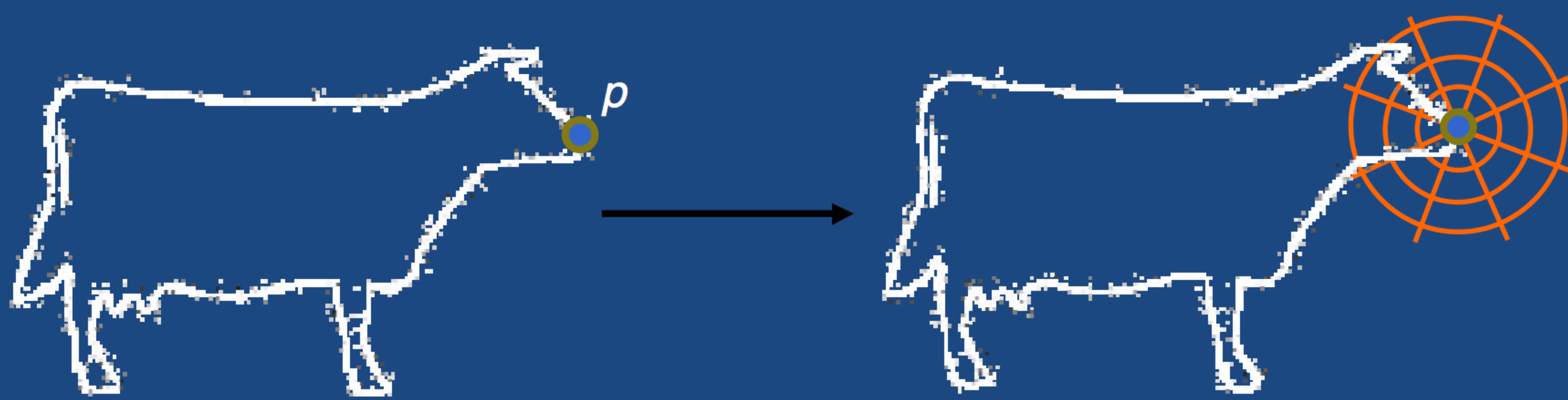
Outline

- **Global Shape Correspondence**
  - Shape Descriptors
  - Alignment
- **Partial Shape Correspondence**
  - From Global to Local
  - Pose Normalization
  - Partial Shape Descriptors
- **Registration**
  - Closed Form Solutions
  - Branch & Bound
  - Random Sample Consensus (RANSAC)

From Global to Local

To characterize the surface about a point p, take global descriptor and:

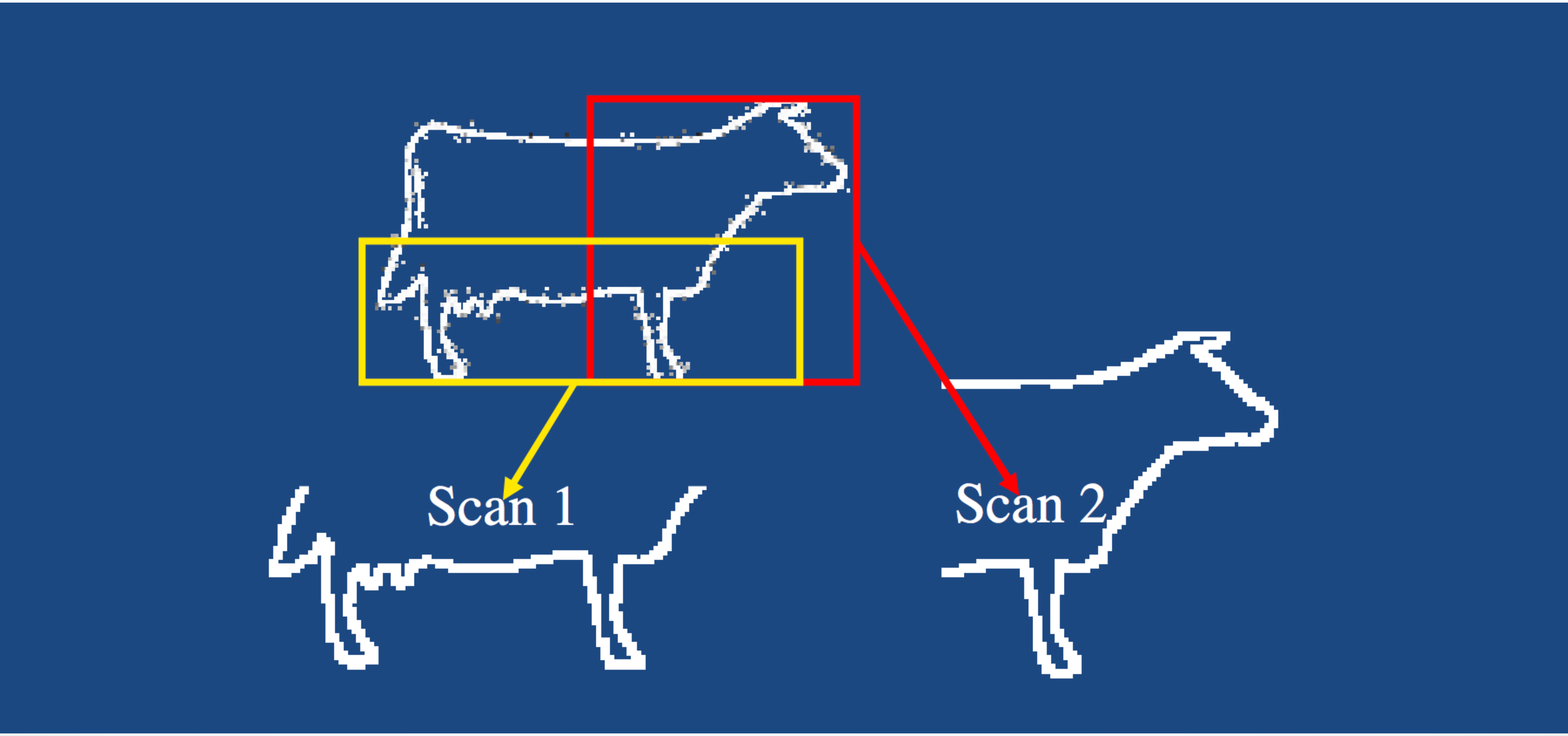
- center it about p (instead of center of mass), and
- restrict the extent to a small region about p



Shape histograms as local shape descriptors

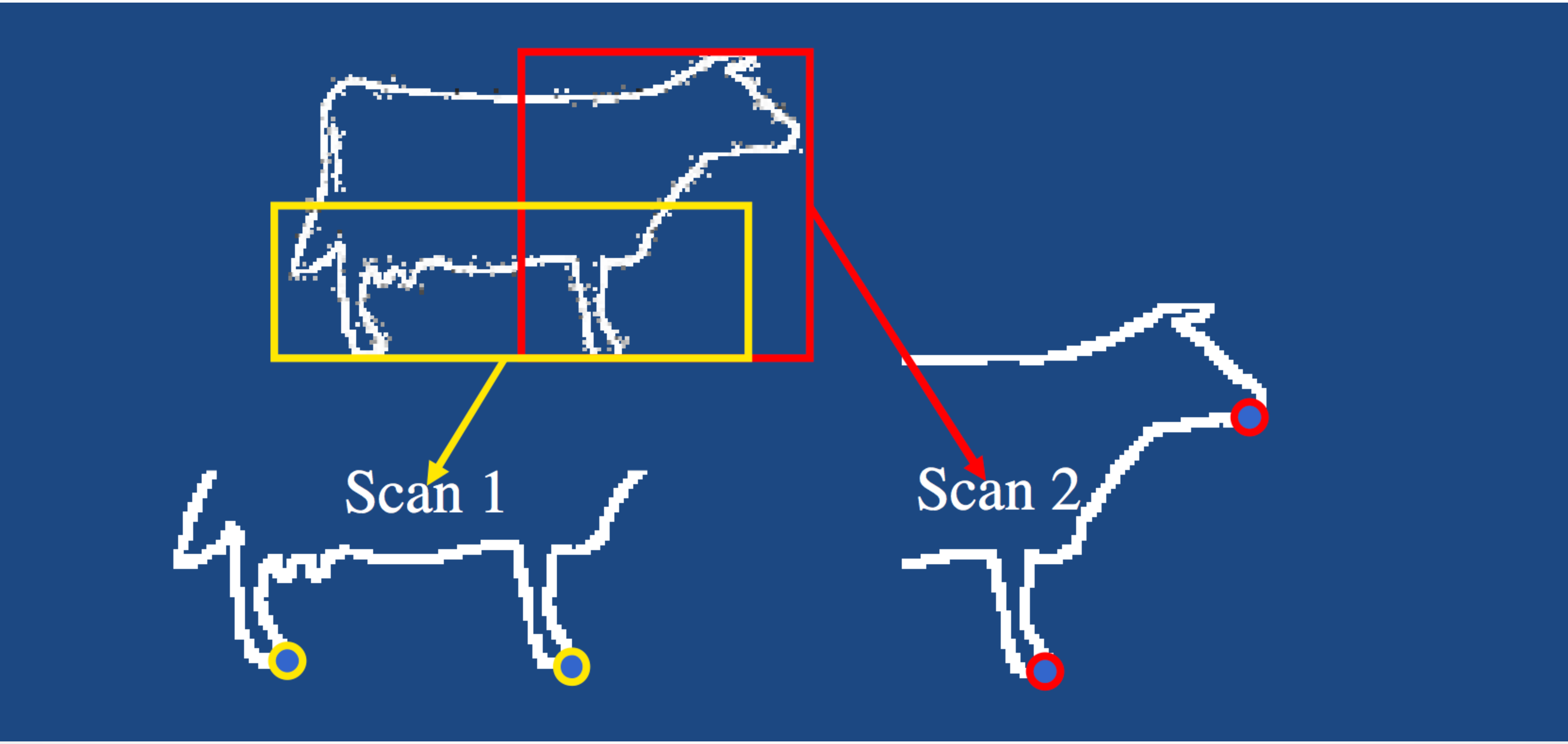
From Global to Local

Given scans of a model:



From Global to Local

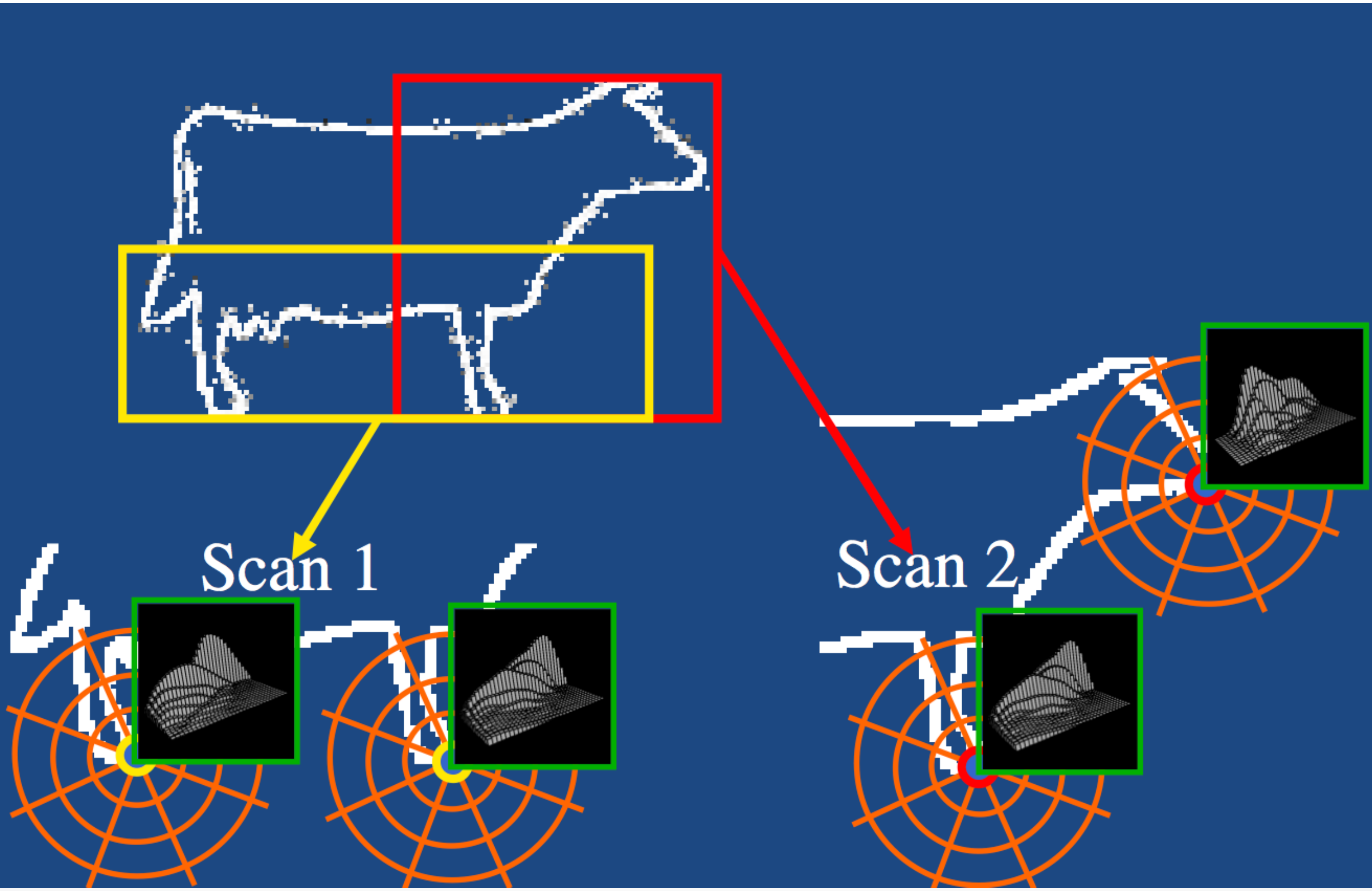
Identify the features



From Global to Local

Identify the features

Computer a local descriptor for each feature

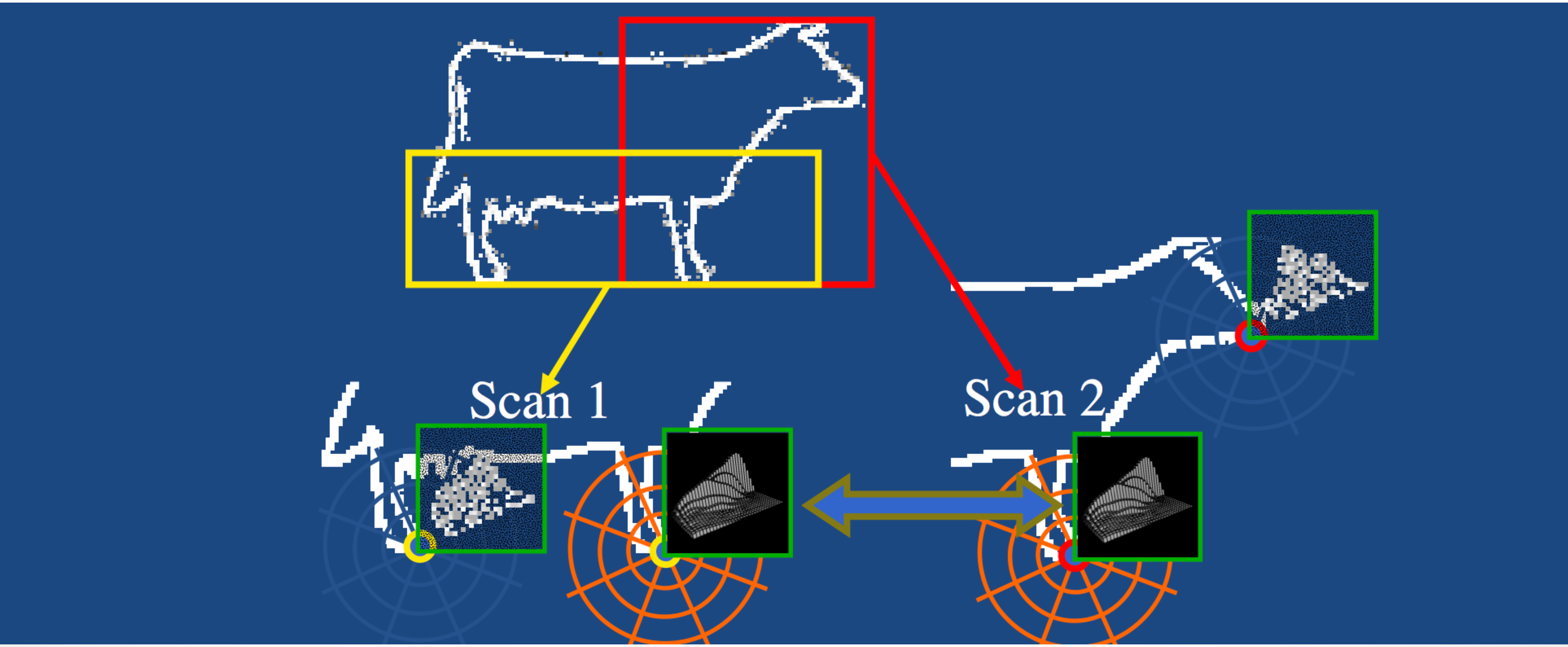


From Global to Local

Identify the features

Computer a local descriptor for each feature

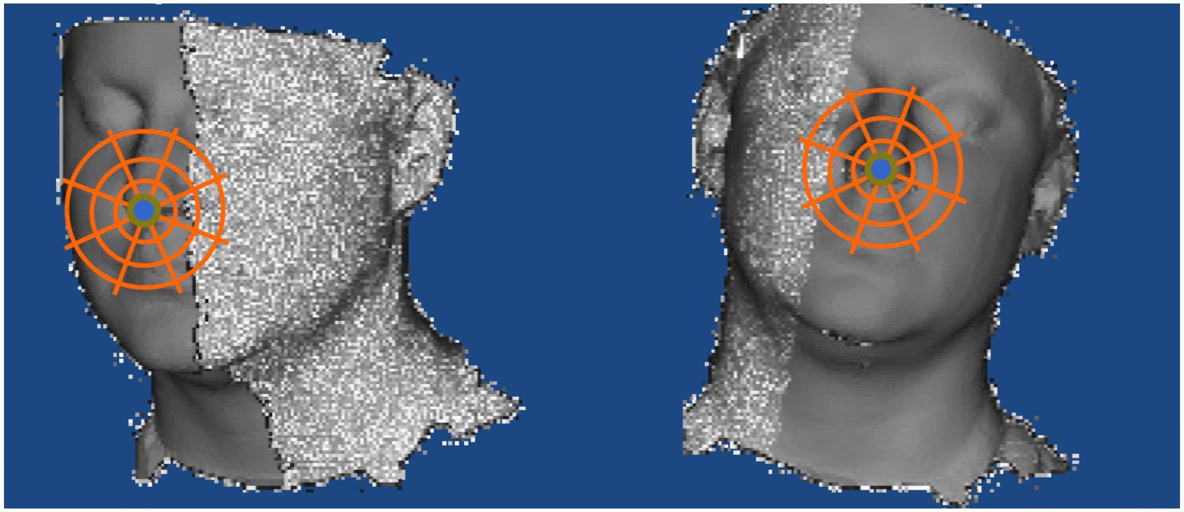
Feature correspond → descriptors are similar



Pose Normalization

From Global to Local

- Translation: Accounted for by centering the descriptor at the point of interest.
- Rotation: We still need to be able to match descriptors across different rotations.



Pose Normalization

Challenge

- Since only parts of the models are given, we cannot use global normalization to align the local descriptors

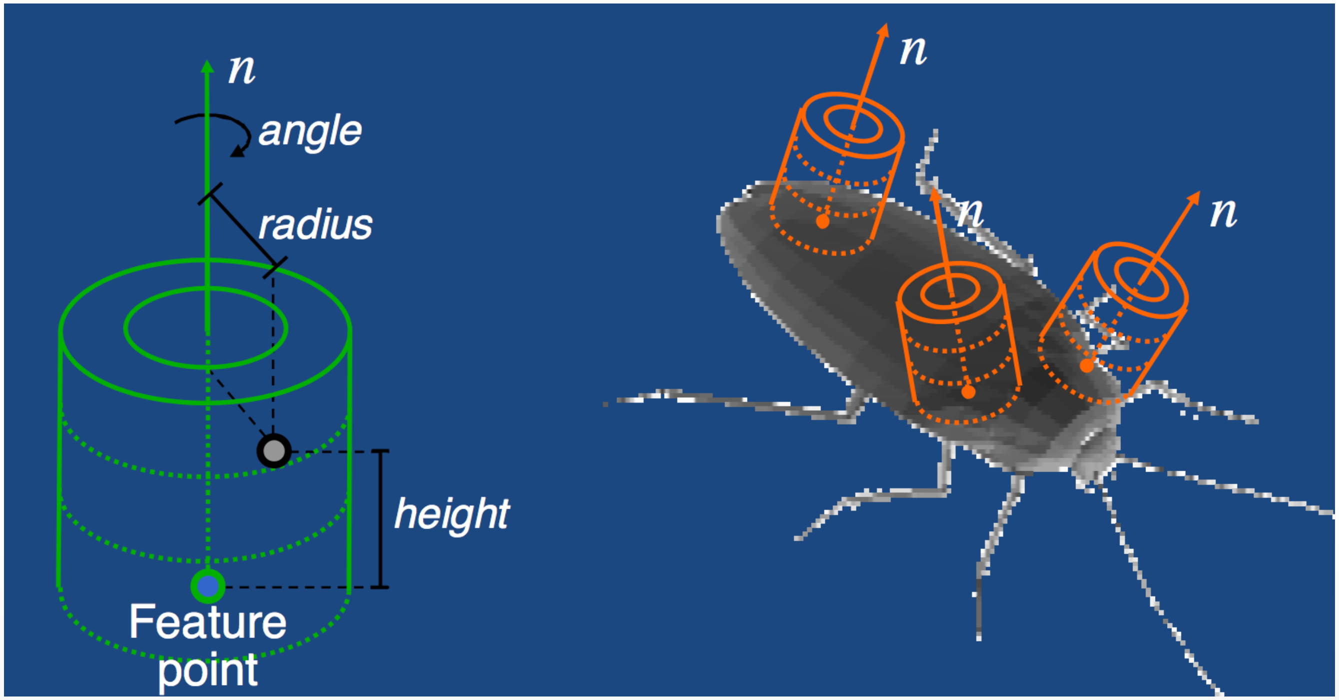
Solutions

- Normalize using **local** information

Local Descriptors: Examples

Variations of Shape Histograms

- For each feature, represent its local geometry in cylindrical coordinates about the normal



Local Descriptors: Examples

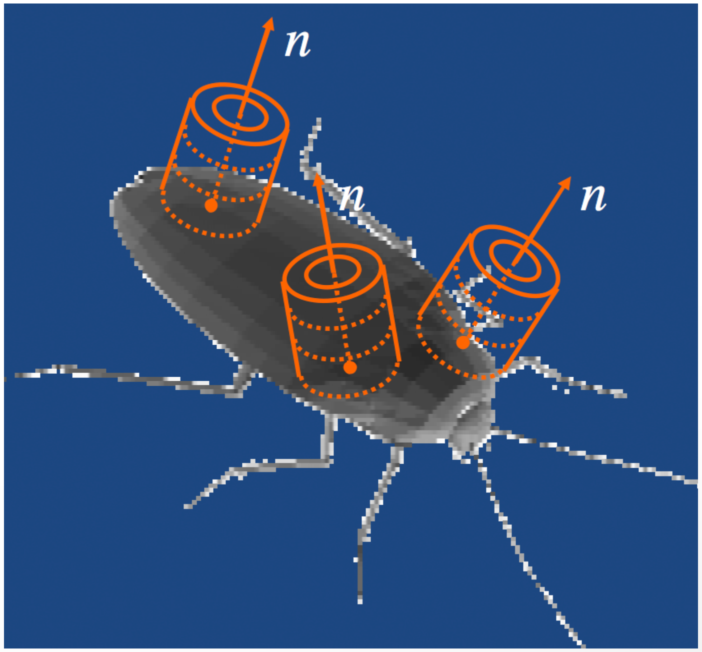
Variations of Shape Histograms

- For each feature, represent its local geometry in cylindrical coordinates about the normal

  - **Spin Images**: Store energy in each normal ring

  - **Harmonic Shape Contexts**: Store power spectrum of each normal ring

  - **3D Shape Contexts**: Search over all rotatinos about the normal for best match



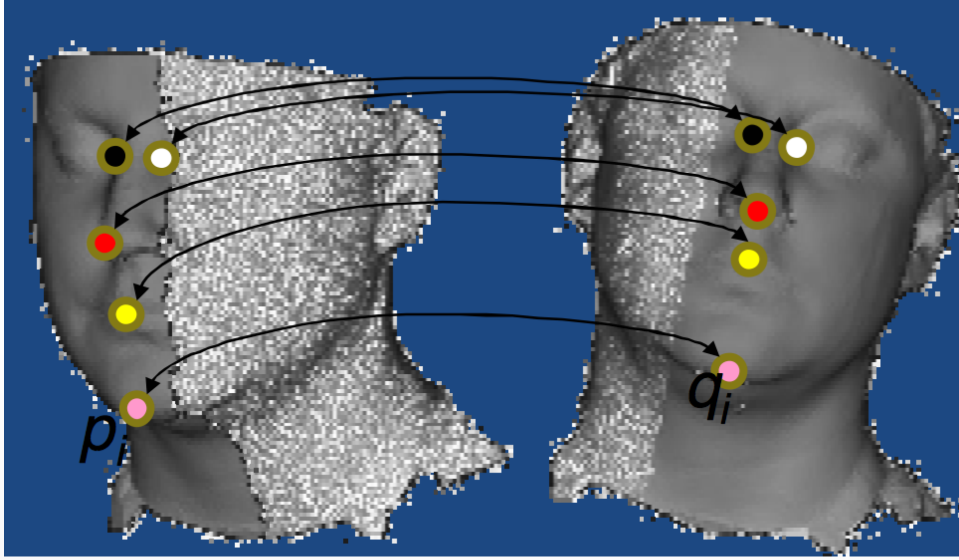
Outline

- **Global Shape Correspondence**
  - Shape Descriptors
  - Alignment
- **Partial Shape Correspondence**
  - From Global to Local
  - Pose Normalization
  - Partial Shape Descriptors
- **Registration**
  - Closed Form Solutions
  - Branch & Bound
  - Random Sample Consensus (RANSAC)

Registration

Ideal Case

- Every feature point on one scan has a **single** corresponding feature on the other.
- Solve for optimal transformation T



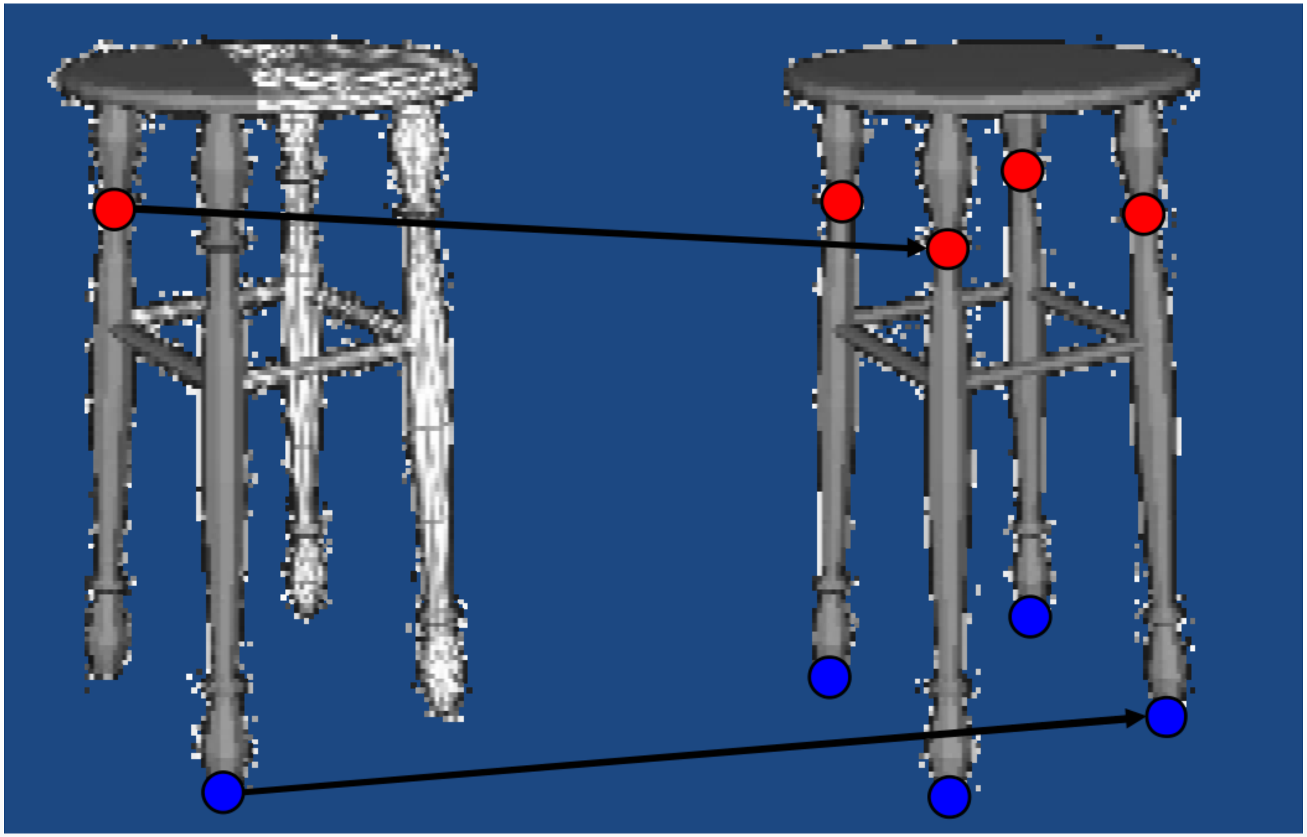
$$\sum_{i=1}^{n} \left\| p_i - T(q_i) \right\|^2$$

[McLachlan, 1979]
[Arun et al., 1987]
[Horn, 1987]
[Horn et al., 1988]

Registration

Challenge:

- Even with good descriptors, symmetries in the model and the locality of descriptors can result in multiple and incorrect correspondences



Registration

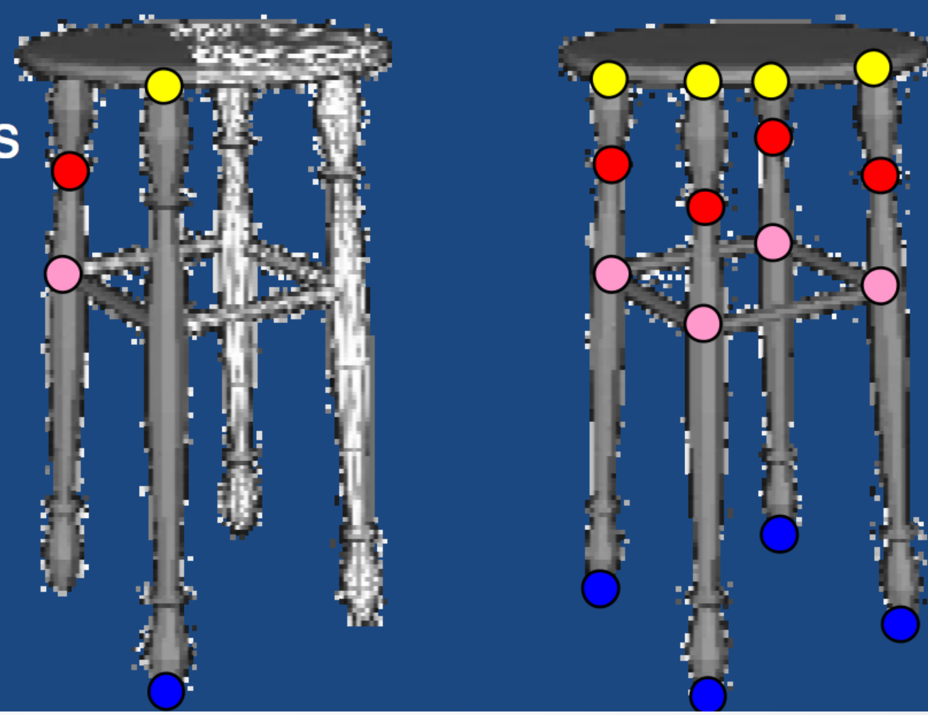
Exhaustive Search

- Compute alignment error at each permutation of correspondences and use the optimal one

$$\text{Error} = \underset{\pi \in \Psi}{\operatorname{argmin}} \left(\underset{T \in E^3}{\operatorname{argmin}} \sum_{i=1}^{n} \left\| p_i - T(\pi(p_i)) \right\|^2 \right)$$

Ψ = Set of possible correspondence

E^3 = Group of rigid body transformations



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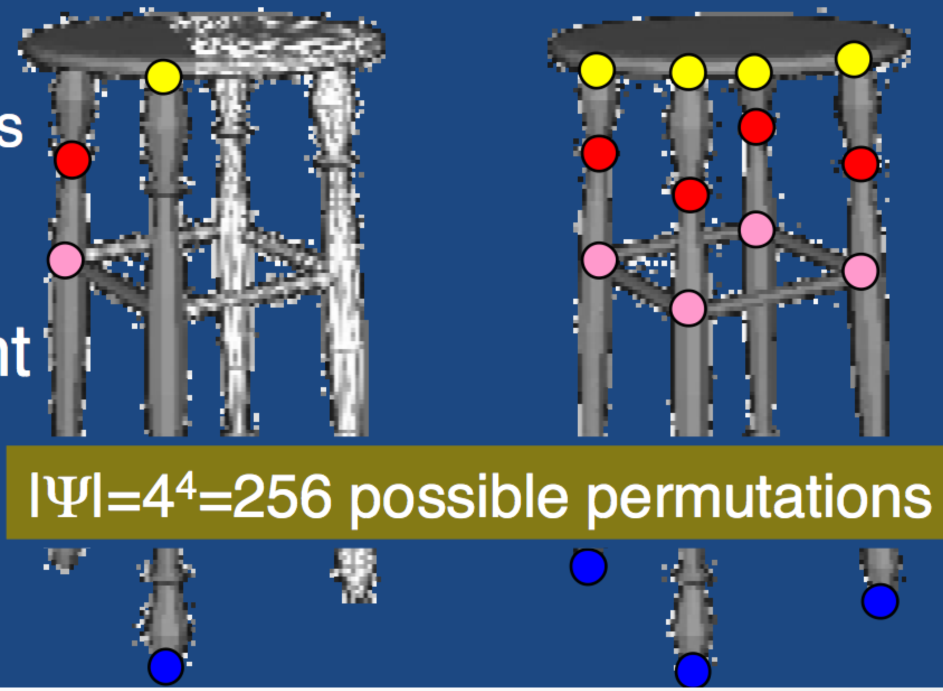
Ψ = Set of possible correspondence

E^3 = Group of rigid body transformations

Given points $\{p_1, \ldots, p_n\}$ on the query, if p_i matches m_i different target points:

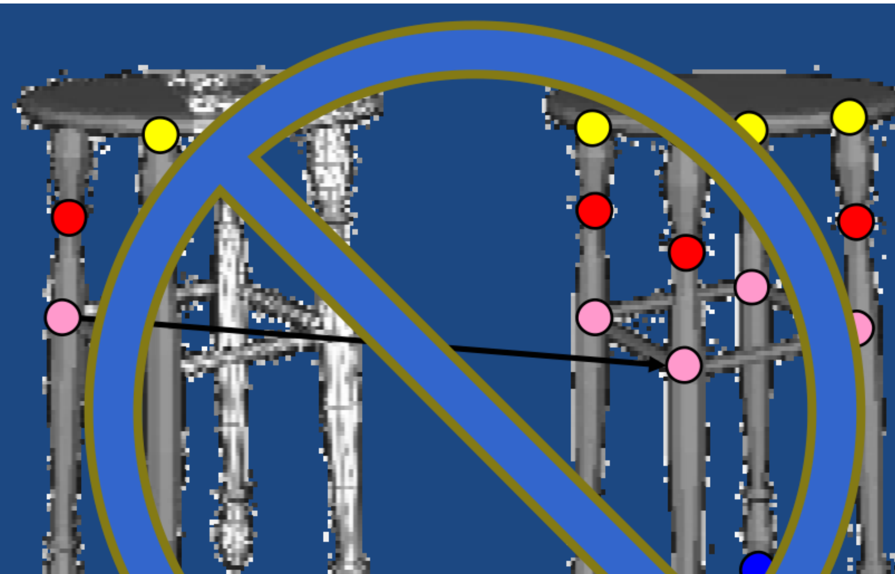
$$|\Psi| = \prod_{i=1}^{n} m_i$$



Registration

Branch & Bound (Decision tree)

- Try all permuations but terminate early if the alignment can be predicted to be bad



By performing two comparisons, it was possible to eliminate 16 different possibilities

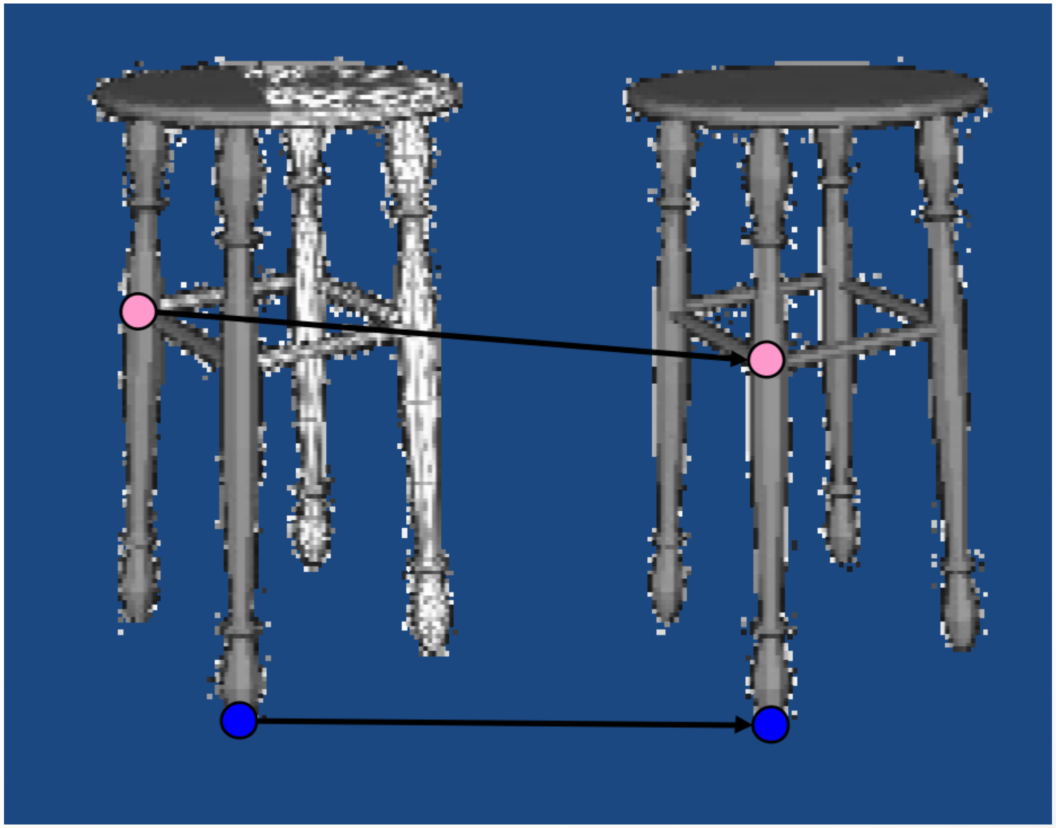
Registration

Goal

- Need to be able to determine if the alignment will be good without knowing all of the correspondences

Observation

- Alignment needs to preserve the lengths between points in a single scan



Registration

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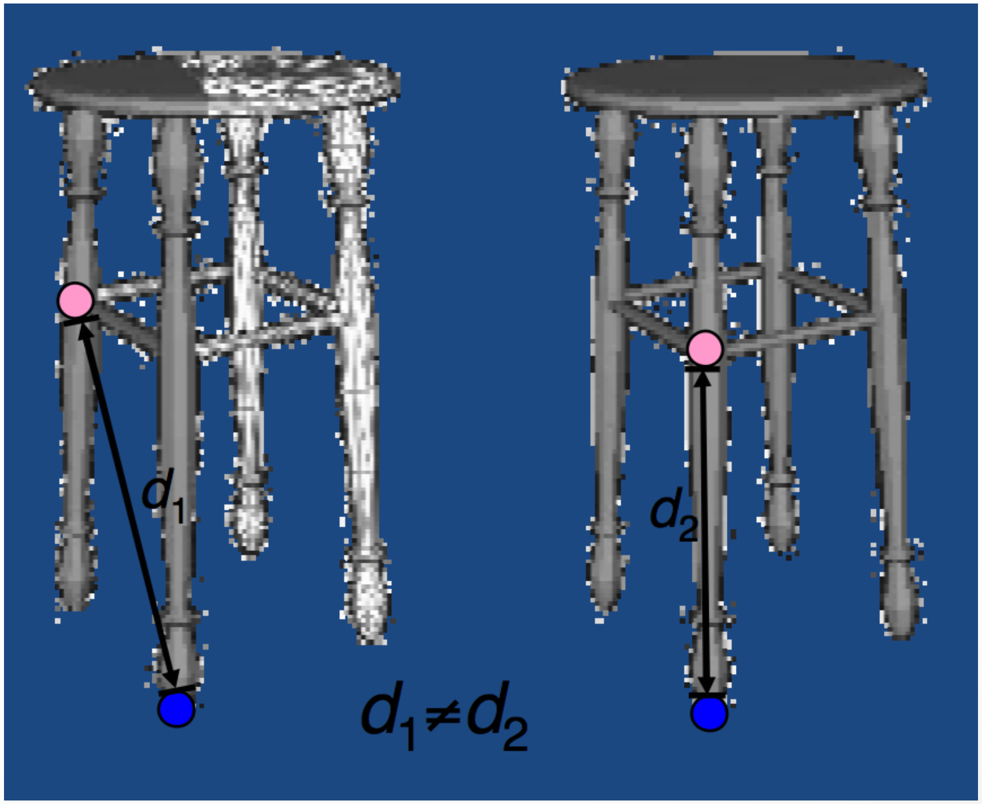
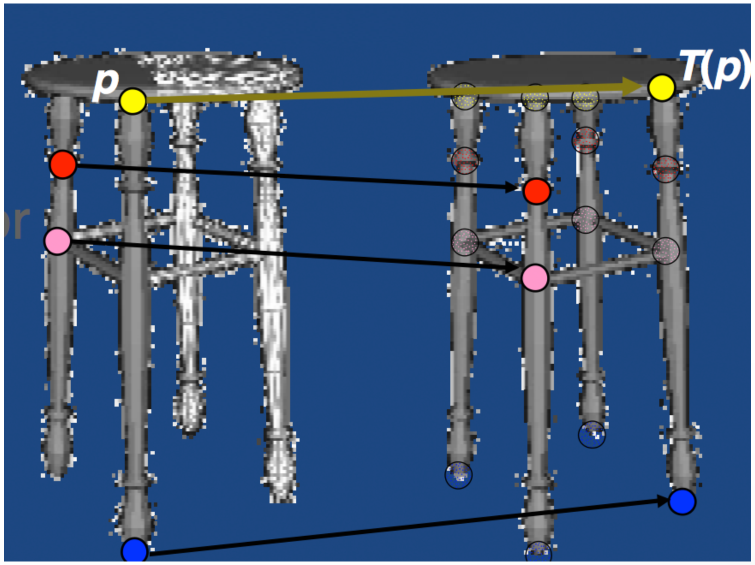


RANdom SAmple Consensus

Algorithm (iterate 100 times)

- Randomly choose 3 points on source
- For all possible correspondences on target:
  - Compute T
  - For every other source p:
    - find closest correspondence T(p)
  - Compute alignment error



Summary

Global Shape Correspondences

- Shape Descriptors
  - Shells (1D)
  - Sectors (2D)
  - Sectors & Shells (3D)
- Alignment
  - Exhaustive Search
  - Normalization
  - Invariance

Summary

Partial-Shape/Point Correspondences

- From Global to Local
  - Center at feature
  - Restrict extent
- Pose Normalization
  - Normal-based alignment
- Partial Shape Descriptors
  - Normalization/invariance
  - Normalization/exhaustive-search

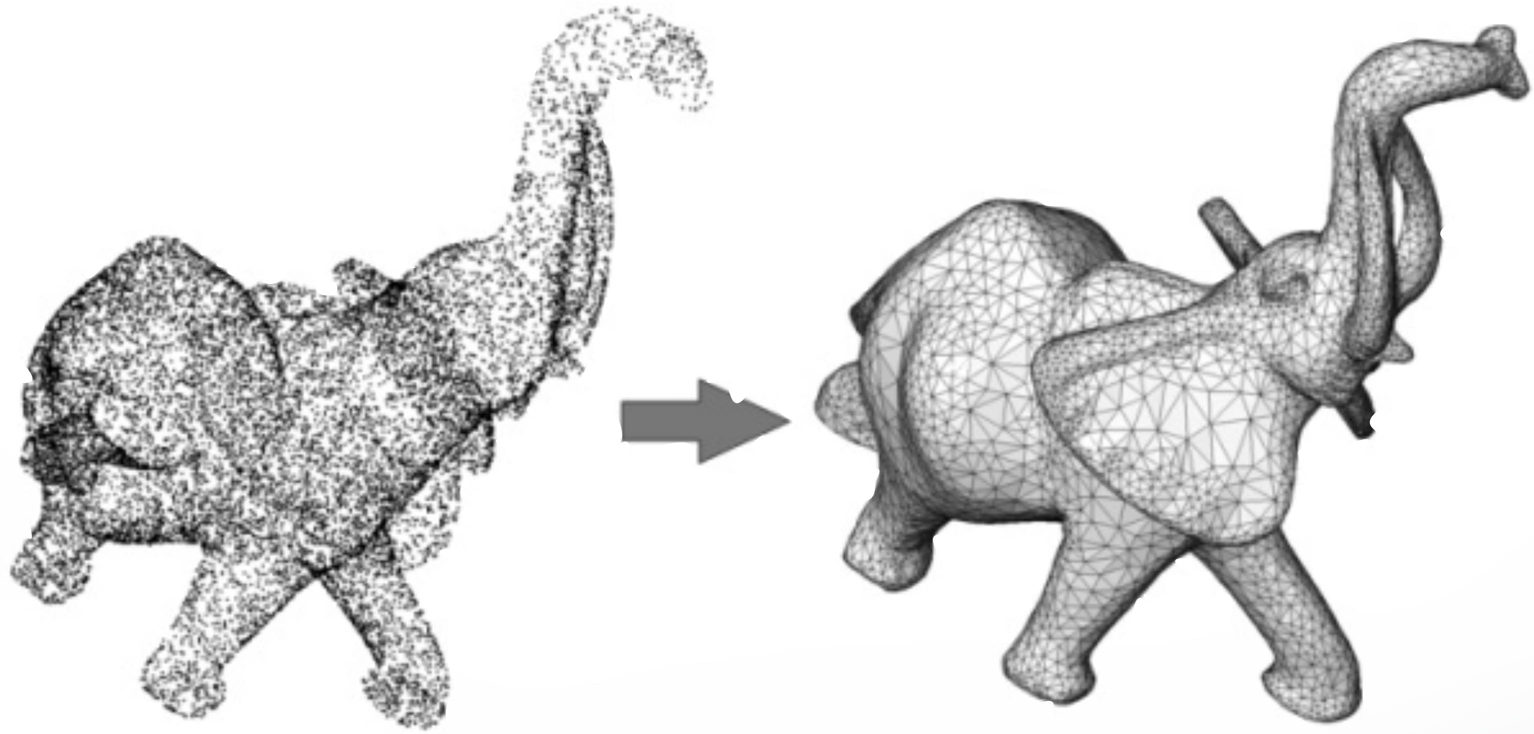
Summary

Registration

- Closed Form Solutions
  - Global symmetry
  - Local self similarity
- Branch & Bound
  - Inter-feature distances for early termination
- RANdom SAmple Consensus
  - Efficient transformation computation

Next Time

Surface Reconstruction



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Thanks!