Exercise 2. Registration
• Selecting source points
• Matching points to the target mesh
• Weighting the correspondences
• Rejecting bad pairs
• Compute error metric
• Minimize error metric
Exercise 2

• Perform rigid registration between 10 scans of the Stanford bunny
Exercise 2

• Demo

• ‘SHIFT’ + mouse controls: manual alignment for an initial transformation
• ‘r’: perform single registration step with point to point distance minimization
• ‘SPACE’: perform single registration step with point to plane distance minimization
• ‘n’: load next scan
Exercise 2

- Getting it compiled
- Subsampling
- Bad pairs rejection
- Point to point optimization
- Point to plane optimization
Getting It Compiled

- CMake, OpenGL, OpenMesh
- ANN (Approximate Nearest Neighbor)
  - efficient closest point lookup using kd-tree
Subsampling

- Uniform subsampling within a given radius
  subsampleRadius

- RegistrationViewer::subsample() in RegistrationViewer.cc
• Closest points are computed using ANN
• Prune correspondences based on
  • distance threshold
  • normal compatibility
• RegistrationViewer::calculate_correspondences() in RegistrationViewer.cc
Point to Point Optimization

- Minimize $E = \sum_{i=1}^{N} \| Rp_i + t - q_i \|^2_2$
  by solving a linear system $Ax = b$

- Registration::register_point2point() in Registration.cc
Euler Angles

- Three elemental rotations:

\[
R_x(\alpha) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{bmatrix}
\quad R_y(\beta) = \begin{bmatrix}
\cos \beta & 0 & \sin \beta \\
0 & 1 & 0 \\
-\sin \beta & 0 & \cos \beta
\end{bmatrix}
\quad R_z(\gamma) = \begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

- Any rotation matrix can be decomposed as a product of elemental three rotation matrix

\[
R = R_z(\gamma)R_y(\beta)R_x(\alpha) = \begin{bmatrix}
c_\gamma c_\beta & -c_\alpha s_\gamma + c_\gamma s_\beta s_\alpha & s_\gamma s_\alpha + c_\gamma c_\alpha s_\beta \\
c_\beta s_\gamma & c_\gamma c_\alpha + s_\gamma s_\beta s_\alpha & c_\alpha s_\gamma s_\beta - c_\gamma s_\alpha \\
-s_\beta & c_\beta s_\alpha & c_\beta c_\alpha
\end{bmatrix}
\]

\[
c_\alpha = \cos \alpha \quad s_\alpha = \sin \alpha
\]
Linearized Transformation

- Linearized Euler angle

(assuming small rotation: $\cos \alpha = 1 \quad \sin \alpha = \alpha$)

$$\mathbf{R} = \begin{bmatrix}
    c_{\gamma}c_{\beta} & -c_{\alpha}s_{\gamma} + c_{\gamma}s_{\beta}s_{\alpha} & s_{\gamma}s_{\alpha} + c_{\gamma}c_{\alpha}s_{\beta} \\
    c_{\beta}s_{\gamma} & c_{\gamma}c_{\alpha} + s_{\gamma}s_{\beta}s_{\alpha} & c_{\alpha}s_{\gamma}s_{\beta} - c_{\gamma}s_{\alpha} \\
    -s_{\beta} & c_{\beta}s_{\alpha} & c_{\beta}c_{\alpha}
\end{bmatrix} = \begin{bmatrix}
    1 & -\gamma & \beta \\
    \gamma & 1 & -\alpha \\
    -\beta & \alpha & 1
\end{bmatrix}$$

- Linearized transformation

$$\mathbf{x} = \begin{bmatrix}
    \alpha & \beta & \gamma & t_x & t_y & t_z
\end{bmatrix}^T$$
Point to Plane Optimization

- Minimize \( E = \sum_{i=1}^{N} \| n_i^\top (Rp_i + t - q_i) \|^2 \) by solving a linear system \( Ax = b \)

- Registration::register_point2surface() in Registration.cc
Results
Submission

• Deadline: **Feb 21, 2017 11:59pm**

• Upload a .zip compressed file named “Exercise2-YourName.zip” to

  • Blackboard

• Include a “read.txt” file describing how you solve each exercise and the encountered problems
• Office Hours: Tuesday 15:00 - 16:00 PHE 108

• email: saitos@usc.edu
Thanks!