Exercise 4. Surface Quality and Smoothing
Surface Smoothing

• Spectral analysis

• **Diffusion flow**
  • Uniform Laplace operator
  • Laplacian-Beltrami operator

• Energy minimization
Uniform Laplacian Surface Smoothing

- Uniform Laplace operator: 
  \[ L_U(v) = \left( \frac{1}{n} \sum_{i} v_i \right) - v \]

- Mesh smoothing: 
  \[ v' = v + \frac{1}{2} \cdot L_U(v) \]

- Implement uniform Laplace operator in 
  QualityViewer::calc_uniform_mean_curvature() in QualityViewer.cc

- Implement uniform Laplacian smoothing in 
  SmoothViewer::uniform_smooth() in SmoothViewer.cc
Uniform Laplacian Surface Smoothing
Triangle Quality

- Assess triangle quality by the circumradius to the minimum edge length ratio
- Circumradius is computed by
- Implement in `QualityViewer::calc_triangle_quality()` in `QualityViewer.cc`

\[ A = \frac{|a \cdot b \cdot c|}{4 \cdot r} = \frac{|a \times b|}{2} \]

\[ \frac{r_1}{e_1} < \frac{r_2}{e_2} \]
Triangle Quality
Laplace-Beltrami curvature and smoothing

- Laplace-Beltrami Operator

\[ L_B(v) = \frac{1}{2A} \sum_i ((\cot \alpha_i + \cot \beta_i)(v_i - v)) \]

- Compute mean curvature using Laplace-Beltrami weights in `QualityViewer::calc_mean_curvature()` in `QualityViewer.cc`

- Implement smoothing in `SmoothViewer::smooth()` in `SmoothViewer.cc`
Laplace-Beltrami curvature and smoothing
3.4 Gaussian curvature

In the lecture you have been presented an easy way to approximate the Gaussian curvature on a triangle mesh. The formula uses the sum of the angles around a vertex and the same associated area which is used in the Laplace-Beltrami operator:

\[ G = \frac{2\pi - \sum_j \theta_j}{A} \]

Implement the `QualityViewer::calc_gauss_curvature()` function in the `QualityViewer` class so that it stores the Gaussian curvature approximations in the `vgausscurvature` vertex property! Note that the `vweight` property already stores \( \frac{1}{2}A \) value for every vertex, you do not need to calculate \( A \) again. For the bunny dataset you should get a Gaussian curvature approximation like on Figure ??.

3.5 For the passionate (optional)

Implement the “tangential smoothing” which moves vertices only in the tangent plane of the vertex, thus focuses on enhancing triangle shapes. For this, you need to project the uniform Laplace approximation back to the tangent plane of the vertex. Use this projection vector to compute the new position of the vertex. Notice that you need to store the original normal of the vertex additionally, in order to keep the vertices always on the original tangent plane, even after several smoothing iterations.
Gaussian Curvature
Submission

- Deadline: **Mar 20, 2018 12:00 pm (noon)**
- Upload a .zip compressed file named “Exercise4-YourName.zip” to Blackboard.
- Include a “read.txt” file describing how you solve each exercise and the encountered problems.
Contact

• Office Hours: Monday 3:00 - 4:00pm, PHE 108
• email: tianyeli@usc.edu
• Highly recommended to post your question on Piazza: https://piazza.com/class/jcyaqIg8zmi3us
http://cs621.hao-li.com

Thanks!