Exercise 2. Registration
Rigid Registration

- **Selecting** source points
- **Matching** points to the target mesh
- **Weighting** the correspondences
- **Rejecting** bad pairs
- Compute **error metric**
- **Minimize** error metric
Exercise 2

• Perform rigid registration between 10 scans of the Stanford bunny
Exercise 2

• Demo

• ‘SHIFT’ + mouse controls: manual alignment for an initial transformation
• ‘r’: perform single registration step with point to point distance minimization
• ‘SPACE’: perform single registration step with point to plane distance minimization
• ‘n’: load next scan
Exercise 2

• Getting it compiled
• Subsampling
• Bad pairs rejection
• Point to point optimization
• Point to plane optimization
Getting It Compiled

- CMake, OpenGL, OpenMesh
- ANN (Approximate Nearest Neighbor)
  - efficient closest point lookup using kd-tree
Subsampling

- Uniform subsampling within a given radius
  \[ \text{subsampleRadius} \]
- \text{RegistrationViewer::subsample()} in \text{RegistrationViewer.cc}
Bad Pairs Rejection

- Closest points are computed using ANN
- Prune correspondences based on
  - distance threshold
  - normal compatibility
- `RegistrationViewer::calculate_correspondences()` in RegistrationViewer.cc
Point to Point Optimization

- Minimize \( E = \sum_{i=1}^{N} \| R p_i + t - q_i \|_2^2 \)
  - by solving a linear system \( Ax = b \)

- Registration::register_point2point() in Registration.cc
Euler Angles

- Three elemental rotations:

\[ R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \quad R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \quad R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

- Any rotation matrix can be decomposed as a product of elemental three rotation matrix

\[
R = R_z(\gamma)R_y(\beta)R_x(\alpha) = \begin{bmatrix}
c_\gamma c_\beta & -c_\alpha s_\gamma + c_\gamma s_\beta s_\alpha & s_\gamma s_\alpha + c_\gamma c_\alpha s_\beta \\
c_\beta s_\gamma & c_\gamma c_\alpha + s_\gamma s_\beta s_\alpha & c_\alpha s_\gamma s_\beta - c_\gamma s_\alpha \\
-s_\beta & c_\beta s_\alpha & c_\beta c_\alpha
\end{bmatrix}
\]

\[
c_\alpha = \cos \alpha \quad s_\alpha = \sin \alpha
\]
Linearized Transformation

- Linearized Euler angle

(assuming small rotation: \( \cos \alpha = 1 \quad \sin \alpha = \alpha \))

\[
R = \begin{bmatrix}
c_\gamma c_\beta & -c_\alpha s_\gamma + c_\gamma s_\beta s_\alpha & s_\gamma s_\alpha + c_\gamma c_\alpha s_\beta \\
c_\beta s_\gamma & c_\gamma c_\alpha + s_\gamma s_\beta s_\alpha & c_\alpha s_\gamma s_\beta - c_\gamma s_\alpha \\
-s_\beta & c_\beta s_\alpha & c_\beta c_\alpha
\end{bmatrix} = \begin{bmatrix}1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1\end{bmatrix}
\]

- Linearized transformation

\[
x = \left[ \begin{array}{cccccc}
\alpha & \beta & \gamma & t_x & t_y & t_z \\
\end{array} \right]^T
\]
Minimize \( E = \sum_{i=1}^{N} \| n_i^\top (R_p i + t - q_i) \|_2^2 \) by solving a linear system \( Ax = b \)

- **Registration::register_point2surface()** in Registration.cc
Results
Submission

• Deadline: **Feb 20, 2018 12:00pm (noon)**

• Upload a .zip compressed file named “Exercise2-YourName.zip” to
  • Blackboard

• Include a “read.txt” file describing how you solve each exercise and the encountered problems
Contact

• Office Hours: Monday 15:00 - 16:00 PHE 108

• email: tianyeli@usc.edu
Thanks!