11.1 Remeshing
• **What is remeshing?**

• **Why remeshing?**

• **How to do remeshing?**
• **What is remeshing?**

• **Why remeshing?**

• **How to do remeshing?**
Definition

Given a 3D mesh
  • Already a manifold mesh

Compute another mesh
  • Satisfy some quality requirements
  • Approximate well the input mesh
Outline

• *What* is remeshing?

• *Why* remeshing?

• *How* to do remeshing?
Motivation

Unsatisfactory “raw” mesh

- By scanning or implicit representations
Motivation

Unsatisfactory “raw” mesh

- By scanning or implicit representations

Improve mesh quality for further use
Motivation

Unsatisfactory “raw” mesh
  • By scanning or implicit representations

Improve mesh quality for further use
  • Modeling: easy processing
  • Simulation: numerical robustness
  • ……

Quality requirements
  • Local structure
  • Global structure
Local structure

Element type

- Triangles vs. quadrangles

- all-triangle mesh
- all-quad mesh
- quad-dominant mesh
Local structure

Element type

• Triangles vs. quadrangles
Local structure

**Element type**
- Triangles vs. quadrangles

**Element shape**
- Isotropic vs. anisotropic
Local structure

Element type
• Triangles vs. quadrangles

Element shape
• Isotropic vs. anisotropic

Element distribution
• Uniform vs. adaptive
Local structure

Element type
- Triangles vs. quadrangles

Element shape
- Isotropic vs. anisotropic

Element distribution
- Uniform vs. adaptive

Element alignment
- Preserve sharp features and curvature lines
Global structure

Valence of a *regular* vertex

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>Interior vertex</th>
<th>Boundary vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle mesh</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Quadrangle mesh</td>
<td>4</td>
<td>3</td>
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Valence of a *regular* vertex

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Different types of mesh structure

- Irregular
- Semi-regular: multi-resolution analysis / modeling
- Highly regular: numerical simulation
- Regular: only possible for special models
• What is remeshing?

• Why remeshing?

• How to do remeshing?
Outline

• What is remeshing?

• Why remeshing?

• How to do remeshing?
  - Isotropic remeshing
  - Anisotropic remeshing
• What is remeshing?

• Why remeshing?

• How to do remeshing?
  - Isotropic remeshing
  - Anisotropic remeshing
Isotropic remeshing

Incremental remeshing
- Simple to implement and robust
- Not need parameterization
- Efficient for high-resolution input

Variational remeshing
- Energy minimization
- Parameterization-based → expensive
- Works for coarse input mesh

Greedy remeshing
Isotropic remeshing

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Greedy remeshing
Local remeshing operators

- Edge Collapse
- Edge Split
- Edge Flip
- Vertex Shift
Incremental remeshing

Specify target edge length \( L \)

\[
L_{\text{max}} = \frac{4}{3} \times L; \quad L_{\text{min}} = \frac{4}{5} \times L;
\]

Iterate:

1. **Split** edges longer than \( L_{\text{max}} \)
2. **Collapse** edges shorter than \( L_{\text{min}} \)
3. **Flip** edges to get closer to optimal valence
4. **Vertex shift** by tangential relaxation
5. **Project** vertices onto reference mesh
Edge split

\[ L_{\text{max}} = \frac{4}{3} L \]

Split edges longer than \( L_{\text{max}} \)
Edge collapse

Edge collapse edges shorter than $L_{\text{min}}$
Optimal valence

- 6 for interior vertices
- 4 for boundary vertices
**Optimal valence**

- 6 for interior vertices
- 4 for boundary vertices

**Improve valences**

- Minimize valence excess

\[
\sum_{i=1}^{4} (\text{valence}(v_i) - \text{opt\_valence}(v_i))^2
\]
Vertex shift

Local “spring” relaxation

- Uniform Laplacian smoothing
- Barycenter of one-ring neighborhood

\[ c_i = \frac{1}{\text{valence}(v_i)} \sum_{j \in N(v_i)} p_j \]
Vertex shift

Local “spring” relaxation

- Uniform Laplacian smoothing
- Barycenter of one-ring neighborhood

\[ c_i = \frac{1}{\text{valence}(v_i)} \sum_{j \in N(v_i)} p_j \]
Vertex shift

Local “spring” relaxation

- Uniform Laplacian smoothing
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\[ c_i = \frac{1}{\text{valence}(v_i)} \sum_{j \in N(v_i)} p_j \]

Keep vertex (approx.) on surface

- Restrict movement to tangent plane

\[ p_i \leftarrow p_i + \lambda (I - n_i n_i^T)(c_i - p_i) \]
Onto original reference mesh

- Find closest triangle
- Use BSP to accelerate $\rightarrow O(\log n)$
- Barycentric interpolation to compute position & normal
Specify target edge length $L$

Iterate:

1. **Split** edges longer than $L_{\text{max}}$
2. **Collapse** edges shorter than $L_{\text{min}}$
3. **Flip** edges to get closer to optimal valence
4. Vertex **shift** by tangential relaxation
5. **Project** vertices onto reference mesh
Adaptive remeshing

- Compute maximum principle curvature on reference mesh
- Determine local target edge length from max-curvature
- Adjust edge split / collapse criteria accordingly
Feature preservation
Feature preservation

Define feature edges / vertices

- Large dihedral angles
- Material boundaries

Adjust local operators

- Do not touch corner vertices
- Do not flip feature edges
- Collapse along features
- Univariante smoothing
- Project to feature curves
Isotropic remeshing

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Greedy remeshing
Voronoi Diagram

Divide space into a number of cells
Voronoi Diagram

Divide space into a number of cells

Dual graph: Delaunay triangulation
For each cell

The generating point $\bullet$ = mass of center $+$

Centroidal Voronoi Diagram

non CVD

CVD
Centroidal Voronoi Diagram

Compute CVD by Lloyd relaxation

1. Compute Voronoi diagram of given points $p_i$
2. Move points $p_i$ to centroids $c_i$ of their Voronoi cells $V_i$
3. Repeat steps 1 and 2 until satisfactory convergence

$$p_i \leftarrow c_i = \frac{\int_{V_i} x \cdot \rho(x) \, dx}{\int_{V_i} \rho(x) \, dx}$$
Centroidal Voronoi Diagram

Compute CVD by Lloyd relaxation

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CVD maximizes compactness

- Minimize the energy:

$$\sum_i \int_{V_i} \rho(x) \|x - p_i\|^2 \, dx \rightarrow \min$$
Variational remeshing

1. Conformal parameterization of input mesh
2. Compute local density
3. Perform in 2D parameter space
   A. Randomly sample according to local density
   B. Compute CVD by Lloyd relaxation
4. Lift 2D Delaunay triangulation to 3D
Variational remeshing
Adaptive remeshing
Feature preservation
• *What* is remeshing?

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  - Isotropic remeshing
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Anisotropic remeshing

Artist-designed models

- Conform to the anisotropy of a surface
Anisotropic remeshing

Anisotropy

Differential geometry

- A local *orthogonal* frame: min/max curvature directions and *normal*
3D curvature tensor

**Isotropic**
- k > 0
- k = 0
- k = 0
- k > 0
- k < 0

**Anisotropic**
- 2 principal directions
- k_{min} > 0
- k_{min} = 0
- k_{min} < 0
- k_{max} > 0

**Types**
- **spherical**
- **planar**
- **elliptic**
- **parabolic**
- **hyperbolic**
Principal direction fields

min curvature → max curvature → overlay
Flattening to 2D

- One 3D tensor per vertex
- Discrete conformal parameterization
- 2D tensor field using barycentric coordinates

Piecewise linear interpolation of 2D tensors
2D direction fields

- Regular case

- minor foliation
- major foliation
- principal foliations
2D direction fields

- Singularities

- umbilic
  (spherical point)
  2D tensor proportional to identity
Lines of curvature

minor net

major net

overlay
Lines of curvature

minor net

major net
• Overlay curvature lines in anisotropic regions
• Add umbilical points in isotropic regions
Vertices

intersect lines of curvatures
Edges

straighten lines of curvatures + Delaunay triangulation near umbilics
Resolve T-junctions
Smoothing

quad-triangle subdivision
Anisotropic remeshing

Remeshing results

- Min curvature
- Max curvature
- Result
- Minor net
- Major net
- Overlay
Tools

MeshLab

- meshlab.sourceforge.net
- open source
- available for Windows, MacOSX, and Linux

Graphite

- available for Windows
- MacOSX or Linux?
“Mesh” → “remesh” → “pliant” →

- [Optional] flag border as feature
- [Optional] flag sharp edges as feature (dihedral angle)
- [Optional] estimate edge size (bounding box divisions)
- remesh (target edge length)
• Textbook: Chapter 6
• Alliez et al, “Interactive geometry remeshing”, SIGGRAPH 2002
• Alliez et al, “Isotropic surface remeshing”, SMI 2003
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• Botsch & Kobbelt, “A remeshing approach to multiresolution modeling”, Symp. on Geometry Processing 2004
• Marinov et al, “Direct anisotropic quad-dominant remeshing”, Pacific Graphics 2004
• Alliez et al, “Recent advances in remeshing of surfaces”, AIM@Shape state of the art report, 2006
http://cs621.hao-li.com

Thanks!