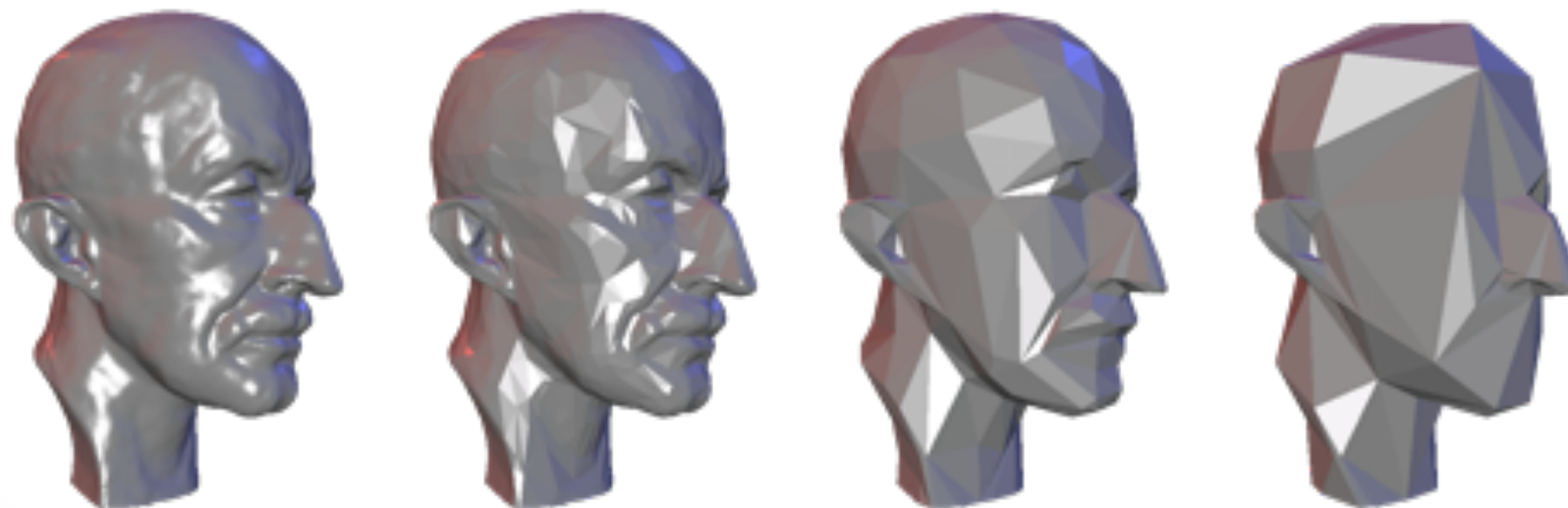


9.2 Decimation



Hao Li

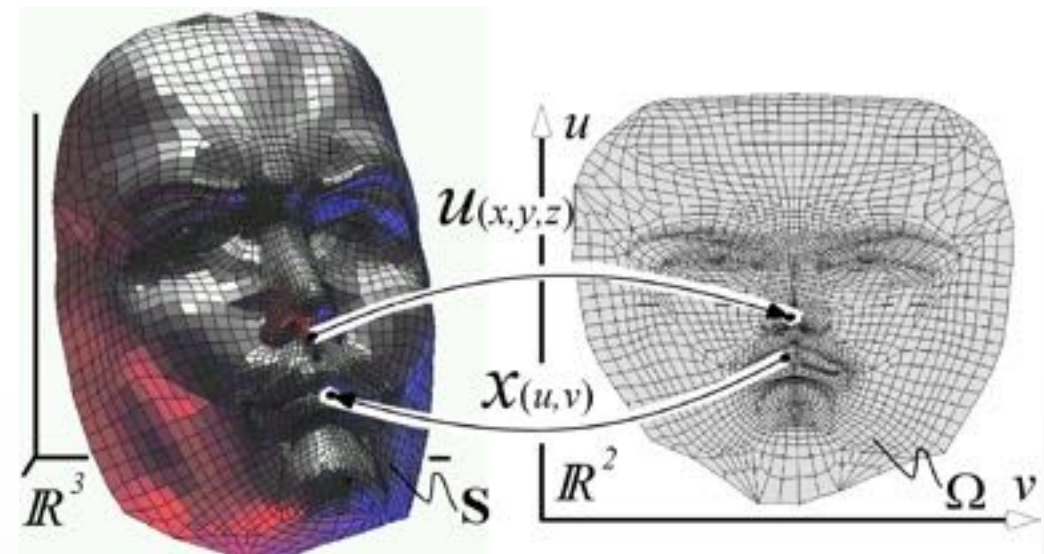
<http://cs621.hao-li.com>

Last Time

Parameterization

- isometric $\mathbf{I}(u, v) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- conformal $\mathbf{I}(u, v) = s(u, v) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- equiareal $\det(\mathbf{I}(u, v)) = 1$

$$\mathbf{I} = \begin{pmatrix} \mathbf{x}_u^T \mathbf{x}_u & \mathbf{x}_u^T \mathbf{x}_v \\ \mathbf{x}_u^T \mathbf{x}_v & \mathbf{x}_v^T \mathbf{x}_v \end{pmatrix}$$



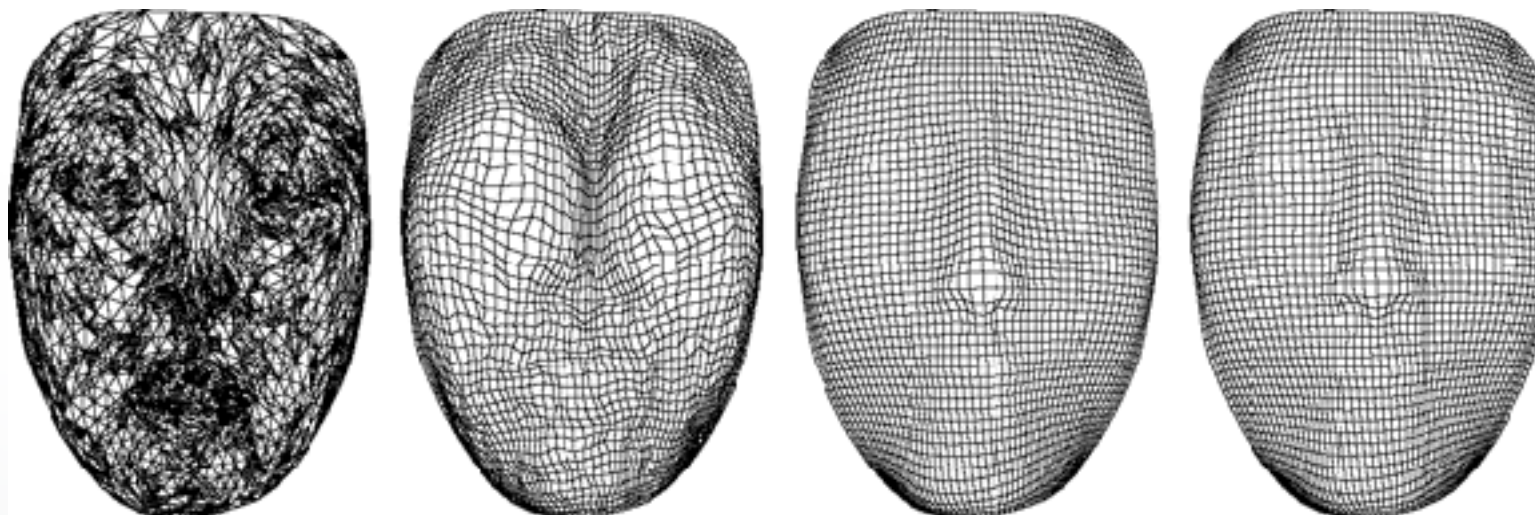
Last Time

Harmonic Maps

- minimize Dirichlet energy: $\int_{\Omega} \|\nabla \mathbf{x}\|^2 = \int_{\Omega} \|\mathbf{x}_u\|^2 + \|\mathbf{x}_v\|^2 \, du \, dv$
- Euler-Lagrange PDE $\Delta \mathbf{x}(u, v) = 0$

Discrete Harmonic Maps

Convex Combination Maps



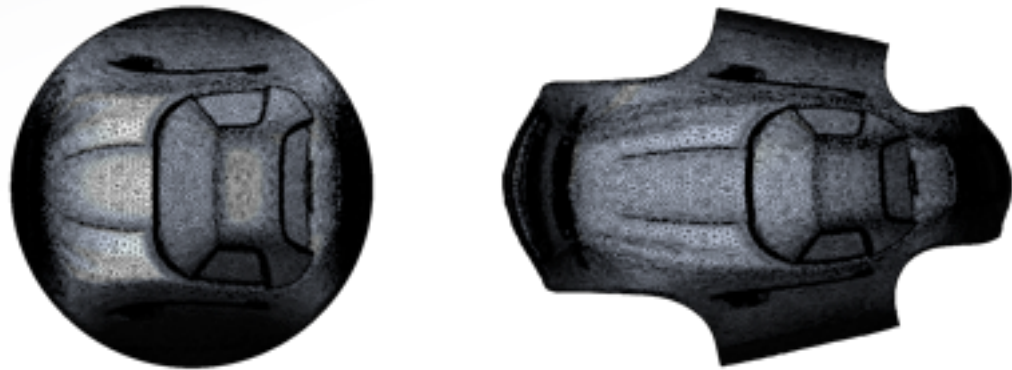
original
mesh

uniform
weights

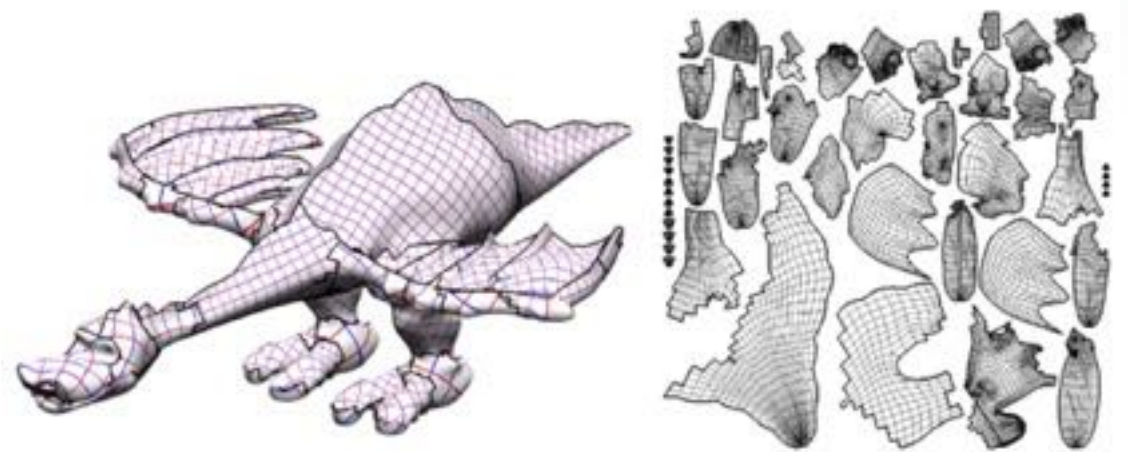
cotan
weights

mean
value

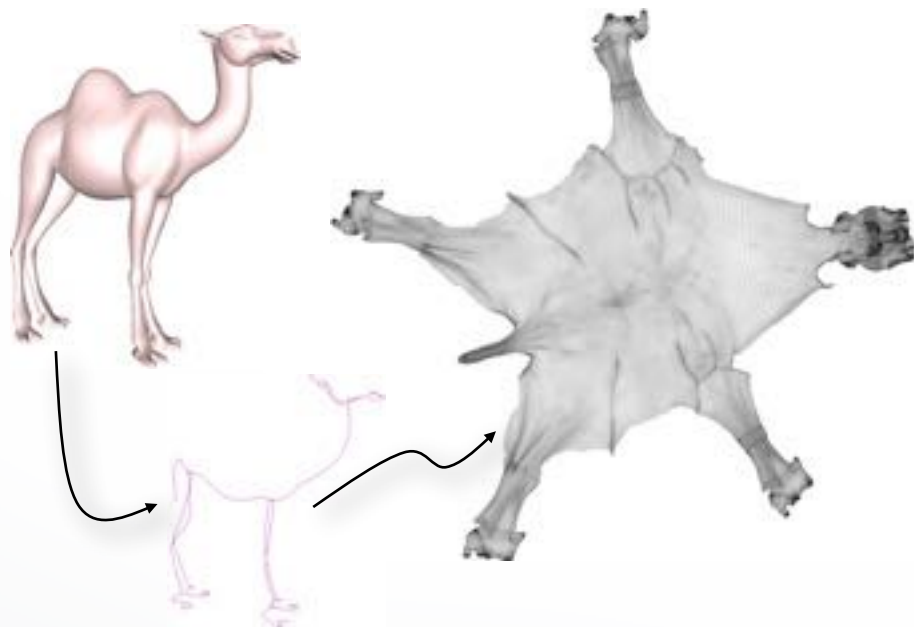
Last Time



fixed vs. open boundaries



texture atlases



cutting the mesh → disk topology



constrained parameterization

Mesh Optimization

Smoothing

- Low geometric noise

Fairing

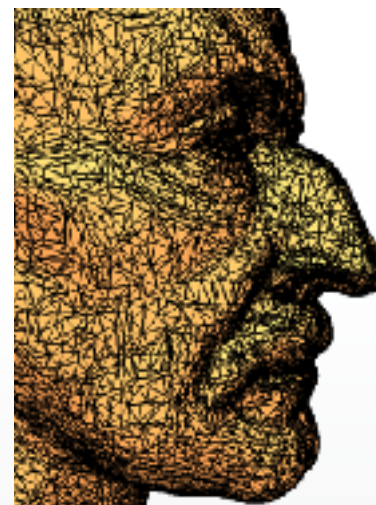
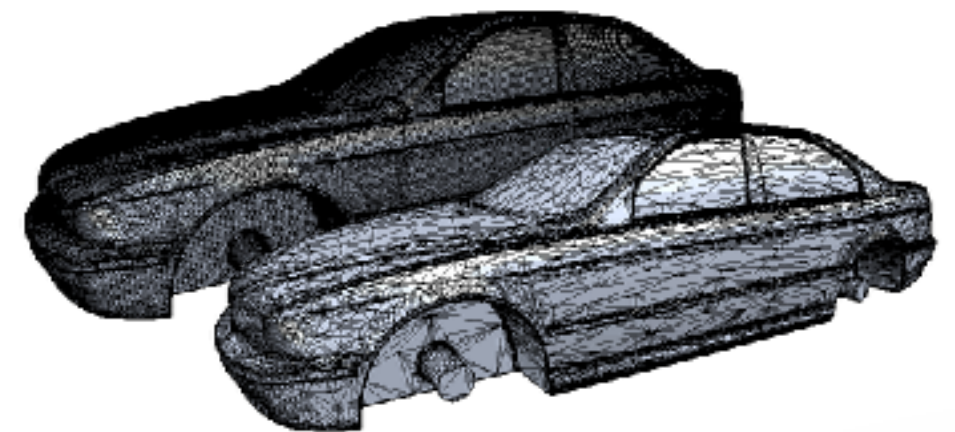
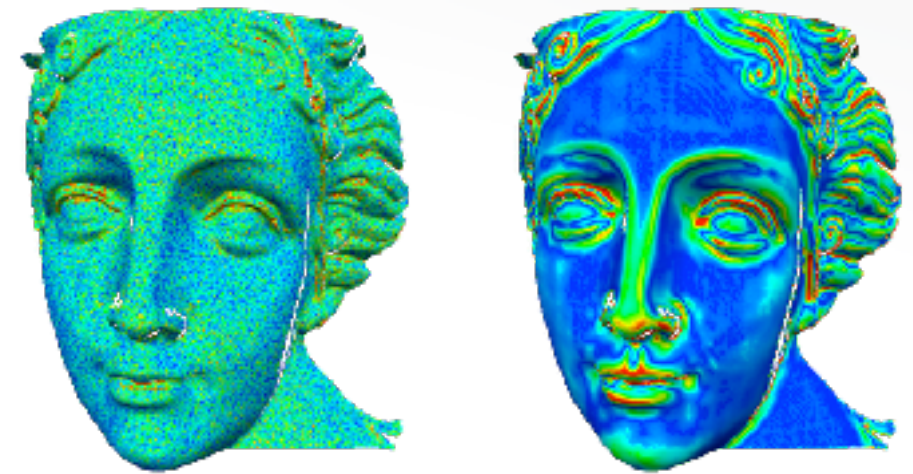
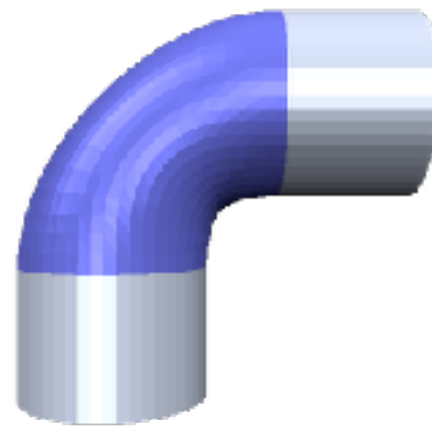
- Simplest shape

Decimation

- Low complexity

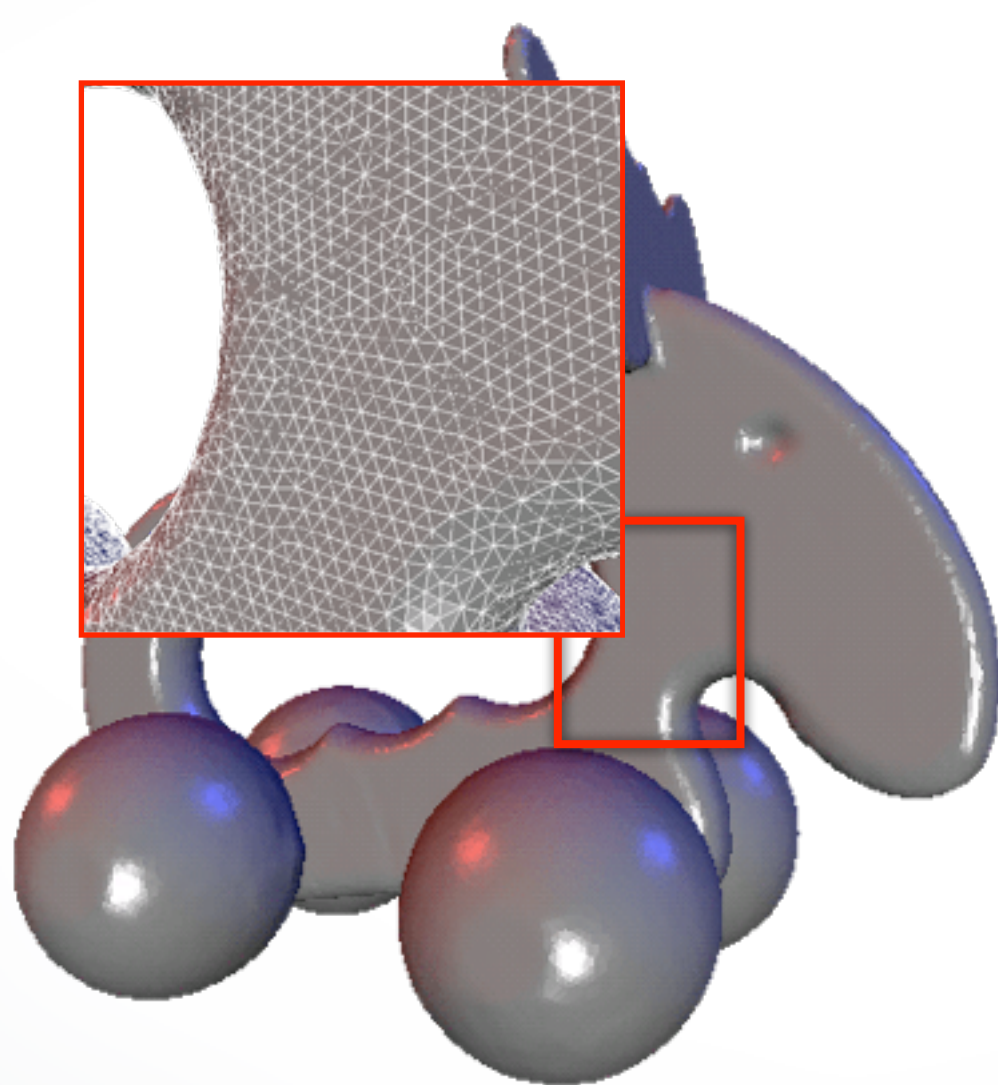
Remeshing

- Triangle Shape

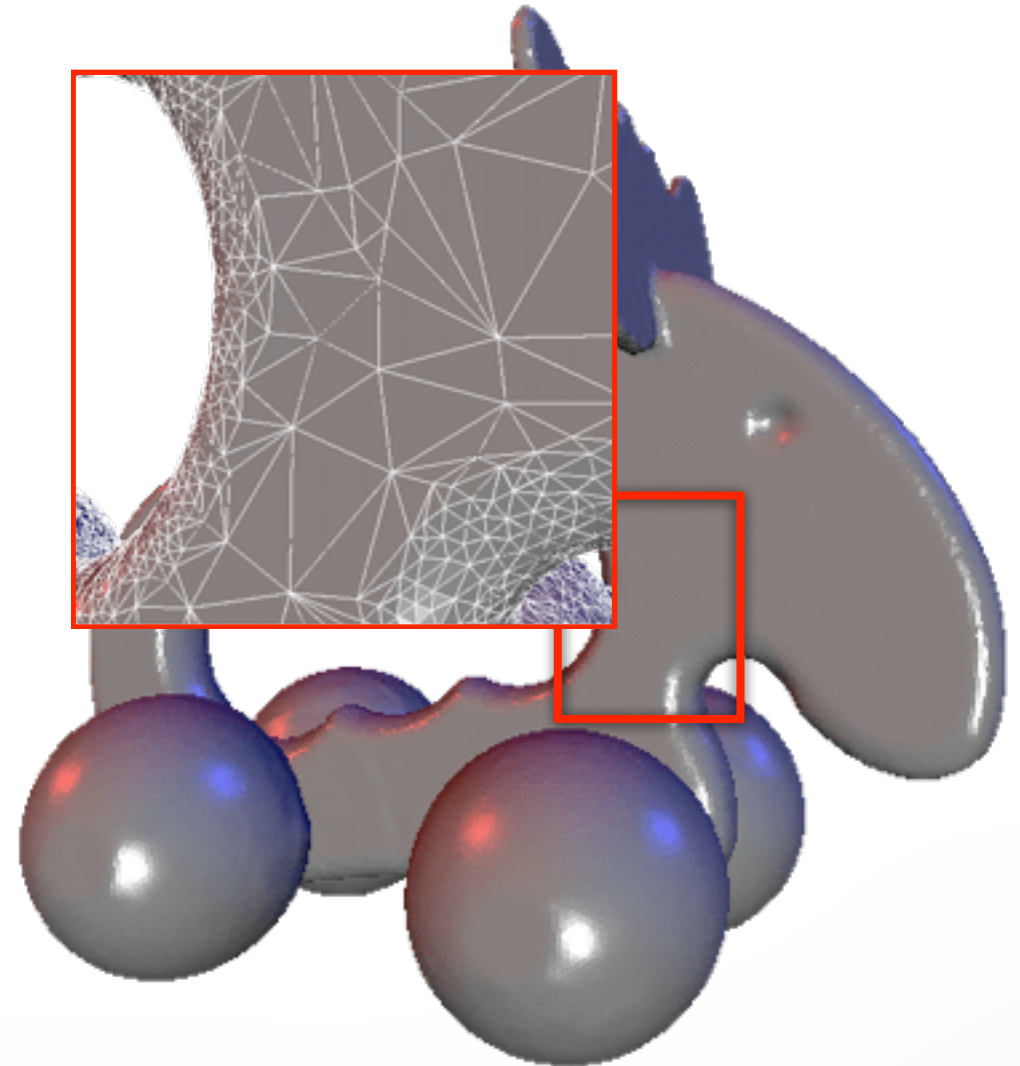


Mesh Decimation

Oversampled 3D scan data



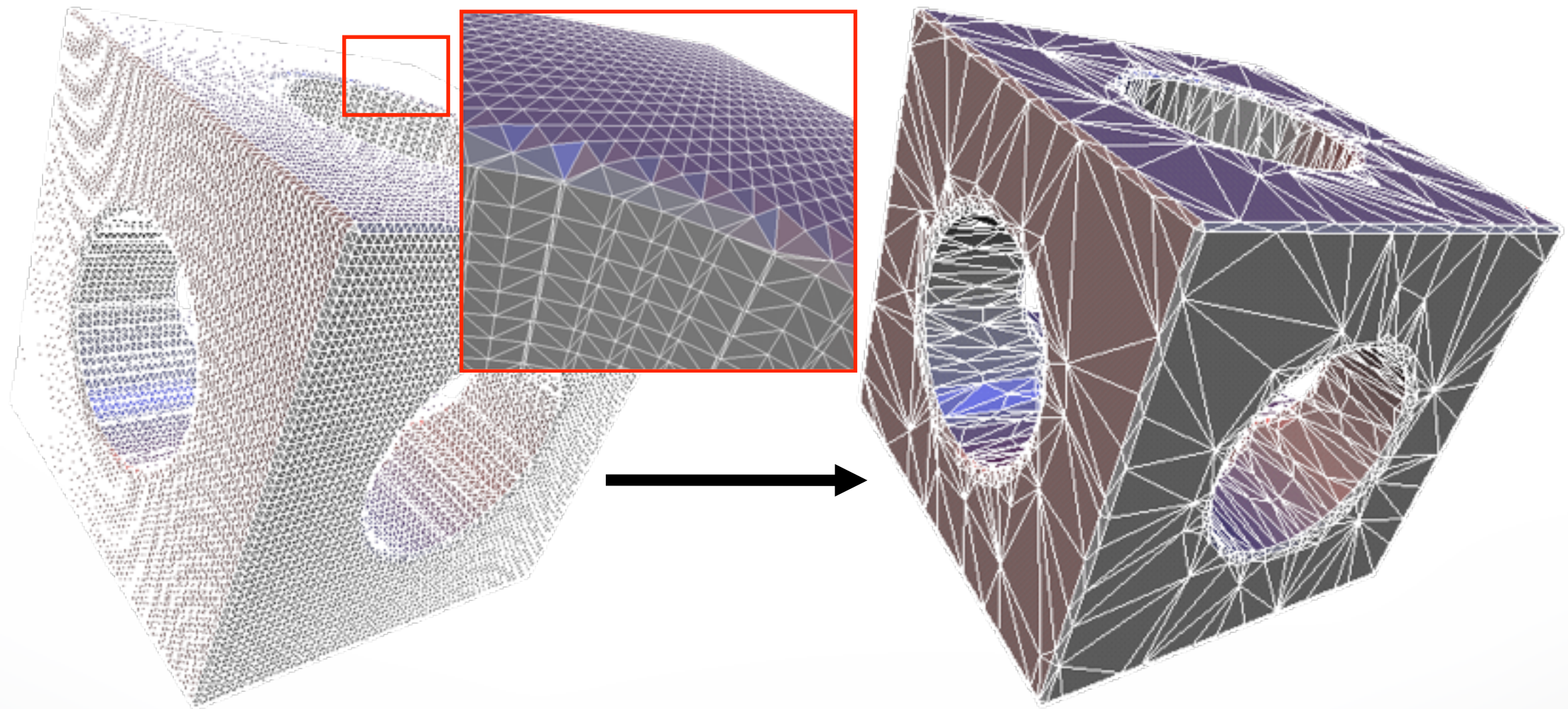
~150k triangles



~80k triangles

Mesh Decimation

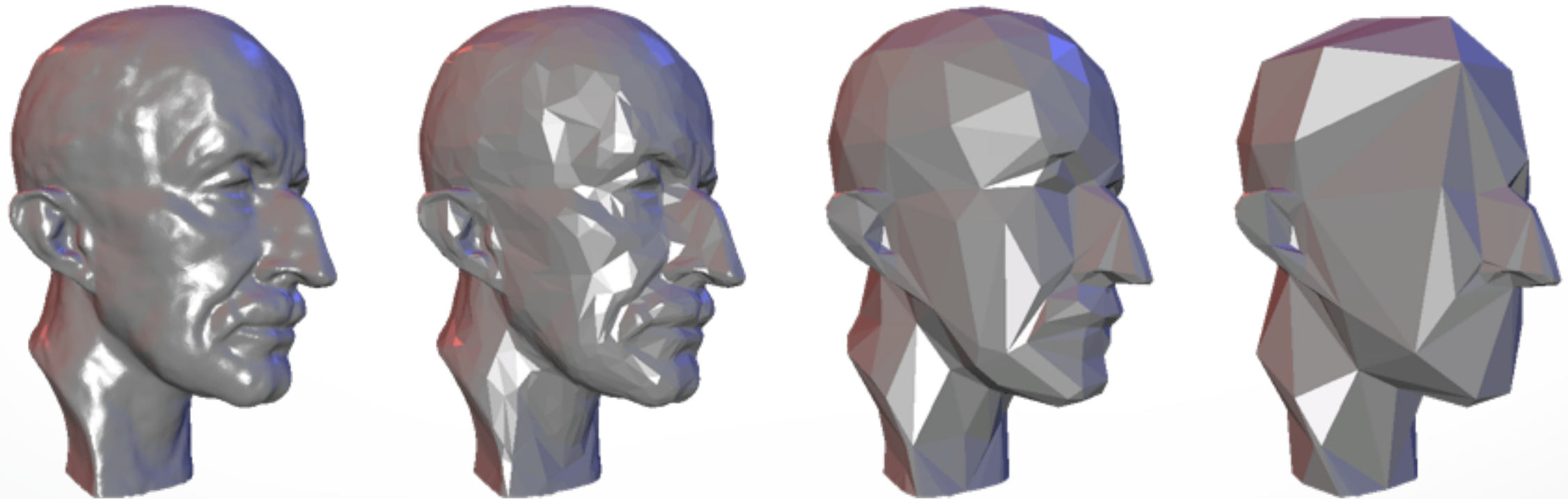
Over tessellation: e.g., Iso-surface extraction



Mesh Decimation

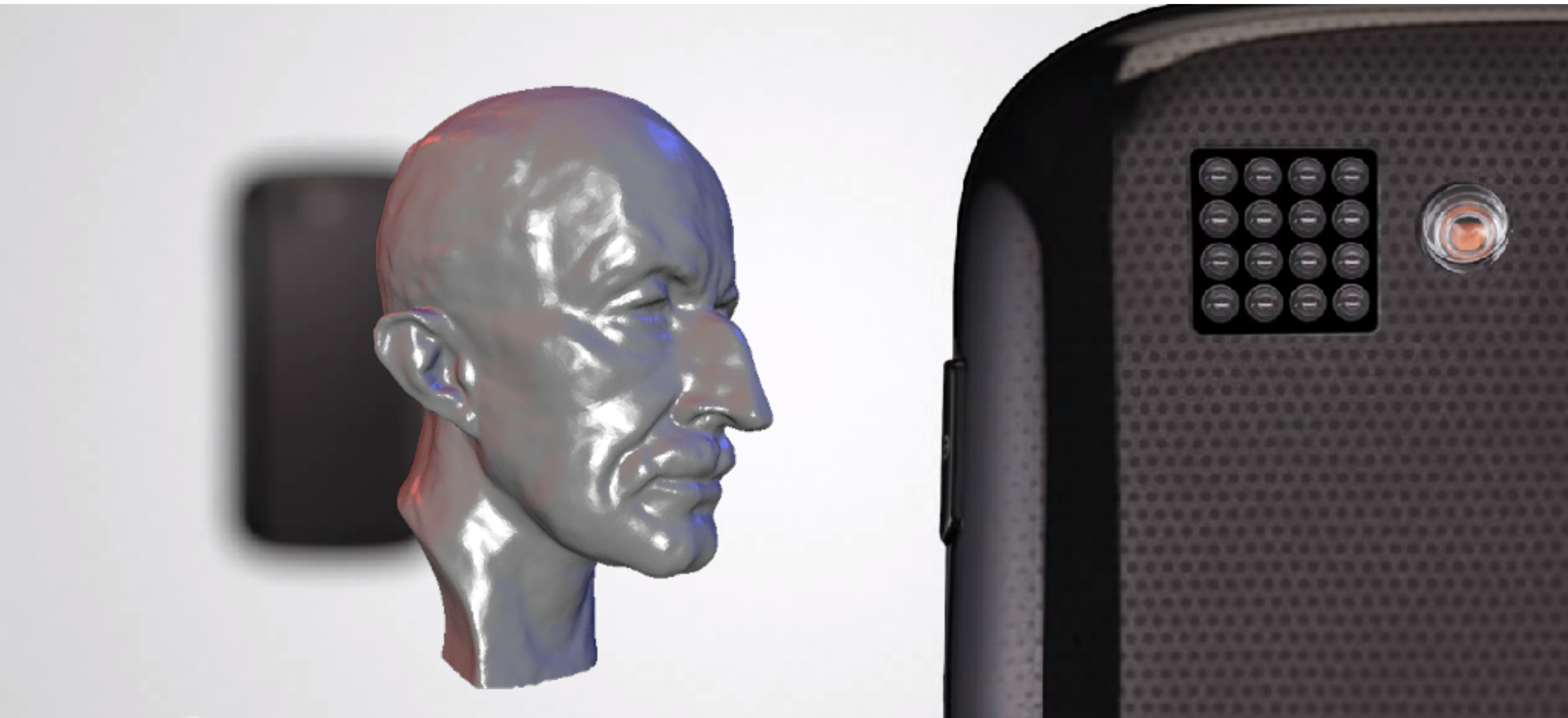
Multi-resolution hierarchies for

- efficient geometry processing
- level-of-detail (LOD) rendering



Mesh Decimation

Adaptation to hardware capabilities

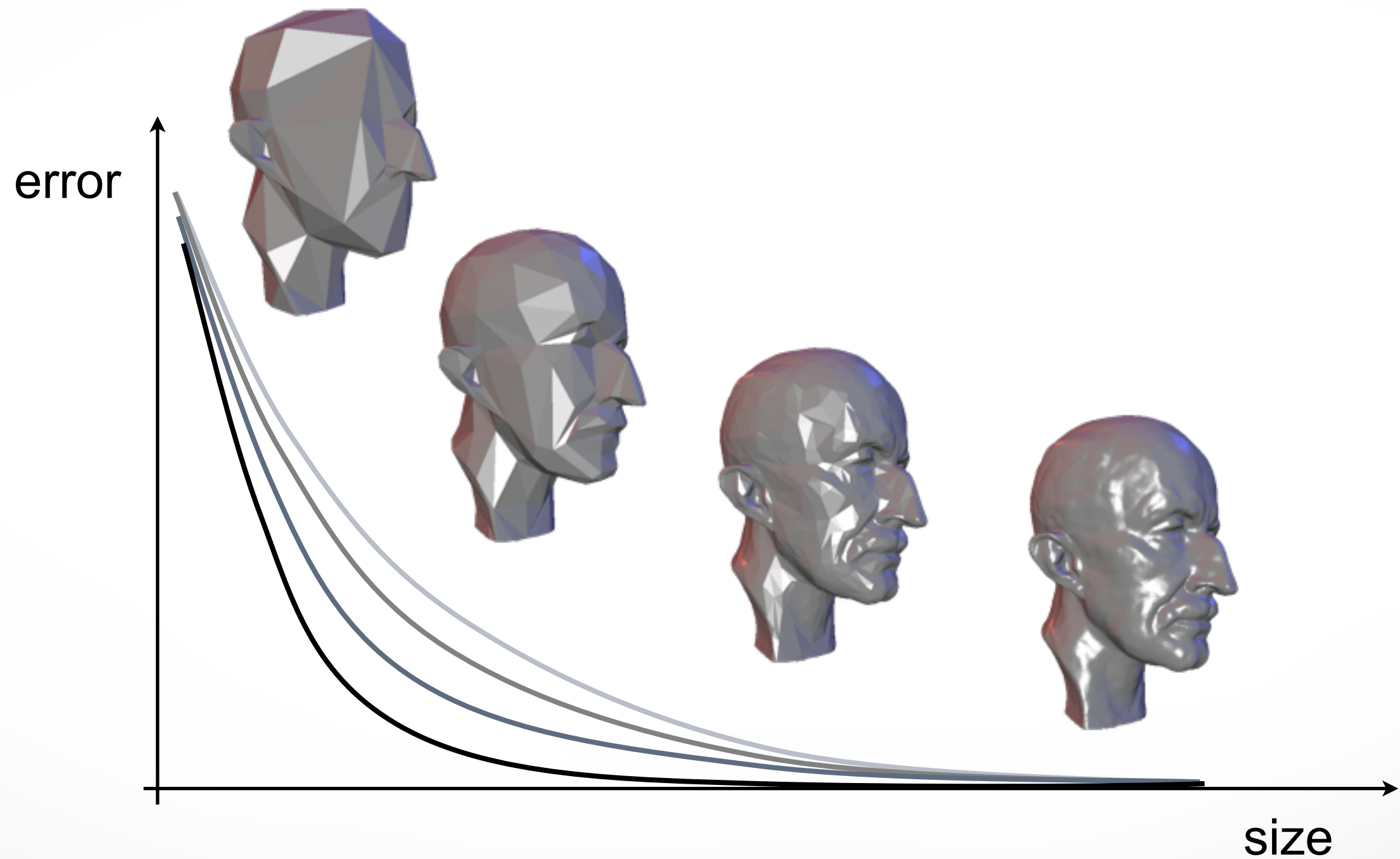


Mesh Decimation

Adaptation to hardware capabilities



Size-Quality Tradeoff



Problem Statement

Given $\mathcal{M} = (\mathcal{V}, \mathcal{F})$, find $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

- $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or
- $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal



$\mathcal{M}$



$\mathcal{M}'$

Problem Statement

Given $\mathcal{M} = (\mathcal{V}, \mathcal{F})$, find $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

- $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or
- $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal

NP hard

- Look for sub-optimal solution

Respect additional fairness criteria

- Normal deviation, triangle shape, colors,...

Outline

Mesh Decimation methods

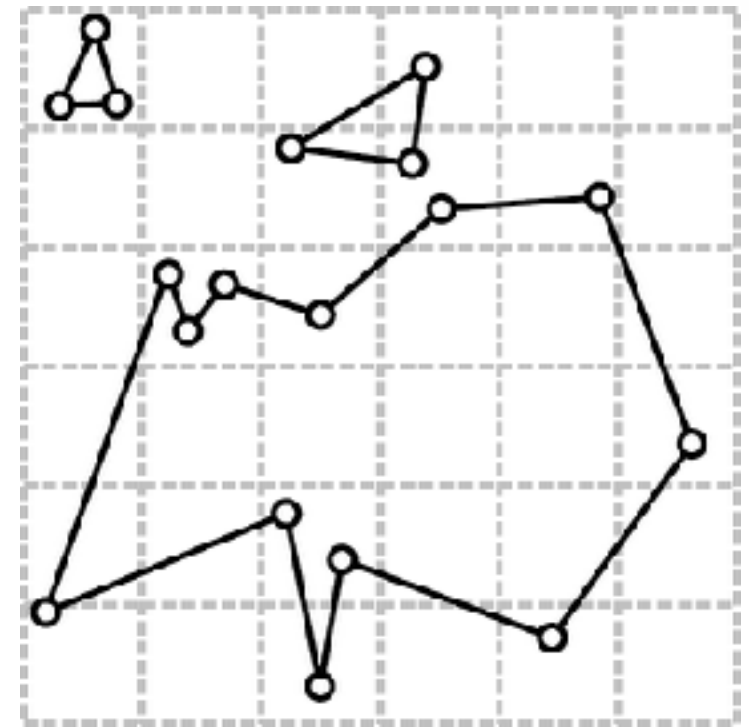
- **Vertex Clustering**
- Iterative Decimation

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes

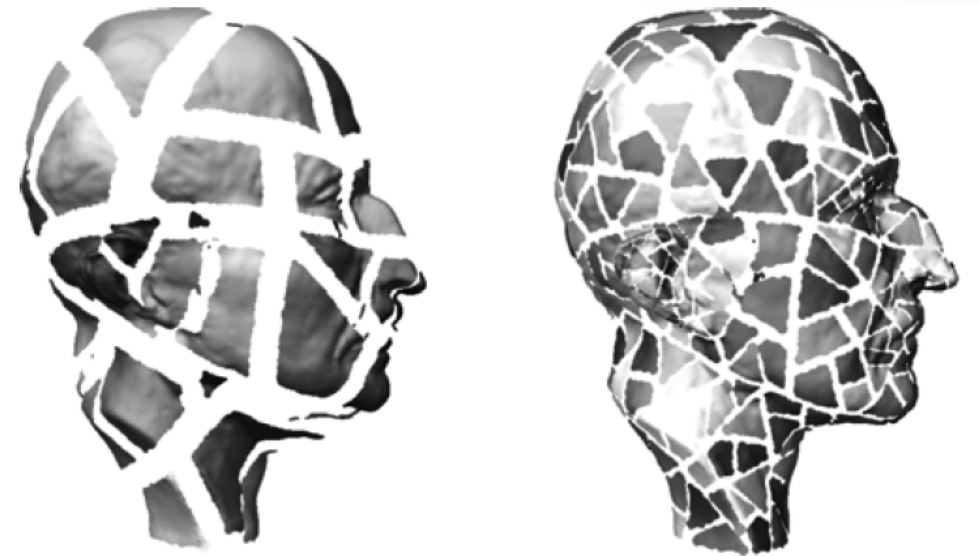
Vertex Clustering

- **Cluster Generation**
 - Uniform 3D grid
 - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

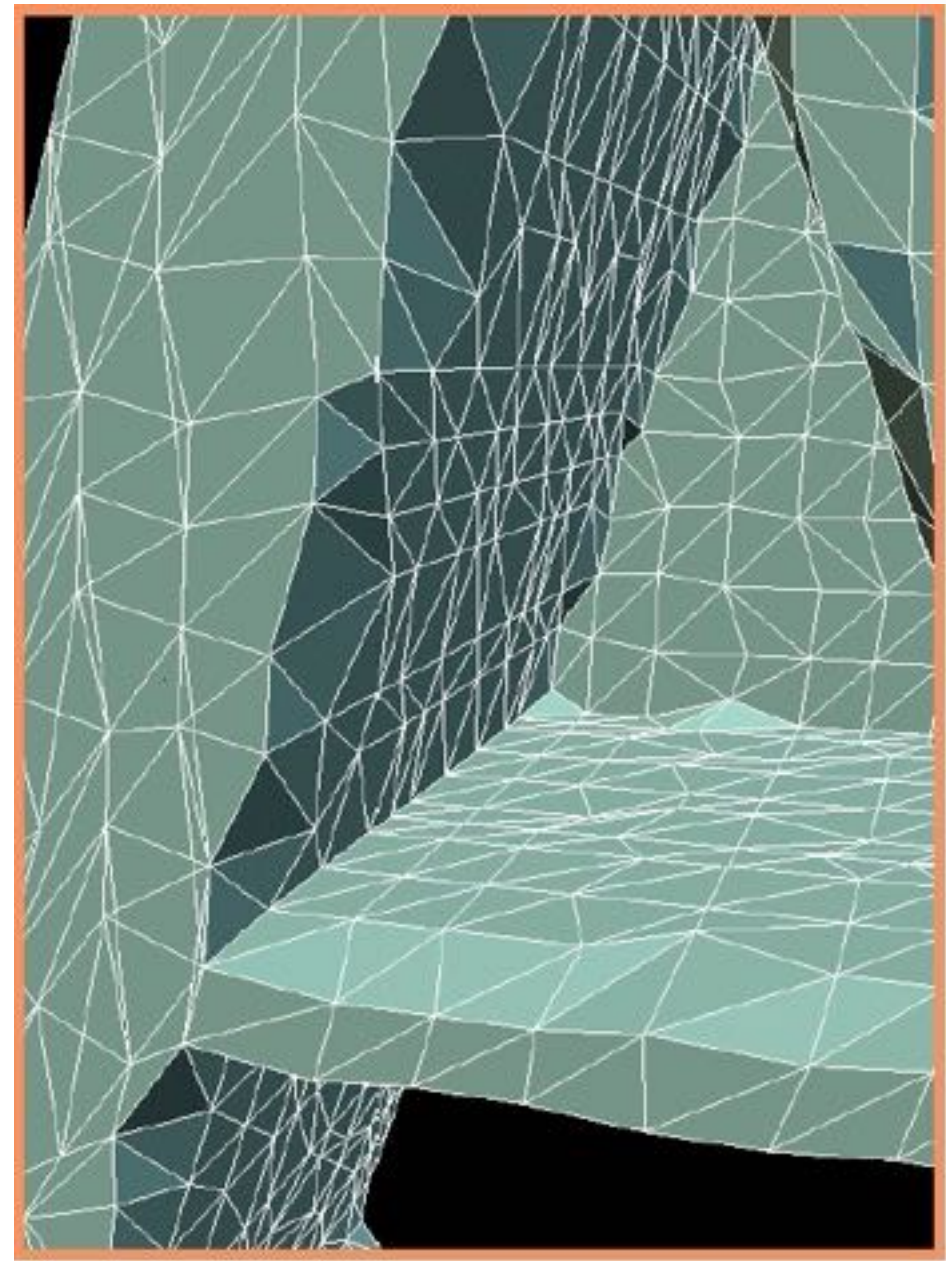
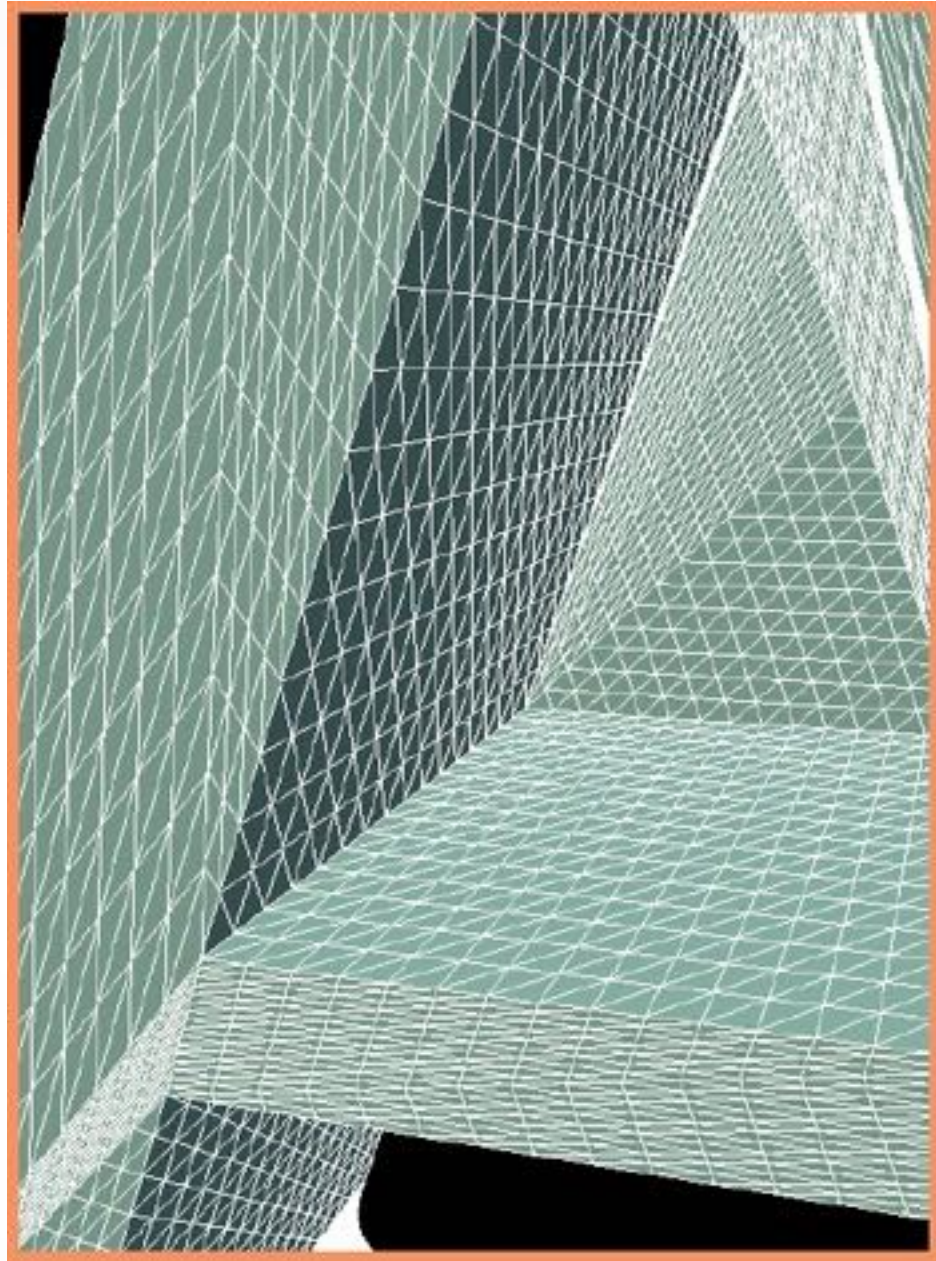
- **Cluster Generation**
 - Hierarchical approach
 - Top-down or bottom-up
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

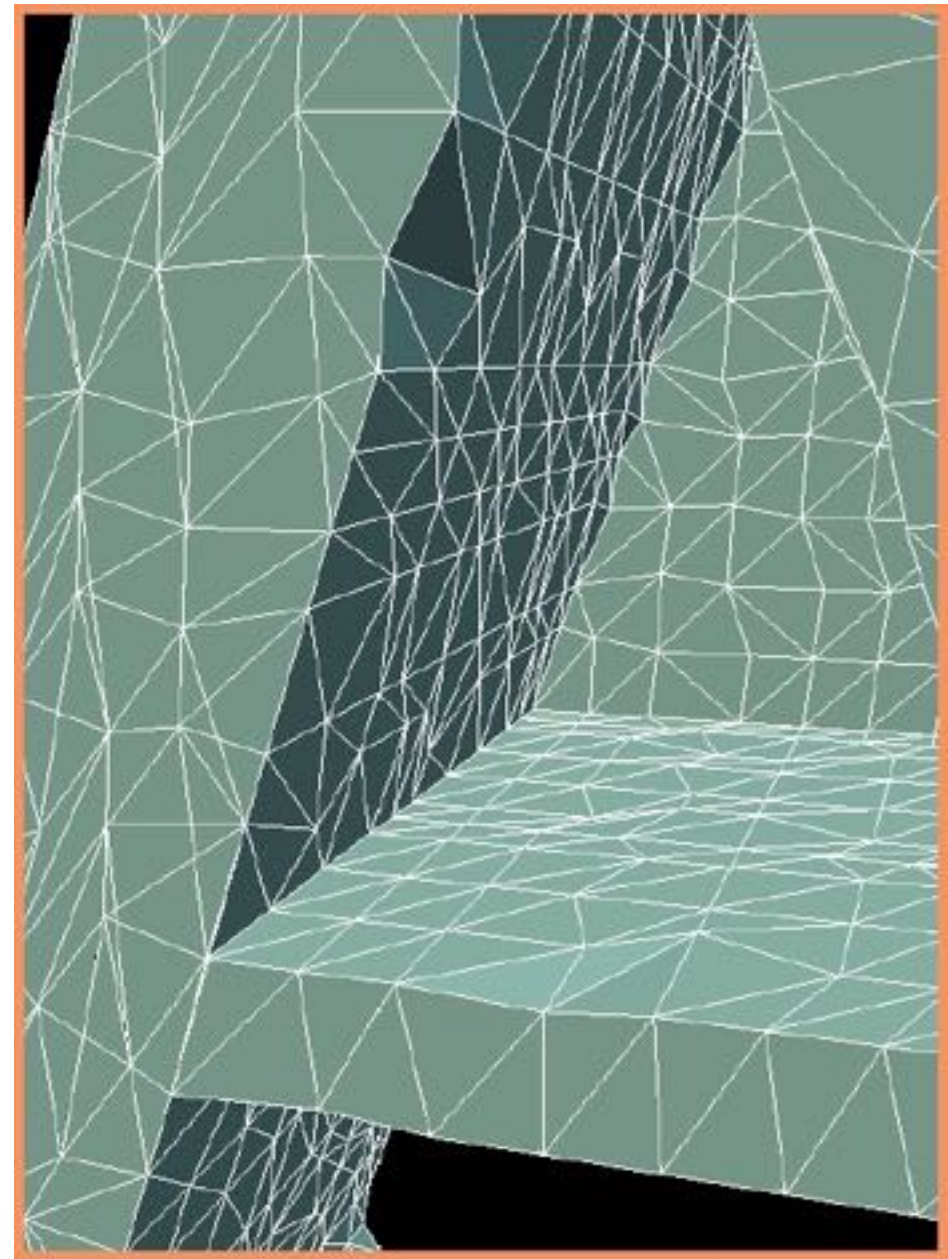
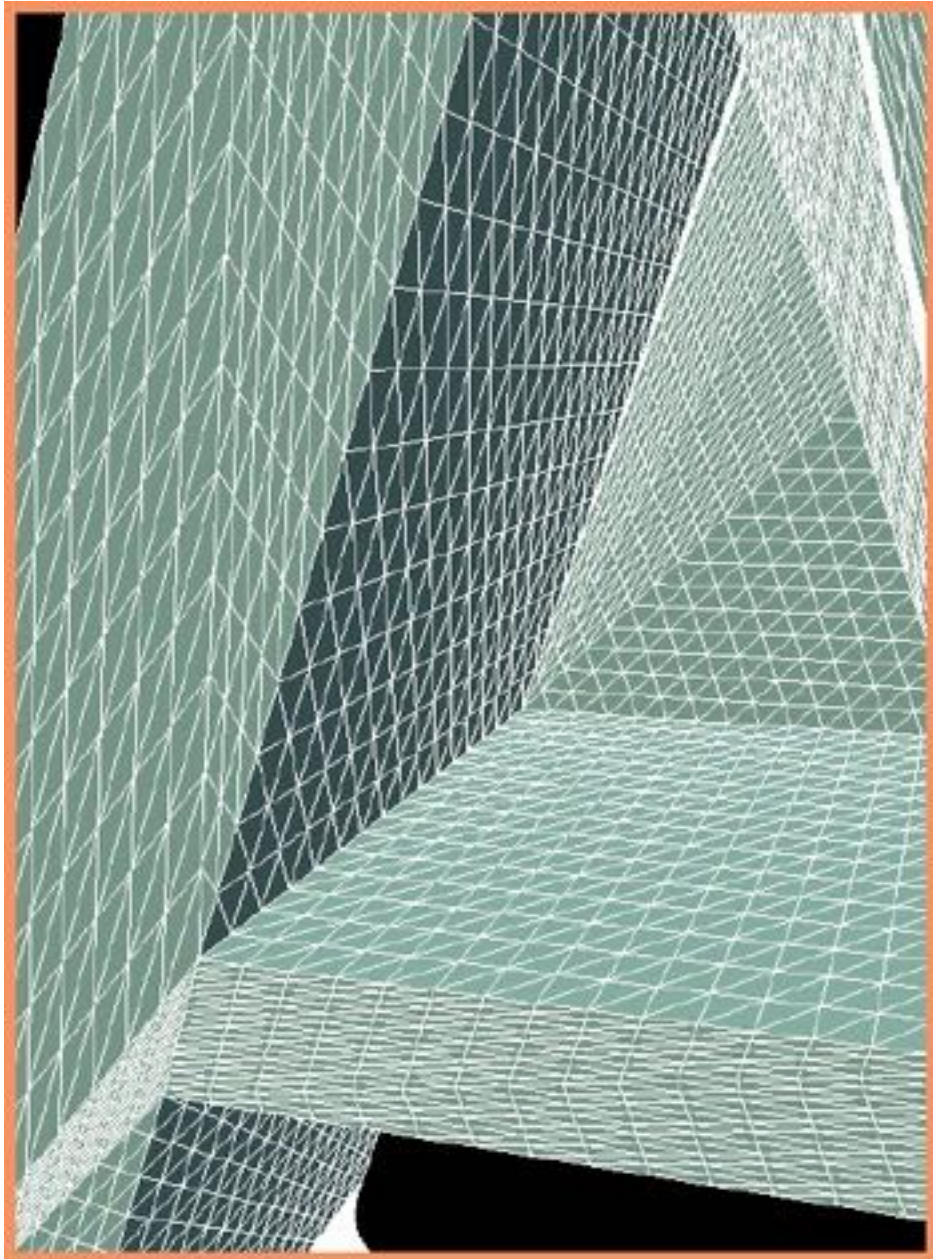
- Cluster Generation
- **Computing a representative**
 - Average/median vertex position
 - Error quadrics
- Mesh generation
- Topology changes

Computing a Representative



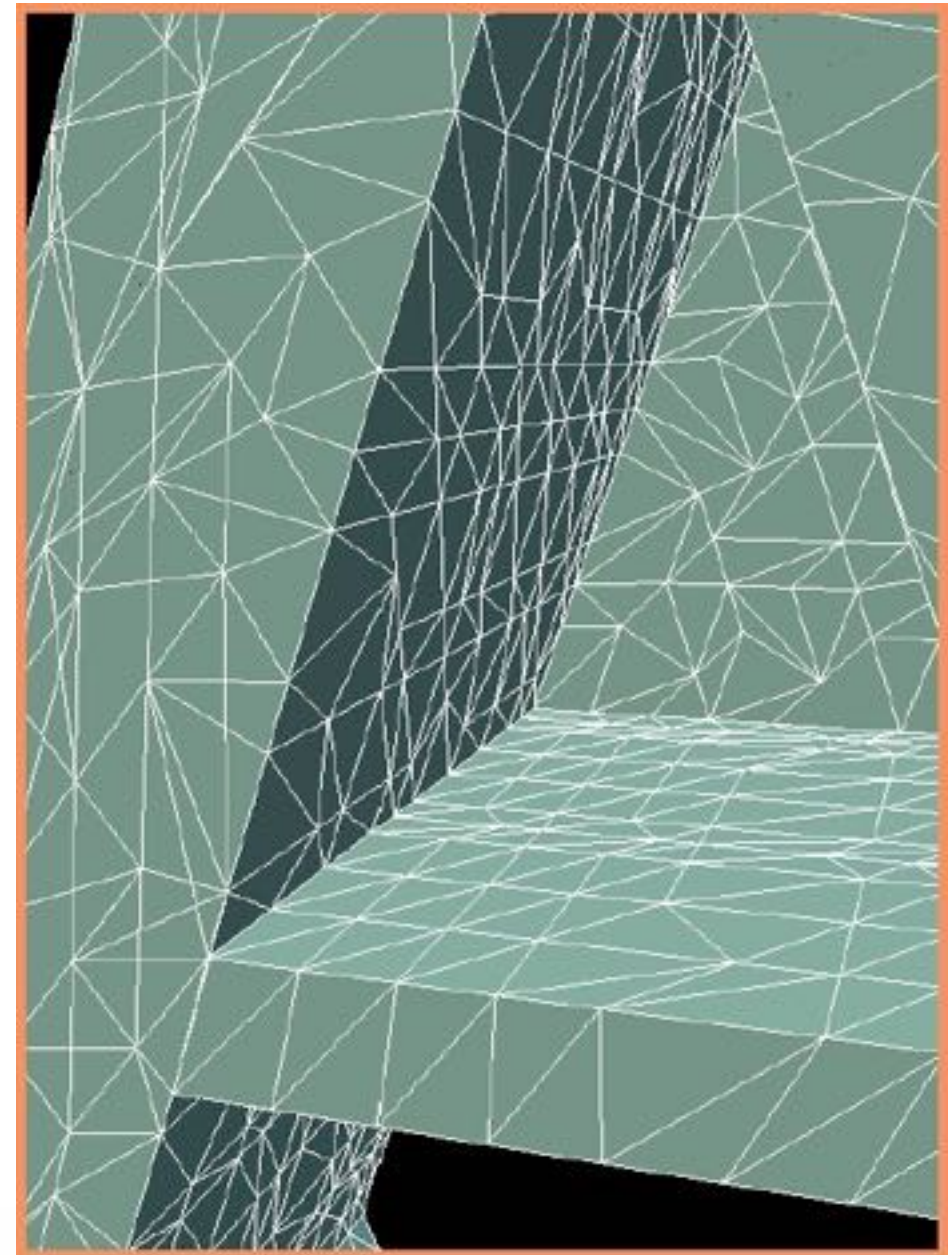
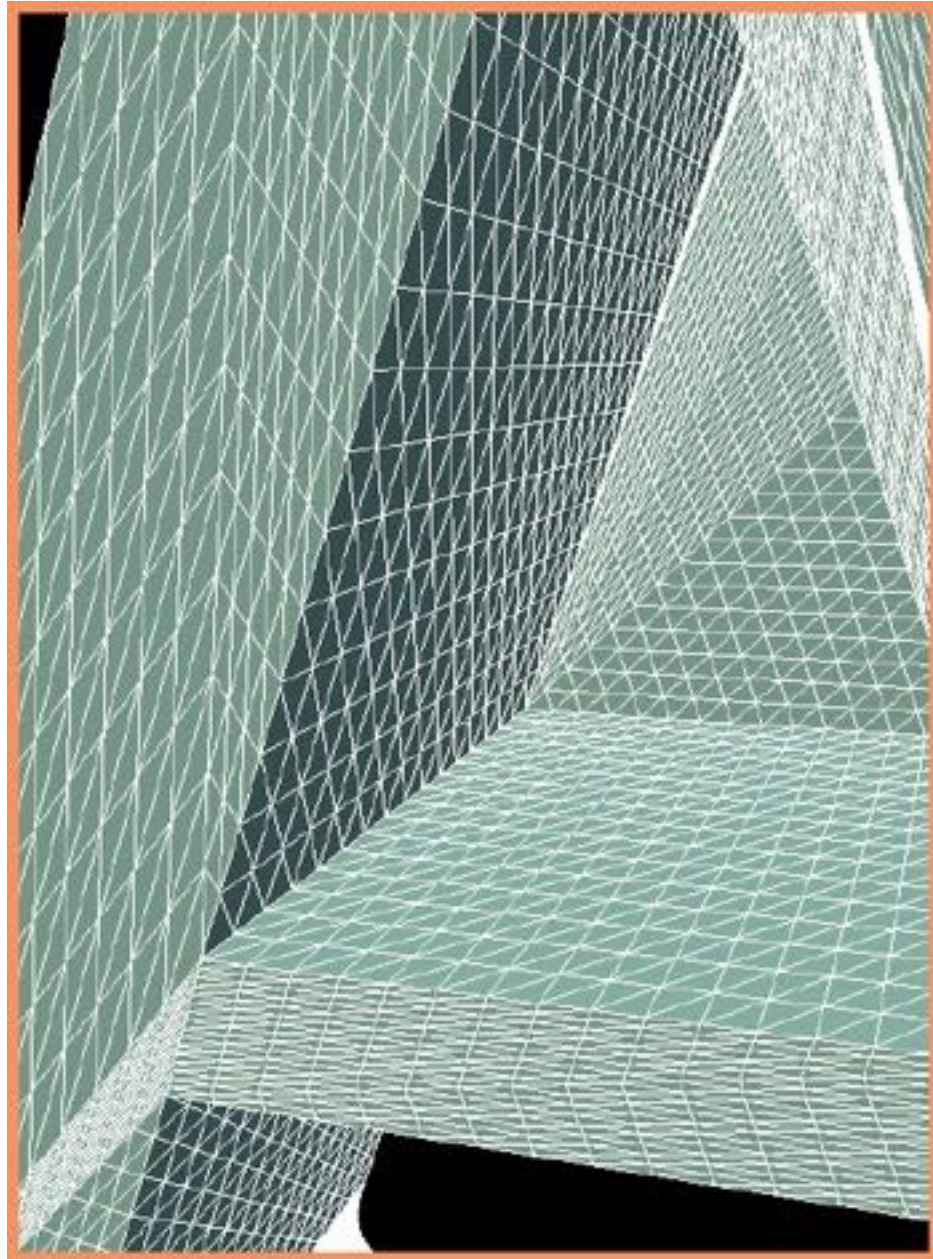
average vertex position → low pass filter

Computing a Representative



median vertex position → sub-sampling

Computing a Representative



error quadrics $\rightarrow$ feature preservation

Error Quadrics

Squared distance to plane

$$\mathbf{p} = (x, y, z, 1)^T, \quad \mathbf{q} = (a, b, c, d)^T$$

$$\text{dist}(\mathbf{q}, \mathbf{p})^2 = (\mathbf{q}^T \mathbf{p})^2 = \mathbf{p}^T (\mathbf{q} \mathbf{q}^T) \mathbf{p} =: \mathbf{p}^T \mathbf{Q}_q \mathbf{p}$$

Error Quadrics

Squared distance to plane

$$\mathbf{p} = (x, y, z, 1)^T, \quad \mathbf{q} = (a, b, c, d)^T$$

$$\text{dist}(\mathbf{q}, \mathbf{p})^2 = (\mathbf{q}^T \mathbf{p})^2 = \mathbf{p}^T (\mathbf{q} \mathbf{q}^T) \mathbf{p} =: \mathbf{p}^T \mathbf{Q}_{\mathbf{q}} \mathbf{p}$$

$$\mathbf{Q}_{\mathbf{q}} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

Sum of distances to vertex planes

$$\sum_i \text{dist}(\mathbf{q}_i, \mathbf{p})^2 =$$

Error Quadrics

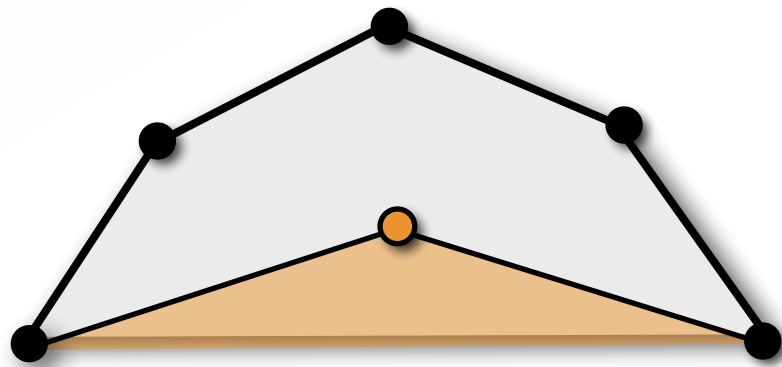
Sum of distances to vertex planes

$$\sum_i \text{dist}(\mathbf{q}_i, \mathbf{p})^2 = \sum_i \mathbf{p}^T \mathbf{Q}_{\mathbf{q}_i} \mathbf{p} = \mathbf{p}^T \left(\sum_i \mathbf{Q}_{\mathbf{q}_i} \right) \mathbf{p} =: \mathbf{p}^T \mathbf{Q}_p \mathbf{p}$$

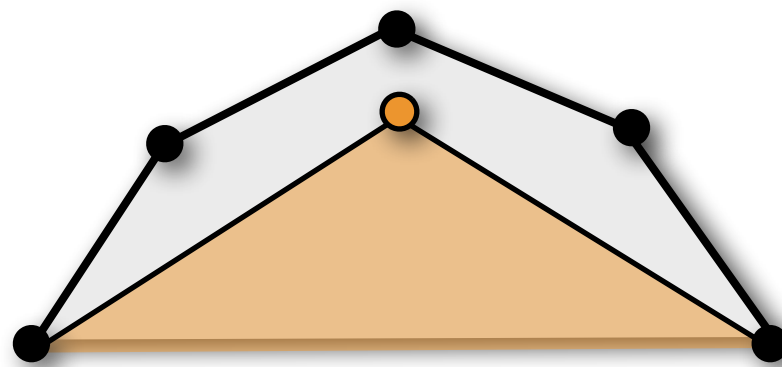
Point that minimizes the error

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{p}^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

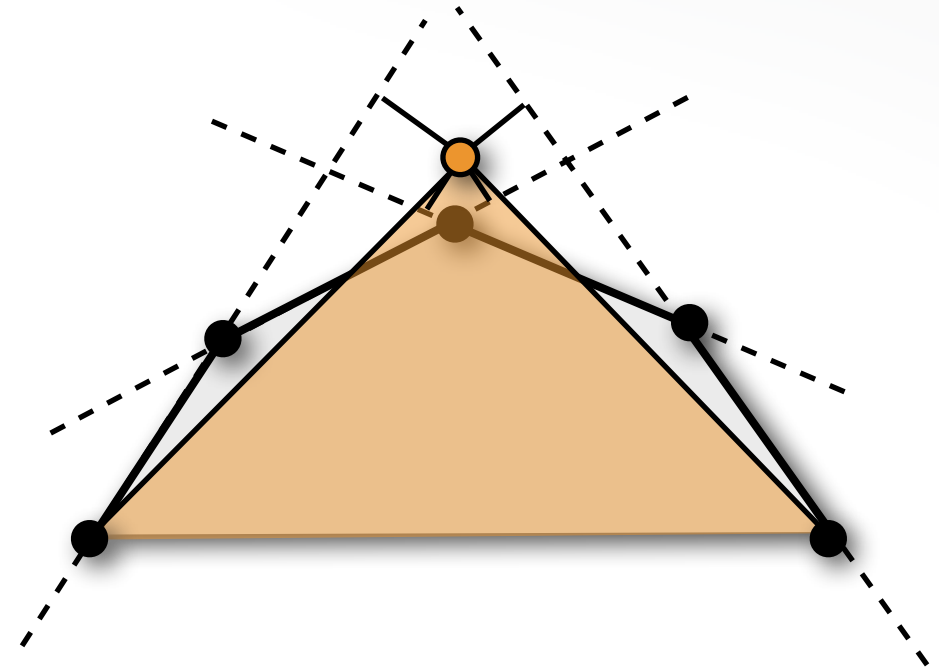
Comparison



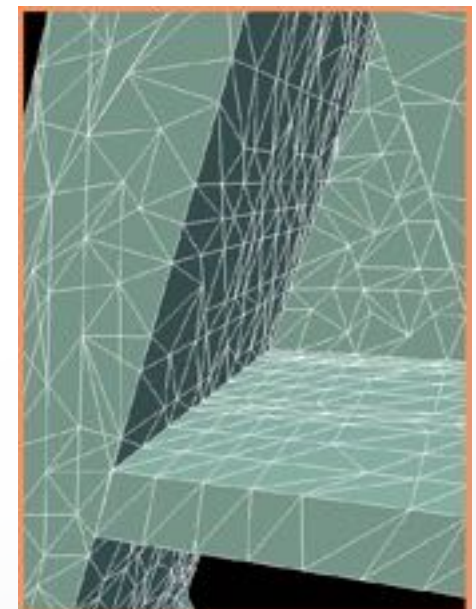
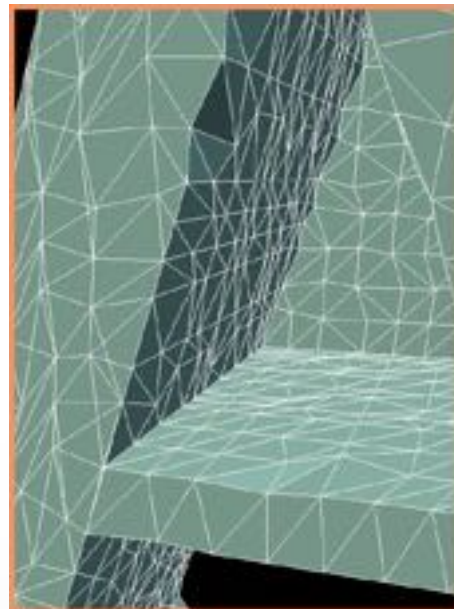
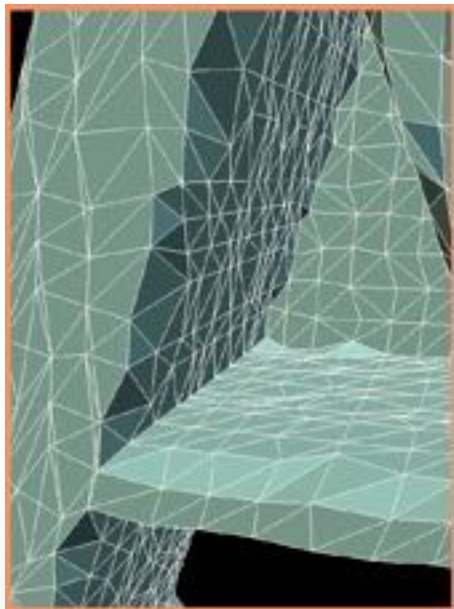
average



median



error quadric

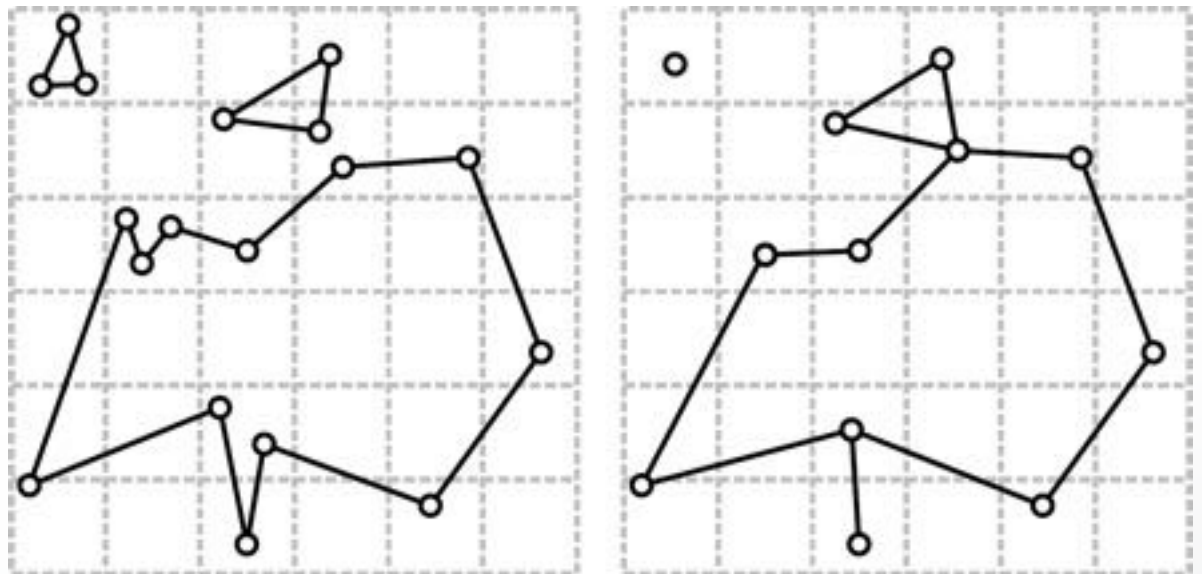


Vertex Clustering

- Cluster Generation
- Computing a representative
- **Mesh generation**
 - Clusters $\mathbf{p} \Leftrightarrow \{\mathbf{p}_0, \dots, \mathbf{p}_n\}, \mathbf{q} \Leftrightarrow \{\mathbf{q}_0, \dots, \mathbf{q}_n\}$
 - Connect $(\mathbf{p}, \mathbf{q})$ if there was an edge $(\mathbf{p}_i, \mathbf{q}_j)$
- Topology changes

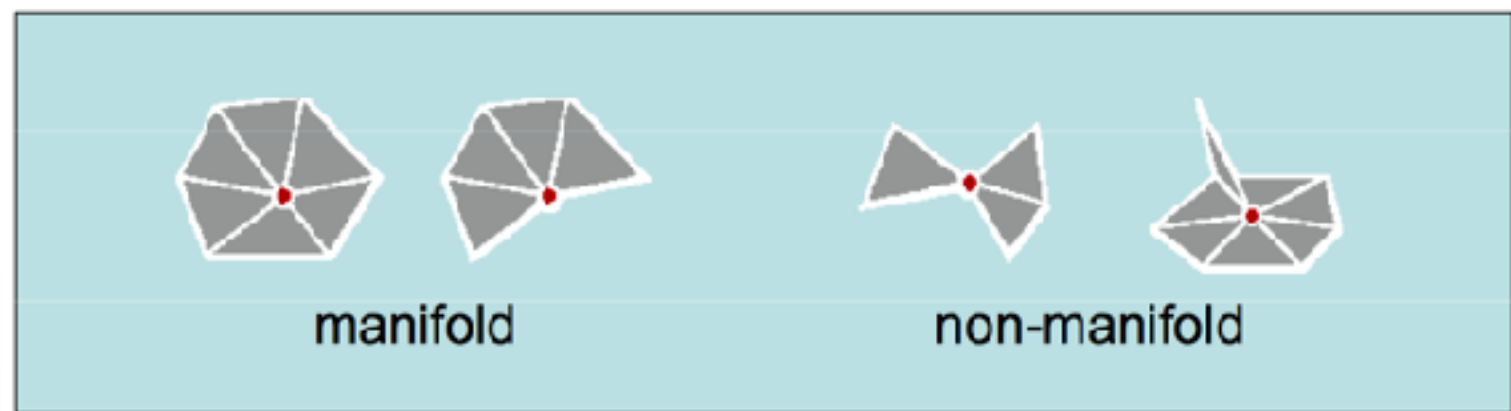
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- **Topology changes**
 - If different sheets pass through on cell
 - Can be non-manifold



Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- **Topology changes**
 - If different sheets pass through on cell
 - Can be non-manifold



Outline

Mesh Decimation methods

- Vertex Clustering
- **Iterative Decimation**

Example



Incremental Decimation

- **General Setup**
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

General Setup

Repeat:

pick mesh region

apply decimation operator

Until no further reduction possible

Greedy Optimization

For each region

 evaluate quality after decimation

 enqueue (quality, region)

Repeat:

 pick best mesh region

 apply decimation operator

 update queue

Until no further reduction possible

Global Error Control

For each region

 evaluate quality after decimation

 enqueue (quality, region)

Repeat:

 pick best mesh region

 if error $< \epsilon$

 apply decimation operator

 update queue

Until no further reduction possible

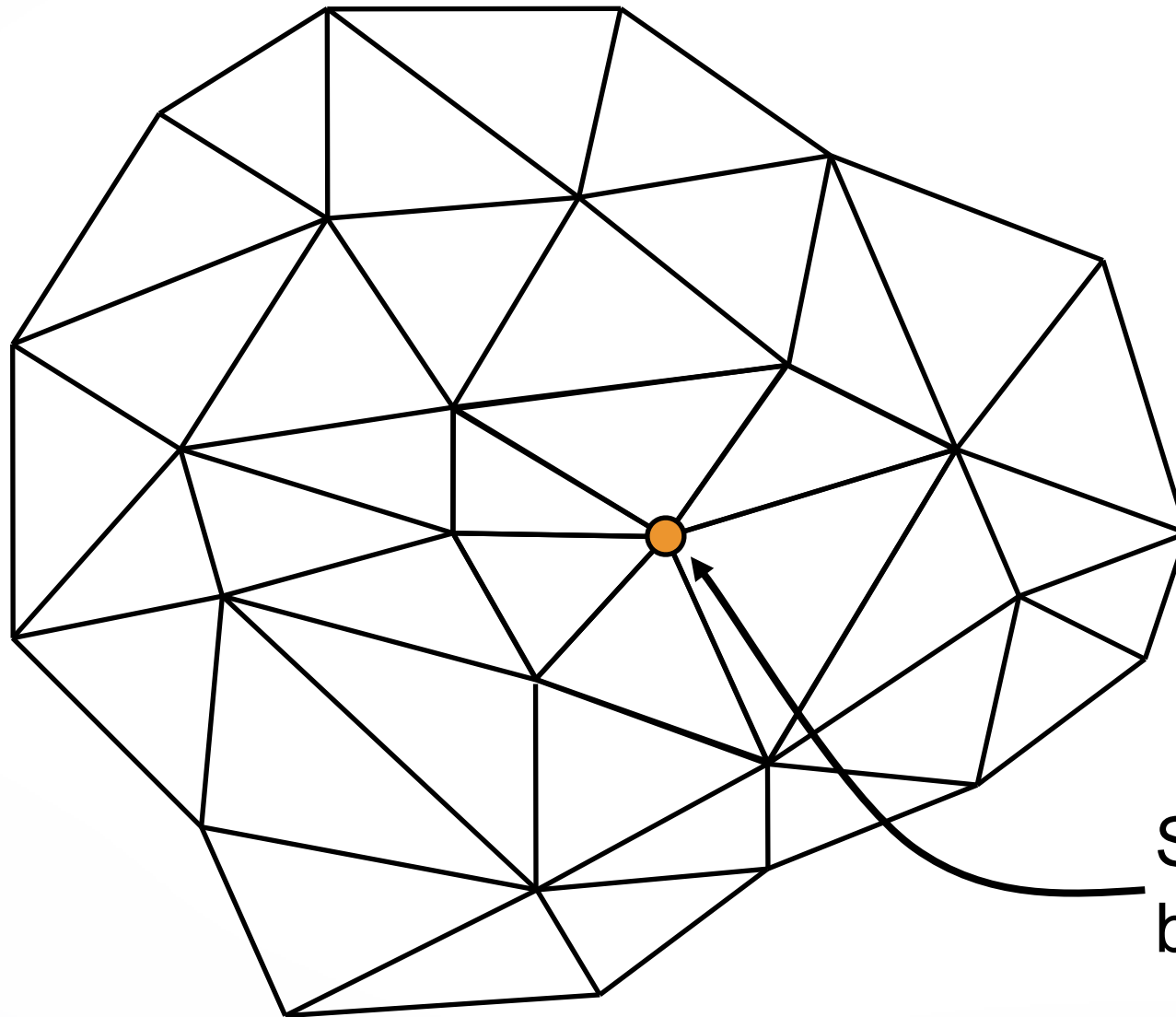
Incremental Decimation

- General Setup
- **Decimation operators**
- Error metrics
- Fairness criteria
- Topology changes

Decimation Operators

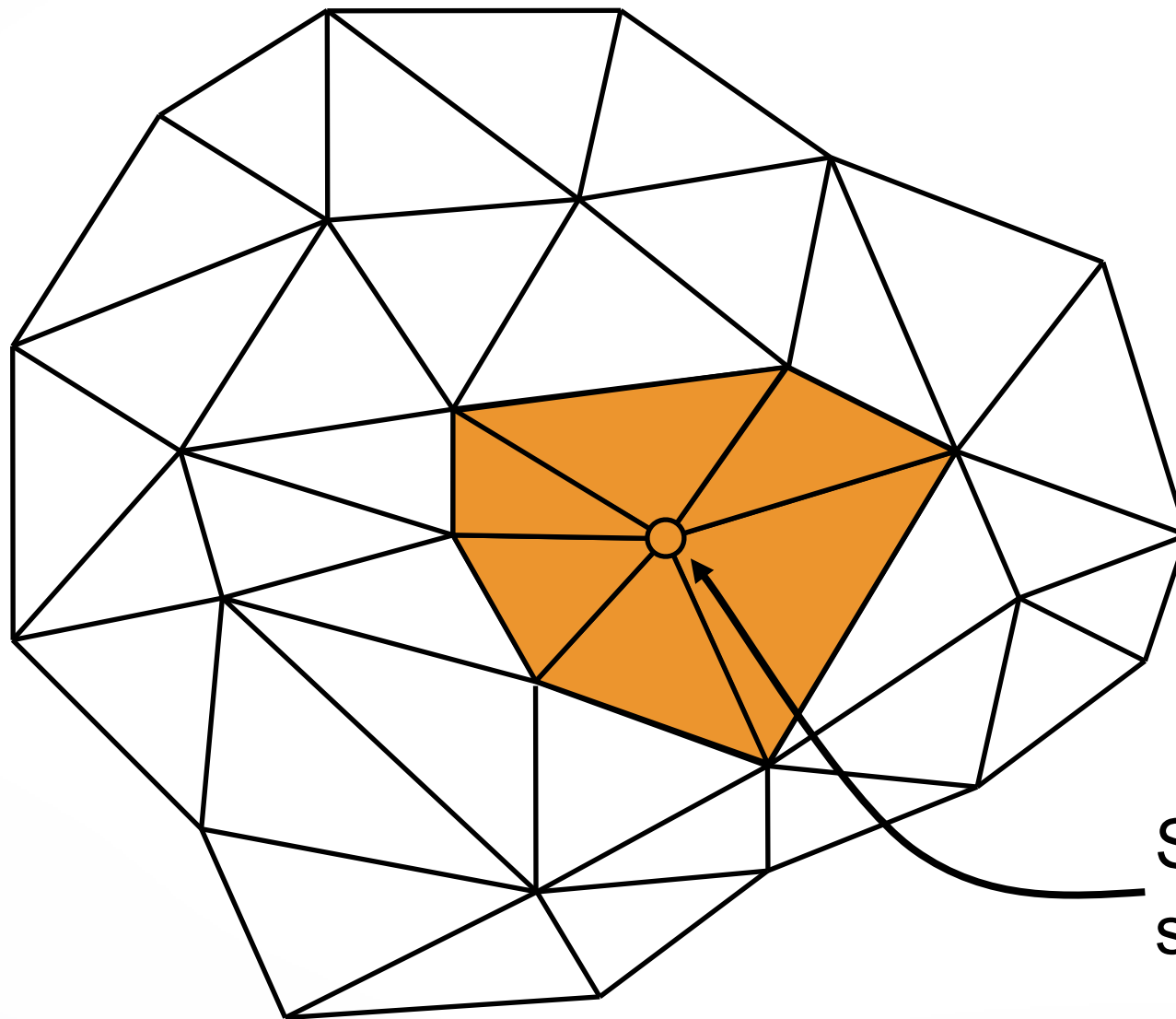
- What is a “region”?
- What are the DOFs for re-triangulation?
- Classification
 - topology-changing vs. topology-preserving
 - subsampling vs. filtering
 - inverse operation → progressive meshes

Vertex Removal



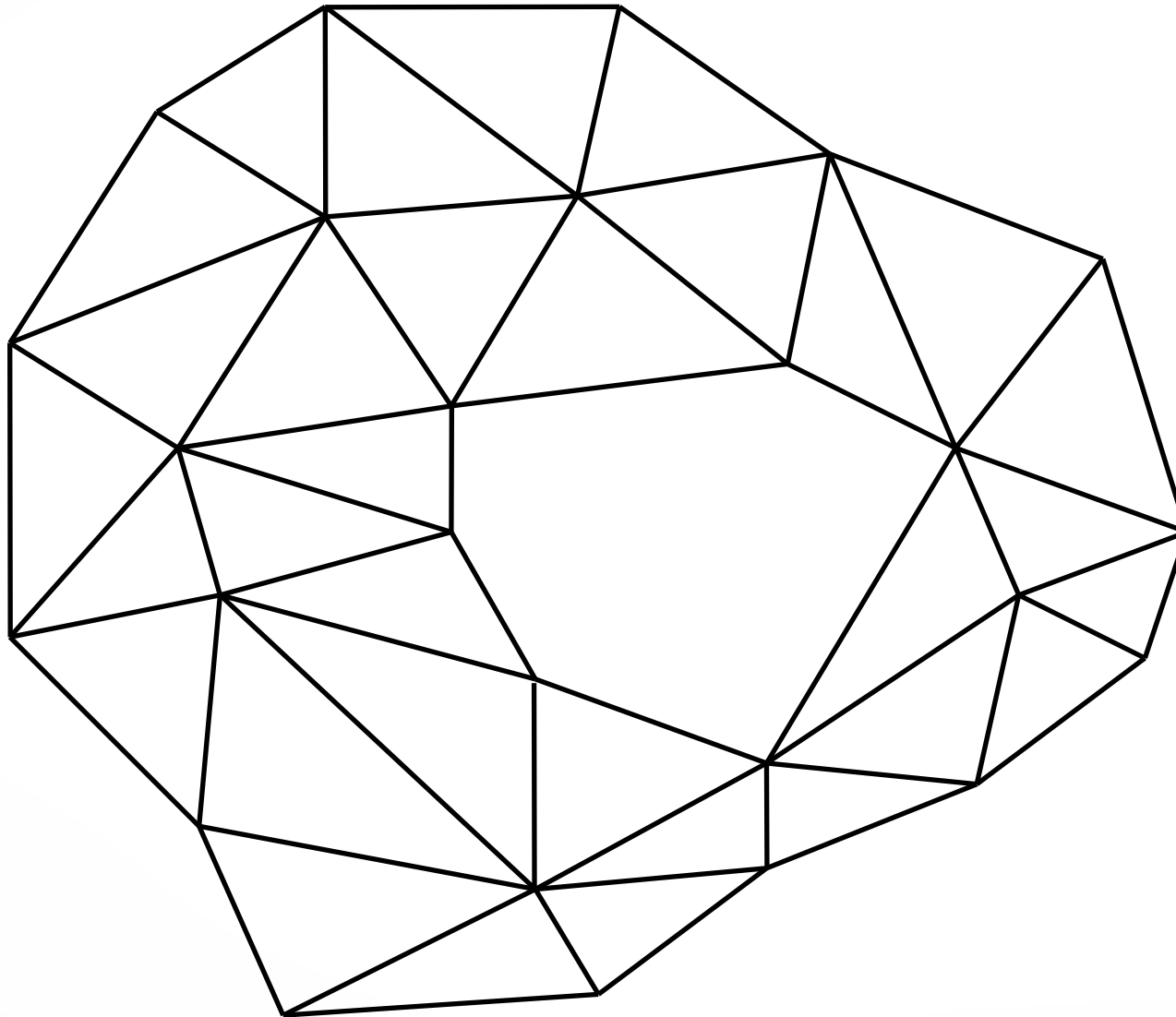
Select a vertex to
be eliminated

Vertex Removal



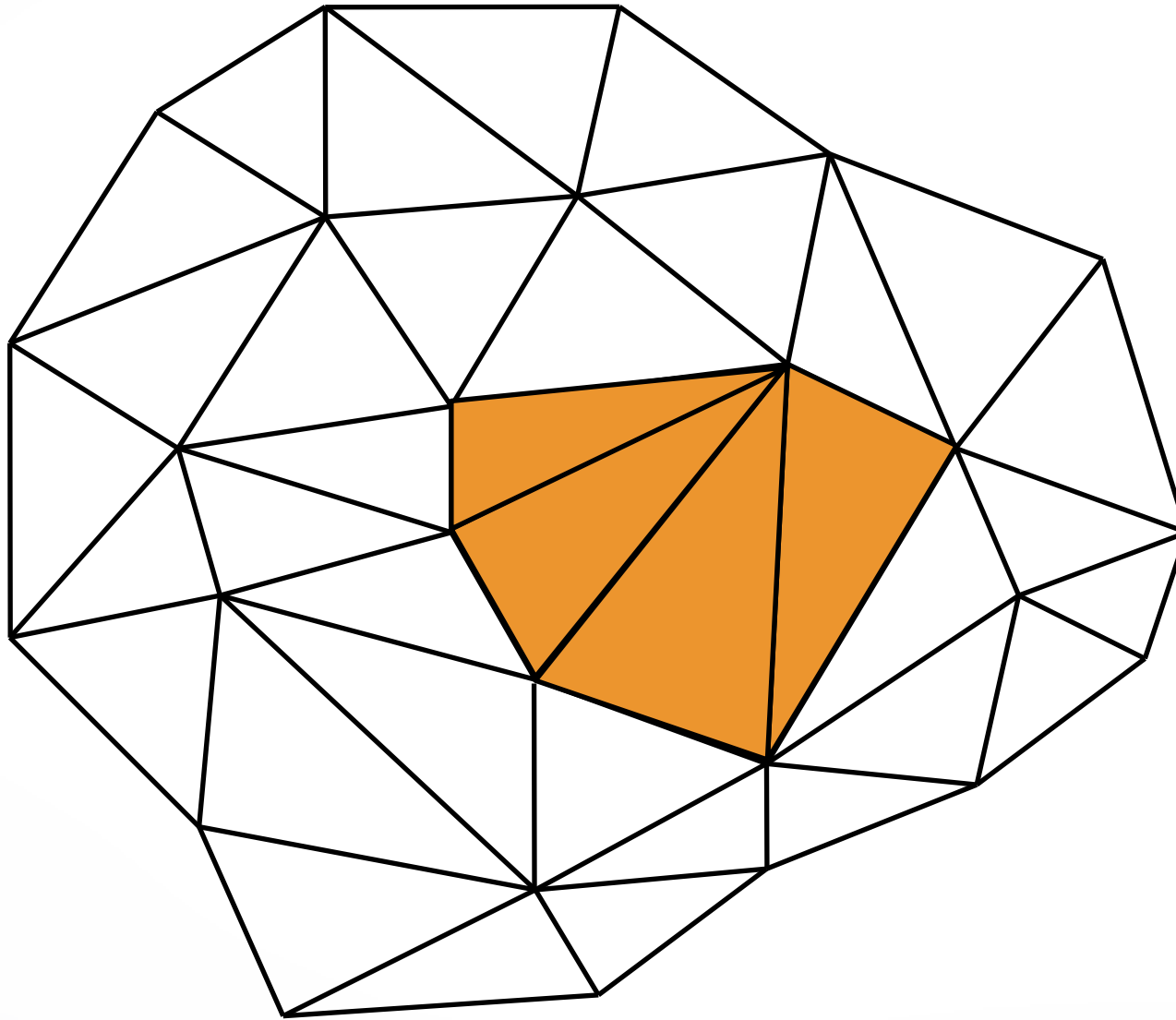
Select all triangles
sharing this vertex

Vertex Removal



Remove the
selected triangles,
creating a hole

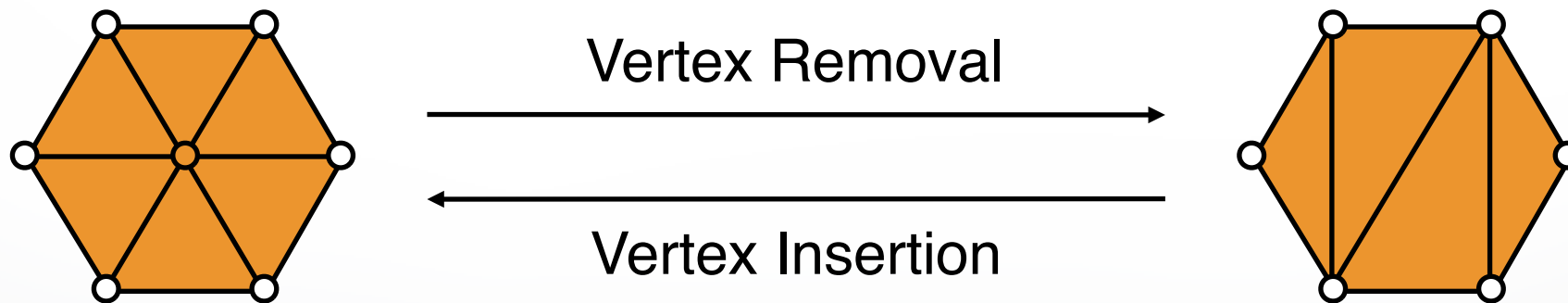
Vertex Removal



Fill the hole
with triangles

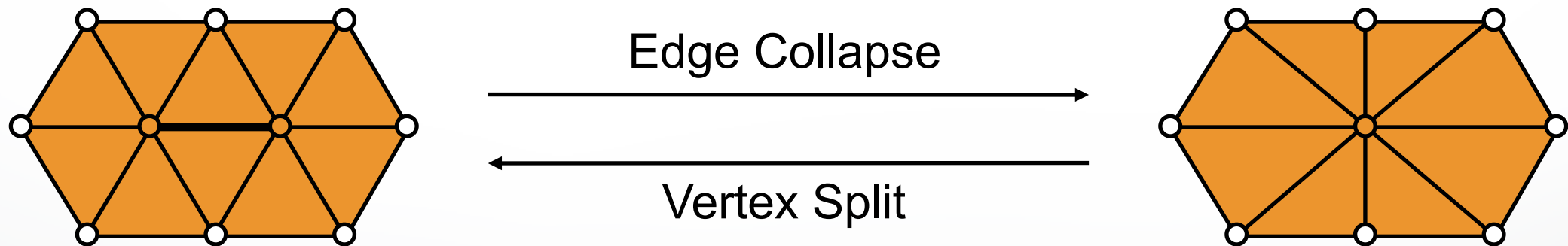
Decimation Operators

- Remove vertex
- Re-triangulate hole
 - Combinatorial DOFs
 - Sub-sampling



Decimation Operators

- Merge two adjacent triangles
- Define new vertex position
 - Continuous DOF
 - Filtering

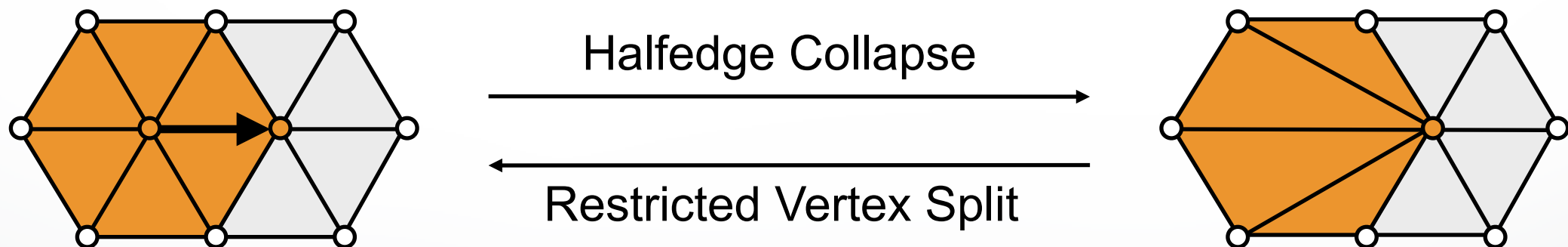


Decimation Operators

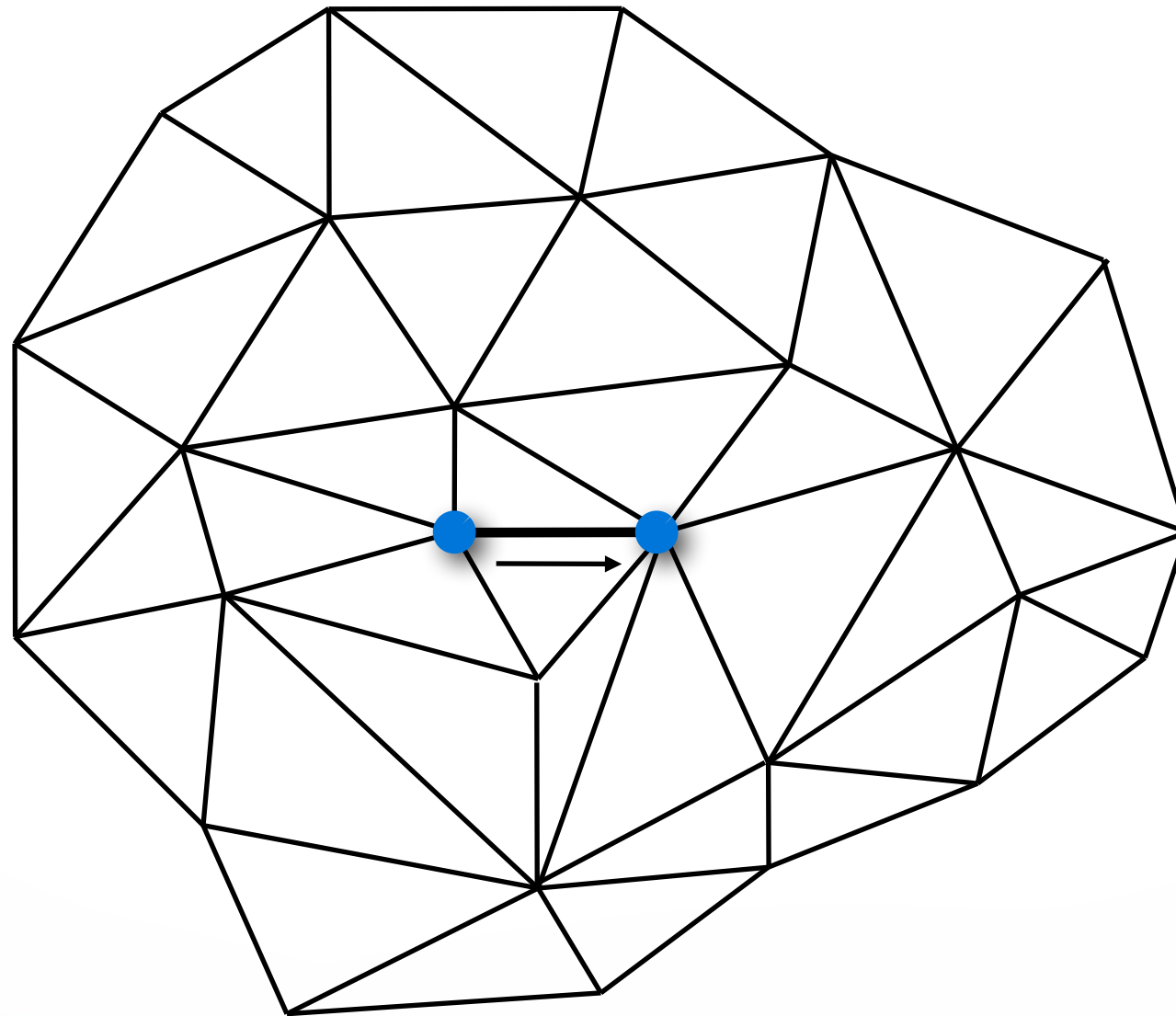
- Collapse edge into one end point
 - Special vertex removal
 - Special edge collapse
- No DOFs
 - One operator per half-edge
 - Sub-sampling



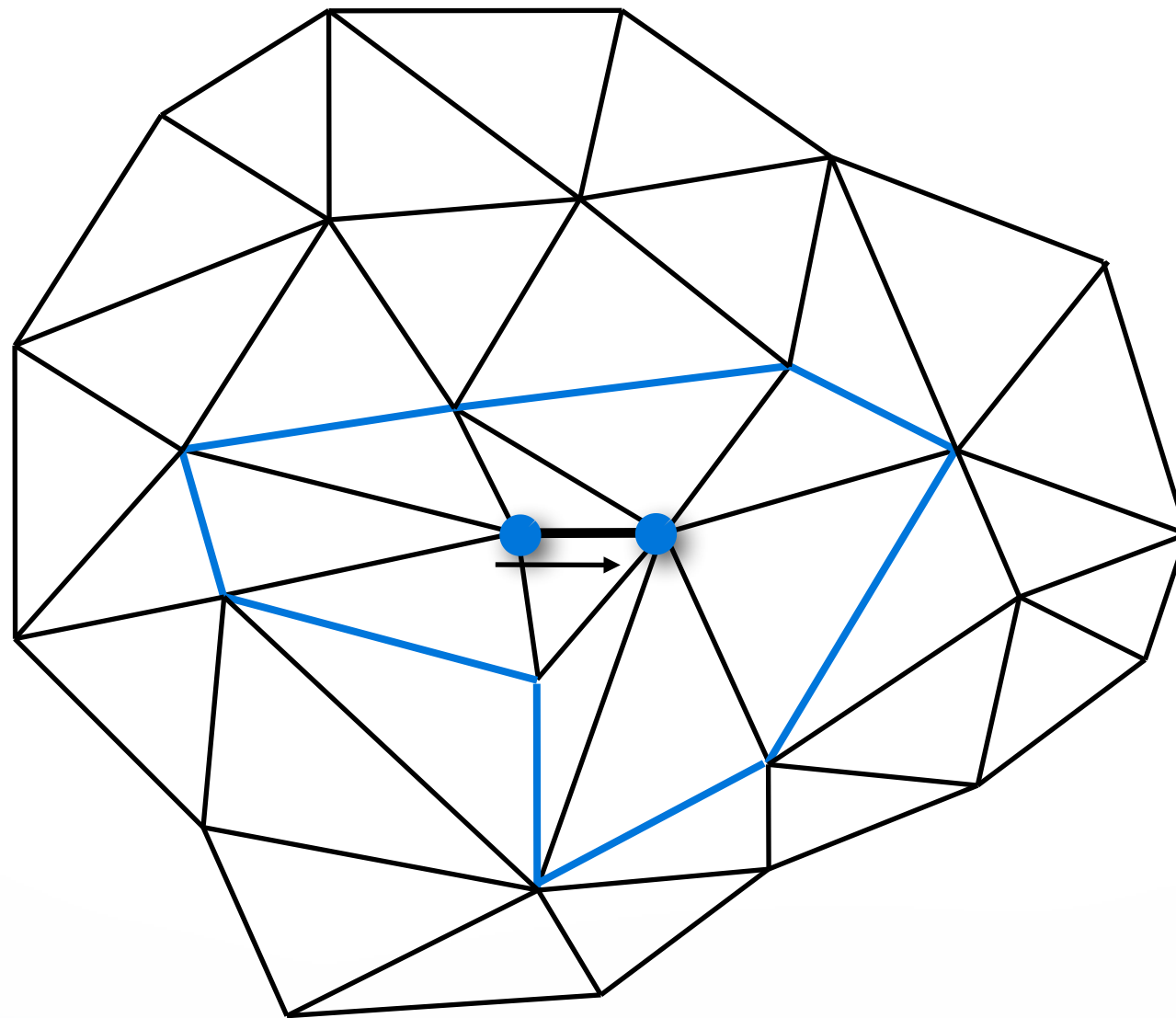
H. Hoppe: Progressive Meshes



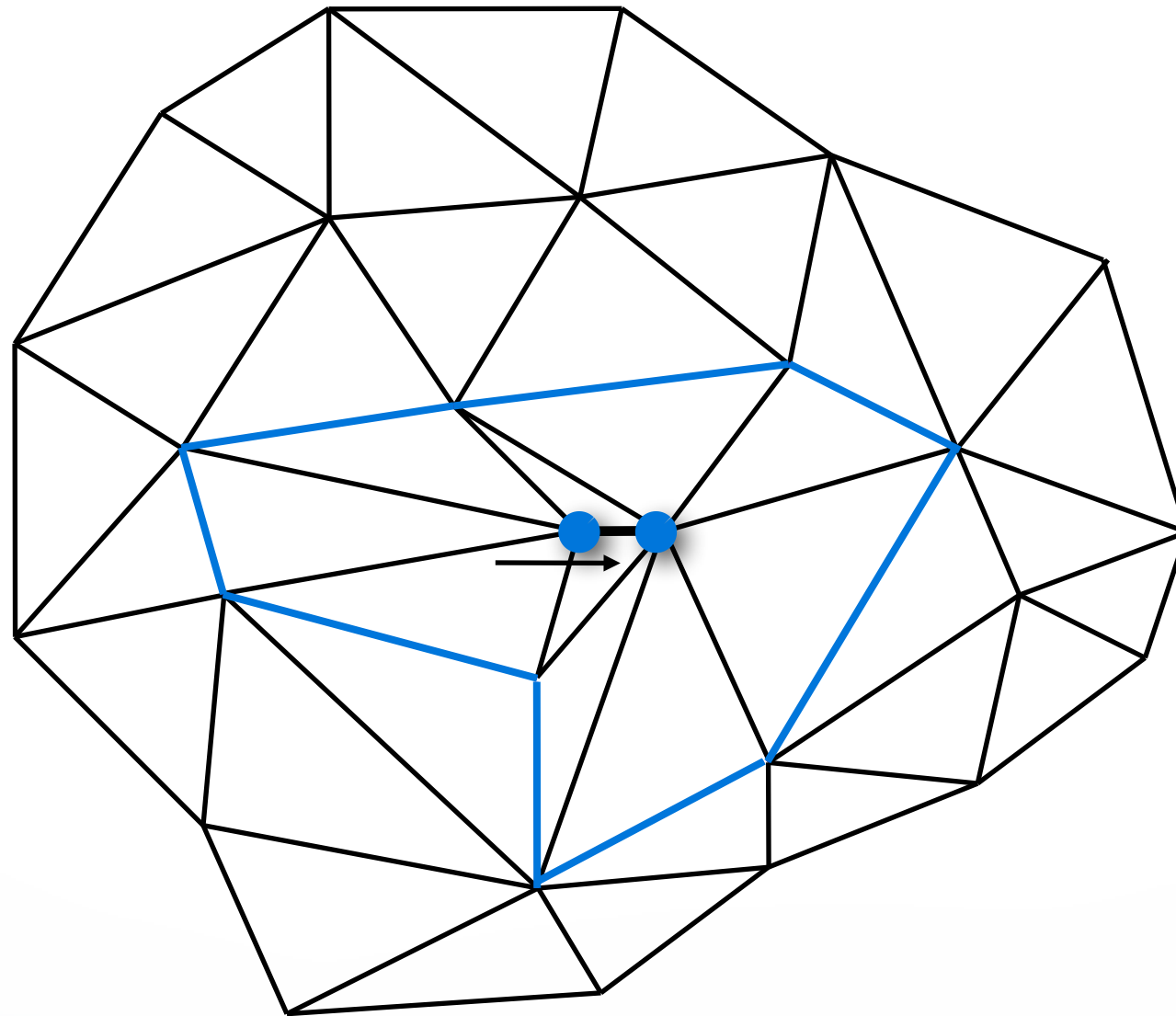
Edge Collapse



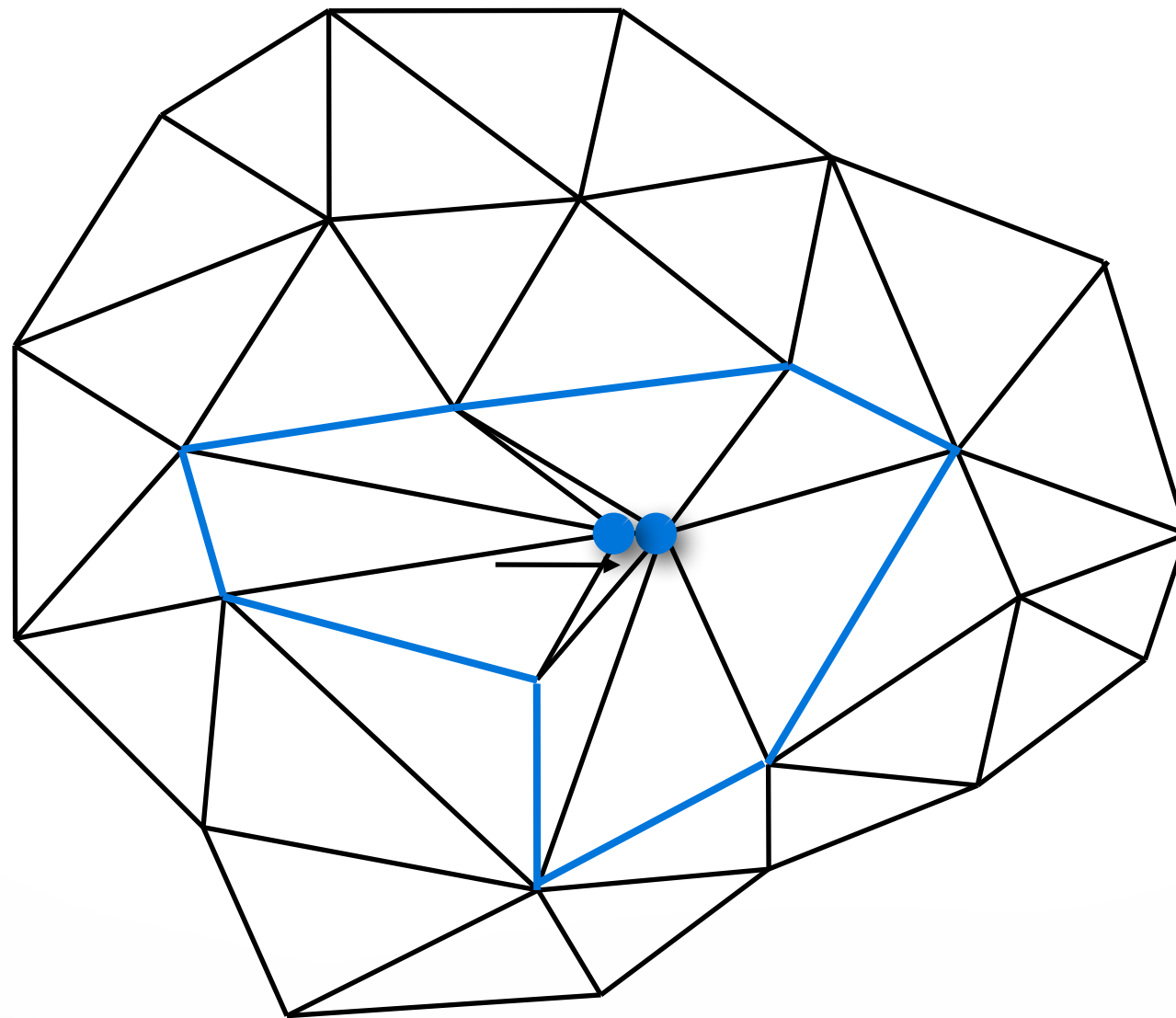
Edge Collapse



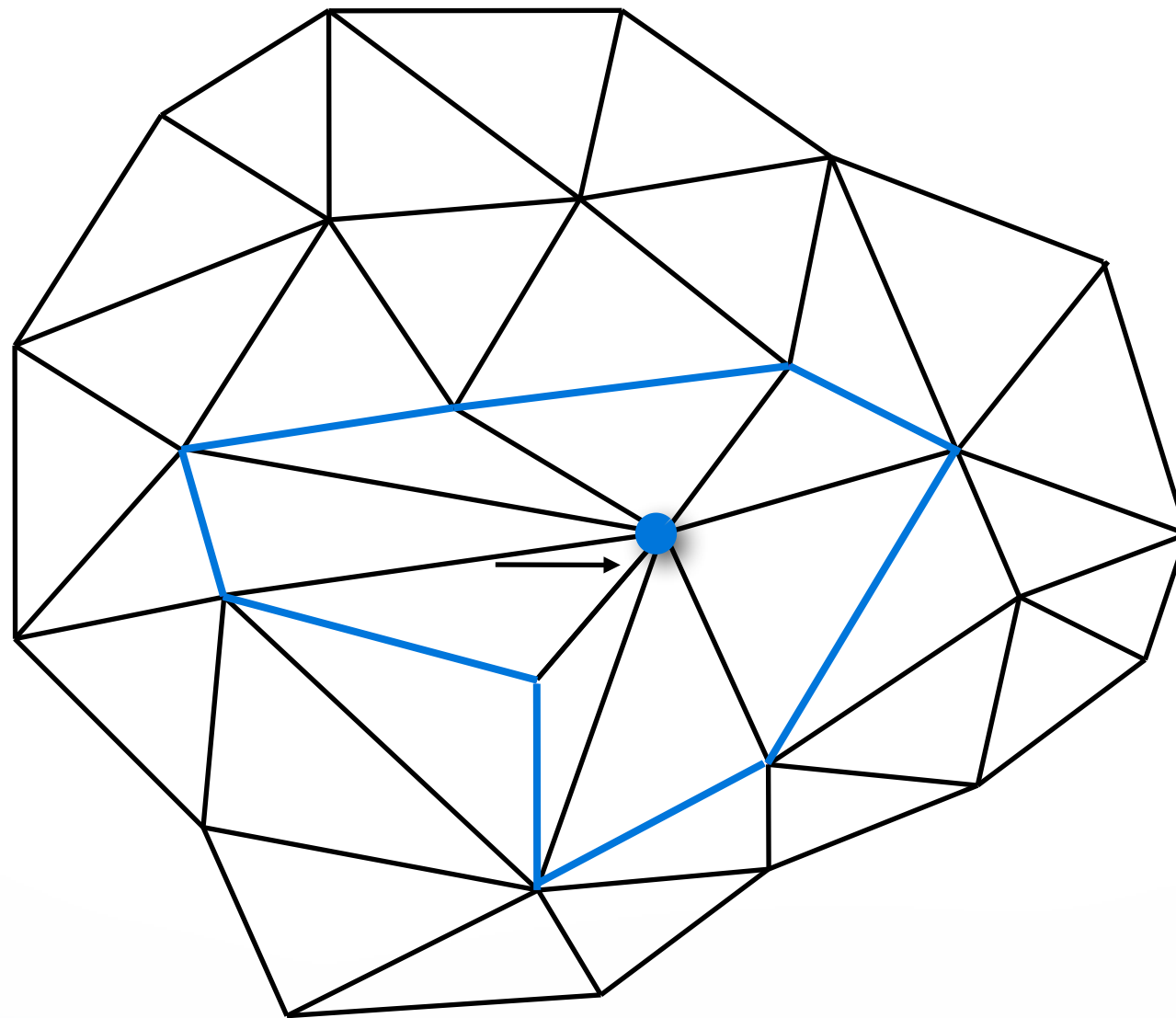
Edge Collapse



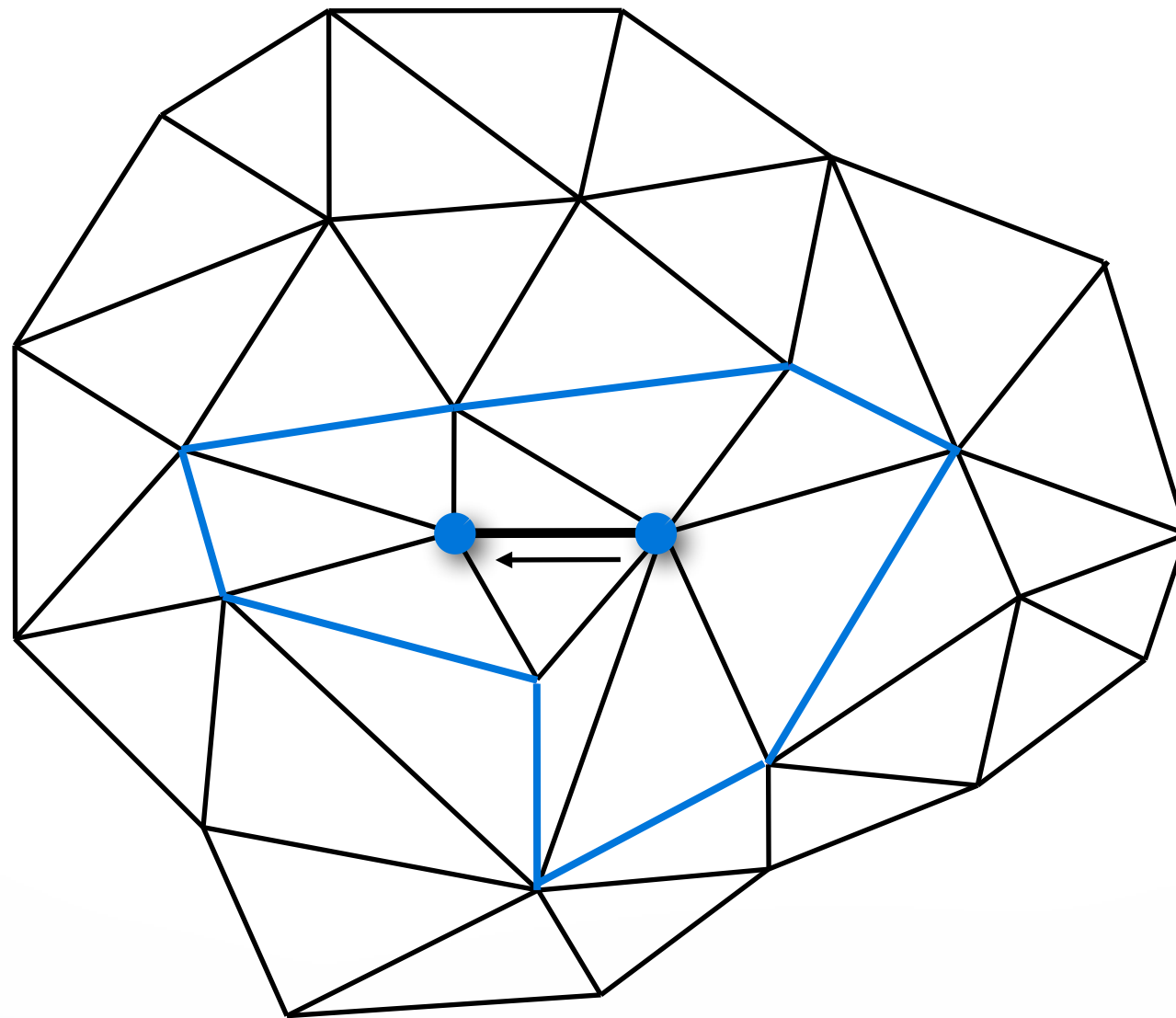
Edge Collapse



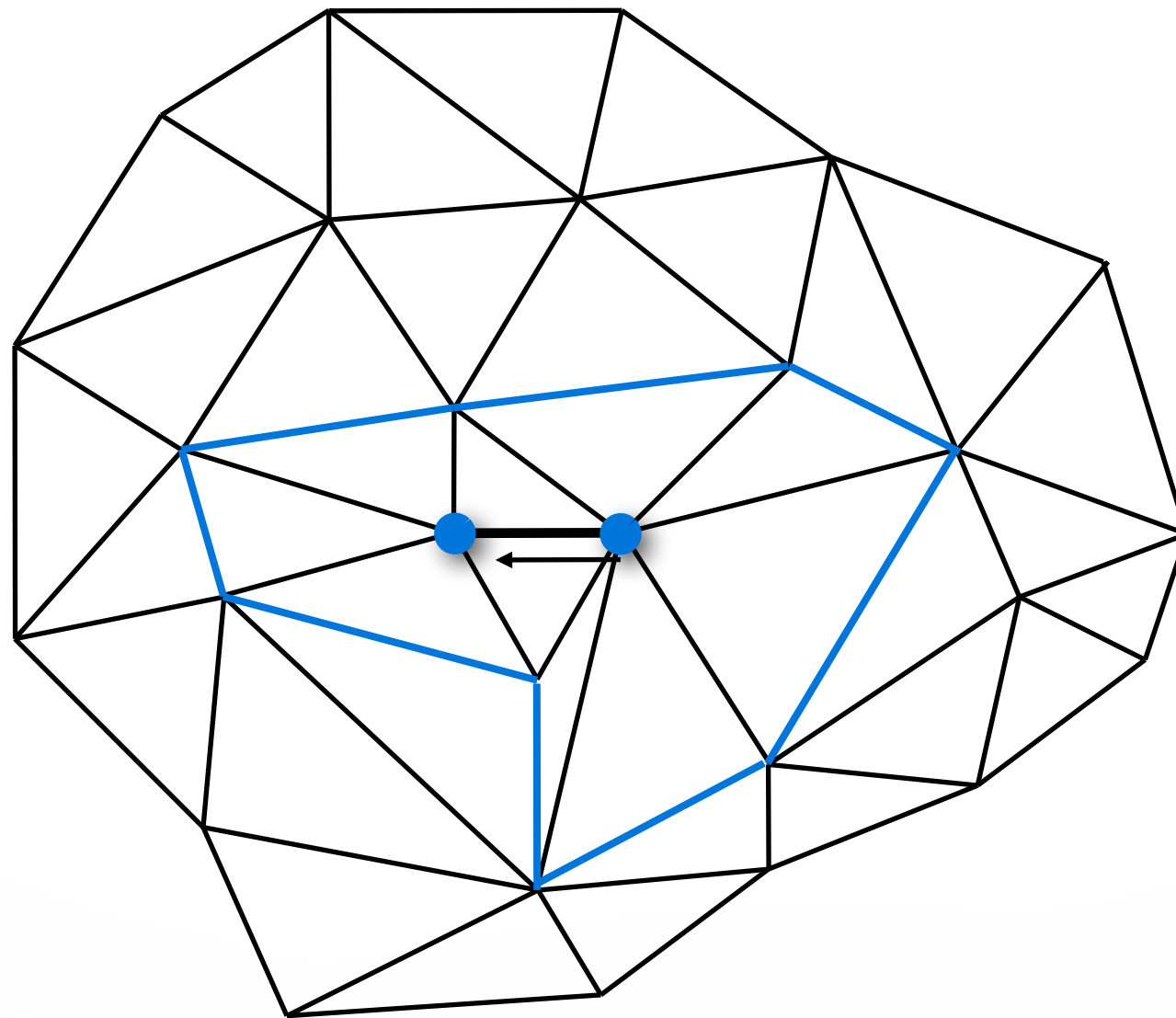
Edge Collapse



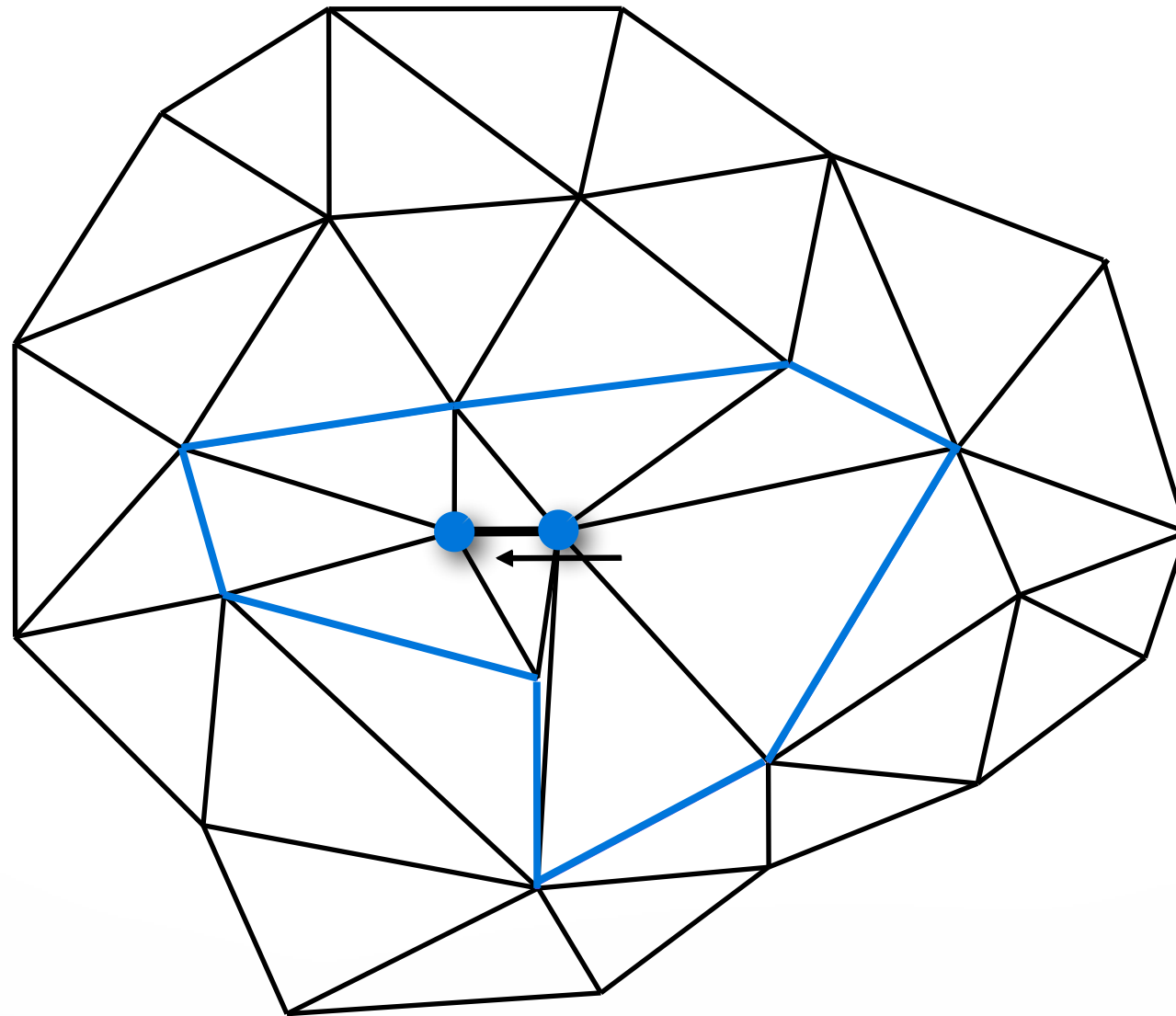
Edge Collapse



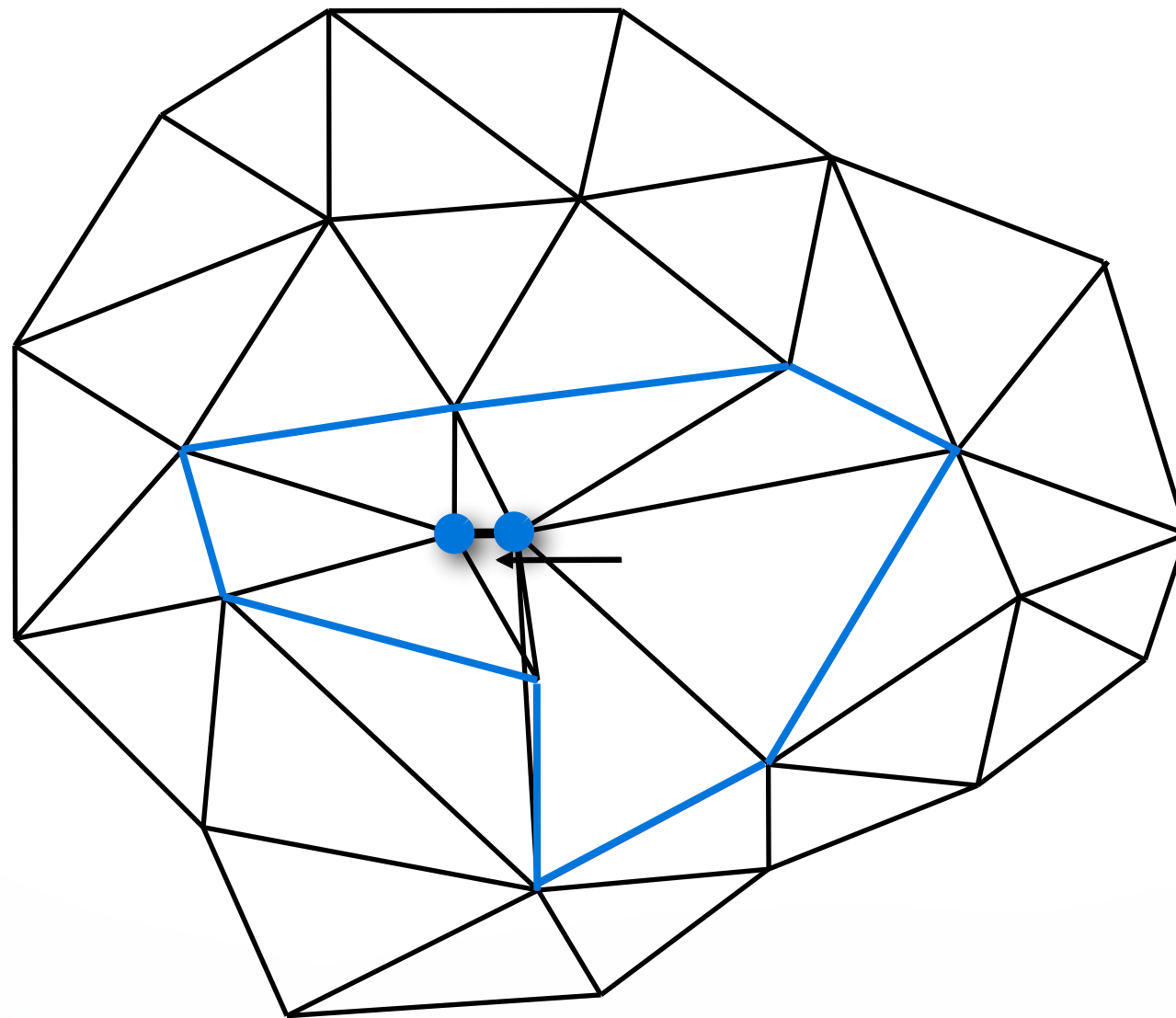
Edge Collapse



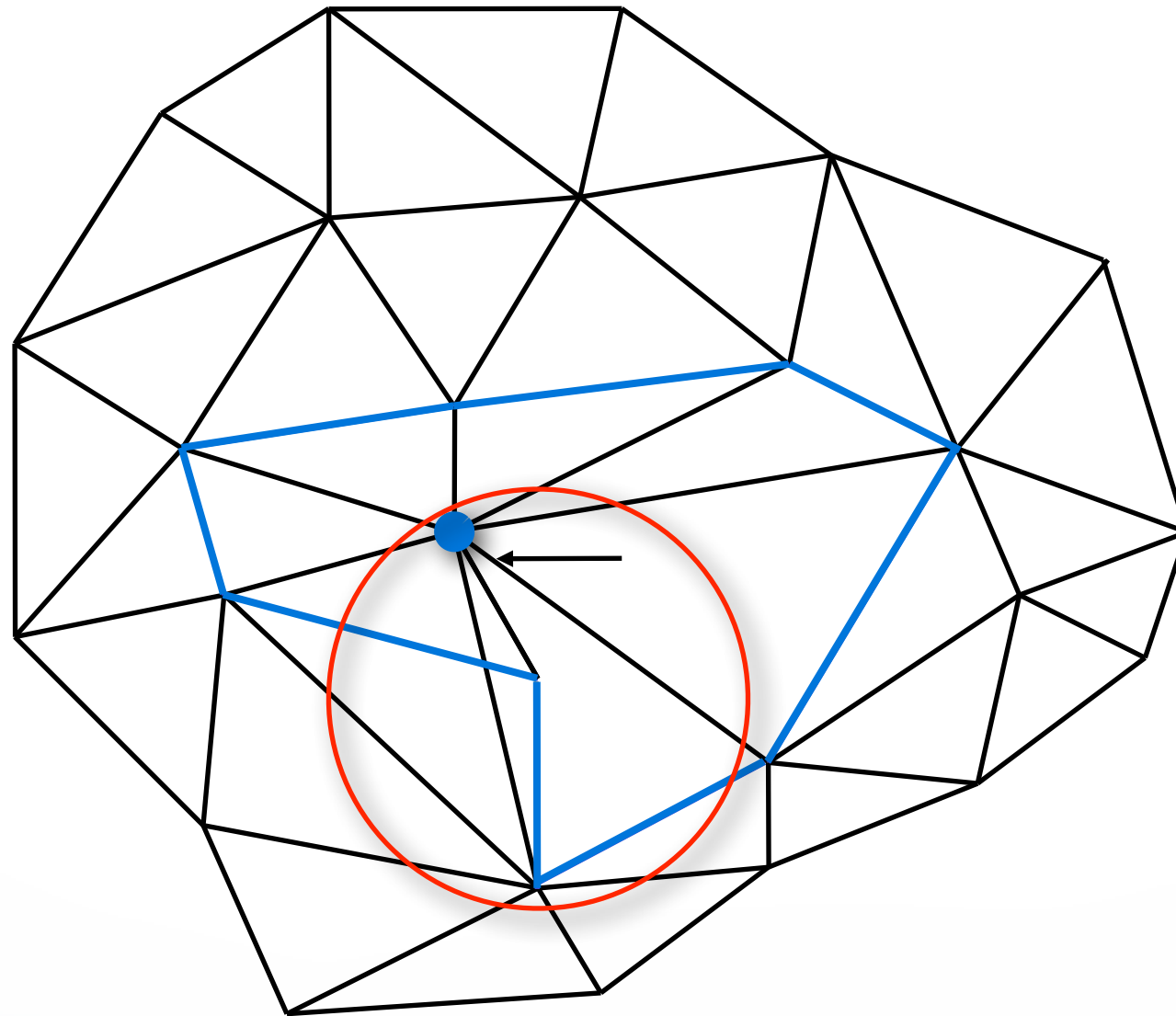
Edge Collapse



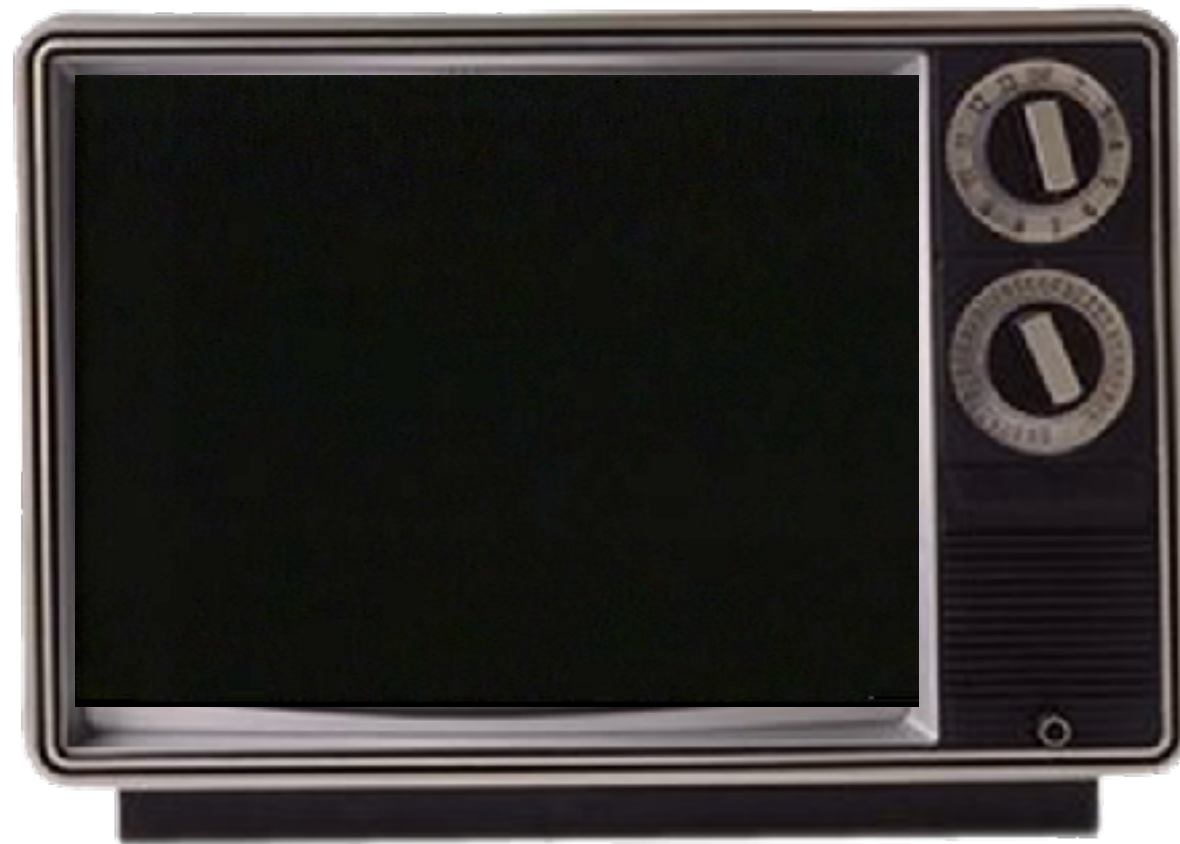
Edge Collapse



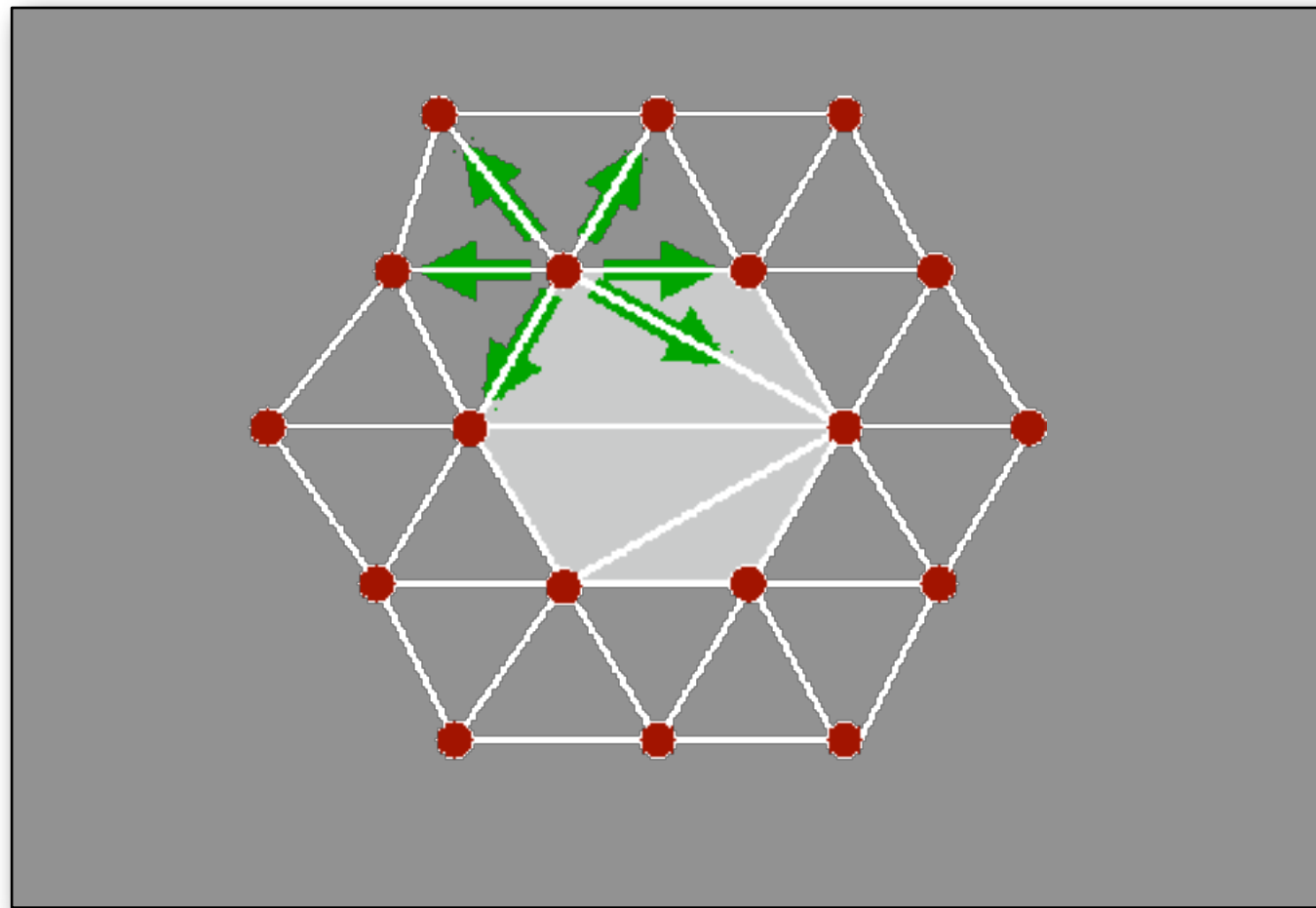
Edge Collapse (Flip!)



Application: **Progressive Meshes**



Priority Queue Updating



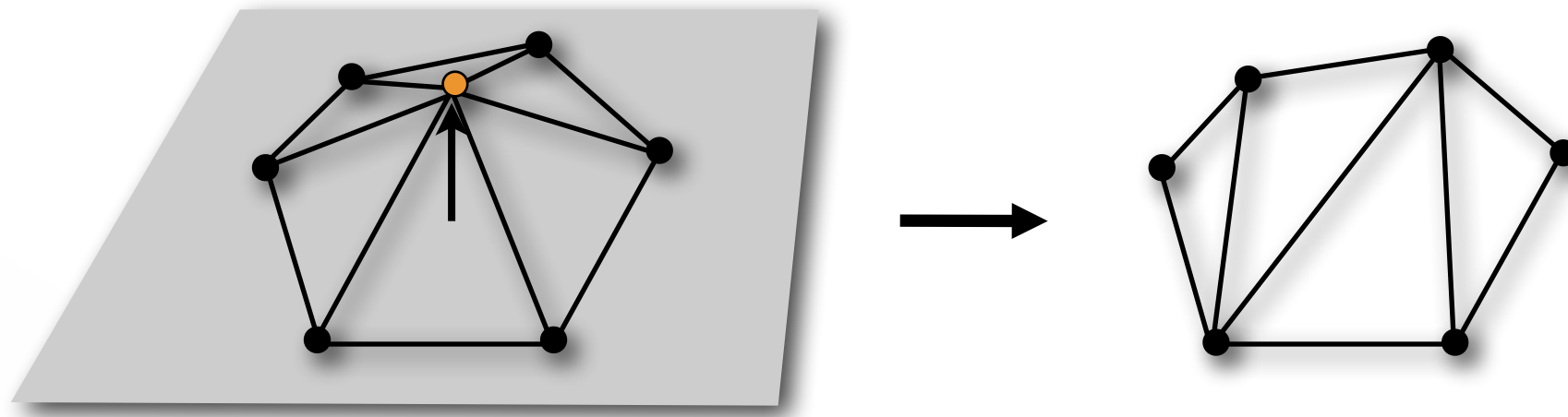
Incremental Decimation

- General Setup
- Decimation operators
- **Error metrics**
- Fairness criteria
- Topology changes

Local Error Metrics

Local distance to mesh [Schröder et al. '92]

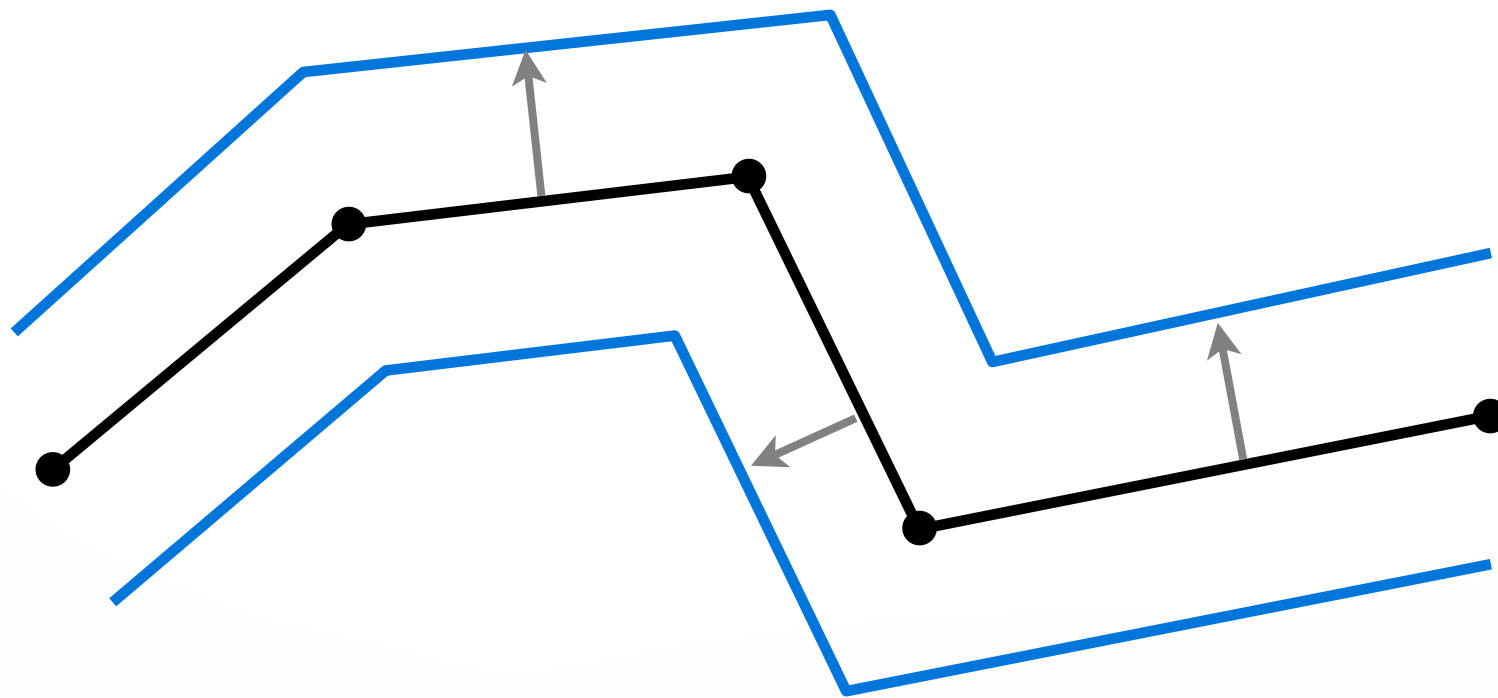
- Compute average plane
- No comparison to original geometry



Global Error Metrics

Simplification envelopes [Cohen al. '96]

- Compute (non-intersecting) offset surfaces
- Simplification guarantees to stay within bounds

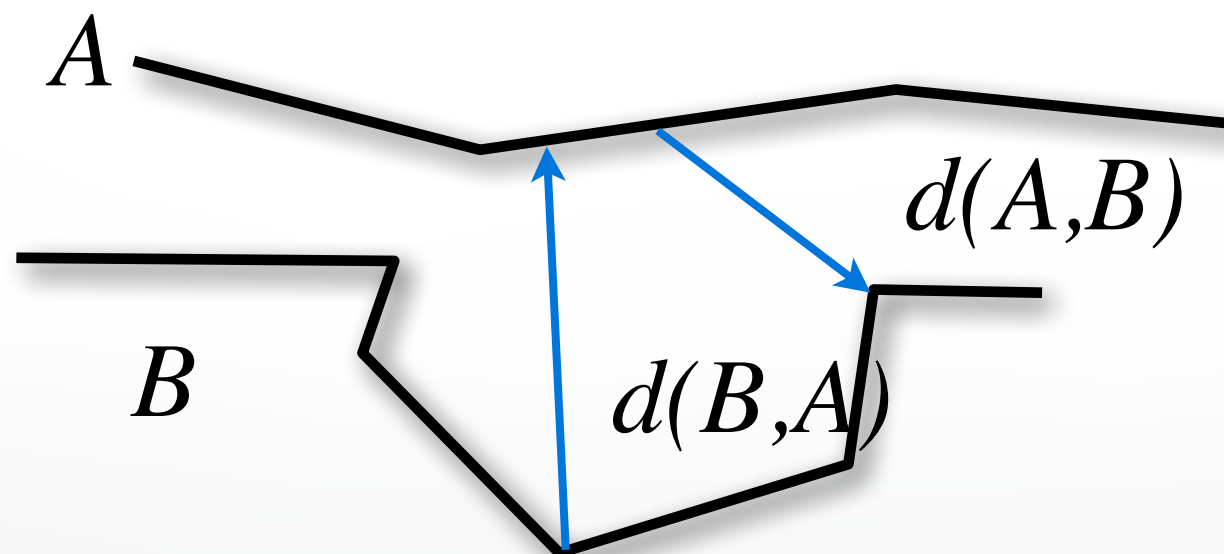


Global Error Metrics

(Two-sided) Hausdorff distance: Maximum distance between two shapes

$$d(A, B) := \max_{\mathbf{a} \in A} \min_{\mathbf{b} \in B} \|\mathbf{a} - \mathbf{b}\|$$

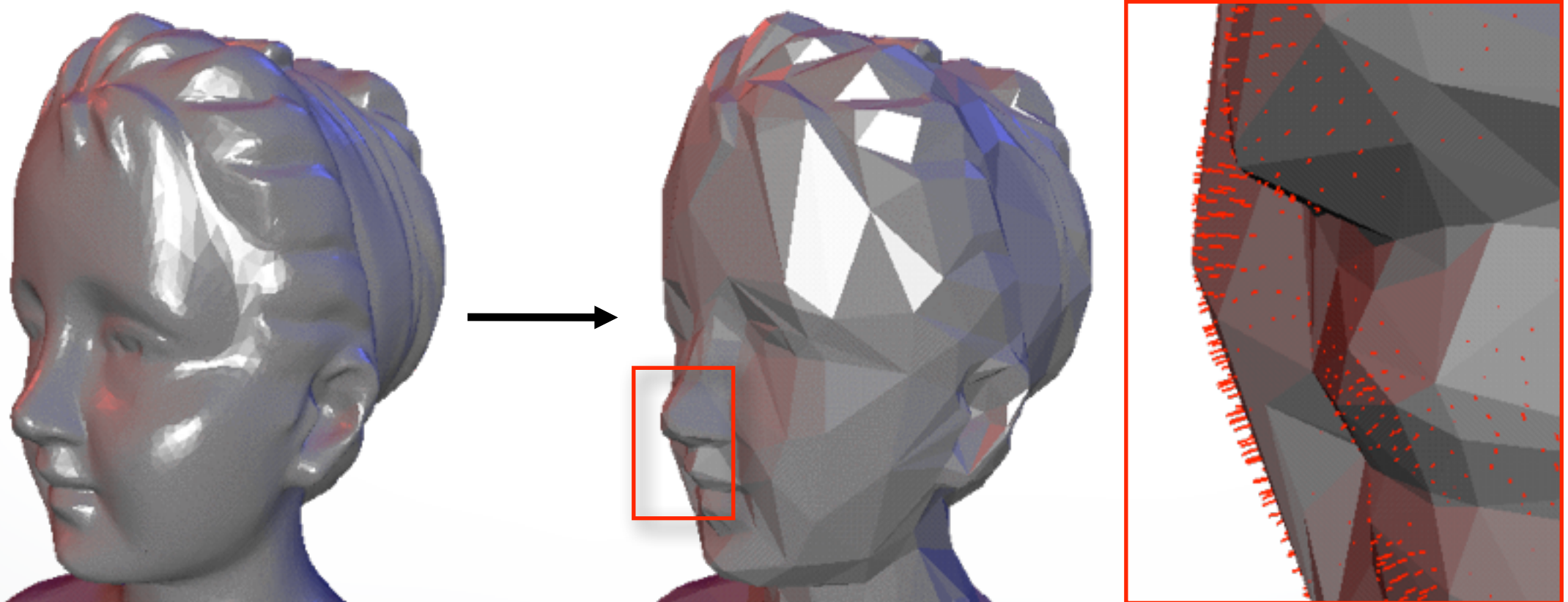
- In general $d(A, B) \neq d(B, A)$
- Computationally involved



Global Error Metrics

Scan data: One-sided Hausdorff distance sufficient

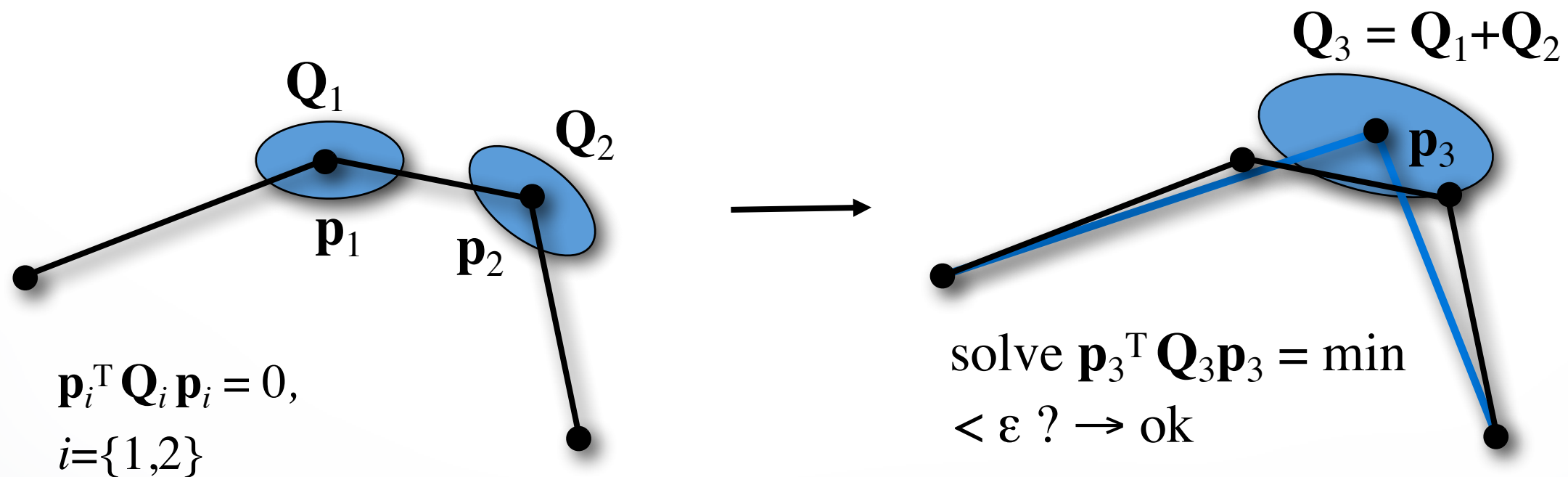
- From original vertices to current surface



Global Error Metrics

Error quadrics [Garland, Heckbert 97]

- Squared distance to planes at vertex
- No bound on true error



Global Error Metrics

Initialization:

- Assign each vertex the quadric built from all its incident triangles' planes

Decimation:

- After collapsing edge $(\mathbf{p}_1, \mathbf{p}_2) \rightarrow \mathbf{p}_3$, simply add the corresponding quadrics: $\mathbf{Q}_3 = \mathbf{Q}_1 + \mathbf{Q}_2$

Memory consumption

- Quasi-global error metric with 10 floats per vertex

Complexity

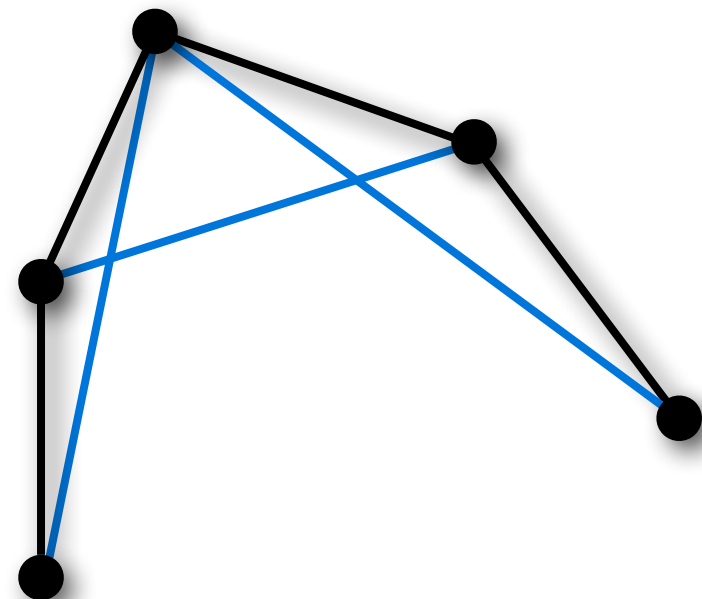
- N = number of vertices
- Priority Queue for half edges
 - $6N \log(6N)$
- Error control
 - Local $O(1) \Rightarrow$ global $O(N)$
 - Local $O(N) \Rightarrow$ global $O(N^2)$

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- **Fairness criteria**
- Topology changes

Fairness Criteria

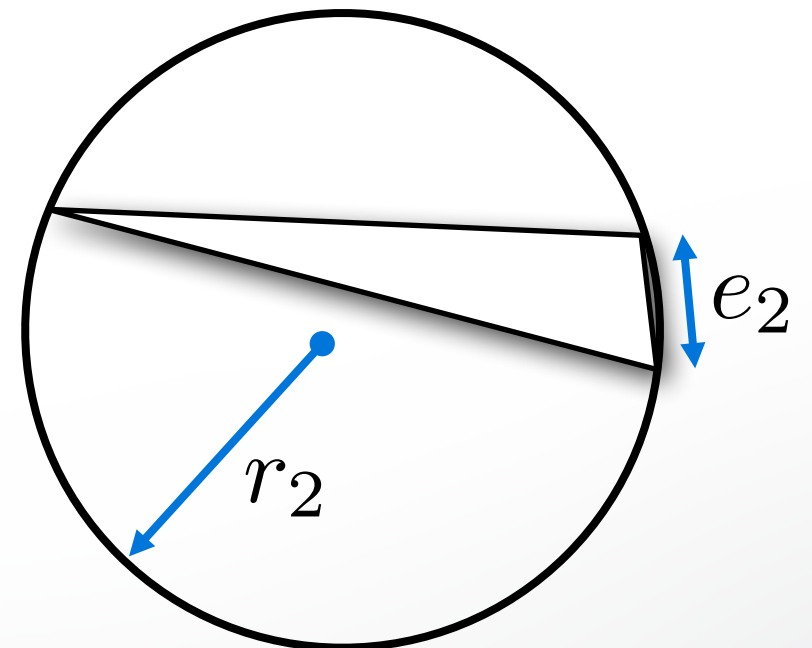
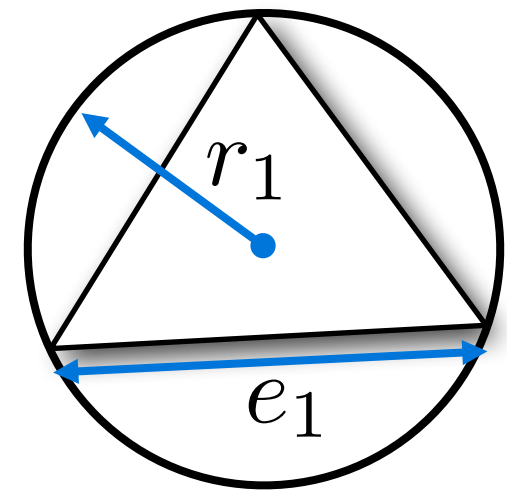
- Rate quality after decimation
 - Approximation error



Fairness Criteria

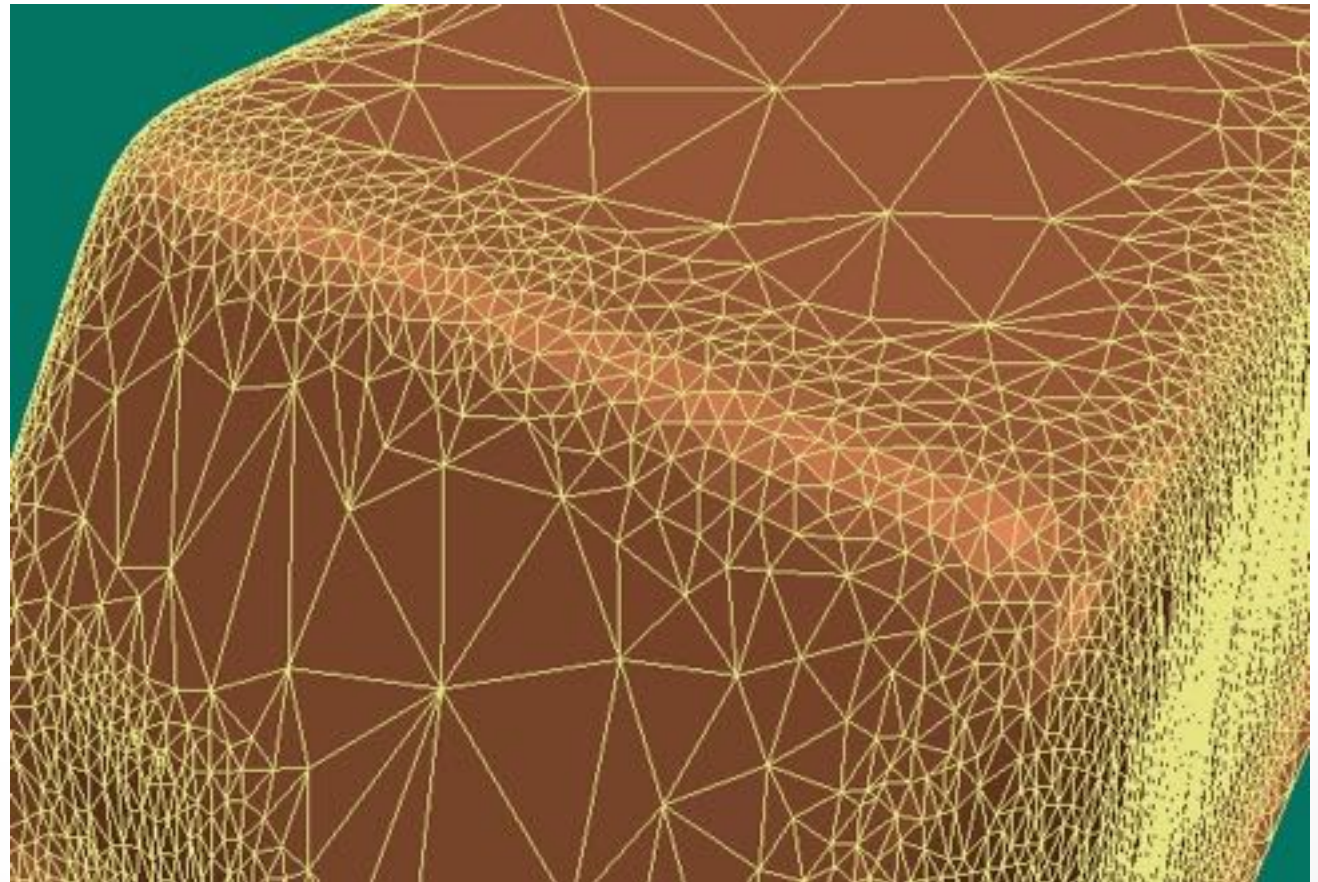
- Rate quality after decimation
 - Approximation error
 - Triangle shape

$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$



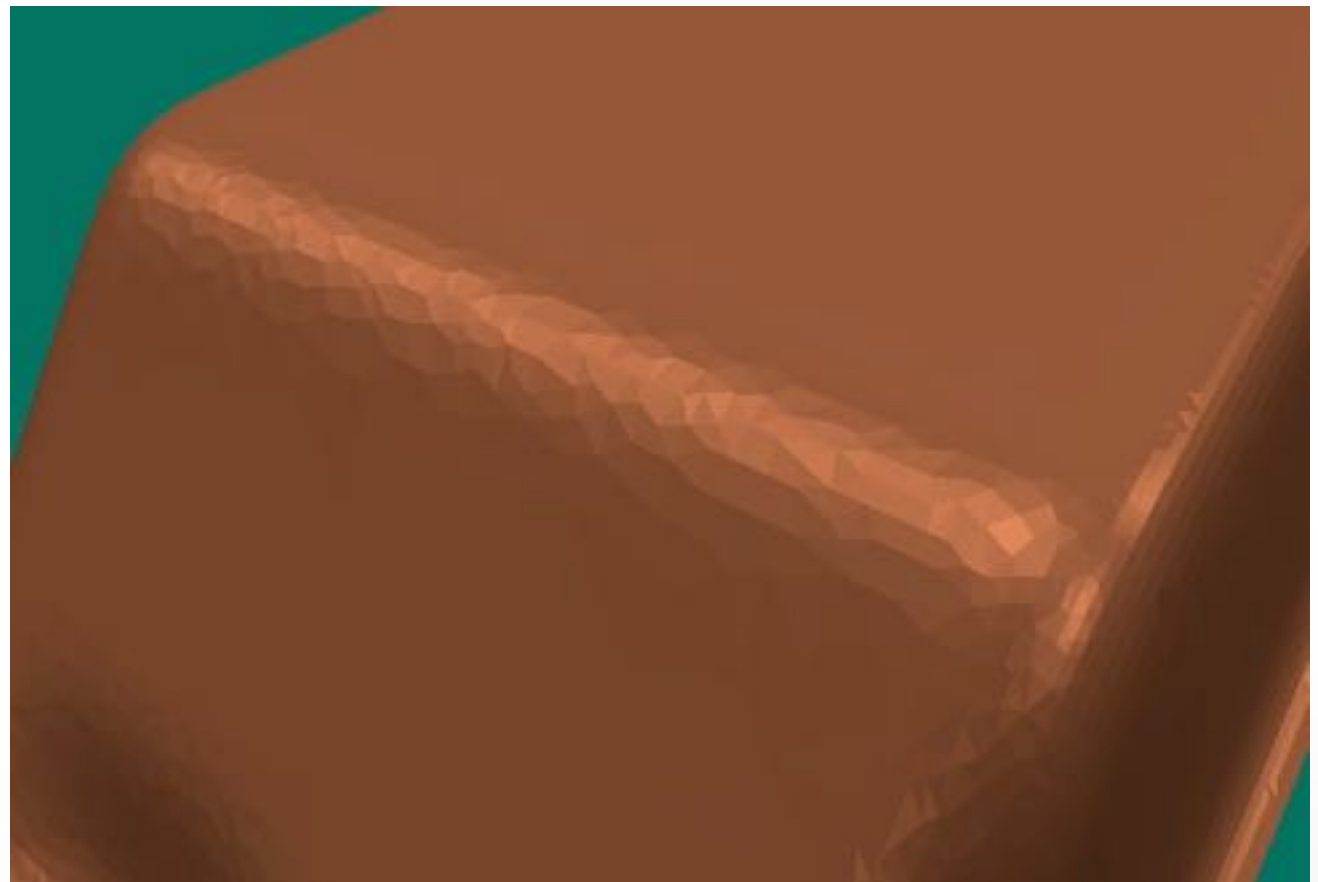
Fairness Criteria

- Rate quality after decimation
 - Approximation error
 - Triangle shape



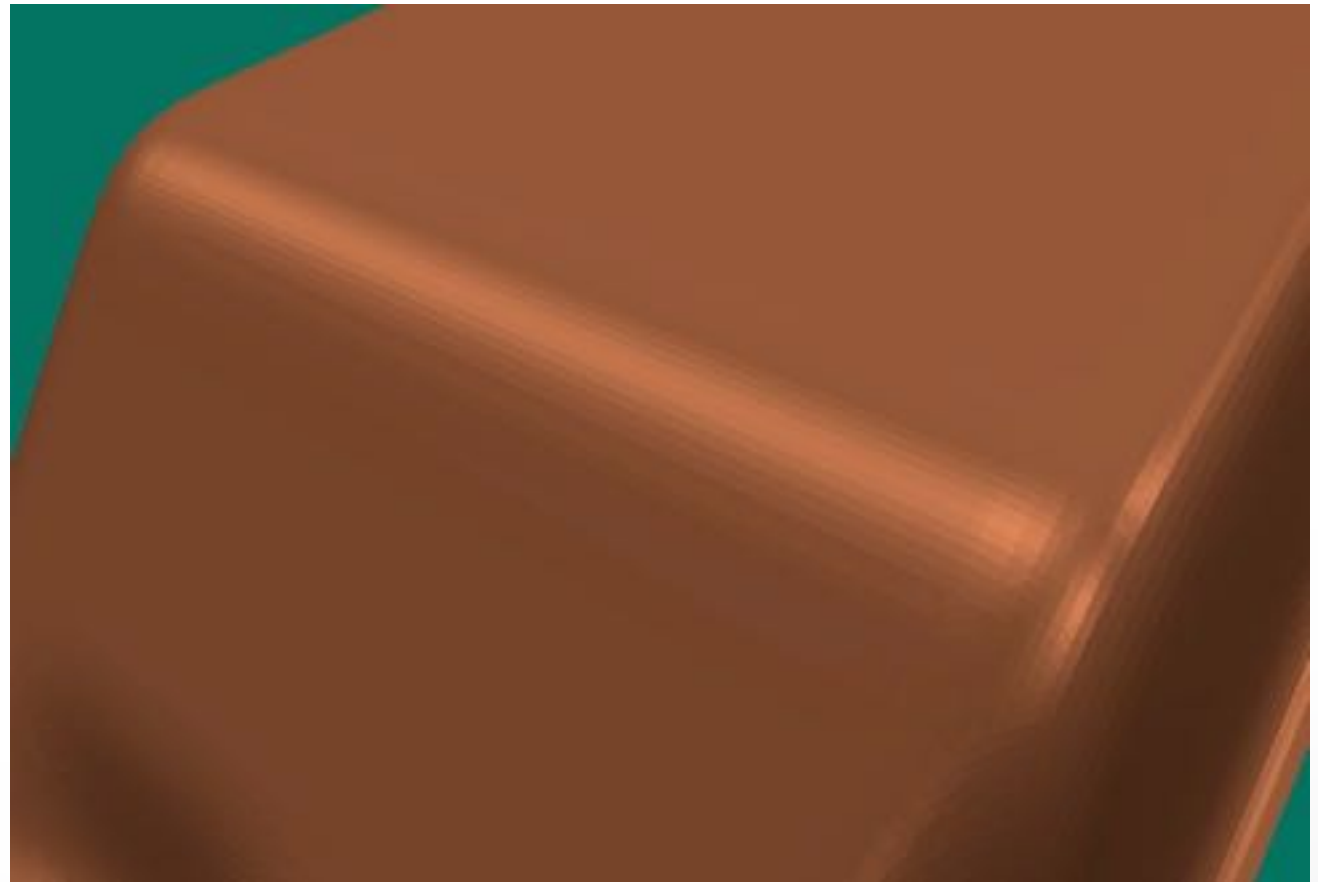
Fairness Criteria

- Rate quality after decimation
 - Approximation error
 - Triangle shape



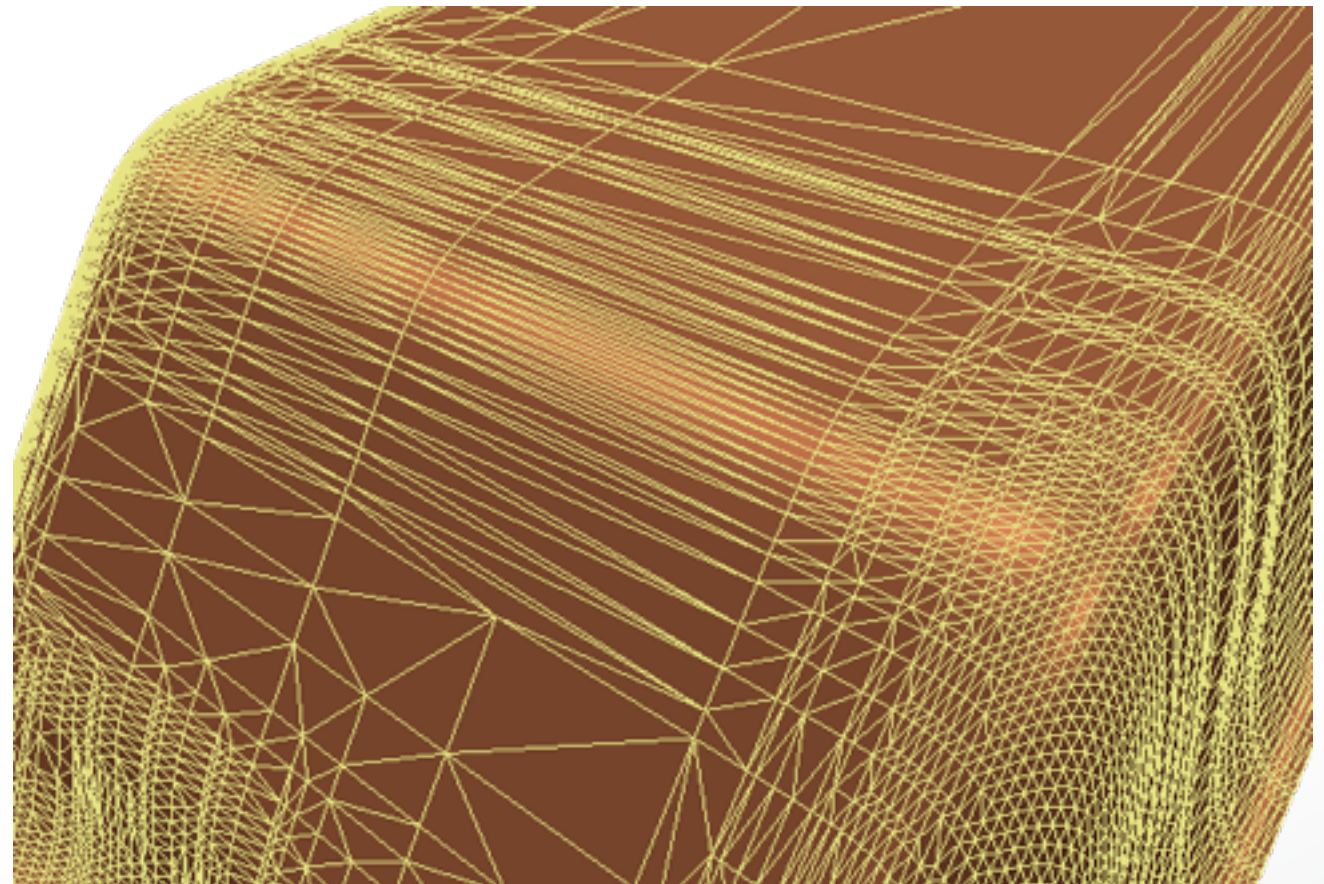
Fairness Criteria

- Rate quality after decimation
 - Approximation error
 - Triangle shape
 - Dihedral angles



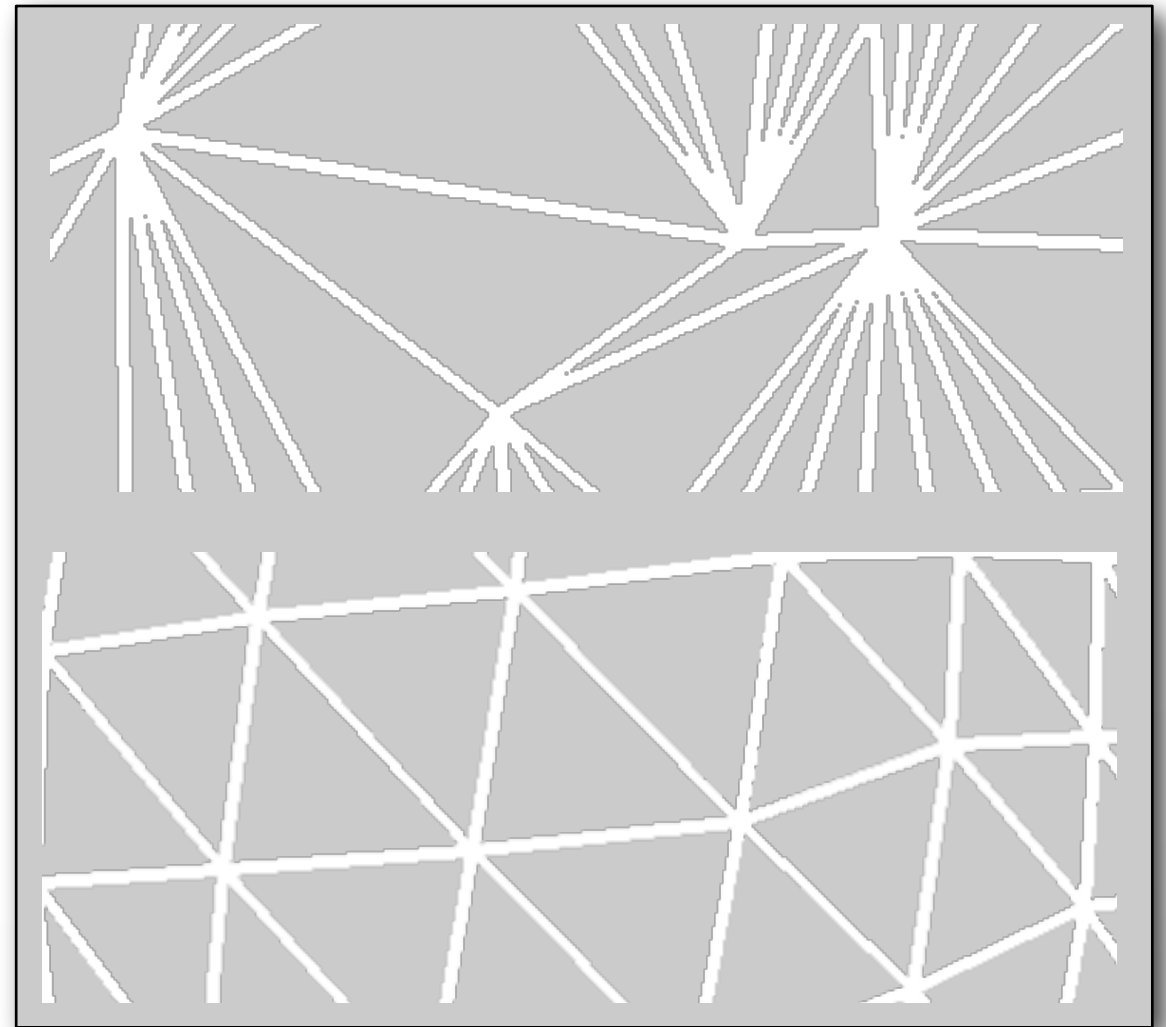
Fairness Criteria

- Rate quality after decimation
 - Approximation error
 - Triangle shape
 - Dihedral angles



Fairness Criteria

- Rate quality after decimation
 - Approximation error
 - Triangle shape
 - Dihedral angles
 - Valence balance



Fairness Criteria

- Rate quality after decimation
 - Approximation error
 - Triangle shape
 - Dihedral angles
 - Valence balance
 - Color differences
 - ...

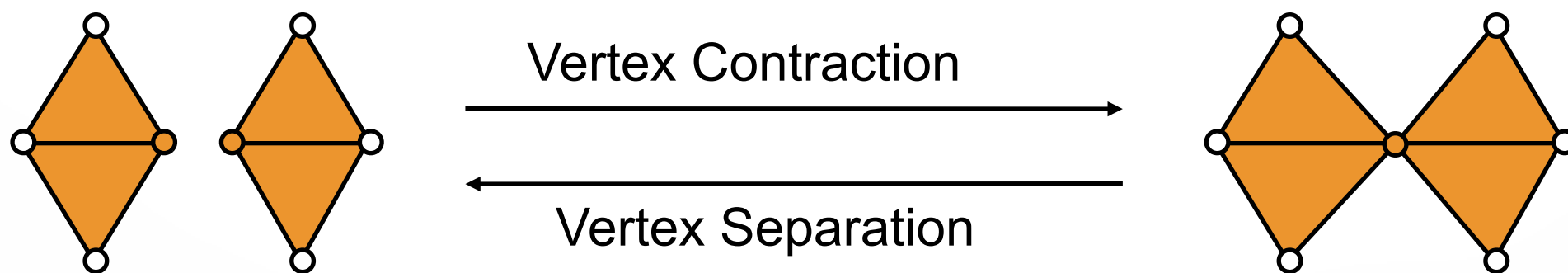


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- **Topology changes**

Fairness Criteria

- Merge vertices across non-edges
 - Changes mesh topology
 - Need spatial *neighborhood* information
 - Generates *non-manifold* meshes



Comparison

- **Vertex clustering**
 - fast but difficult to control simplified mesh
 - topology changes, non-manifold meshes
 - global error bound, but often not close to optimum
- **Iterative decimation with quadric error metrics**
 - good trade-off between mesh-quality and speed
 - explicit control over mesh topology
 - restricting normal deviation improves mesh quality

Literature

- Quadric-based simplification
 - <http://graphics.cs.uiuc.edu/~garland/software/qslim.html>
 - <http://www.openmesh.org>
- Garland, Heckbert: Surface simplification using quadric error metrics, SIGGRAPH 1997.
- Kobbelt et al., A general framework for mesh decimation, Graphics Interface 1998.

Next Time



Remeshing

<http://cs621.hao-li.com>

Thanks!

