

Spring 2017

CSCI 621: **Digital Geometry Processing**

Exercise 4. Surface Quality and Smoothing



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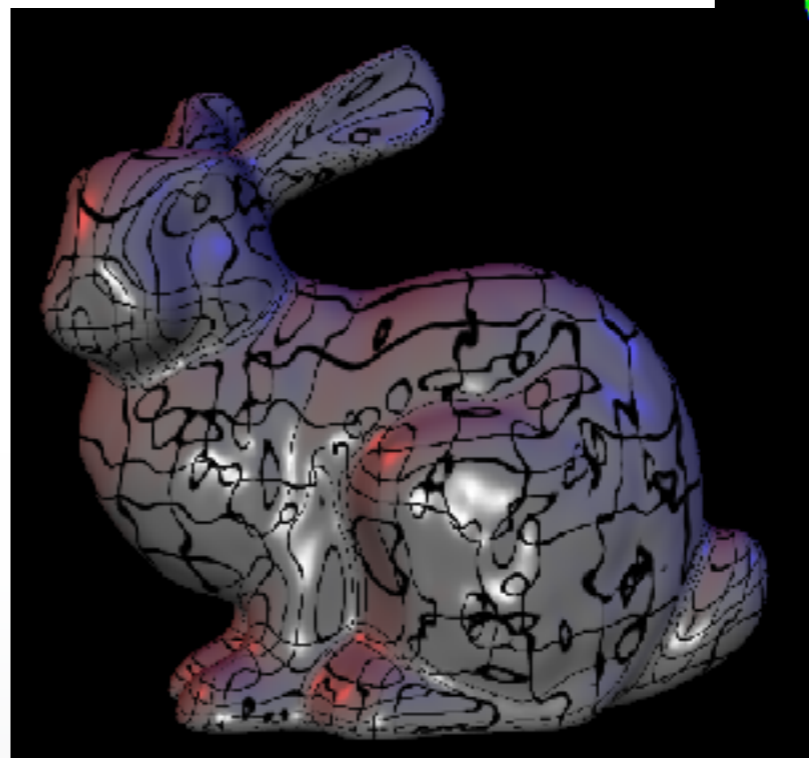
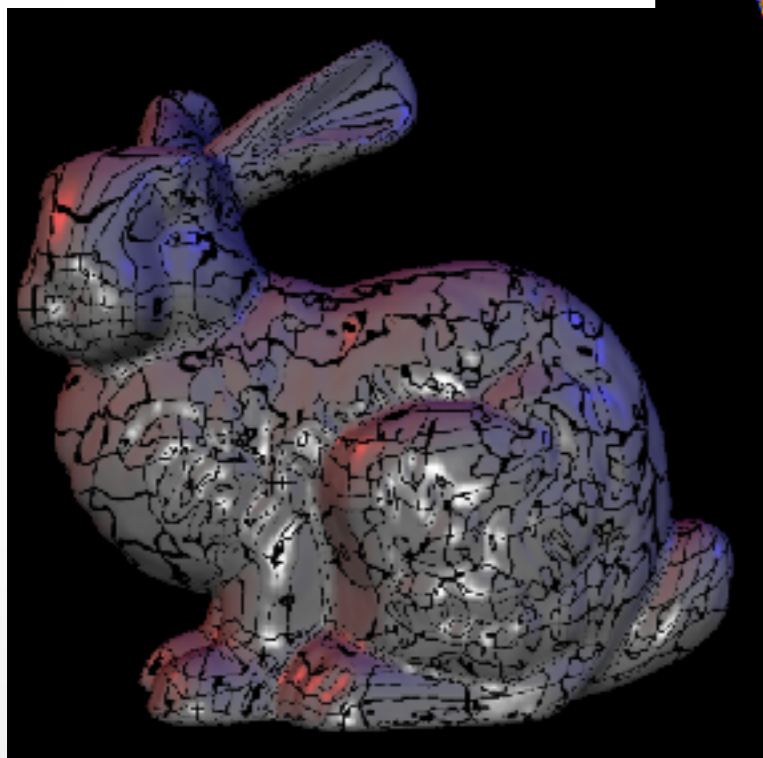
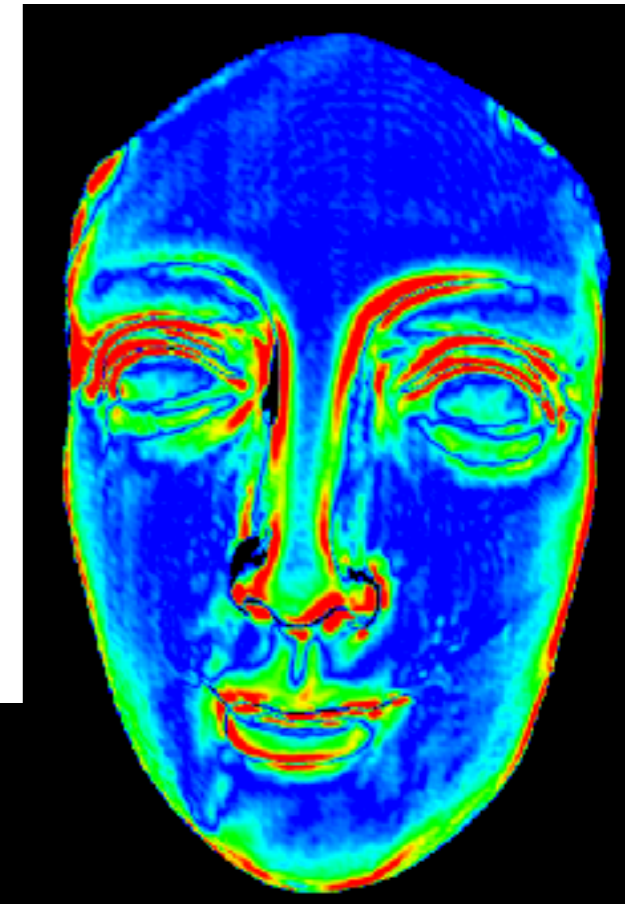
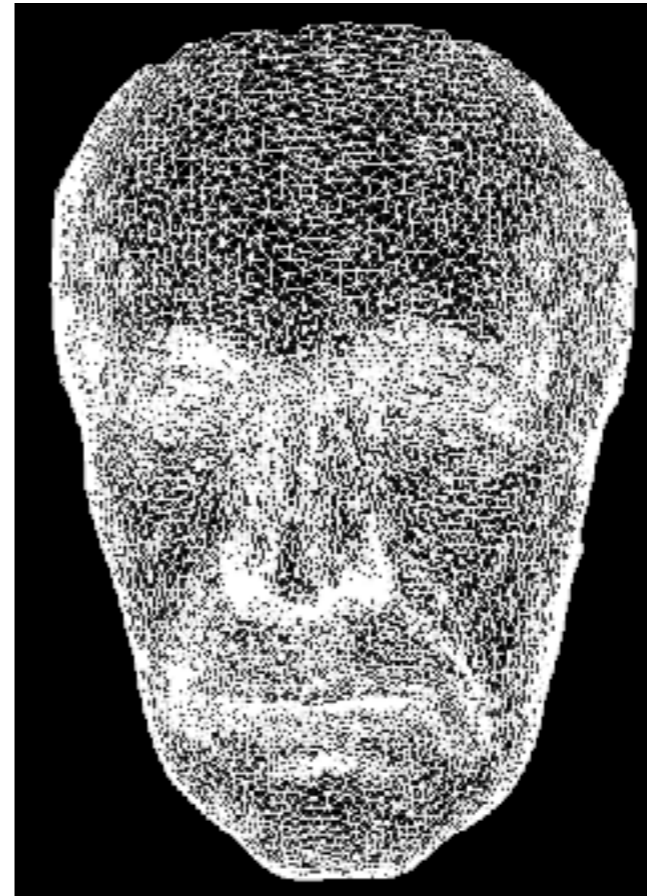
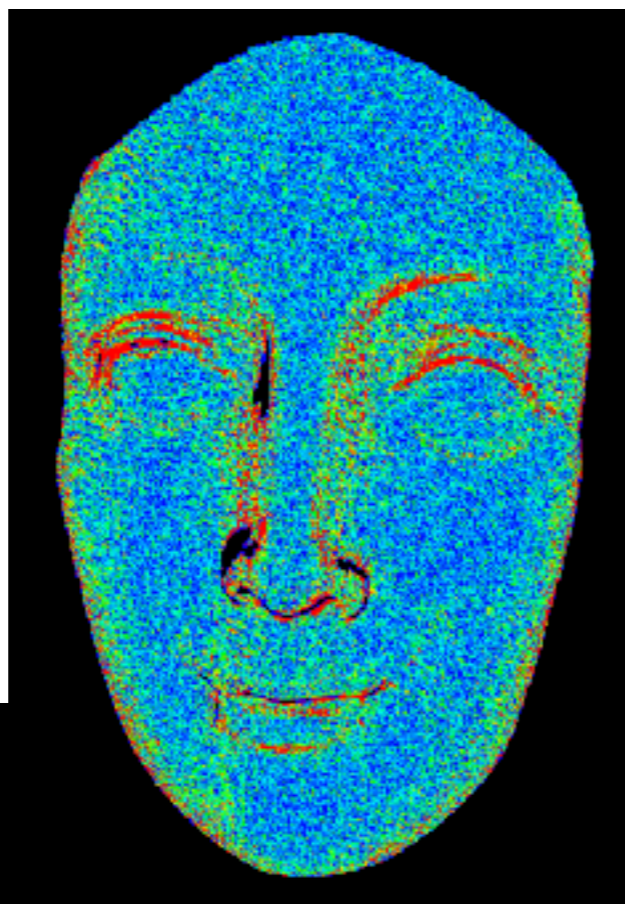
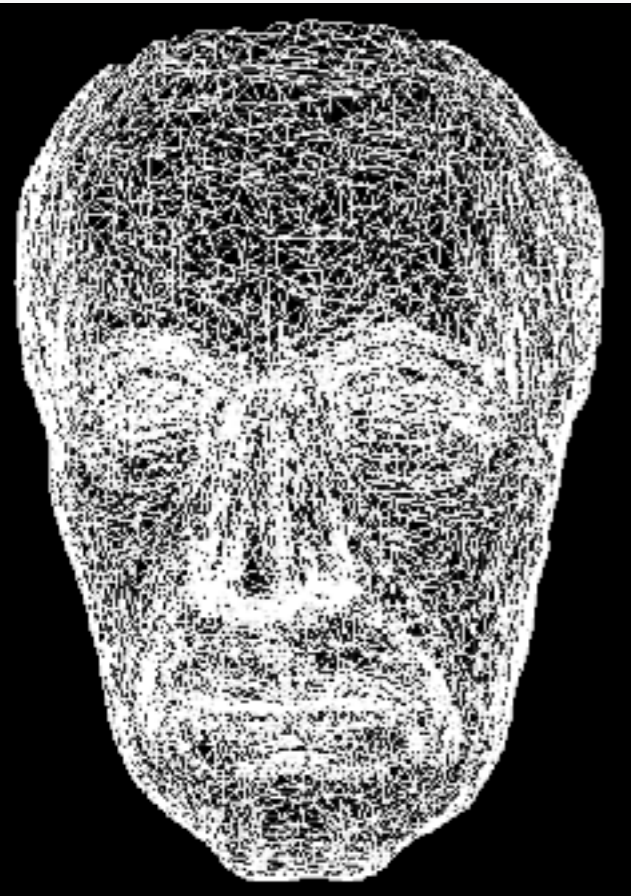
Surface Smoothing

- Spectral analysis
- **Diffusion flow**
 - **Uniform Laplace operator**
 - **Laplacian-Beltrami operator**
- Energy minimization

Uniform Laplacian Surface Smoothing

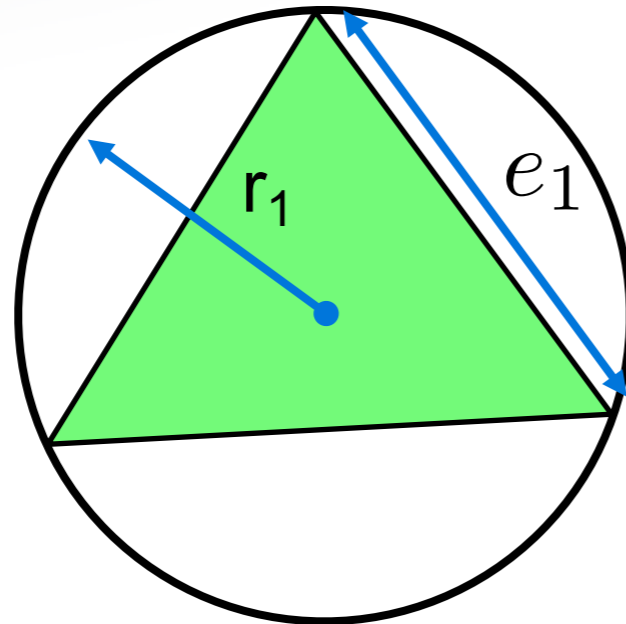
- Uniform Laplace operator $L_U(v) = \left(\frac{1}{n} \sum_i v_i\right) - v$
- Mesh smoothing $v' = v + \frac{1}{2} \cdot L_U(v)$
- Implement uniform Laplace operator in `QualityViewer::calc_uniform_mean_curvature()` in `QualityViewer.cc`
- Implement uniform Laplacian smoothing `SmoothViewer::uniform_smooth()` in `SmoothViewer.cc`

Uniform Laplacian Surface Smoothing

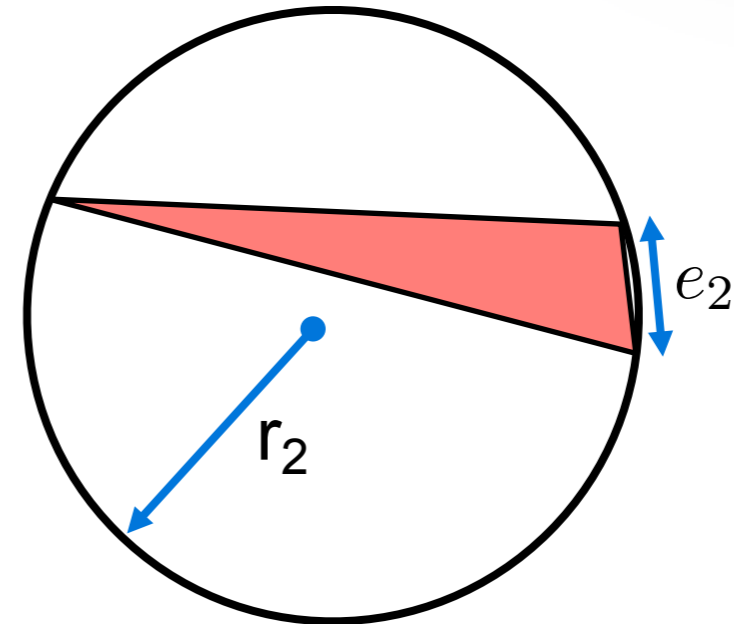


Triangle Quality

good triangle



bad triangle



$$\frac{r_1}{e_1} < \frac{r_2}{e_2}$$

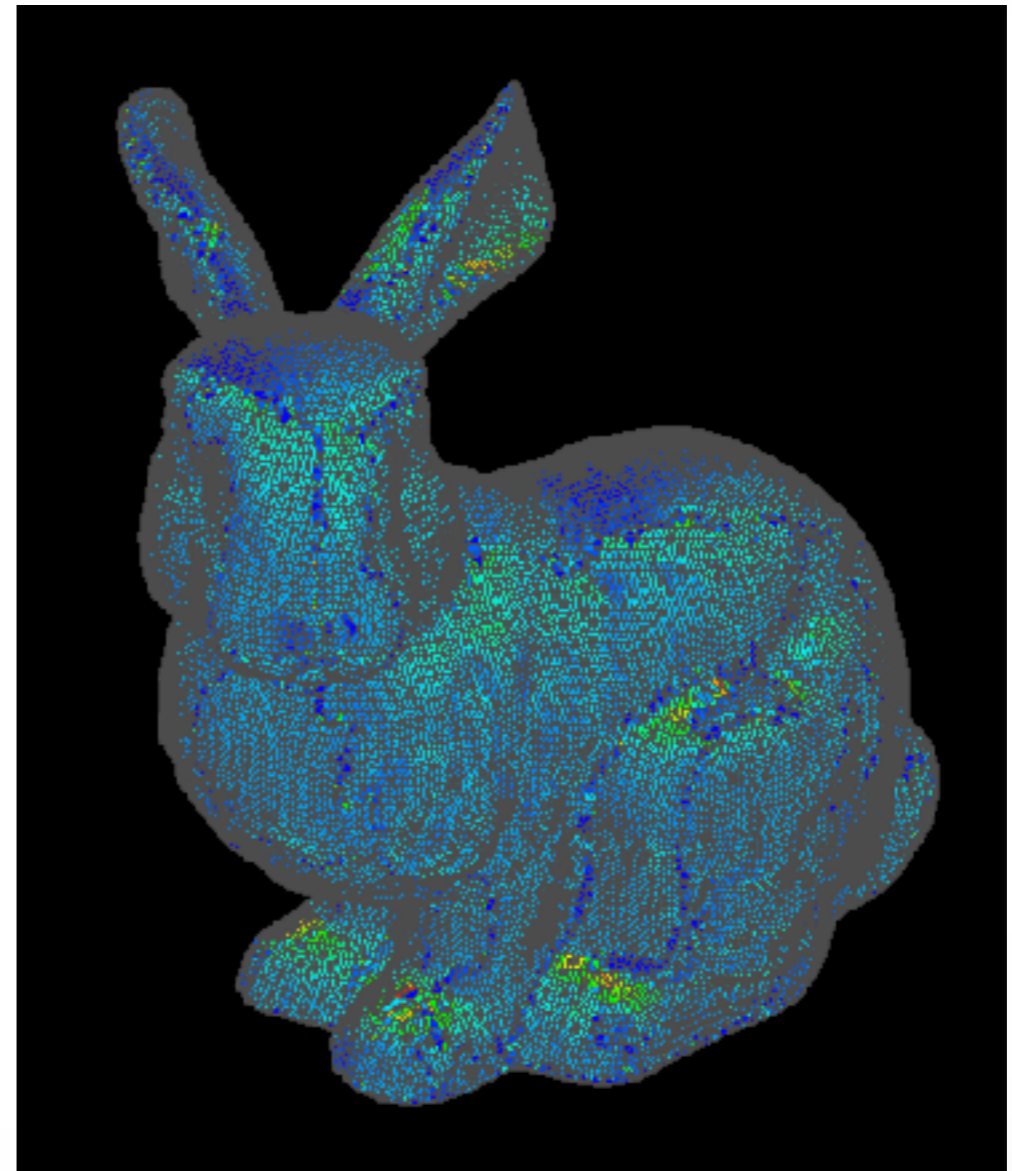
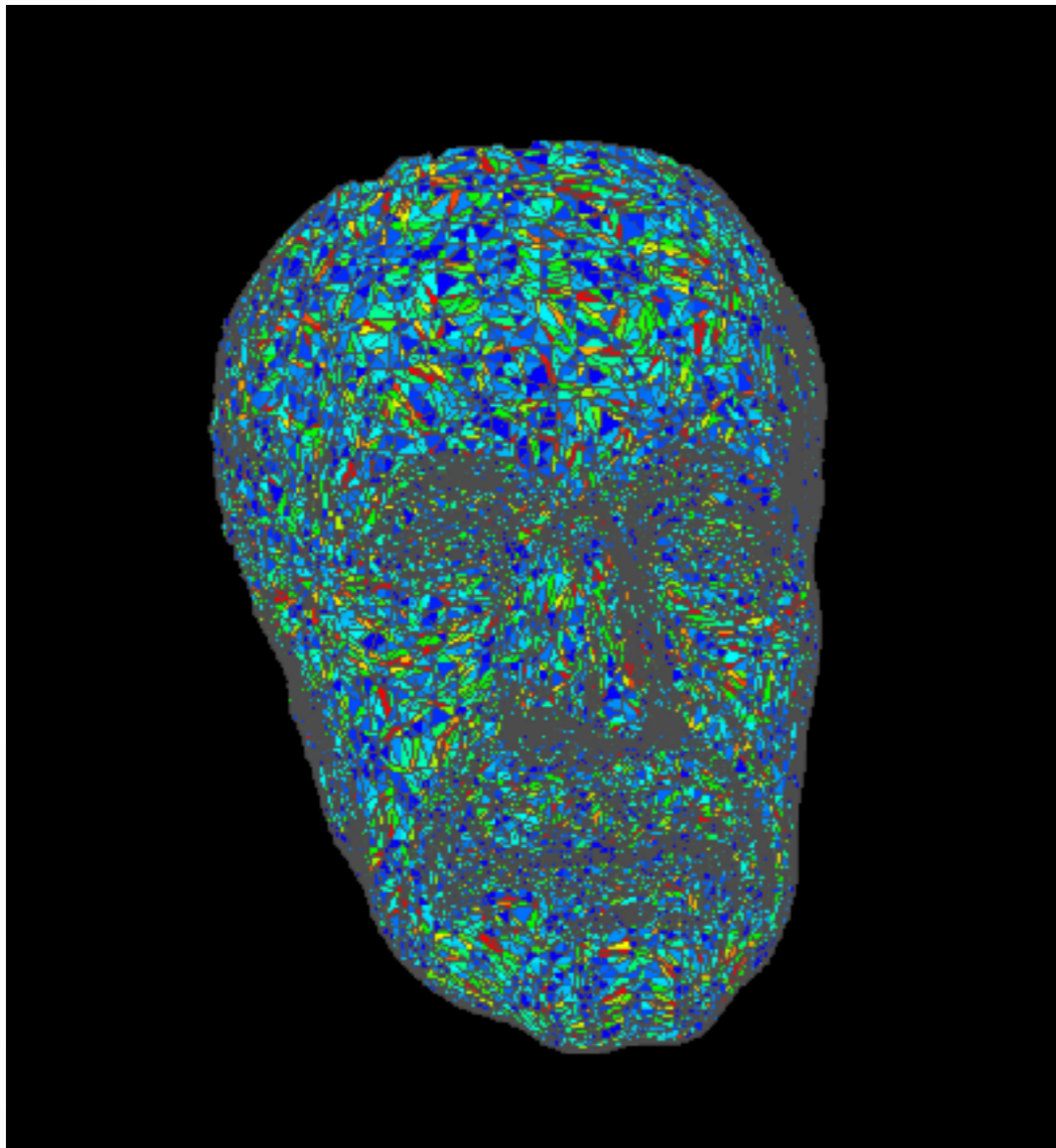
- Assess triangle quality by the circumradius to the minimum edge length ratio

- Circumradius is computed by $A = \frac{|a| \cdot |b| \cdot |c|}{4 \cdot r} = \frac{|a \times b|}{2}$

- Implement in `QualityViewer::`

`calc_triangle_quality()` in `QualityViewer.cc`

Triangle Quality



Laplace-Beltrami curvature and smoothing

- Laplace-Beltrami Operator

$$L_B(v) = \frac{1}{2A} \sum_i ((\cot \alpha_i + \cot \beta_i)(v_i - v))$$

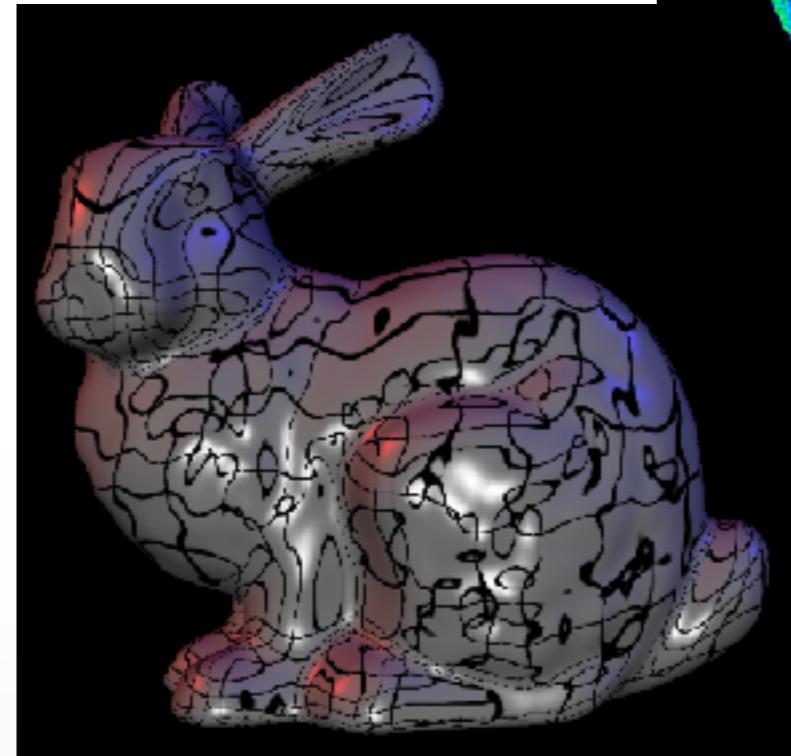
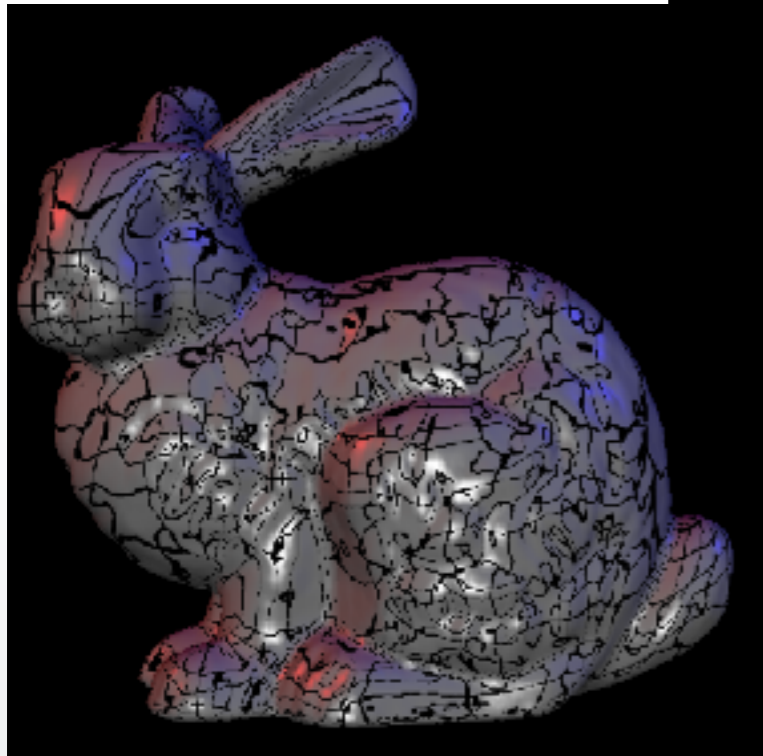
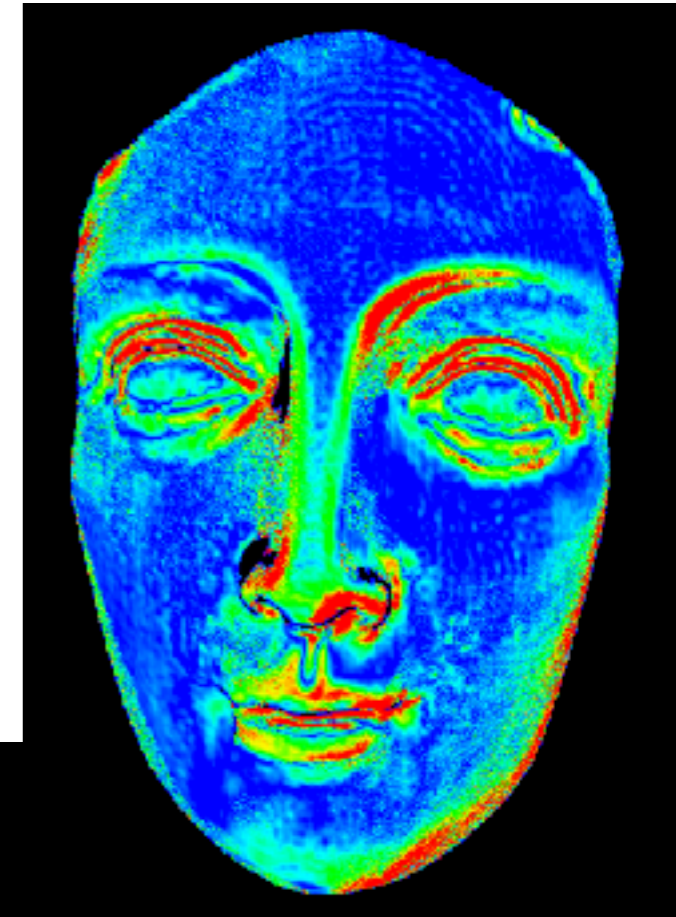
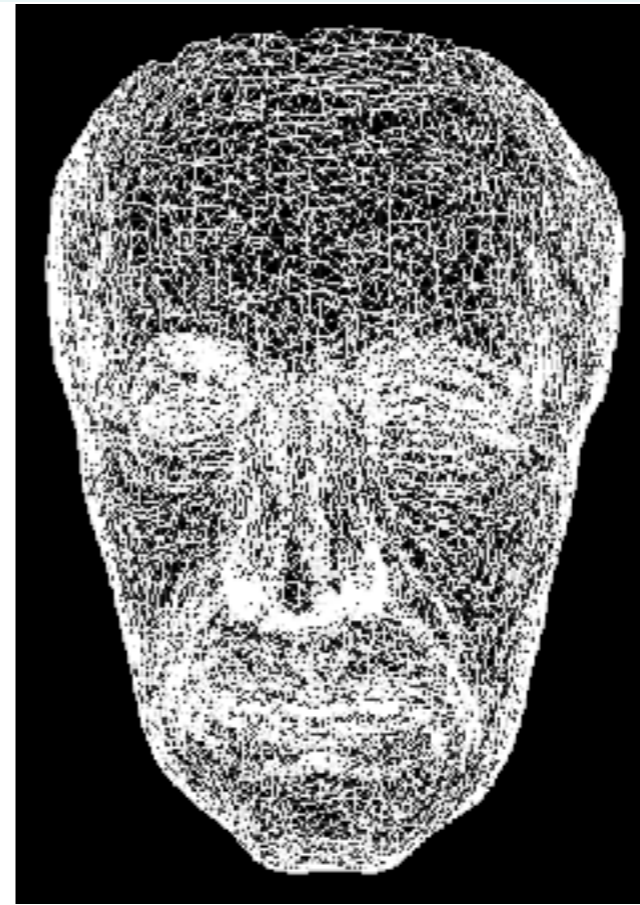
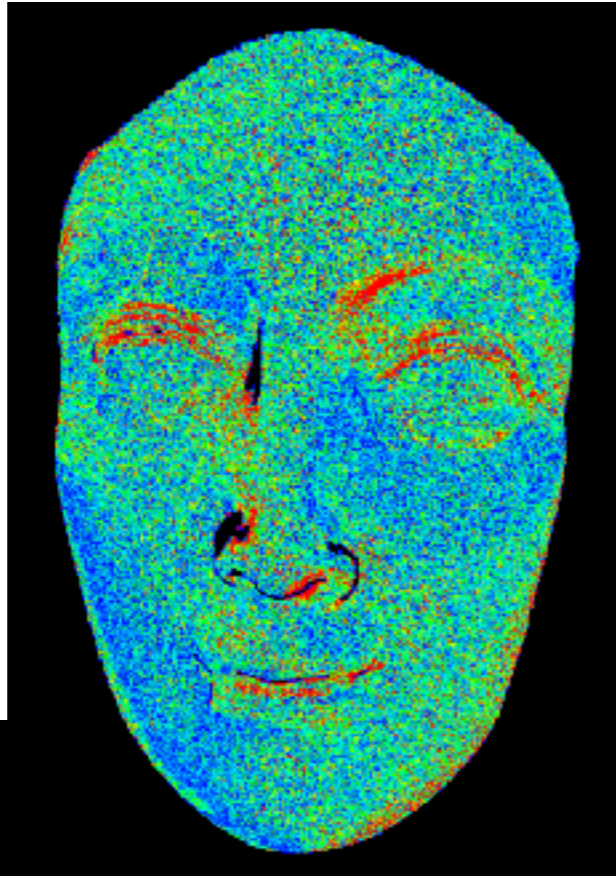
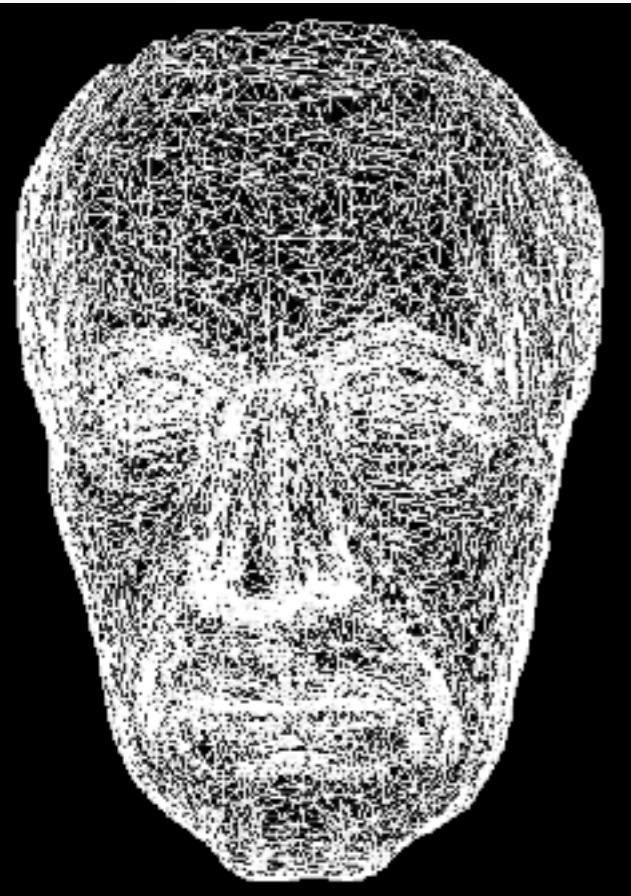
- Compute mean curvature using Laplace-Beltrami weights in `QualityViewer::`

`calc_mean_curvature()` in `QualityViewer.cc`

- Implement smoothing in `SmoothViewer::`

`smooth()` in `SmoothViewer.cc`

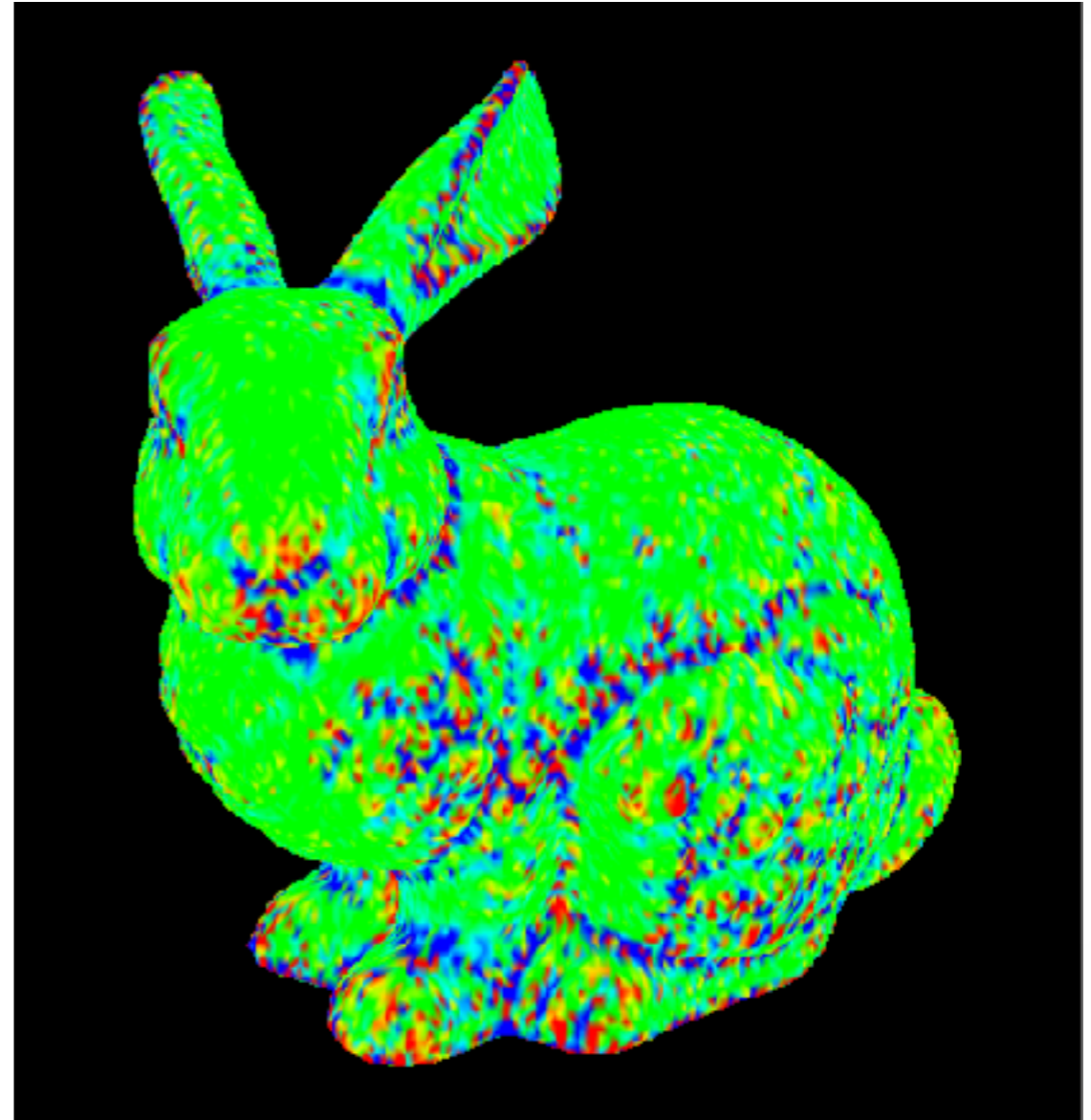
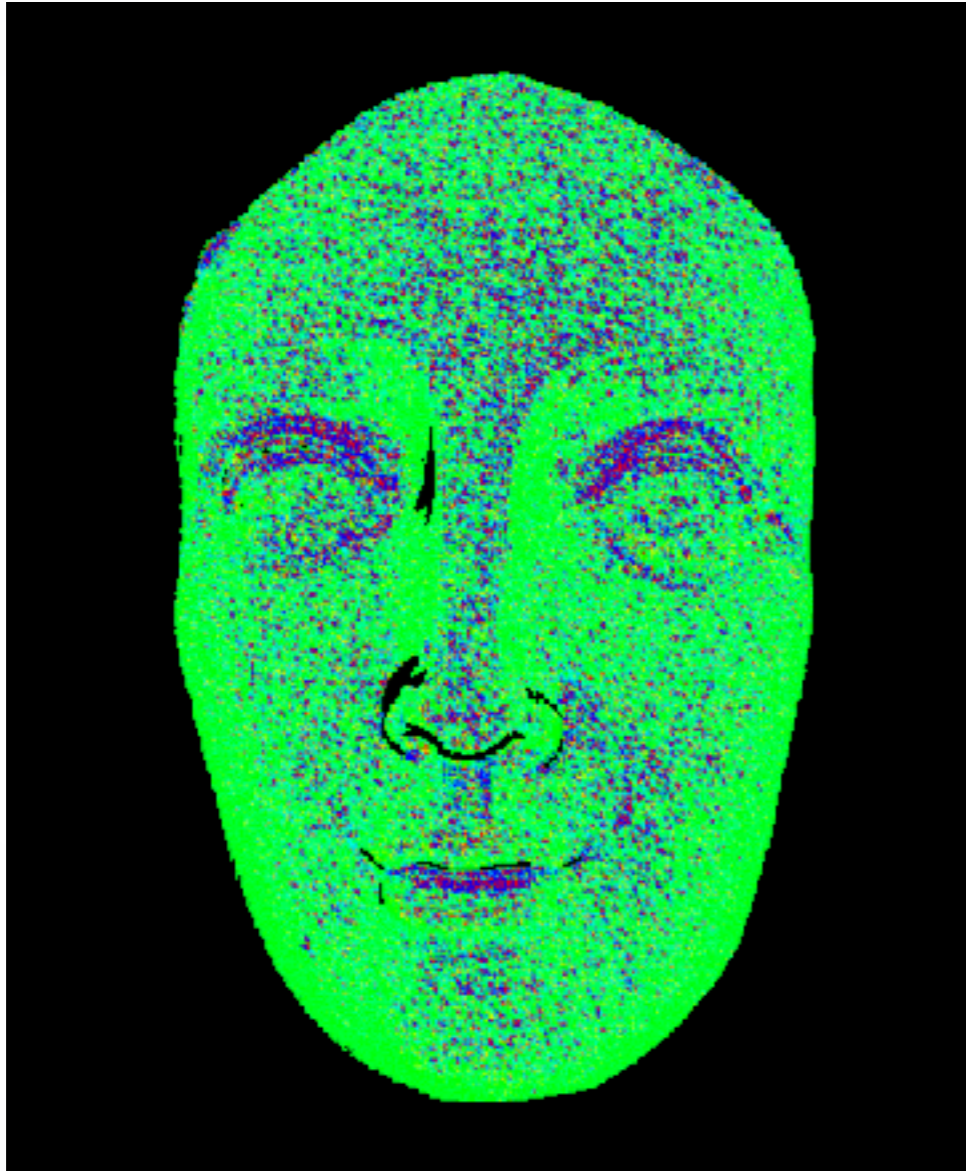
Laplace-Beltrami curvature and smoothing



Gaussian Curvature

- Gaussian curvature $G = (2\pi - \sum_j \theta_j) / A$
- `QualityViewer::calc_gauss_curvature()` in `QualityViewer.cc`

Gaussian Curvature



Submission

- Deadline: **Mar 21, 2017 11:59pm**
- Upload a .zip compressed file named “Exercise4-YourName.zip” to
 - <http://blackboard.usc.edu>
- Include a “read.txt” file describing how you solve each exercise and the encountered problems

Contact

- Office Hours: Tuesday 15:00 - 16:00 PHE 108
- email: saitos@usc.edu

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Thanks!

