6.1 **Shape Matching**

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Acknowledgement

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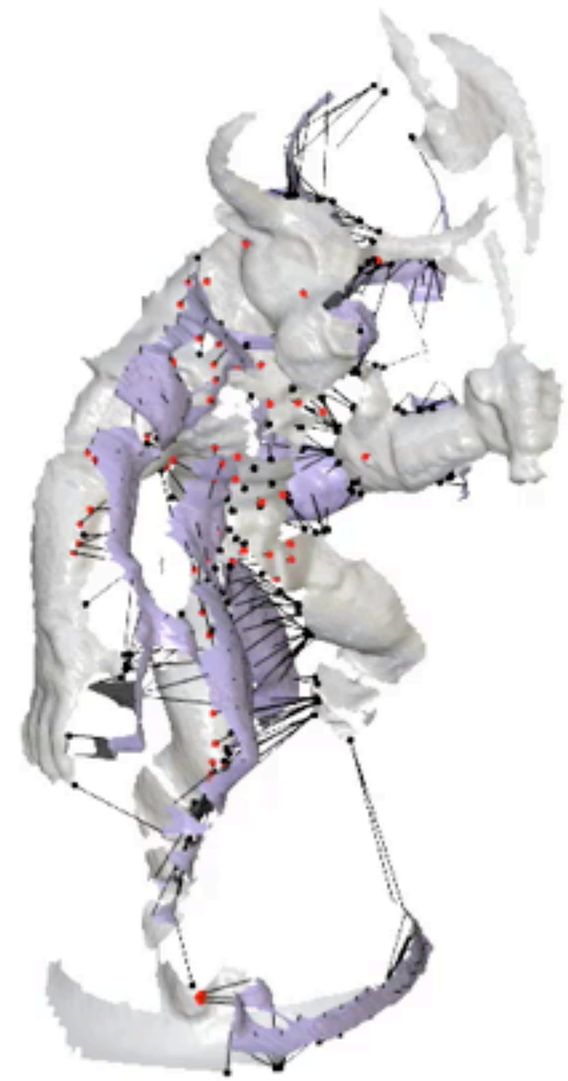
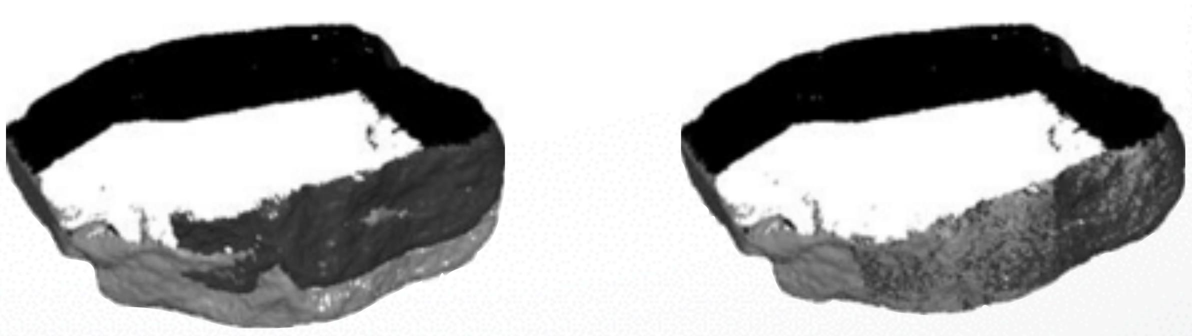
- ICCV Course 2005: http://www.cis.upenn.edu/~bjbrown/iccv05_course/

Last Time

Surface Registration

- Pairwise ICP & Variants
- Point-to-point/plane metric
- BSP closes point search
- Stability Analysis
- Global Registration

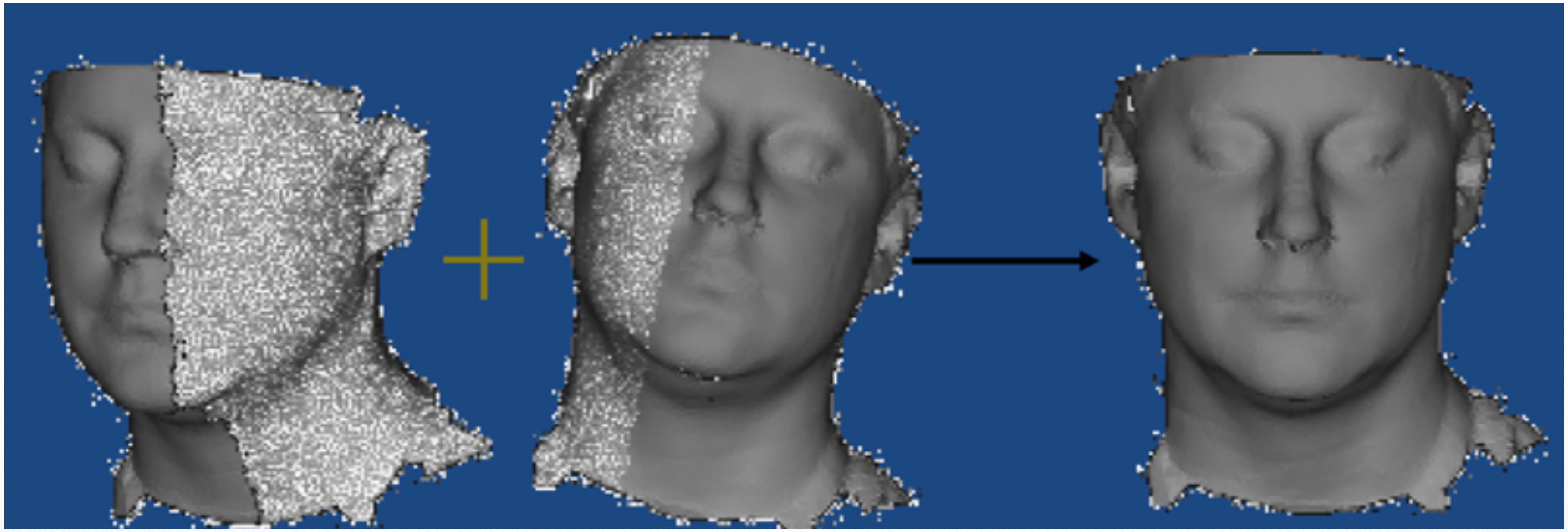




Shape Matching for Model Alignment

Goal

- Given two partially overlapping scans, compute transformation that aligns the two.
- **No assumption** about rough initial alignment

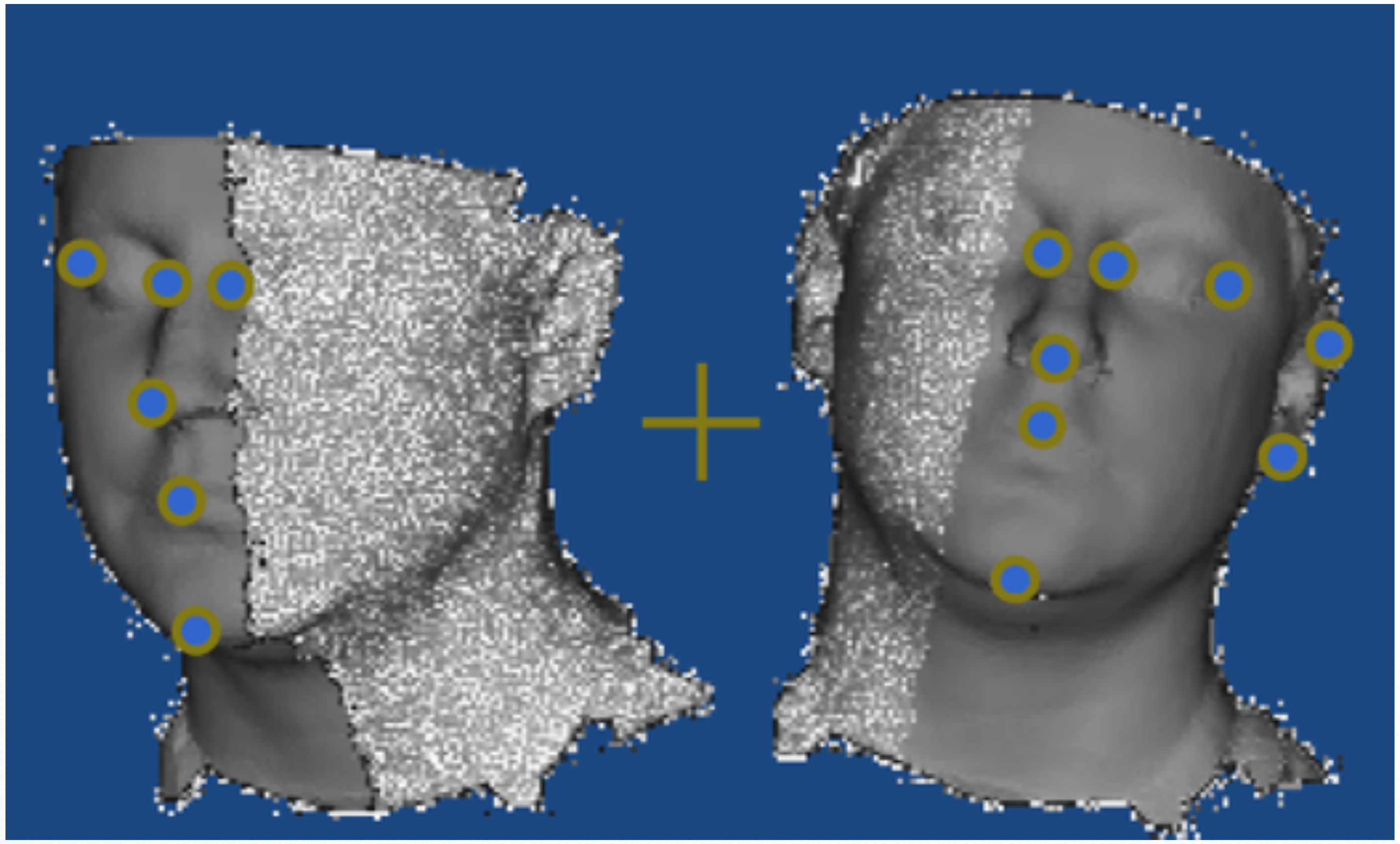


Partially Overlapping Scans **Aligned Scans**

Shape Matching for Model Alignment

Approach

- Find feature points on the two scans

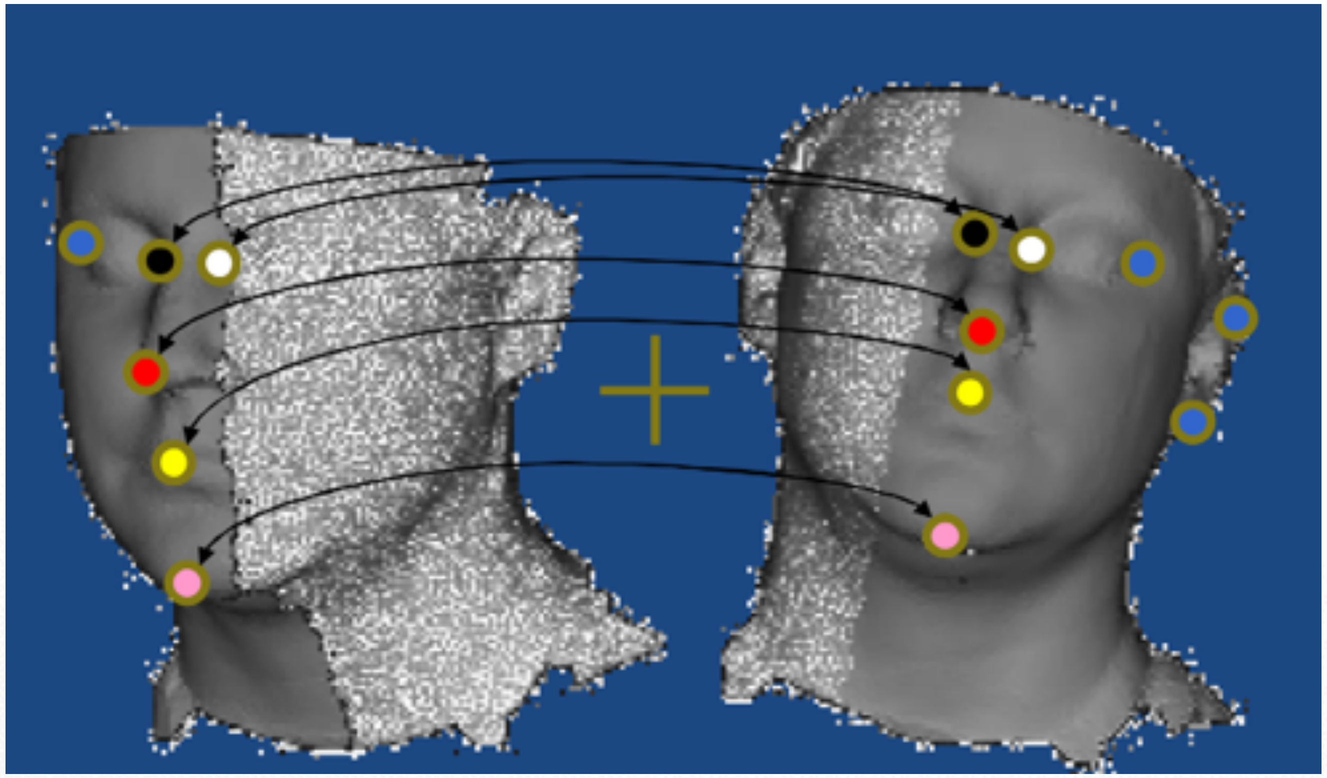


Partially Overlapping Scans

Shape Matching for Model Alignment

Approach

- Find feature points on the two scans
- Establish correspondences

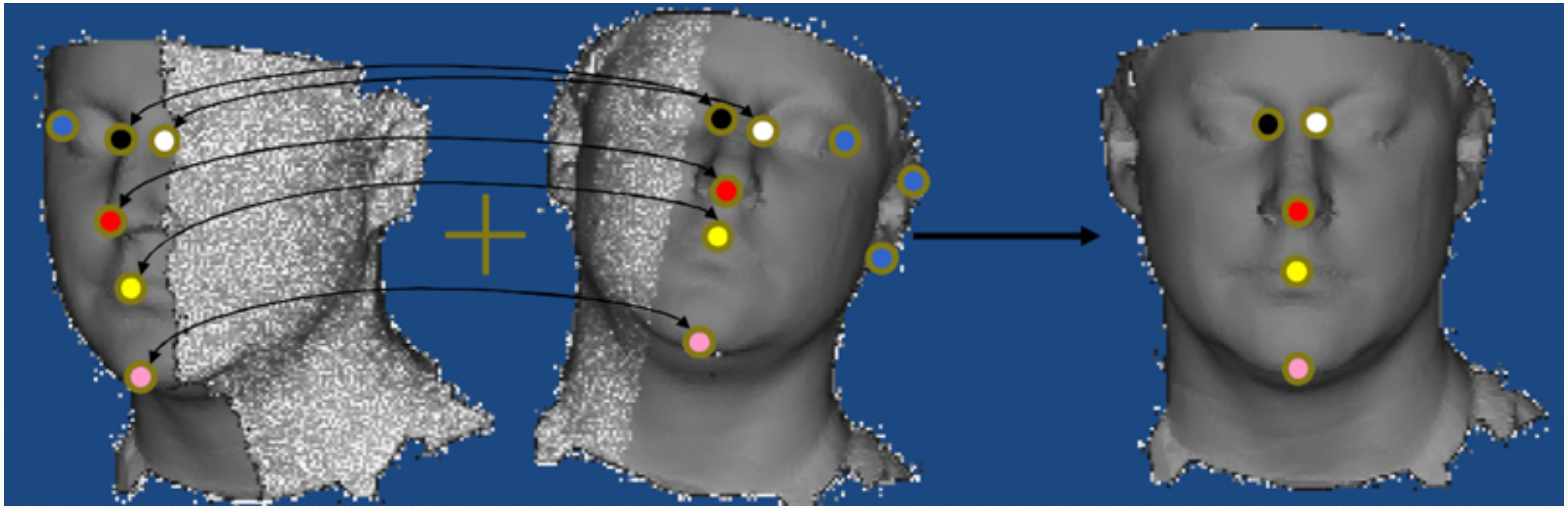


Partially Overlapping Scans

Shape Matching for Model Alignment

Approach

- Find feature points on the two scans
- Establish correspondences
- Compute the alignment



Partially Overlapping Scans **Aligned Scans**

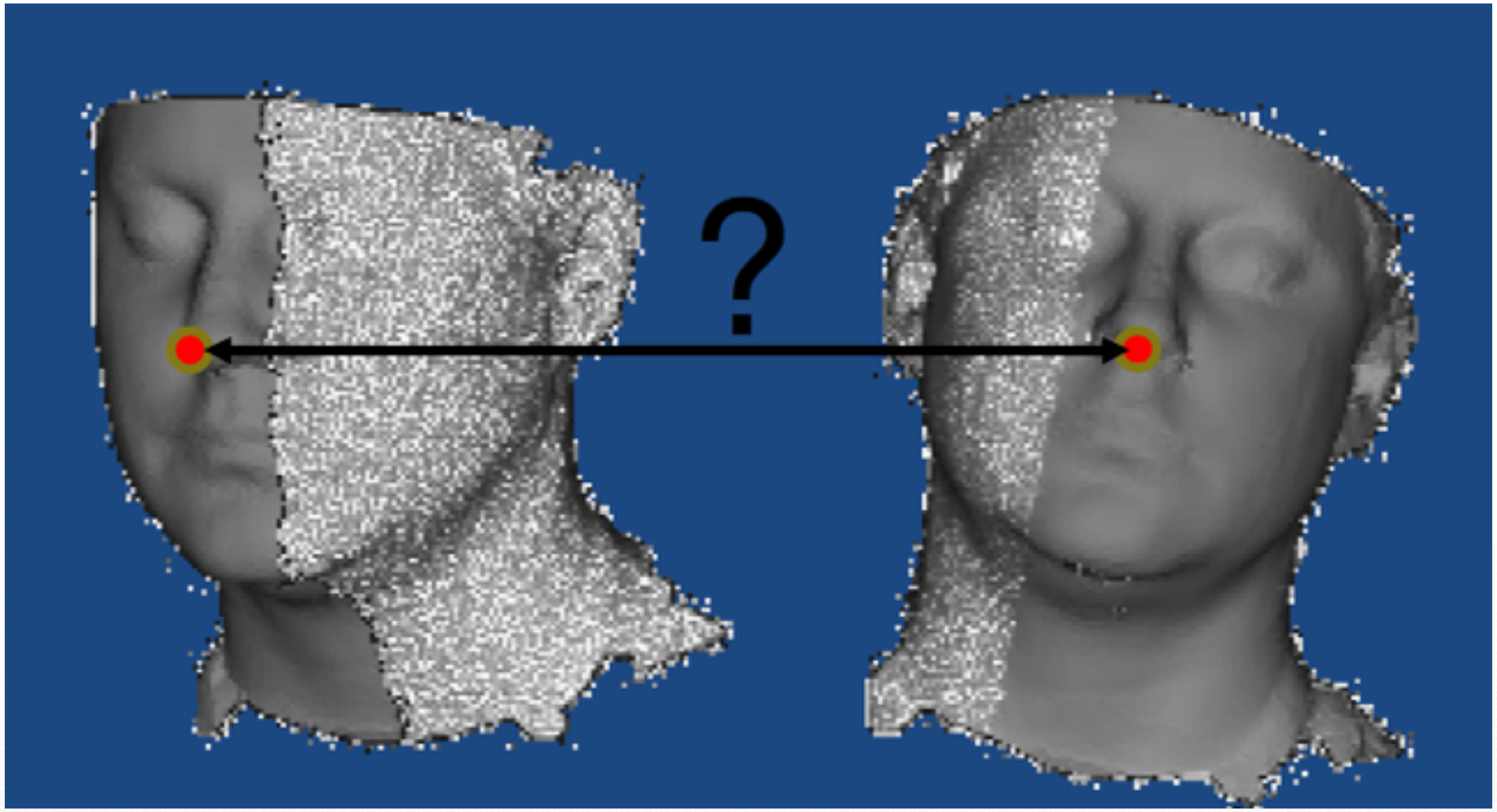
Outline

- **Global Shape Correspondence**
  - Shape Descriptors
  - Alignment
- **Partial Shape Correspondence**
  - From Global to Local
  - Pose Normalization
  - Partial Shape Descriptors
- **Registration**
  - Closed Form Solutions
  - Branch & Bound
  - Random Sample Consensus (RANSAC)

Correspondence

Goal

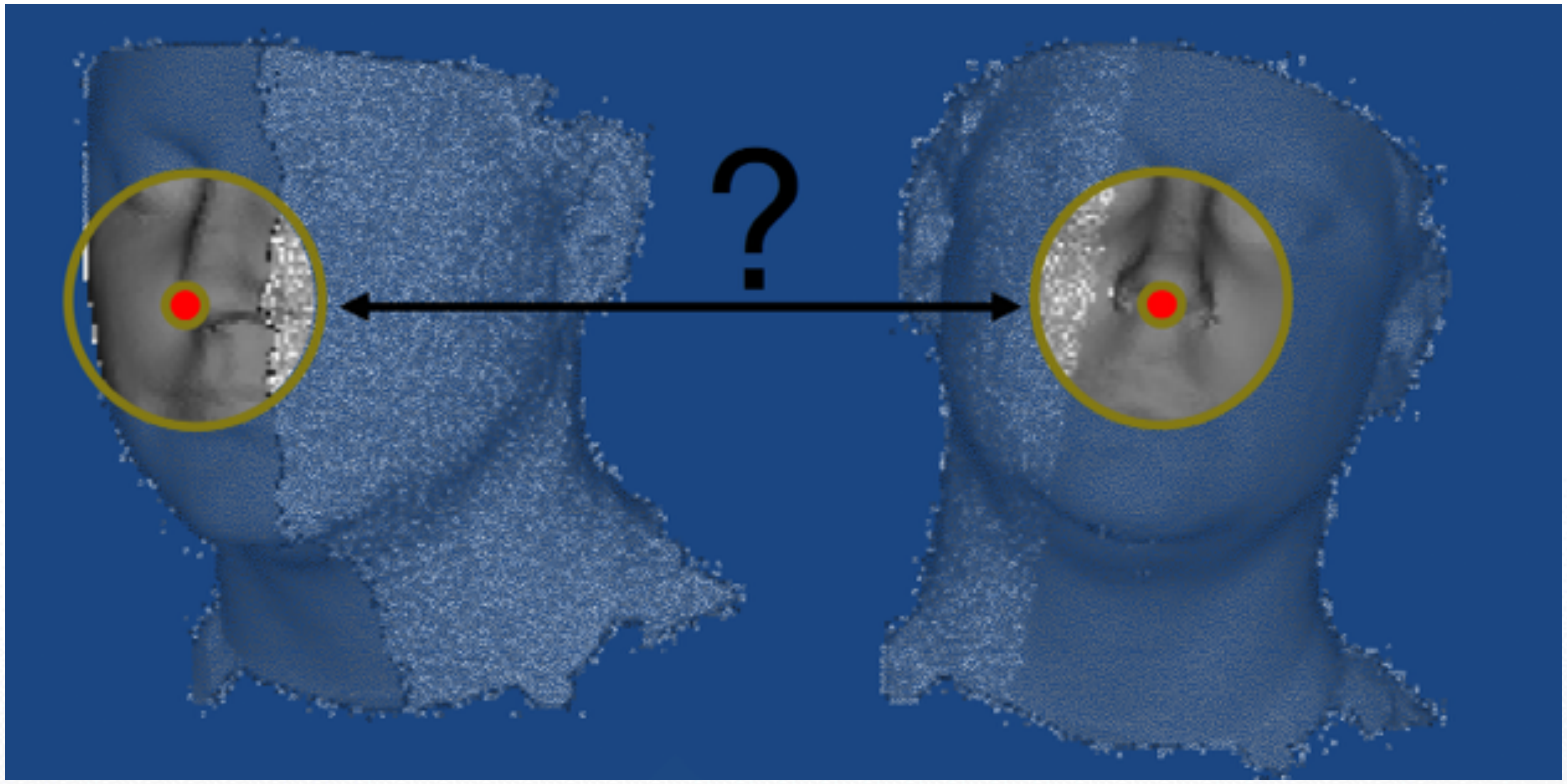
- Identify when two points on different scans represent the same feature



Local **Correspondence**

Goal

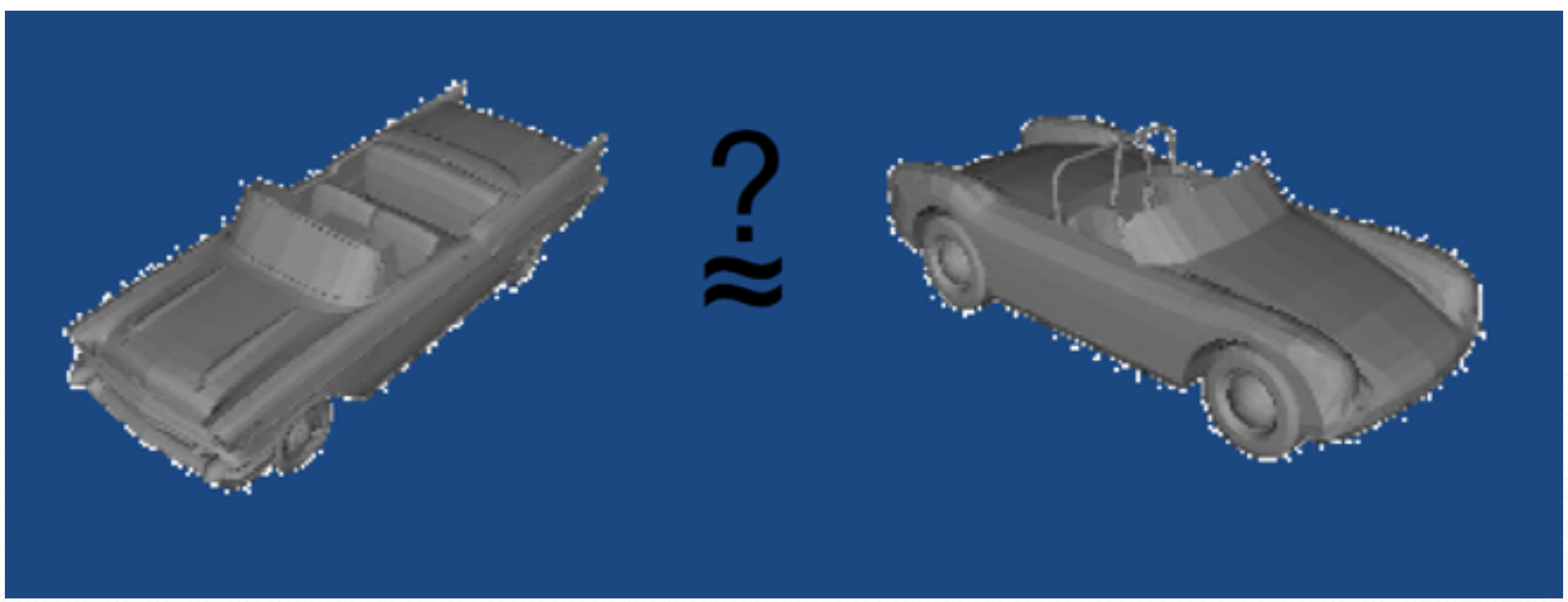
- Identify when two points on different scans represent the same feature
  - Are the surrounding regions similar?



Global **Correspondence**

More Generally:

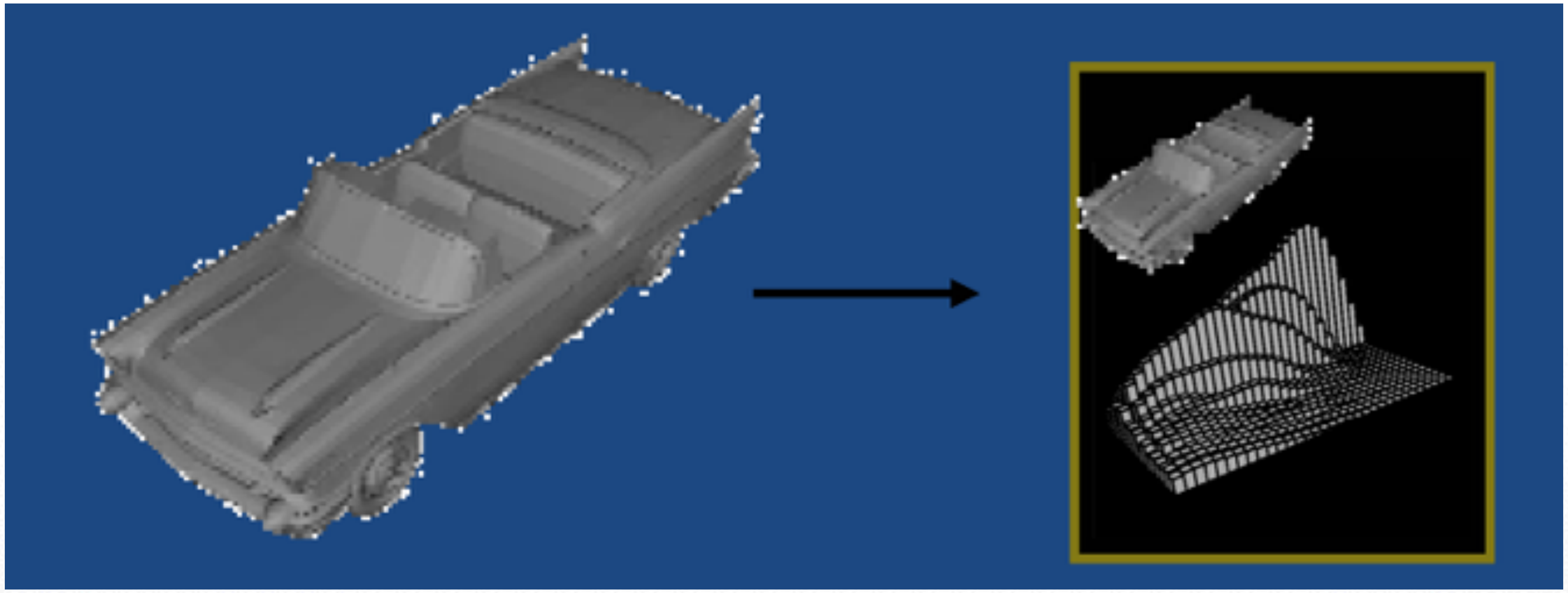
- Given two models, determine if they represent the same/ similar shapes
  - models can have different representations, tesselations, topologies, etc.



Global **Correspondence**

Approach:

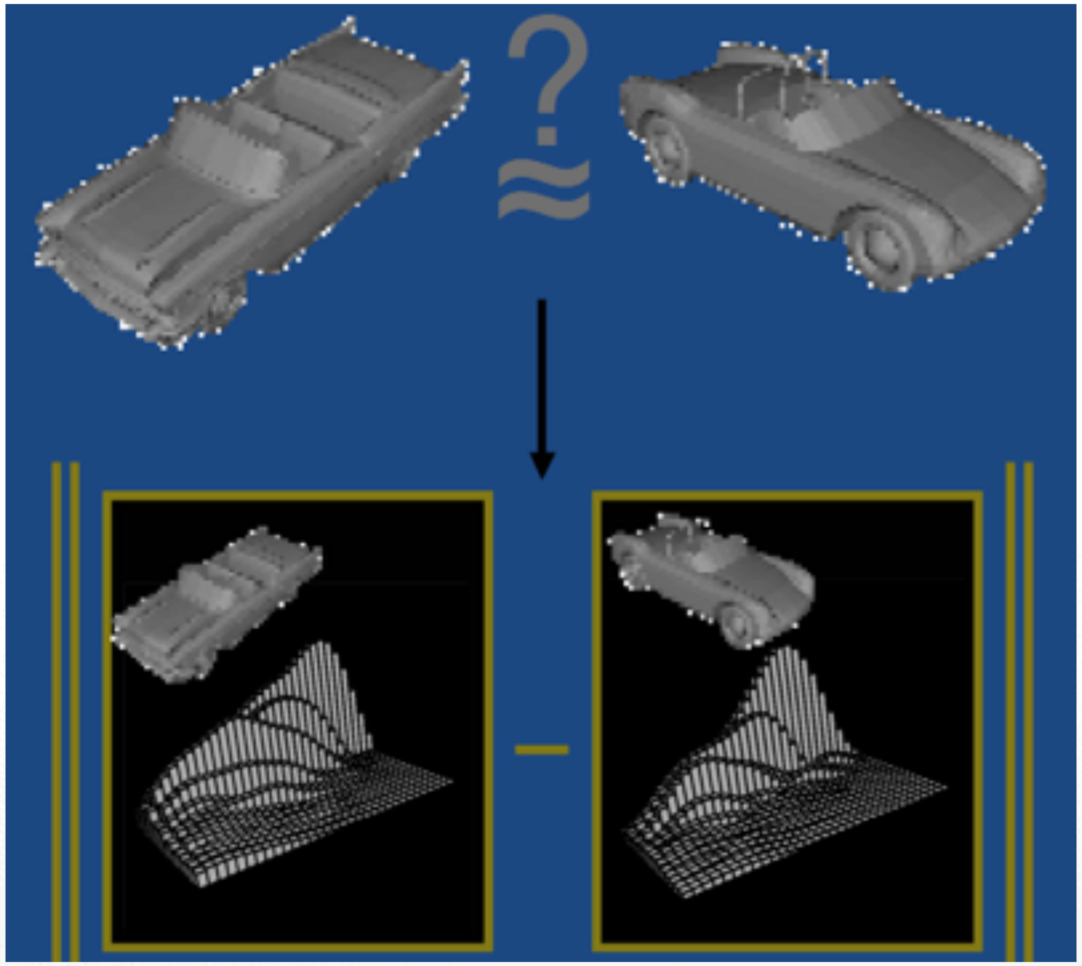
- Represent each model by a shape descriptor:
  - A **structured** abstraction of a 3D model
  - that captures **salient** shape information



Global **Correspondence**

Approach:

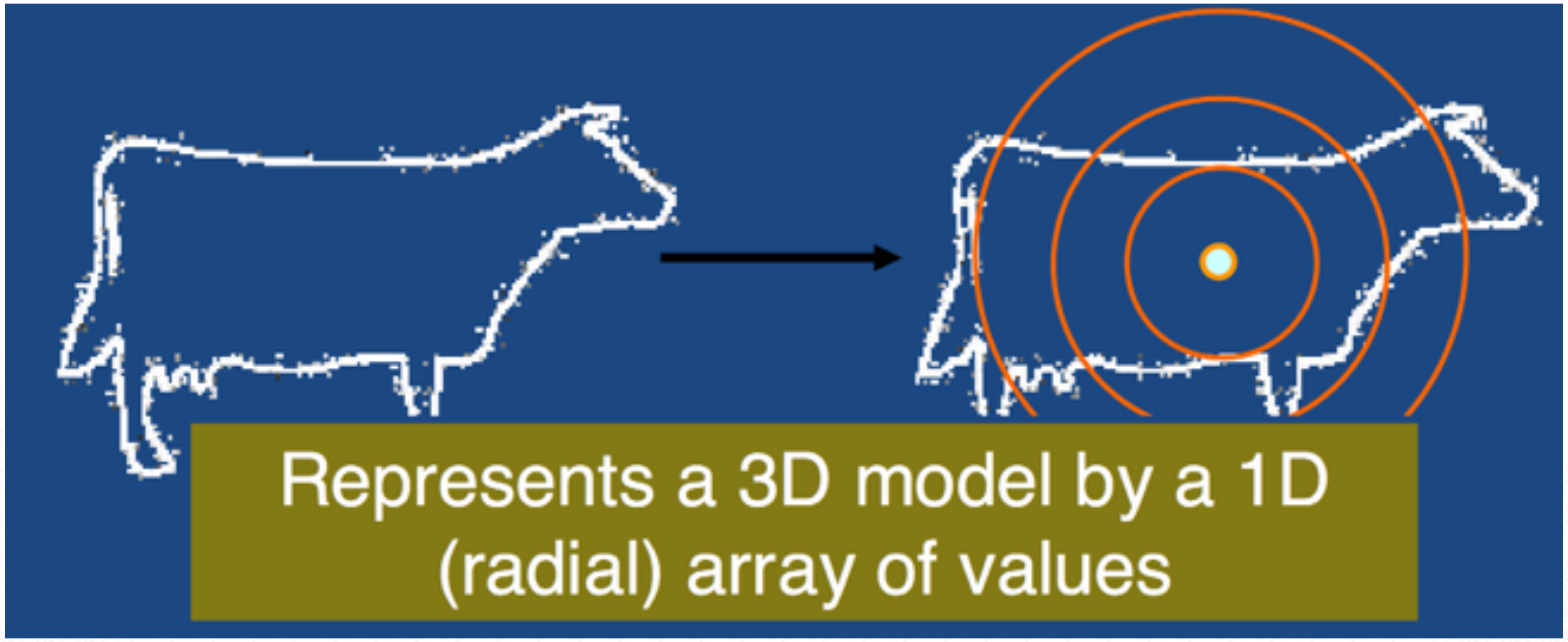
- Represent each model by a shape descriptor:
- Compare shapes by comparing their shape descriptors



Shape Descriptors: Examples

Shape Histograms

- Shape descriptor stores a histogram of how much surface **area** resides within different **concentric shells** in space

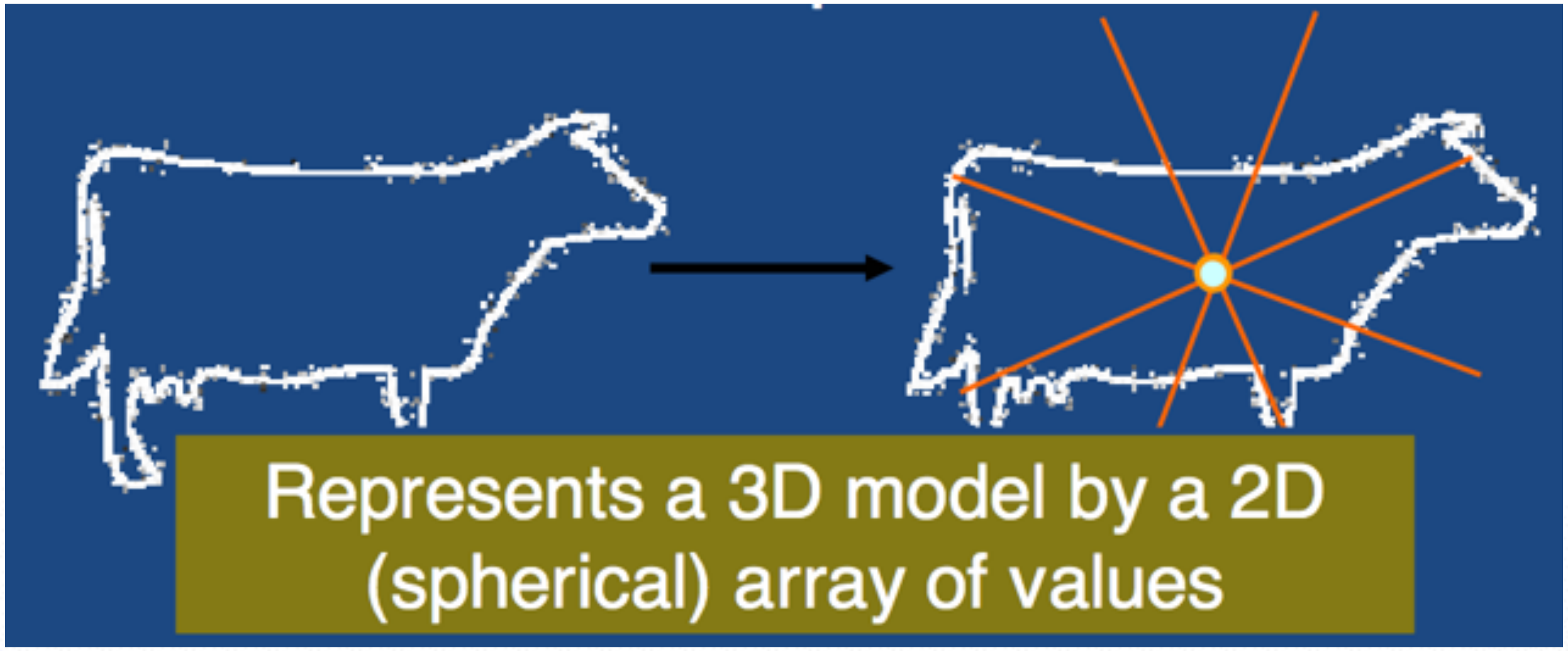


[Ankerst et al. 1999]

Shape Descriptors: Examples

Shape Histograms

- Shape descriptor stores a histogram of how much surface **area** resides within different **sectors** in space

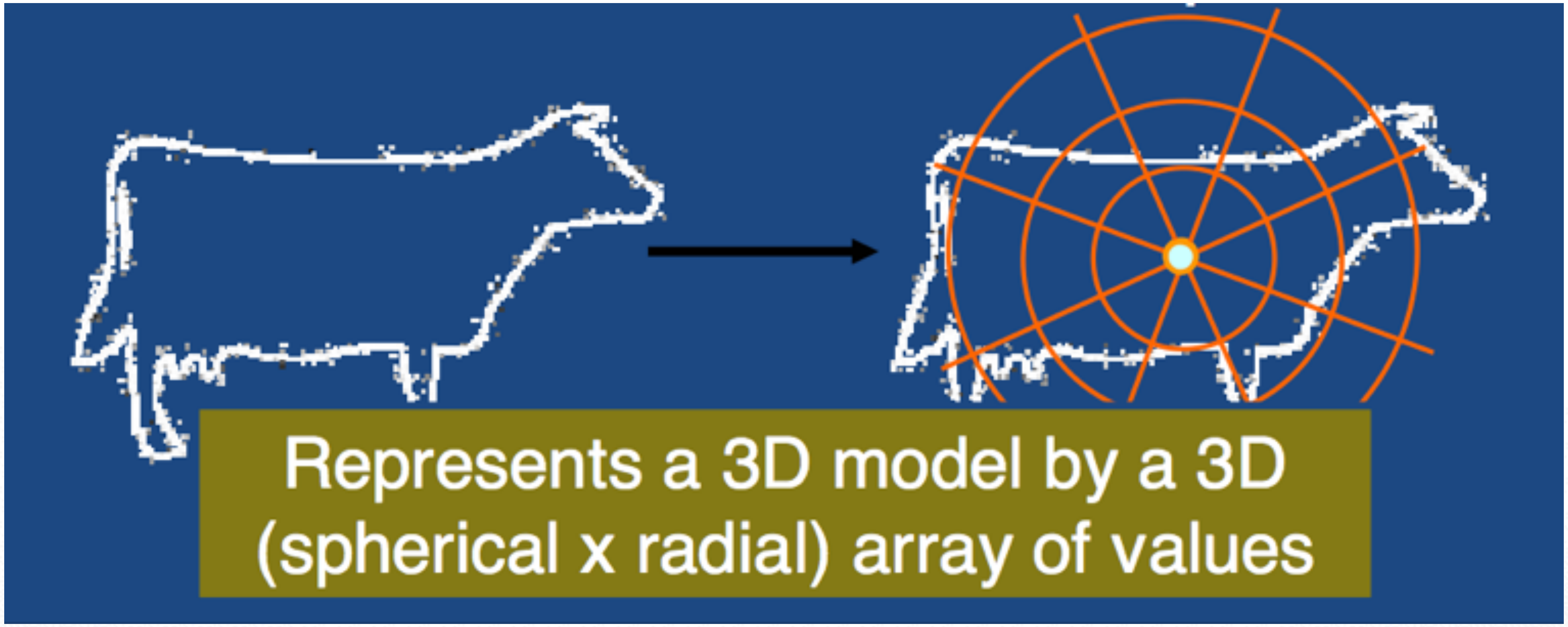


[Ankerst et al. 1999]

Shape Descriptors: Examples

Shape Histograms

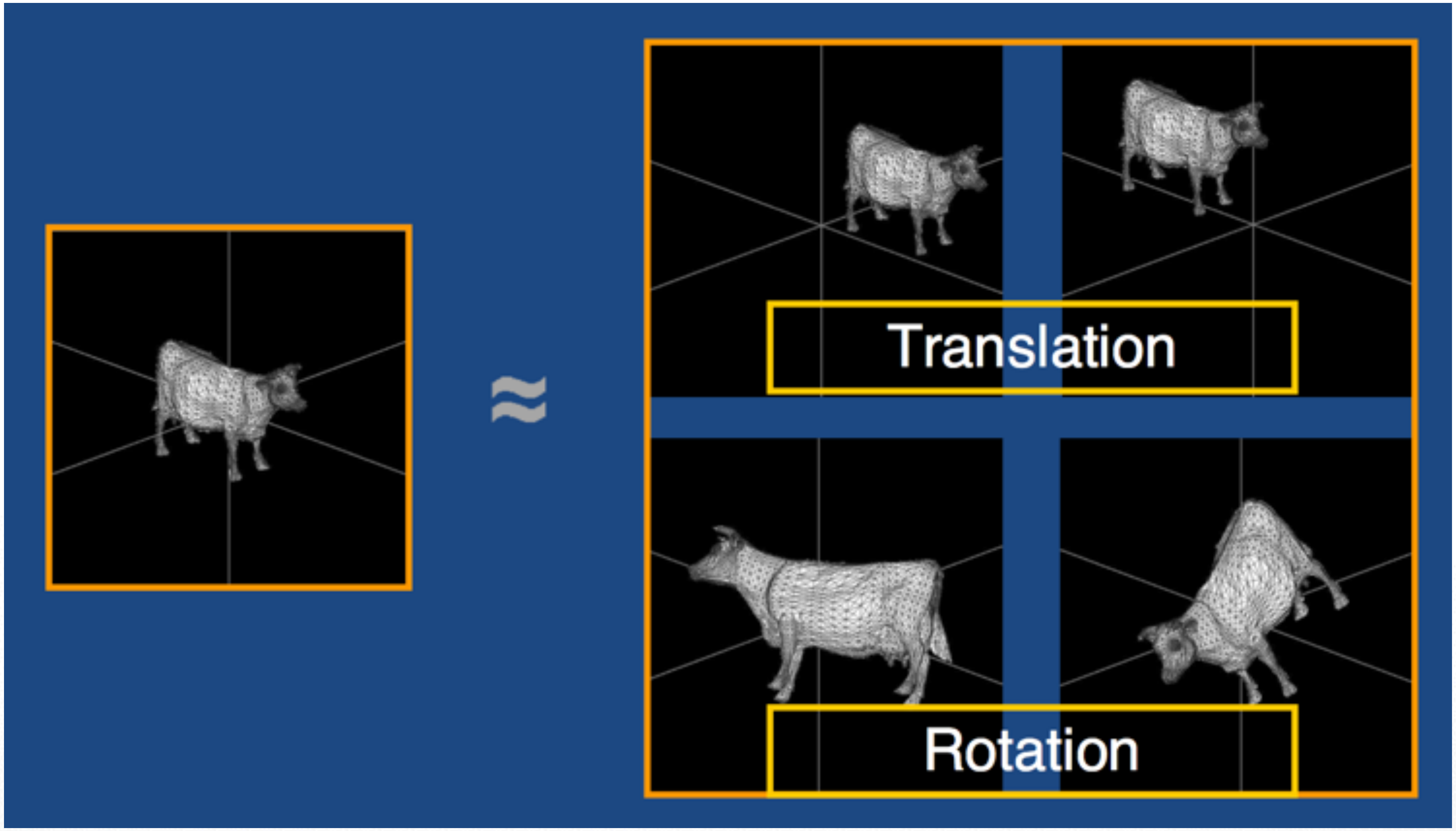
- Shape descriptor stores a histogram of how much surface **area** resides within different **shells** and **sectors** in space



[Ankerst et al. 1999]

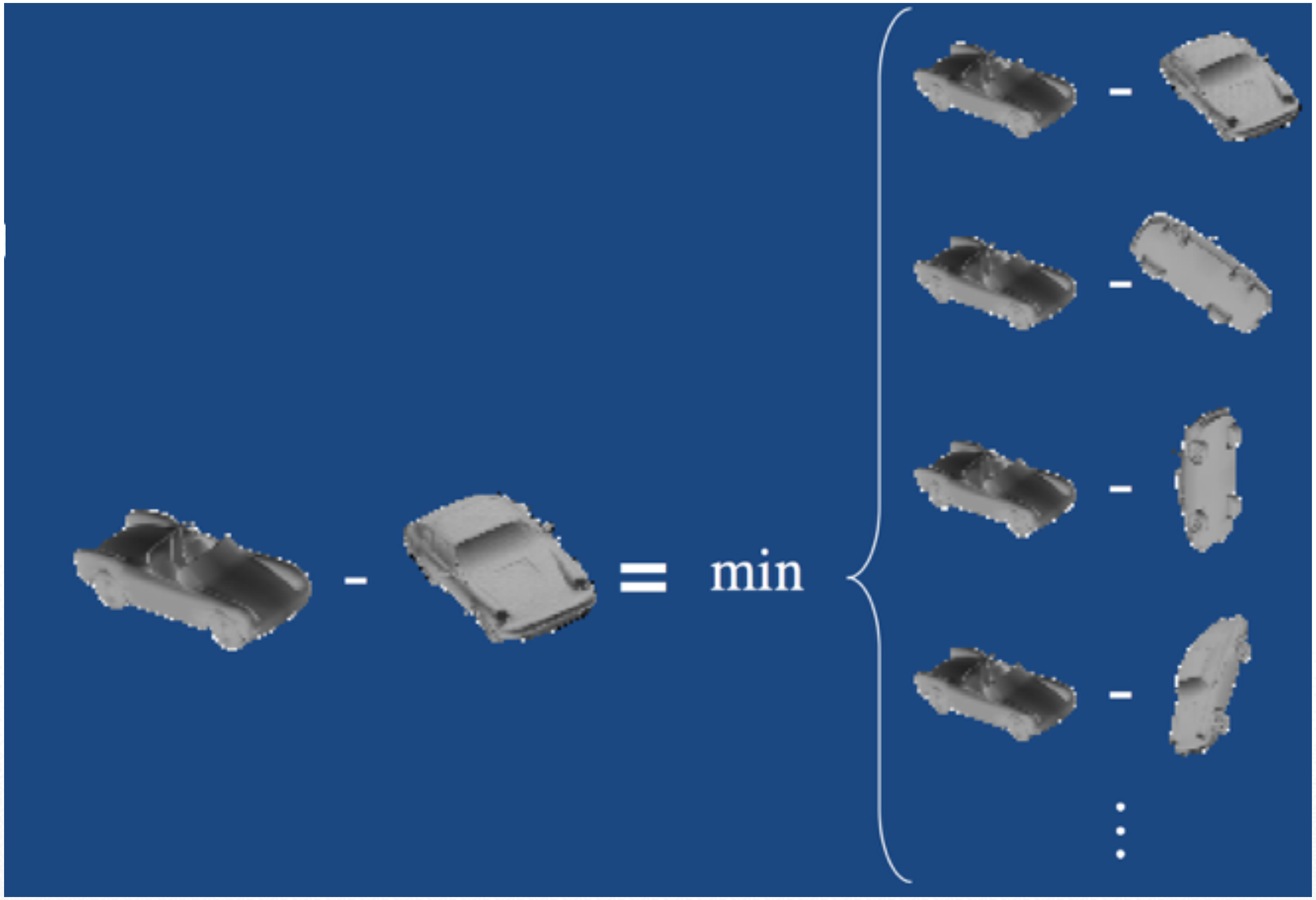
Shape Descriptors: Challenge

- The **shape** of a model does not change when a rigid body transformation is applied to the model.



Shape Descriptors: Challenge

- To compare two models, we need them at their optimal alignment



Shape Descriptors: Alignment

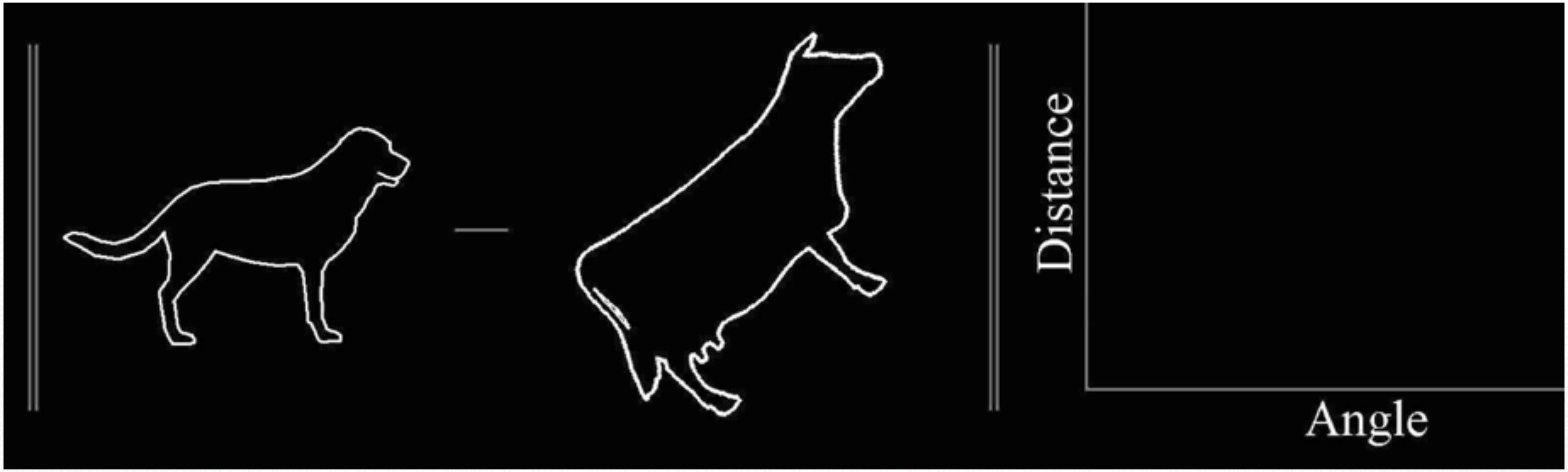
Three general methods:

- Exhaustive Search
- Normalization
- Invariance

Shape Descriptors: Alignment

Exhaustive Search:

- Compare at all alignments

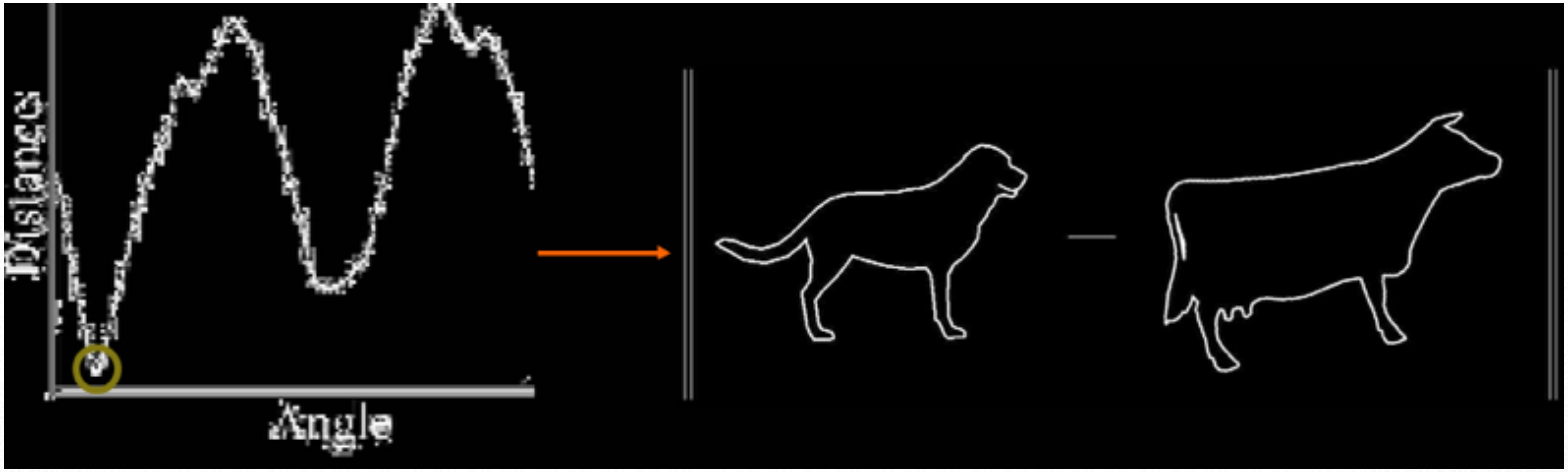


Exhaustive search for optimal rotation

Shape Descriptors: Alignment

Exhaustive Search:

- Compare at all alignments
- Correspondence is determined by the alignment at which the models are closest



Exhaustive search for optimal rotation

Shape Descriptors: Alignment

Exhaustive Search:

- Compare at all alignments
- Correspondence is determined by the alignment at which the models are closest

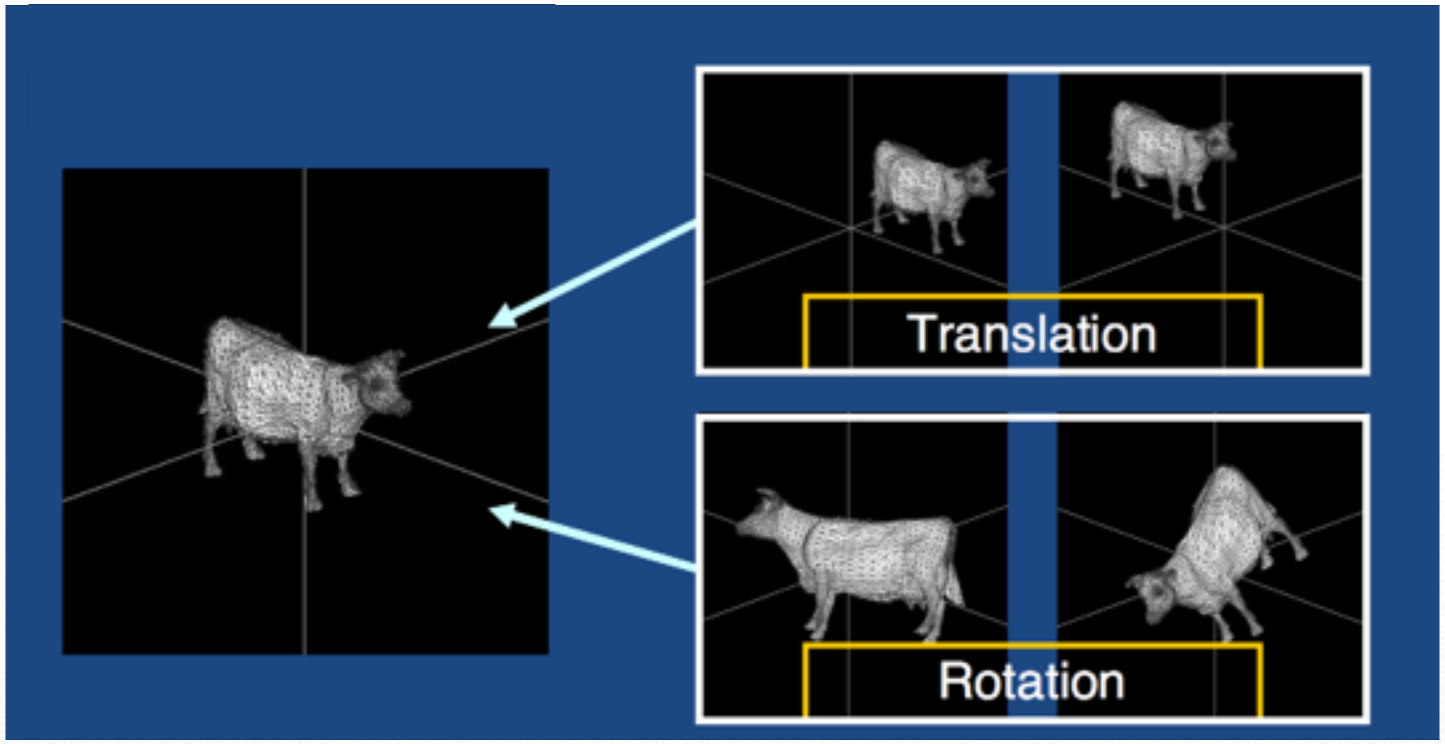
Properties:

- Gives the correct answer (w.r.t. the metric)
- While slow on a single processor, it can be parallelized (Clusters? Multi-Threading? GPU?)

Shape Descriptors: Alignment

Normalization:

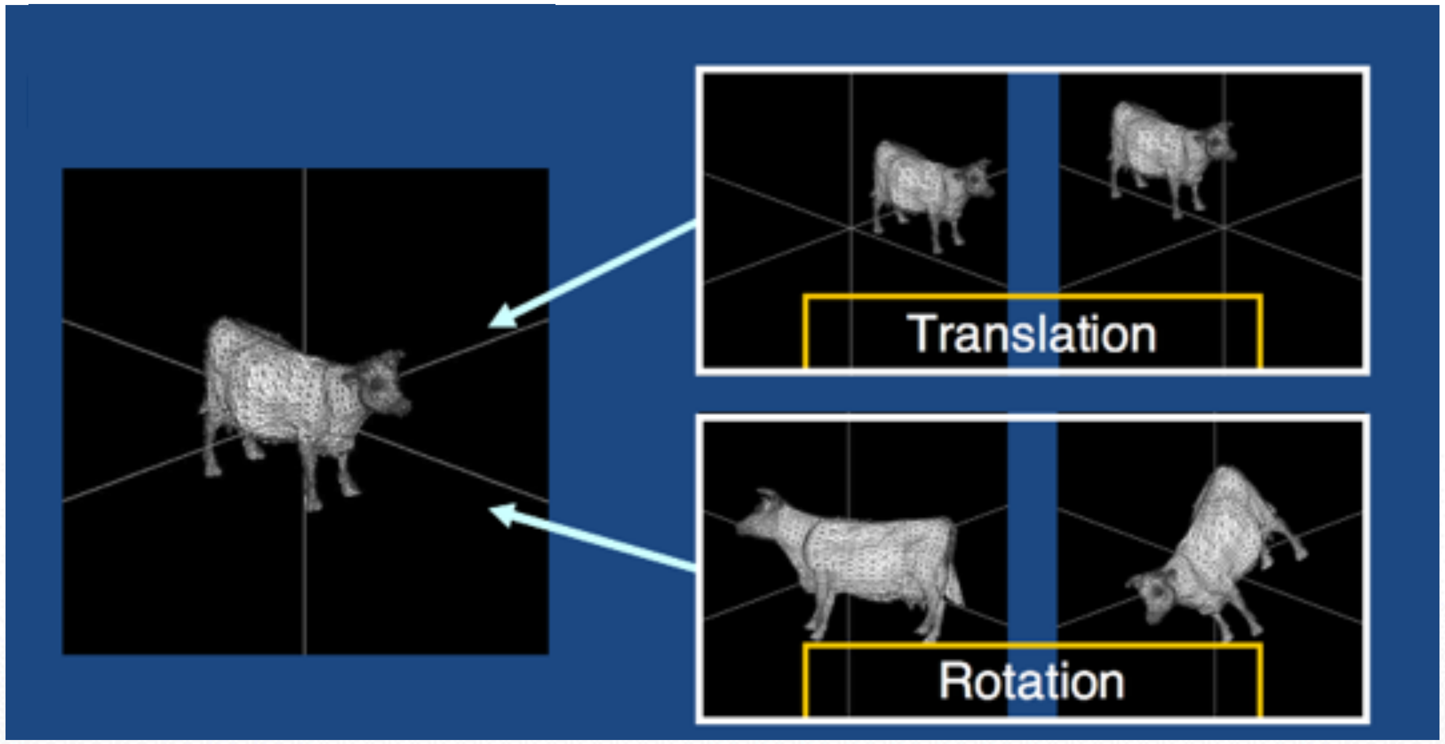
- Put each model into a canonical frame:
  - Translation
  - Rotation



Shape Descriptors: Alignment

Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
  - Rotation



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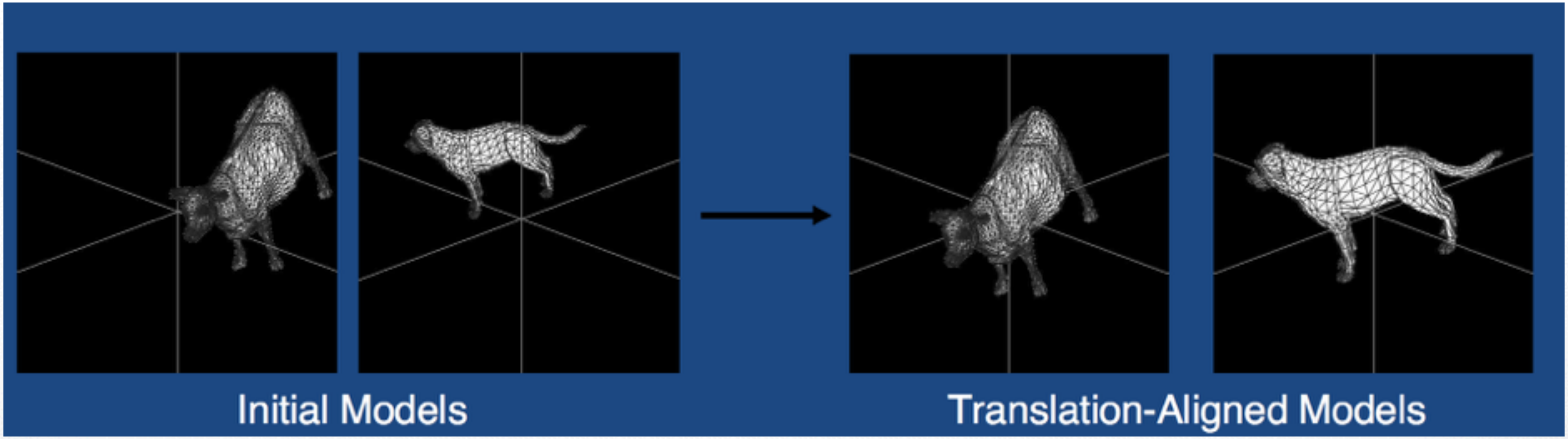
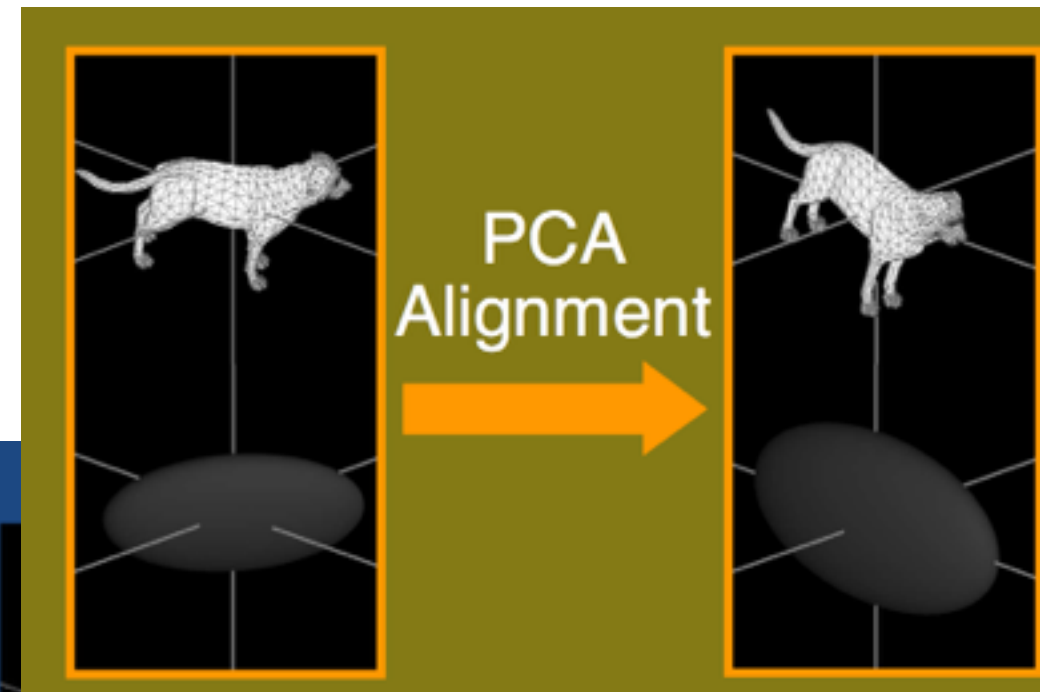
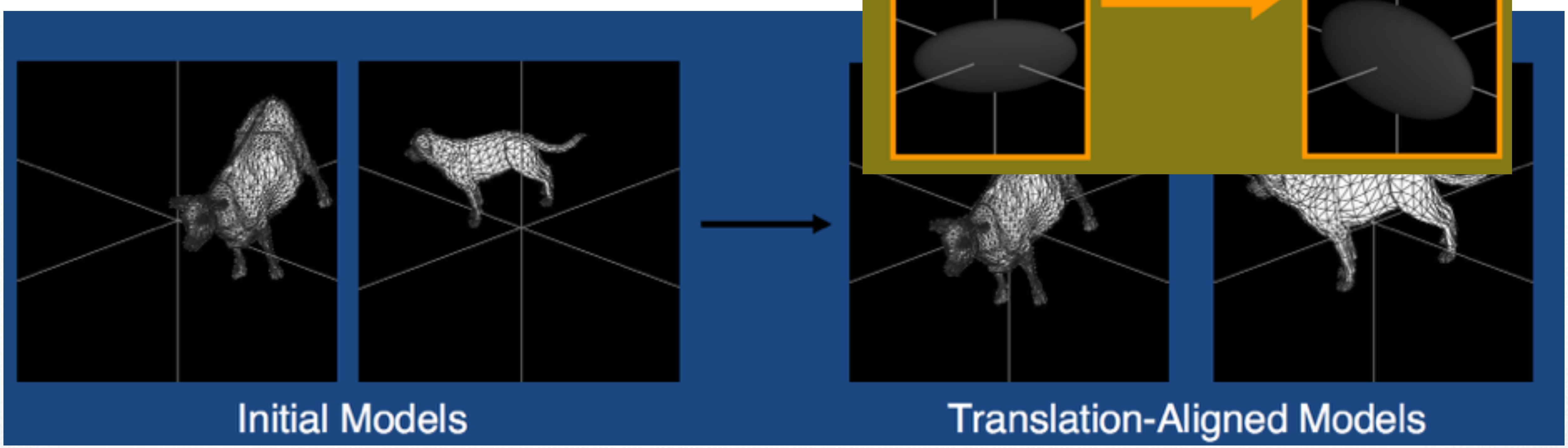


Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
  - Rotation: PCA alignment





Shape Descriptors: Alignment

Normalization:

- Put each model into a canonical frame:
  - Translation: Center of Mass
  - Rotation: PCA alignment

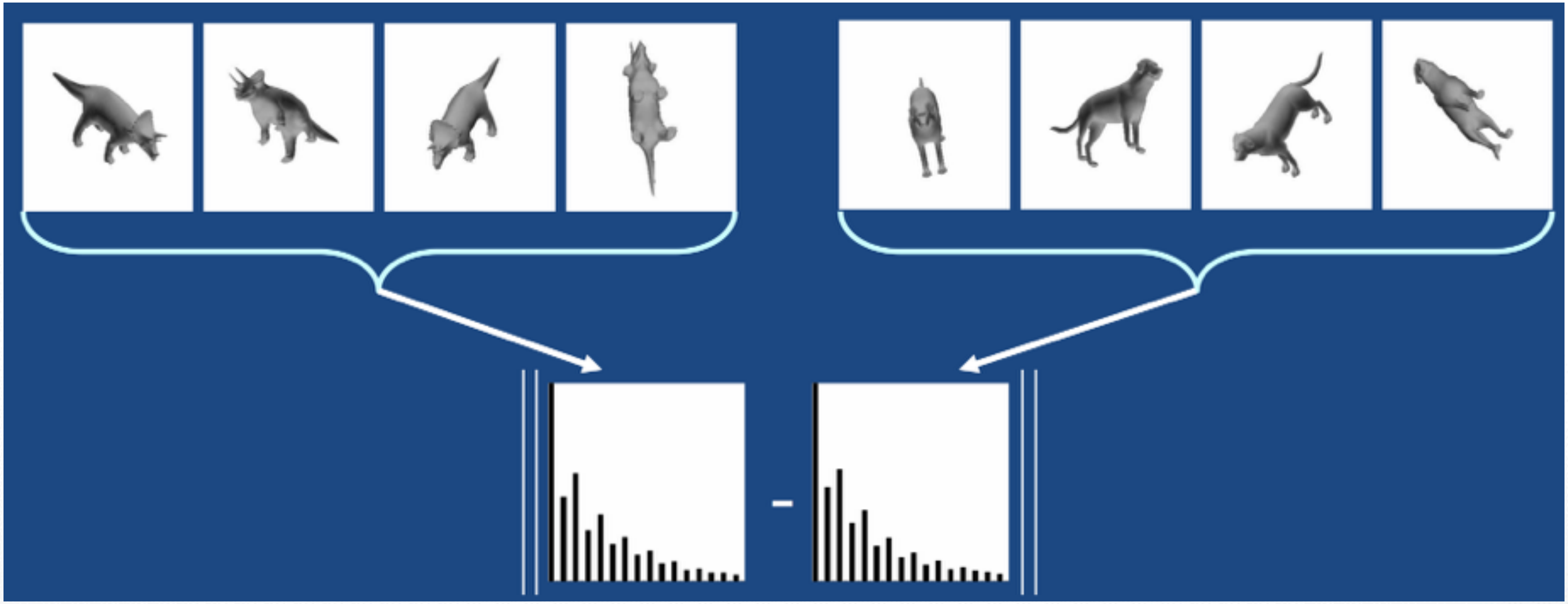
Properties:

- Efficient
- Not always robust
- Not suitable for local feature matching

Shape Descriptors: Alignment

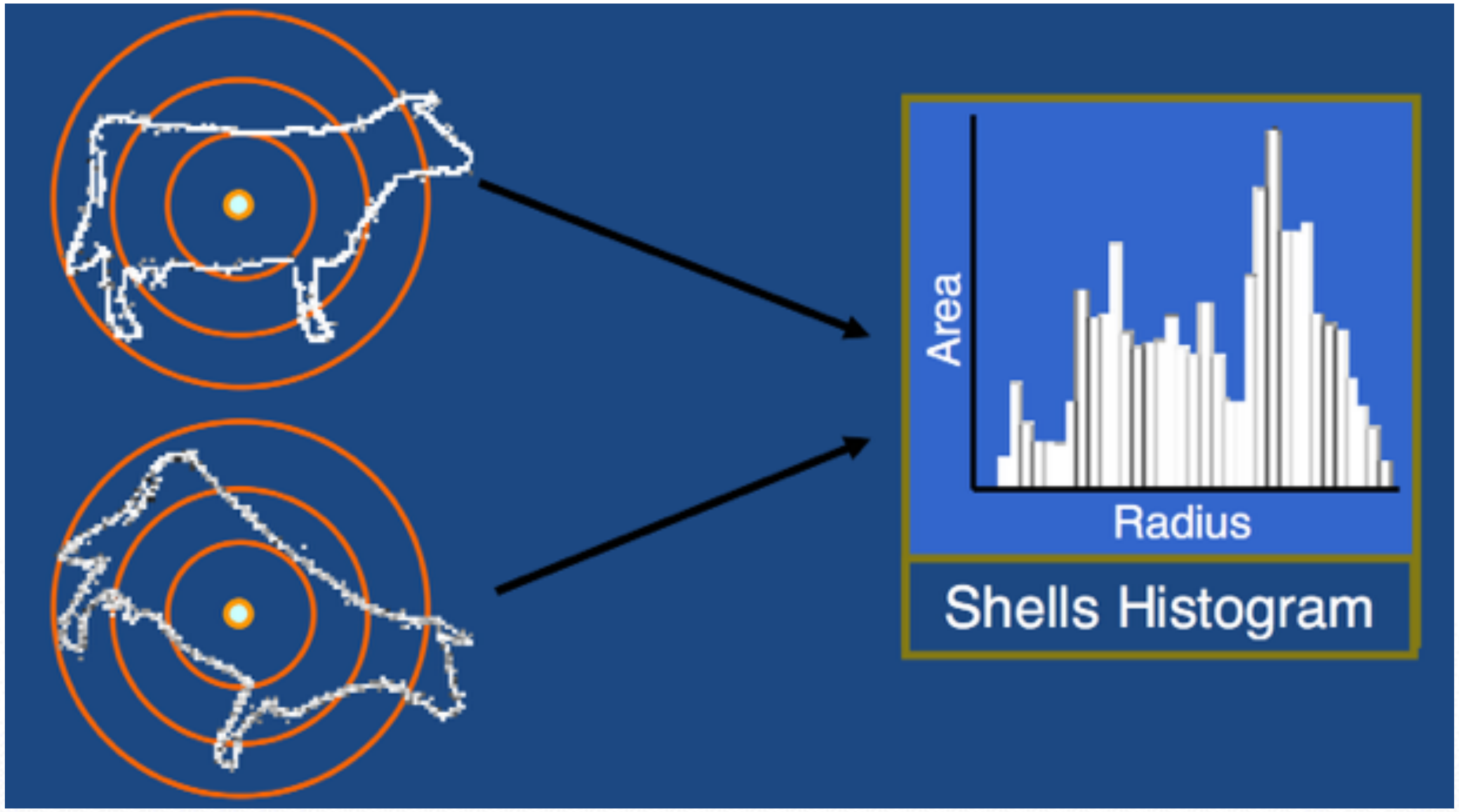
Invariance:

- Represent a model by a shape descriptor that is independent of the pose.



Example: Ankerst's Shells

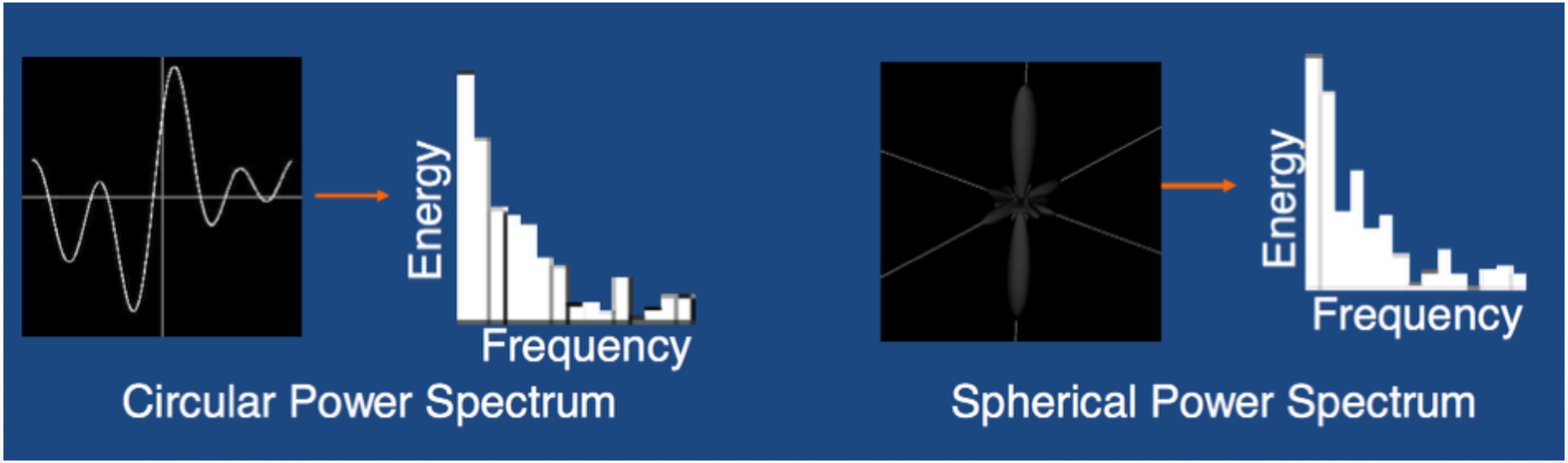
- A histogram of the radial distribution of surface area



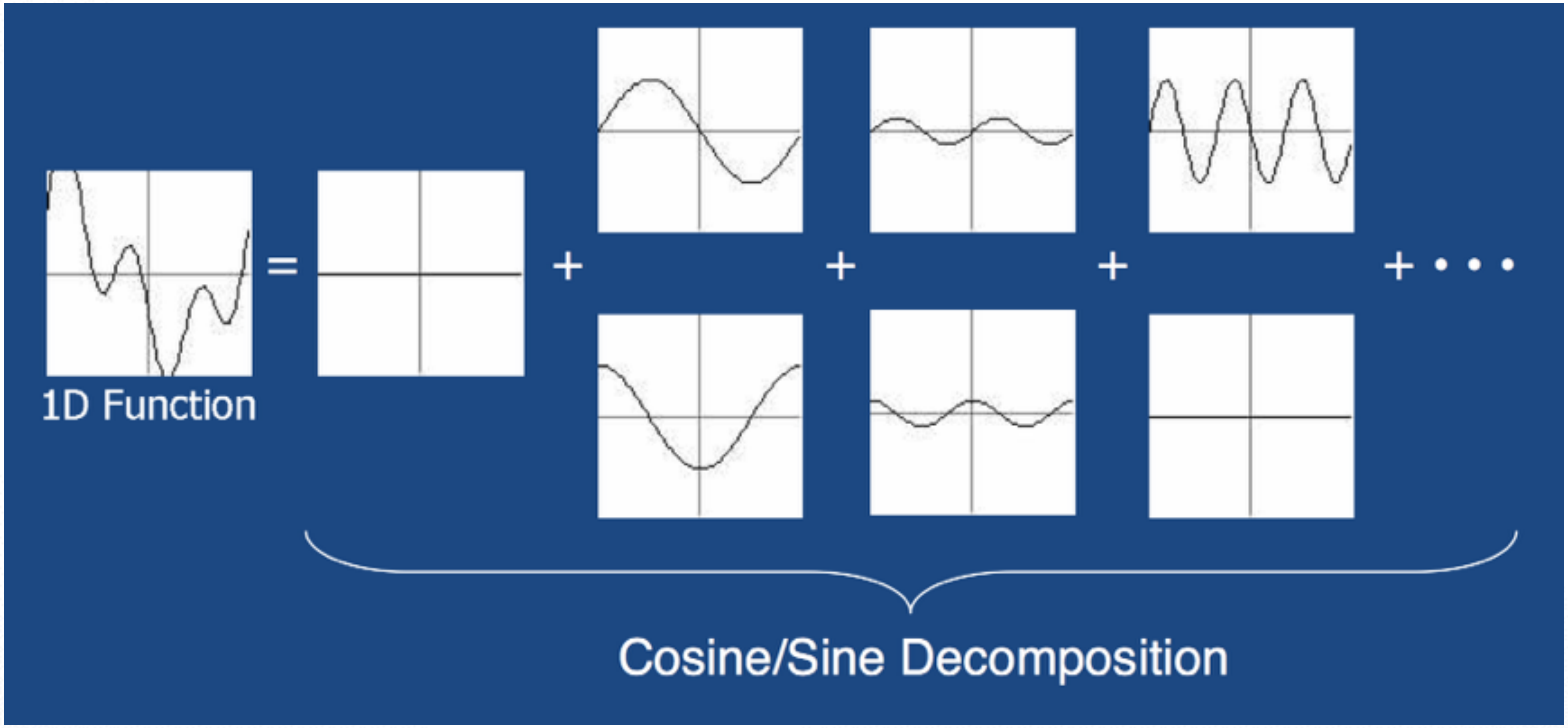
Shape Descriptors: Alignment

Invariance

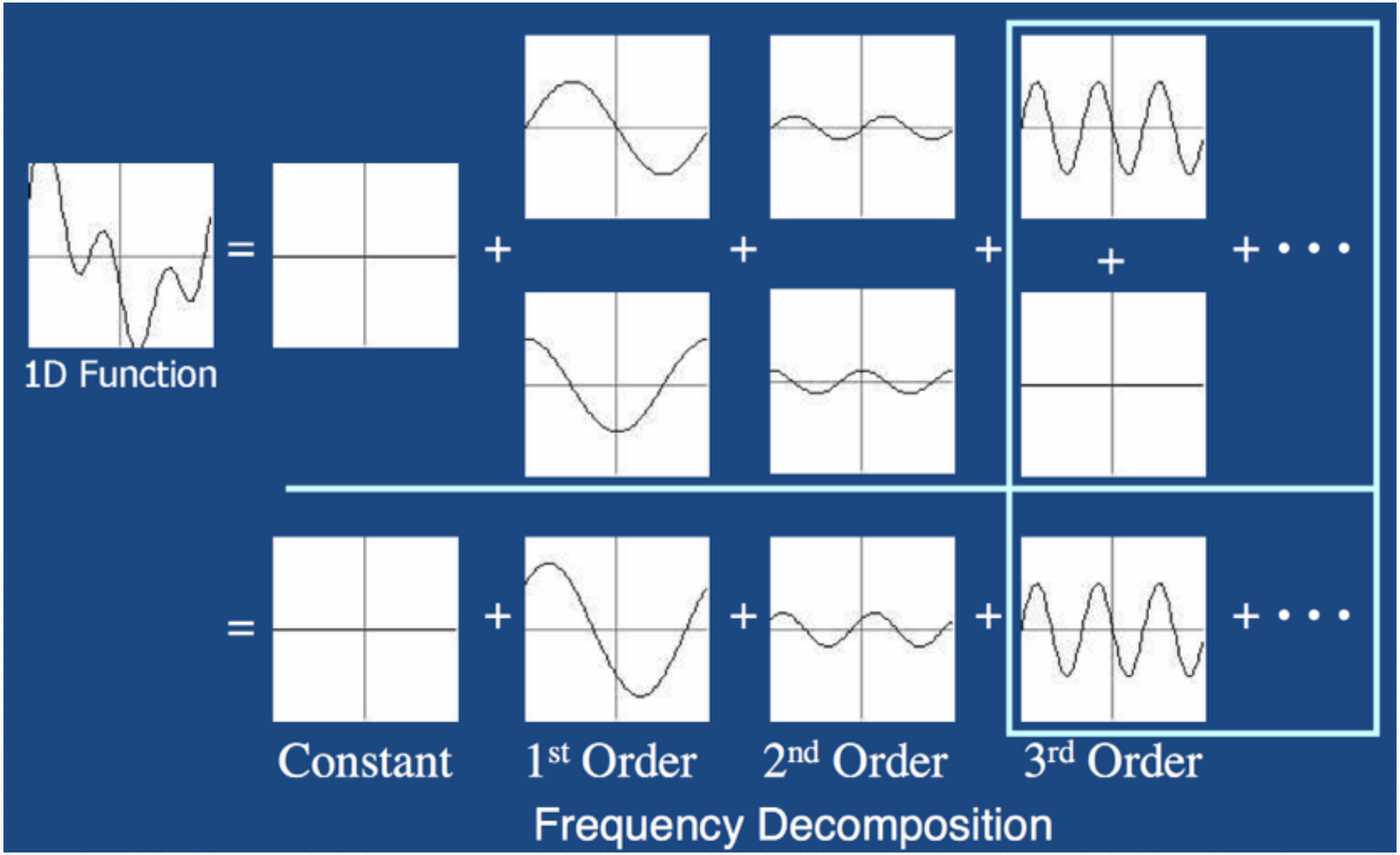
- Power spectrum representation
  - Fourier transform for translations
  - Spherical harmonic transform for rotations



Translation Invariance

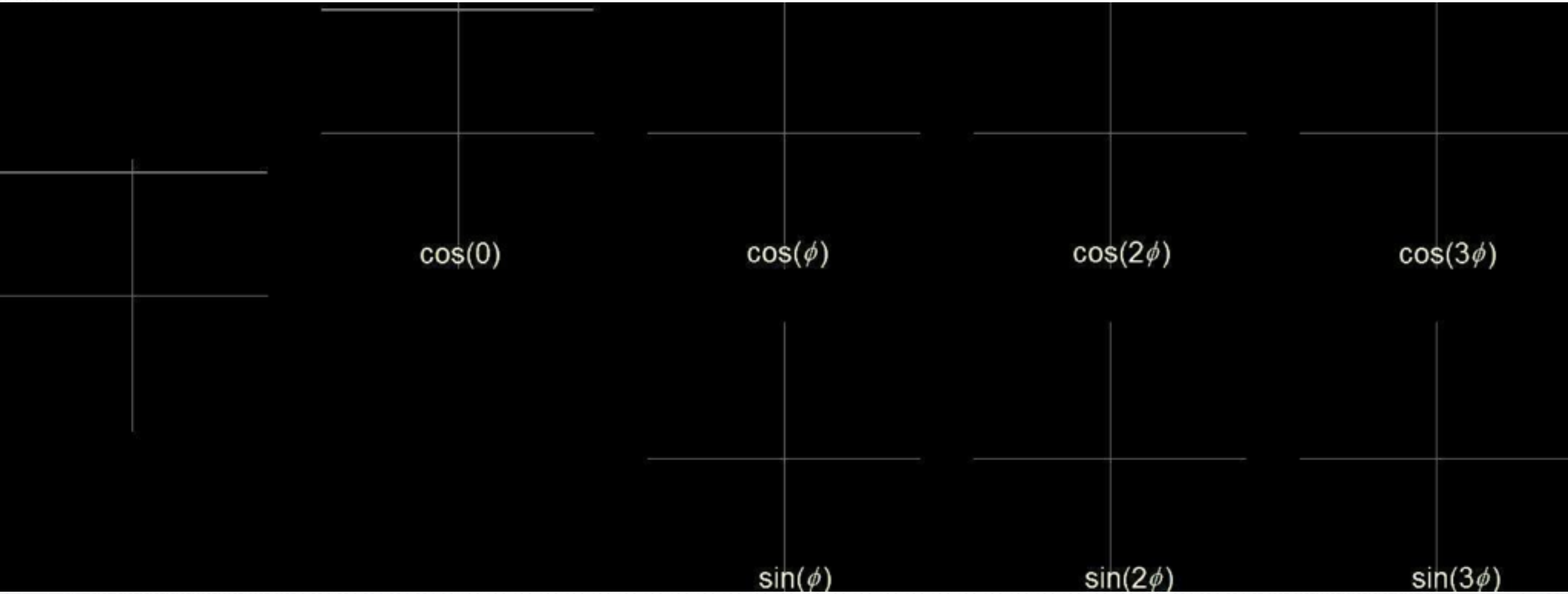


Translation Invariance



Translation Invariance

Frequency subspaces are fixed by rotations:



Translation Invariance

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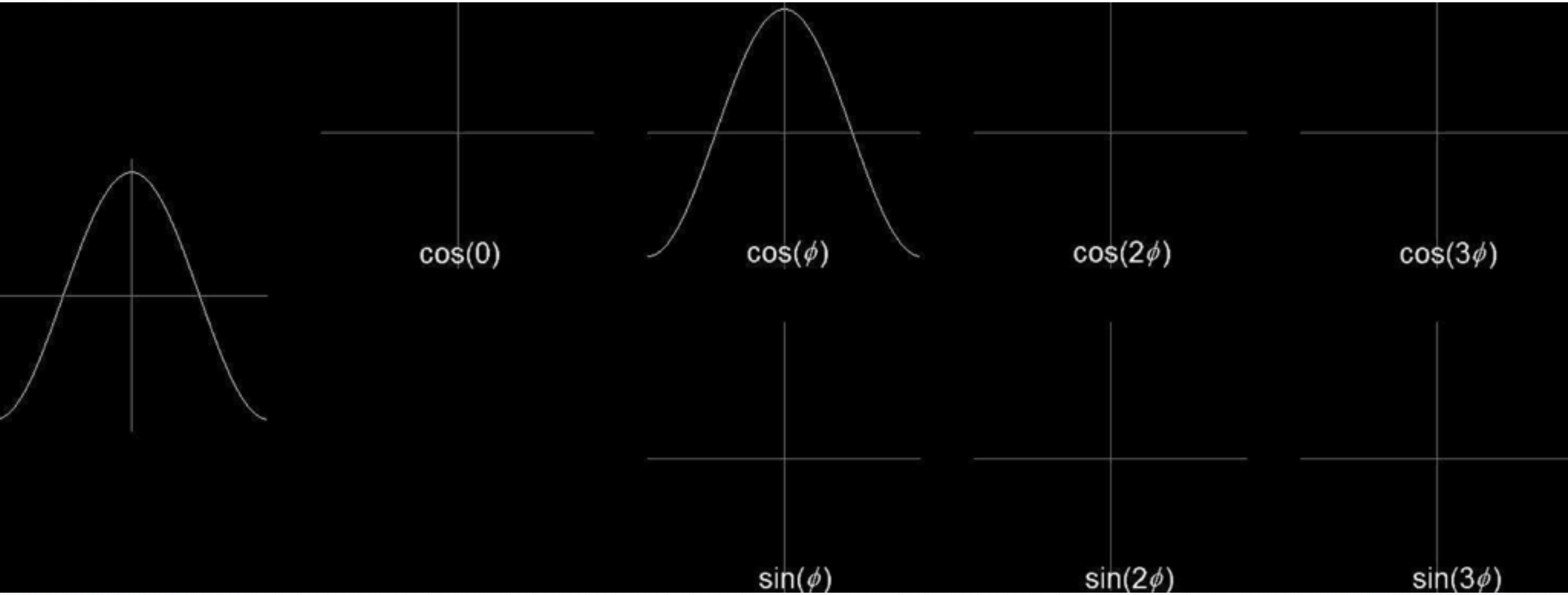

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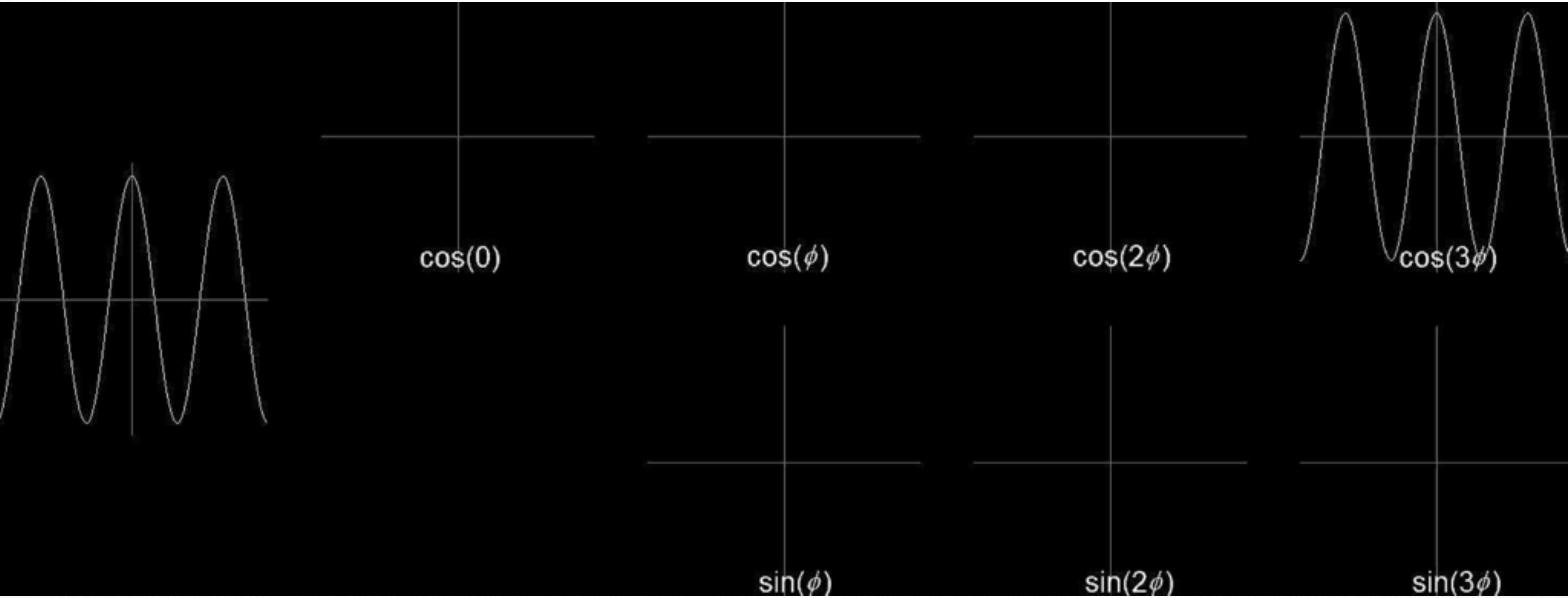
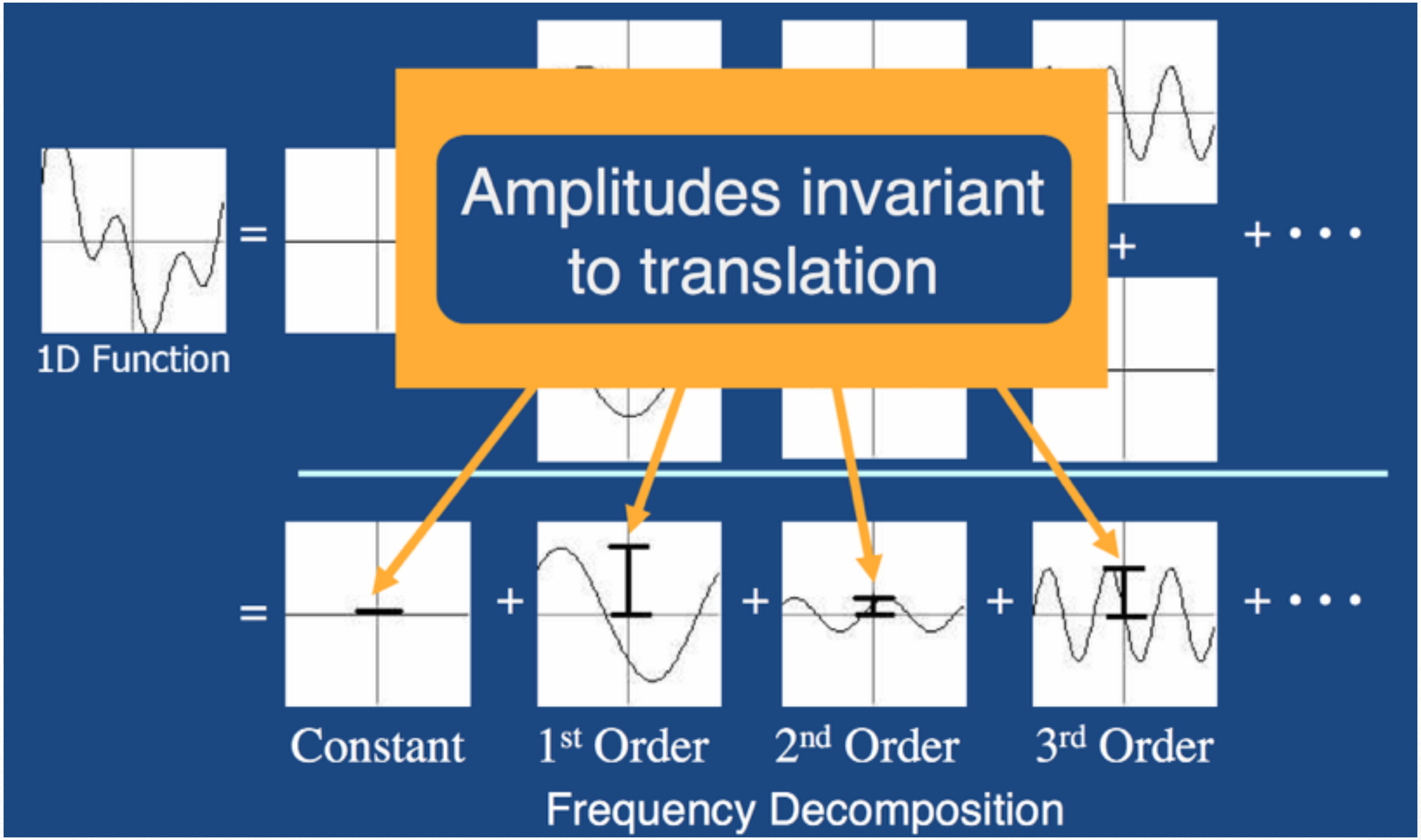
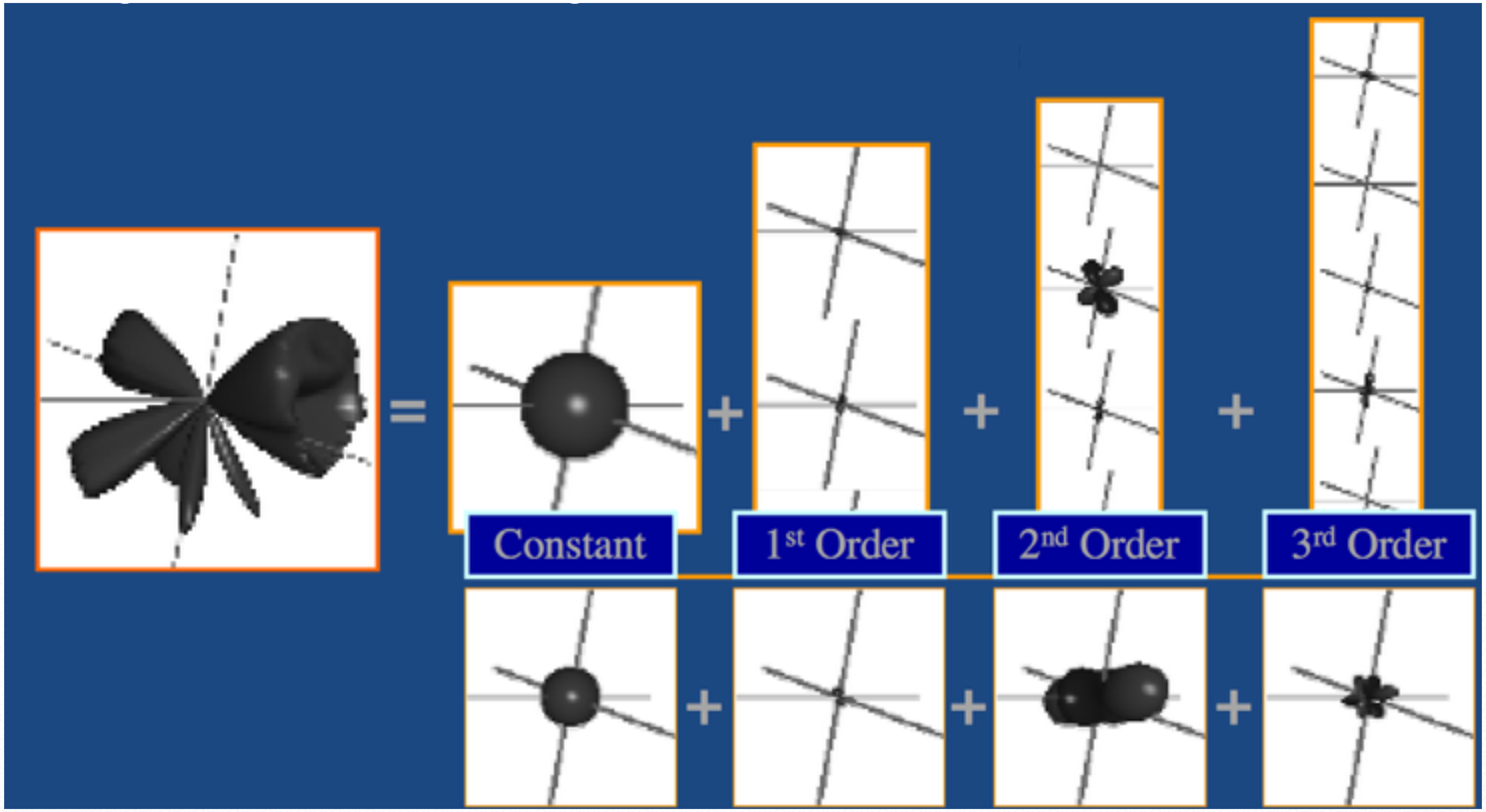


Translation Invariance



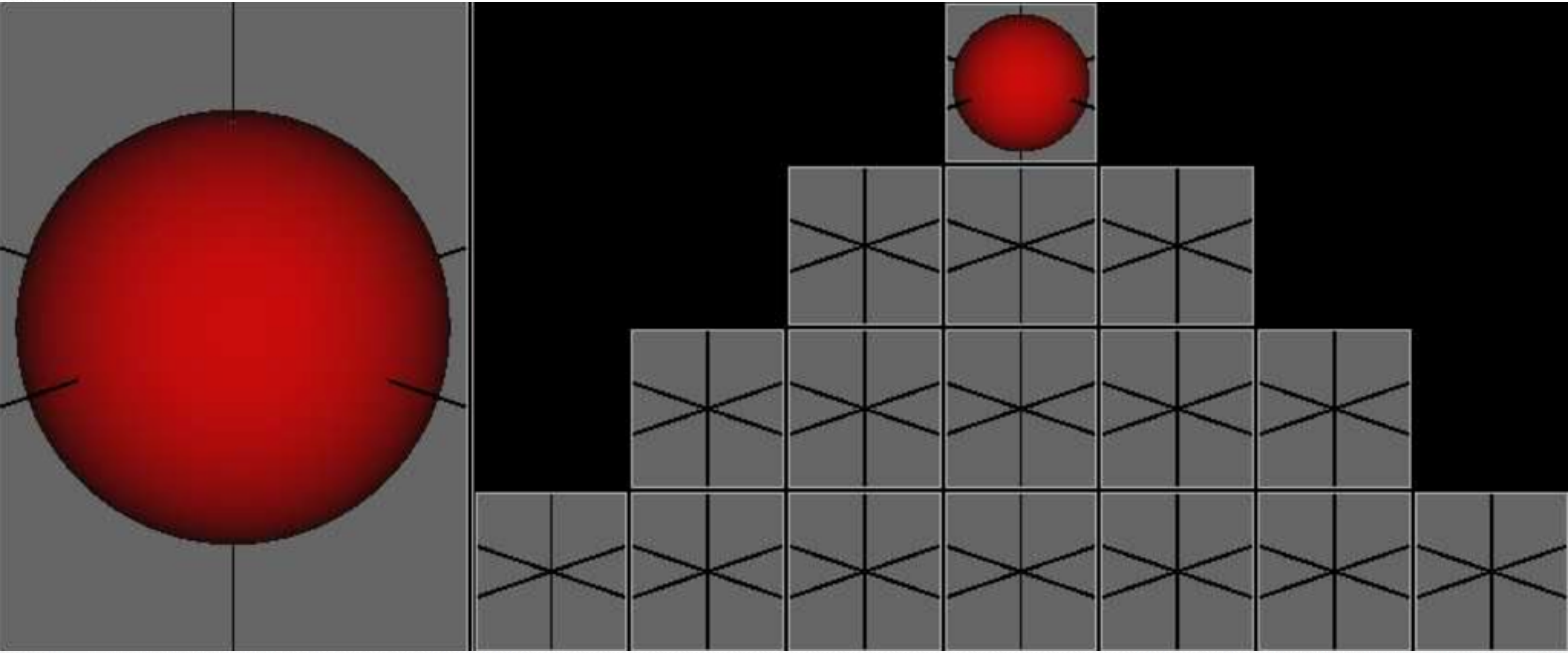
Rotation Invariance

Represent each spherical function as a sum of harmonic frequencies (orders)



Rotation Invariance

Frequency subspaces are fixed by rotations



Rotation Invariance

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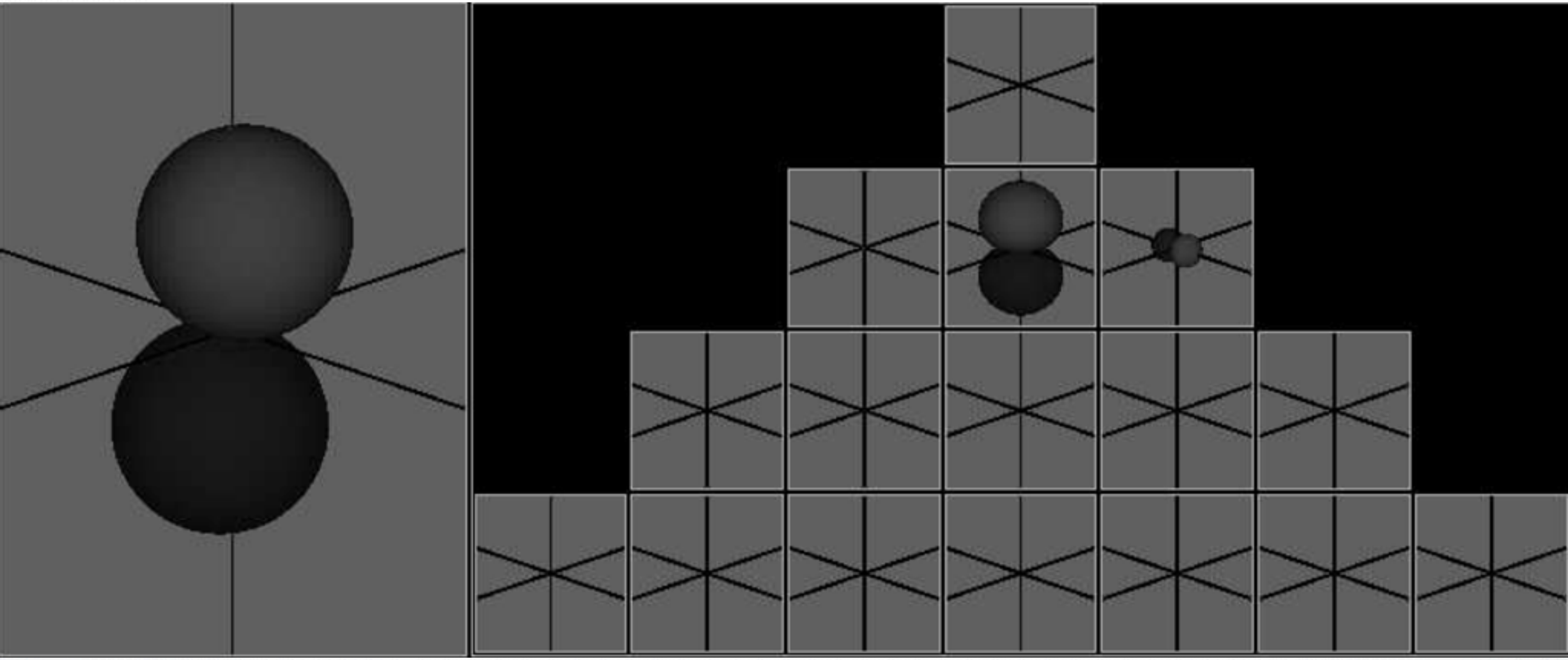
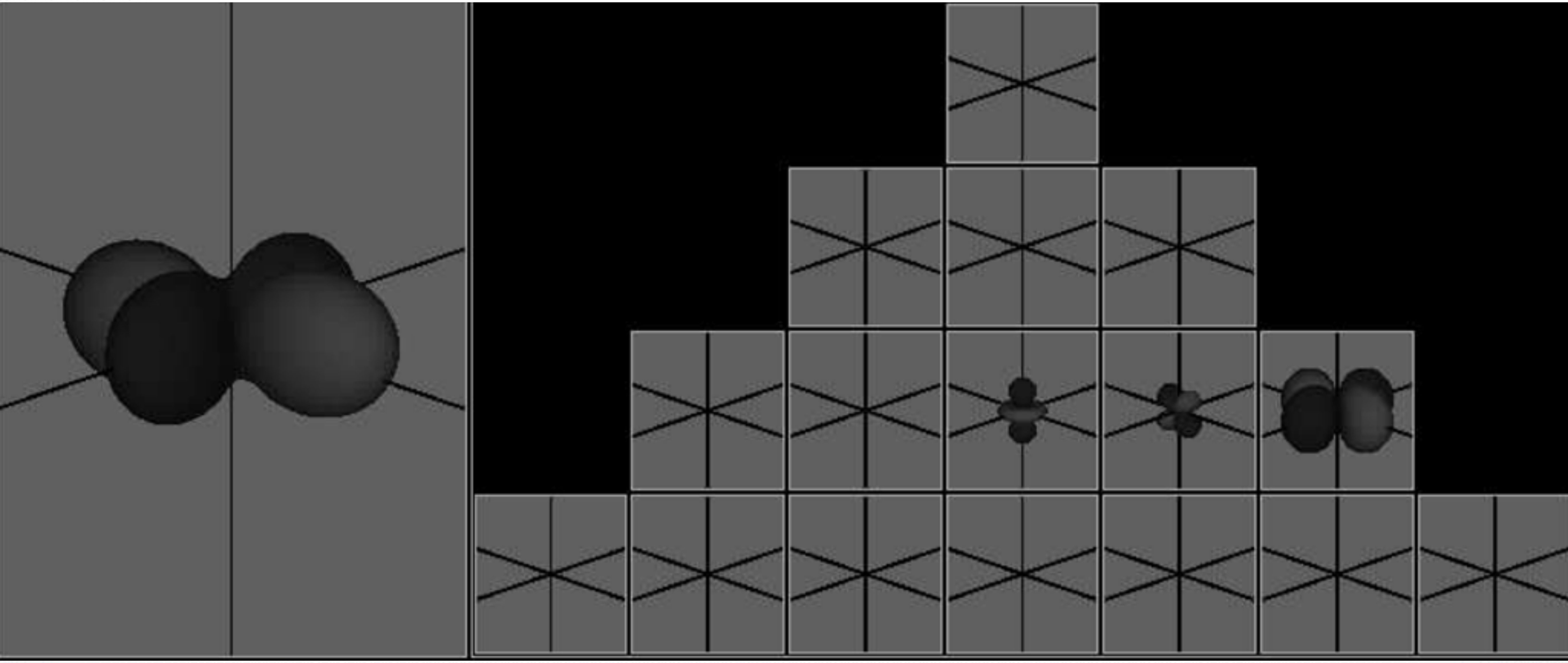


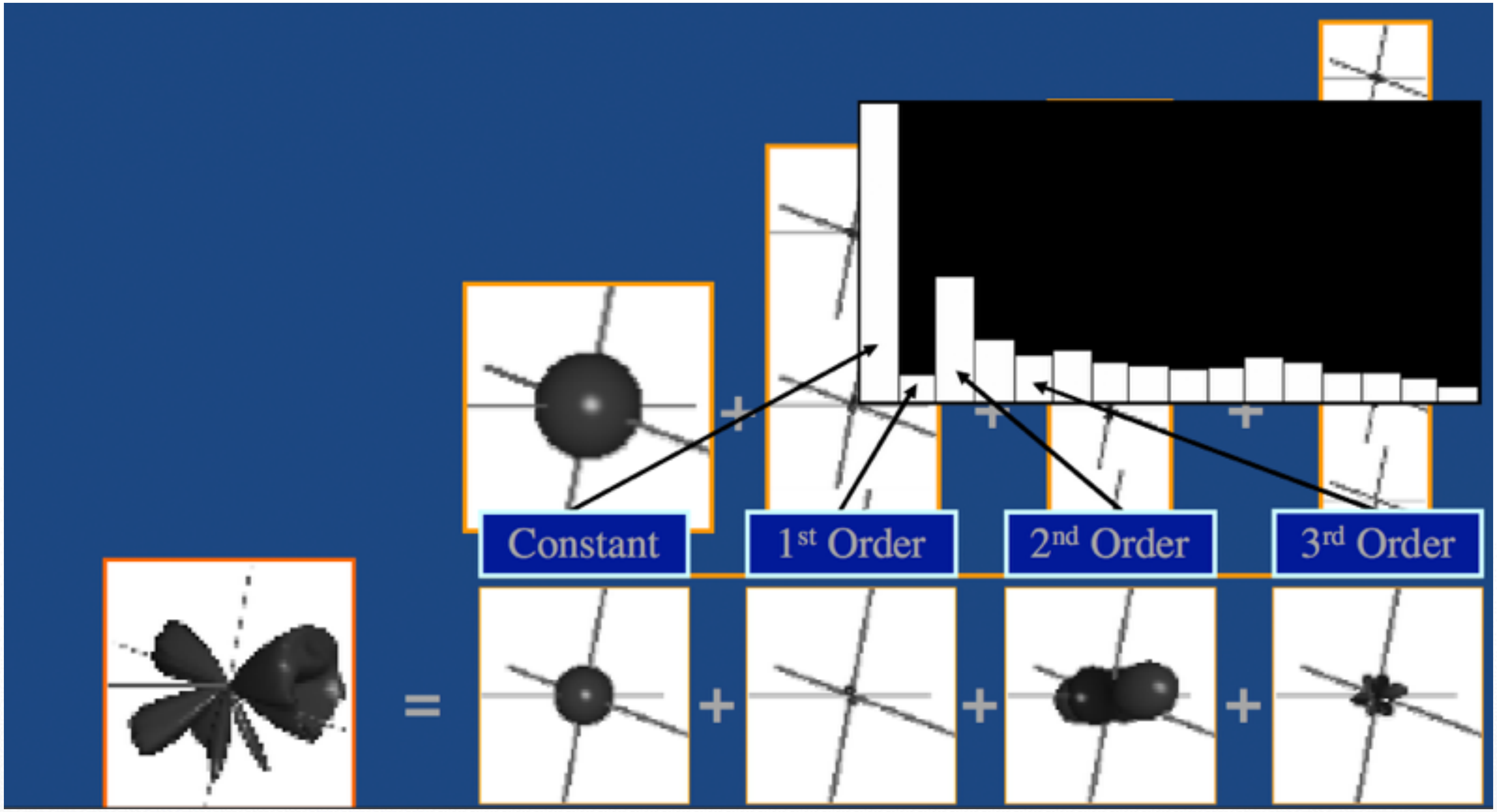
Rotation Invariance

Frequency subspaces are fixed by rotations



Rotation Invariance

Store “how much” (L2-norm) of the shape resides in each frequency to get a rotatin invariant representation



Shape Descriptors: Alignment

Invariance:

- Represent a model by a shape descriptor that is independent of the pose

Properties:

- Compact representation
- Not always discriminating

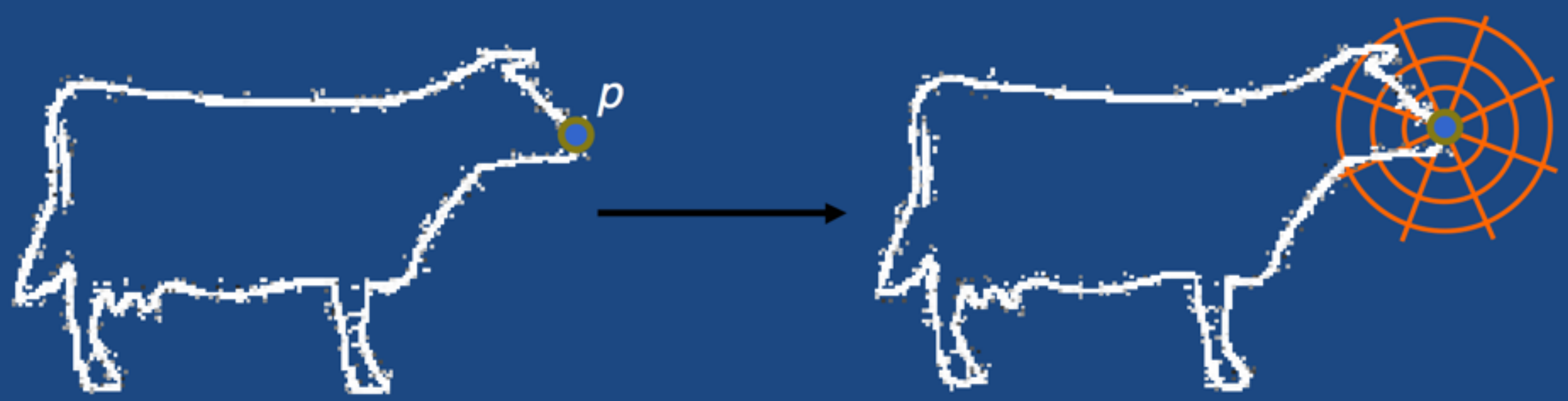
Outline

- **Global Shape Correspondence**
  - Shape Descriptors
  - Alignment
- **Partial Shape Correspondence**
  - From Global to Local
  - Pose Normalization
  - Partial Shape Descriptors
- **Registration**
  - Closed Form Solutions
  - Branch & Bound
  - Random Sample Consensus (RANSAC)

From Global to Local

To characterize the surface about a point p, take global descriptor and:

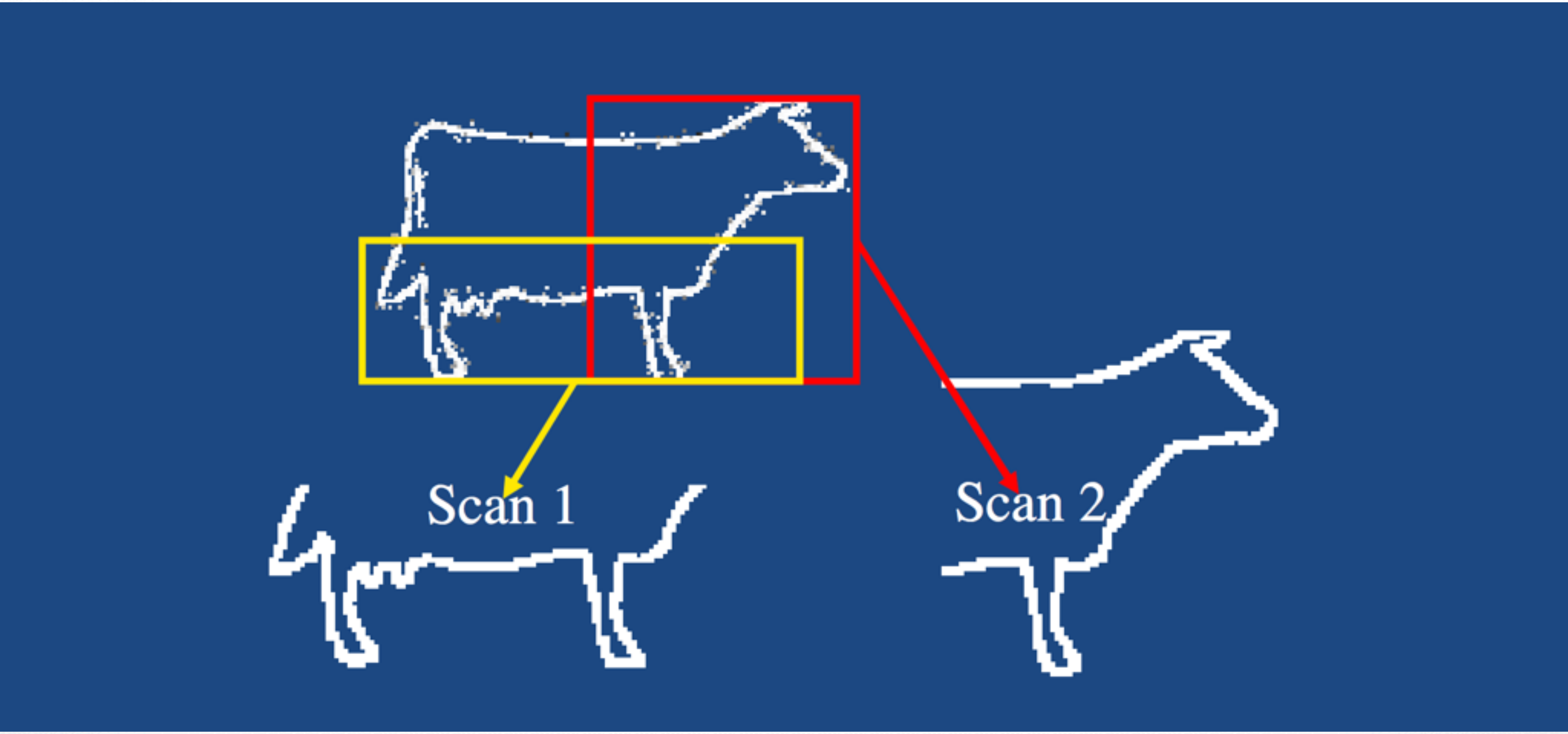
- center it about p (instead of center of mass), and
- restrict the extent to a small region about p



Shape histograms as local shape descriptors

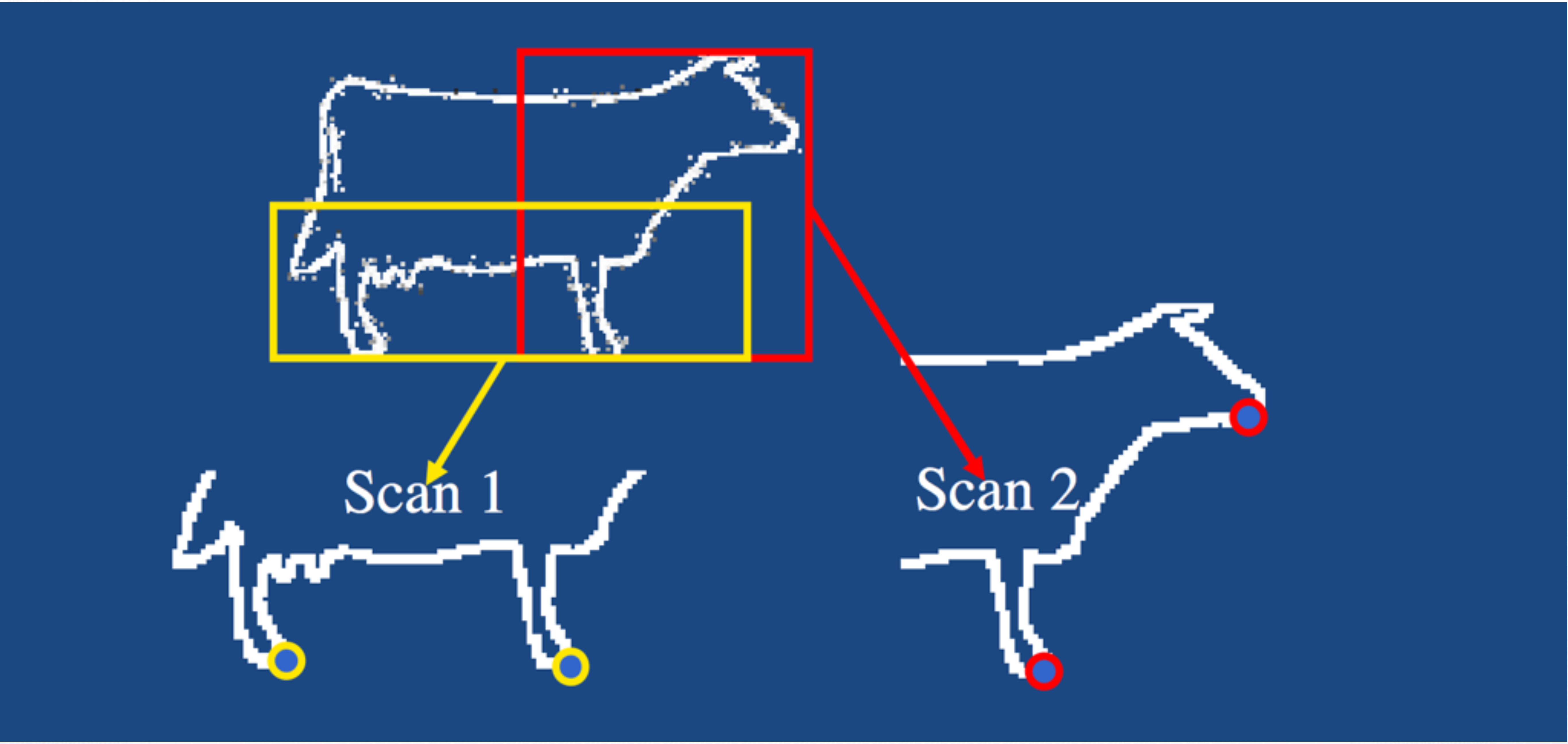
From Global to Local

Given scans of a model:



From Global to Local

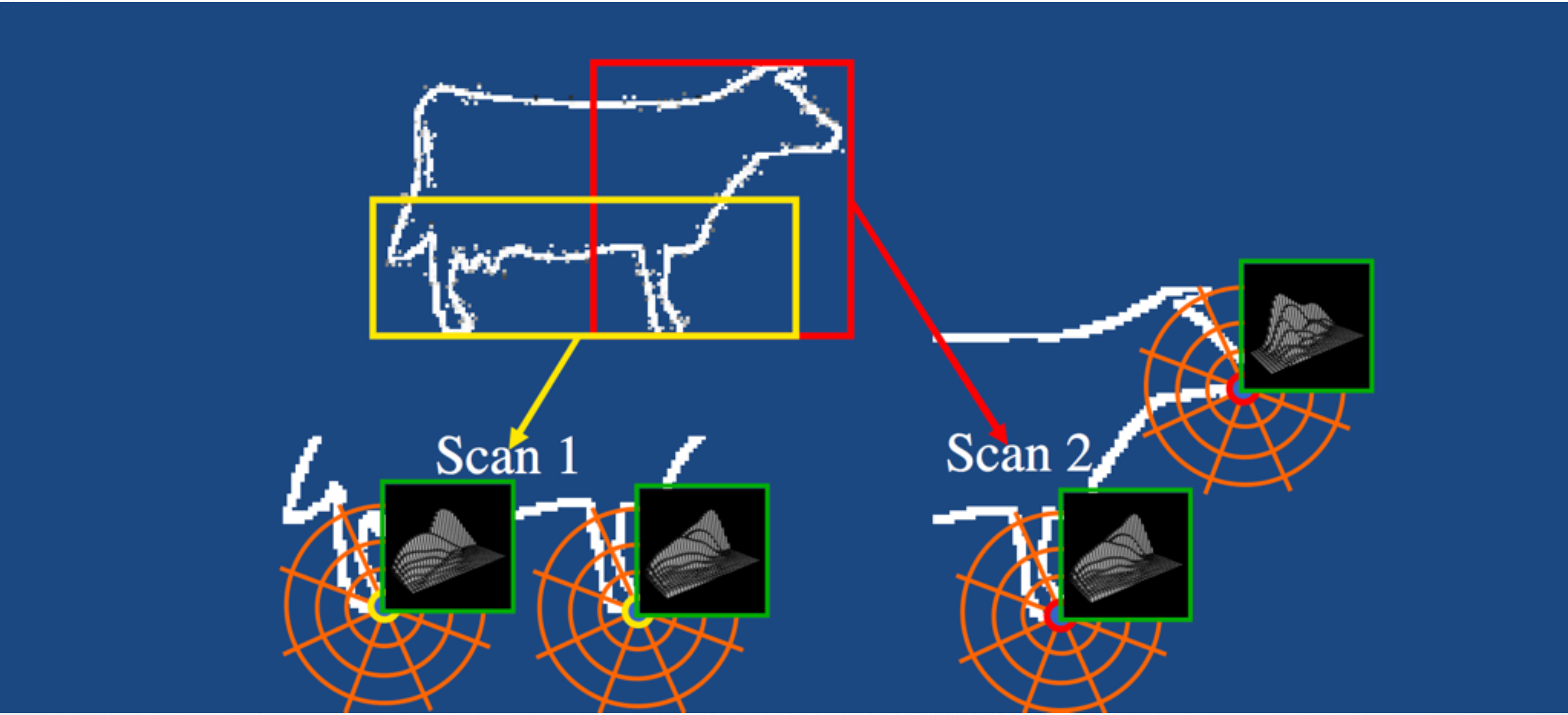
Identify the features



From Global to Local

Identify the features

Computer a local descriptor for each feature

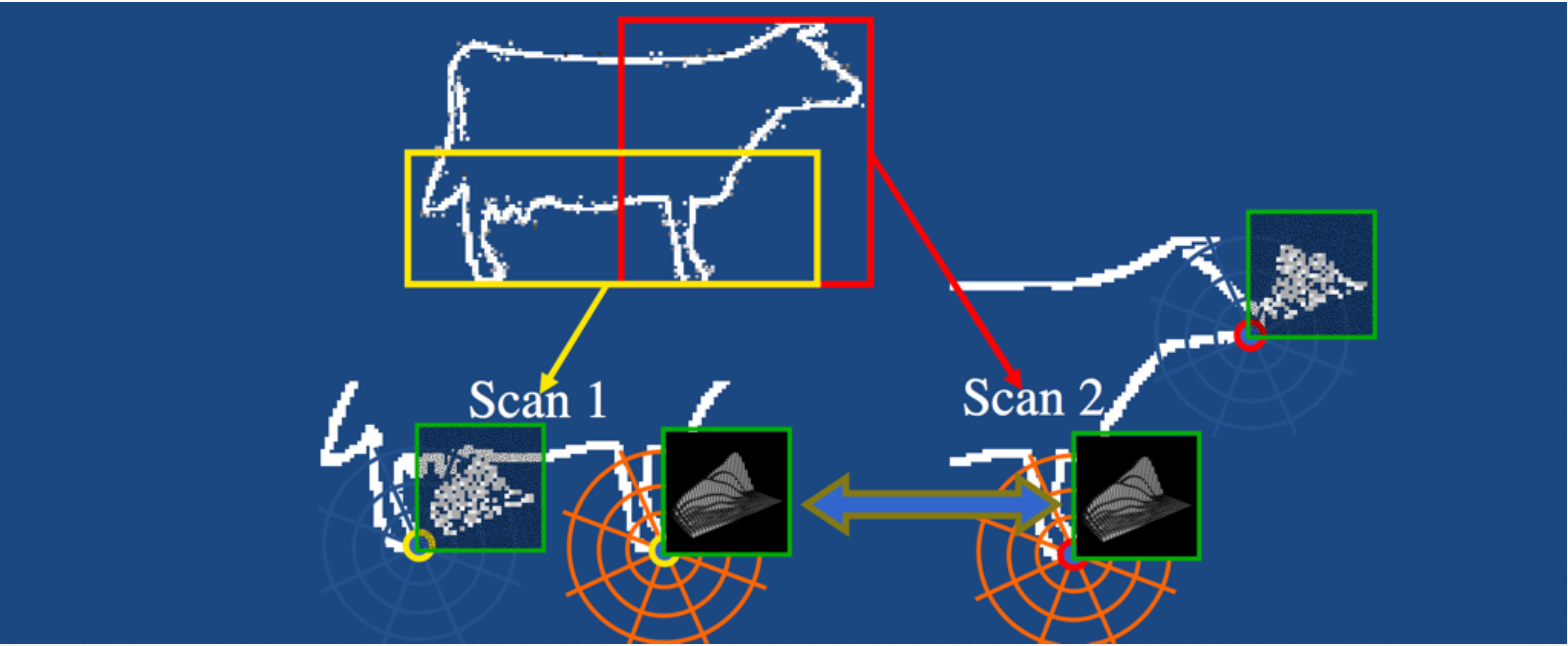


From Global to Local

Identify the features

Computer a local descriptor for each feature

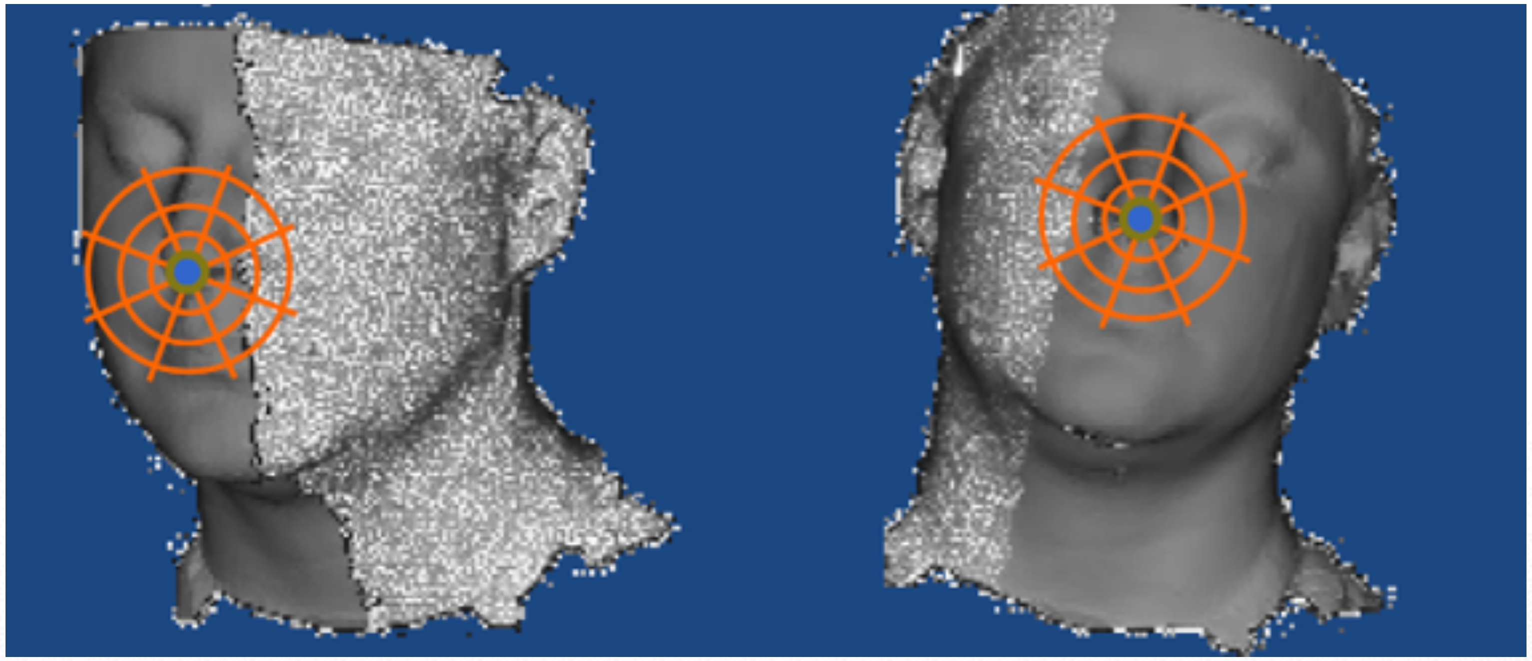
Feature correspond → descriptors are similar



Pose Normalization

From Global to Local

- Translation: Accounted for by centering the descriptor at the point of interest.
- Rotation: We still need to be able to match descriptors across different rotations.



Pose Normalization

Challenge

- Since only parts of the models are given, we cannot use global normalization to align the local descriptors

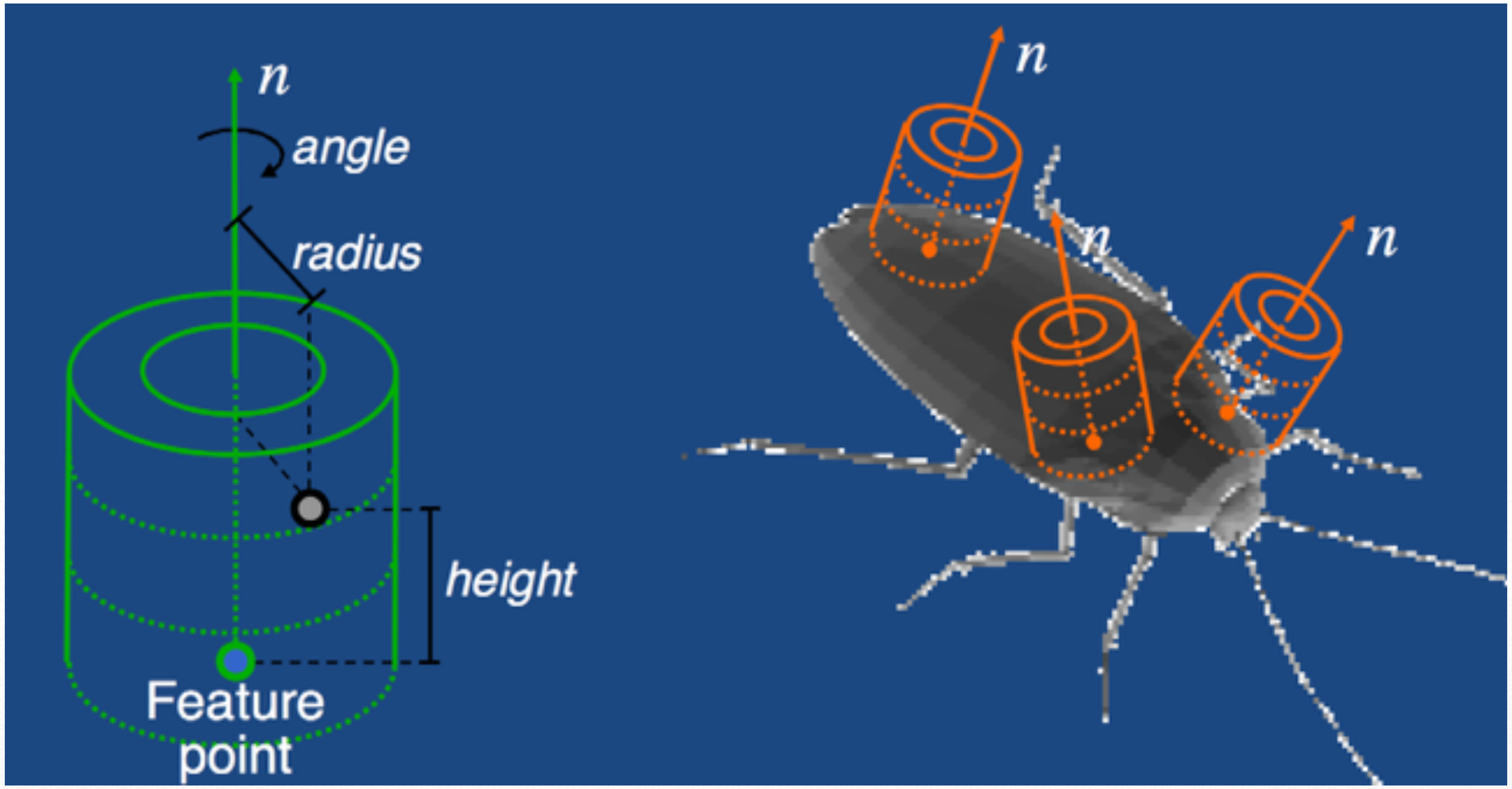
Solutions

- Normalize using **local** information

Local Descriptors: Examples

Variations of Shape Histograms

- For each feature, represent its local geometry in cylindrical coordinates about the normal



Local Descriptors: Examples

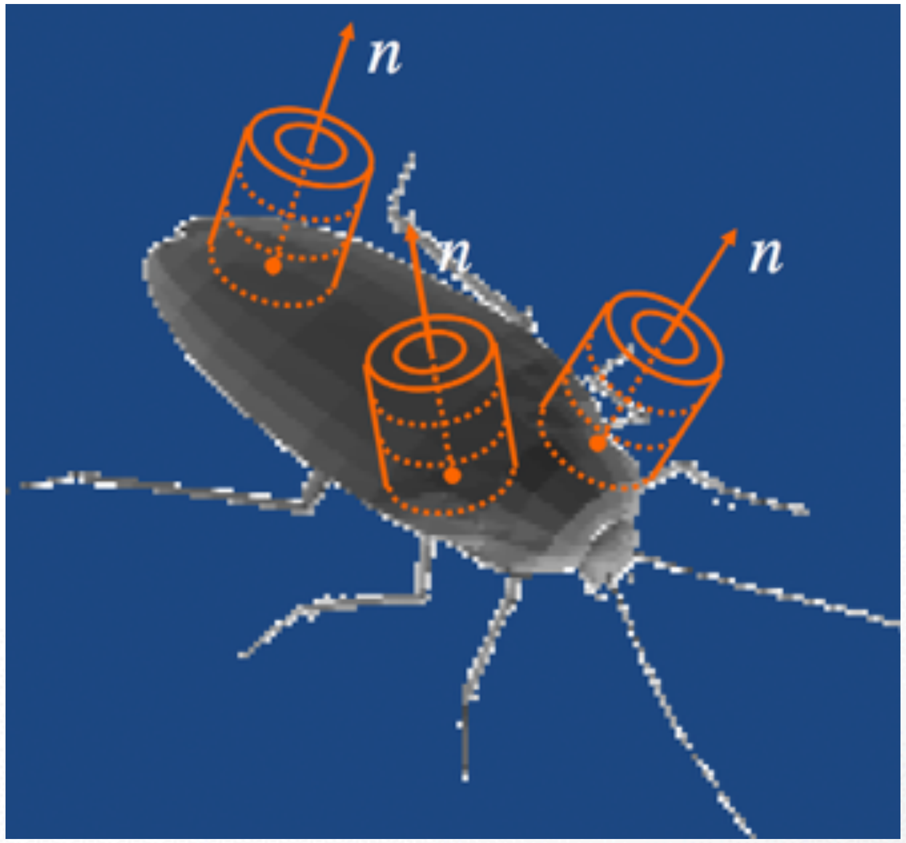
Variations of Shape Histograms

- For each feature, represent its local geometry in cylindrical coordinates about the normal

  - **Spin Images**: Store energy in each normal ring

  - **Harmonic Shape Contexts**: Store power spectrum of each normal ring

  - **3D Shape Contexts**: Search over all rotatinos about the normal for best match



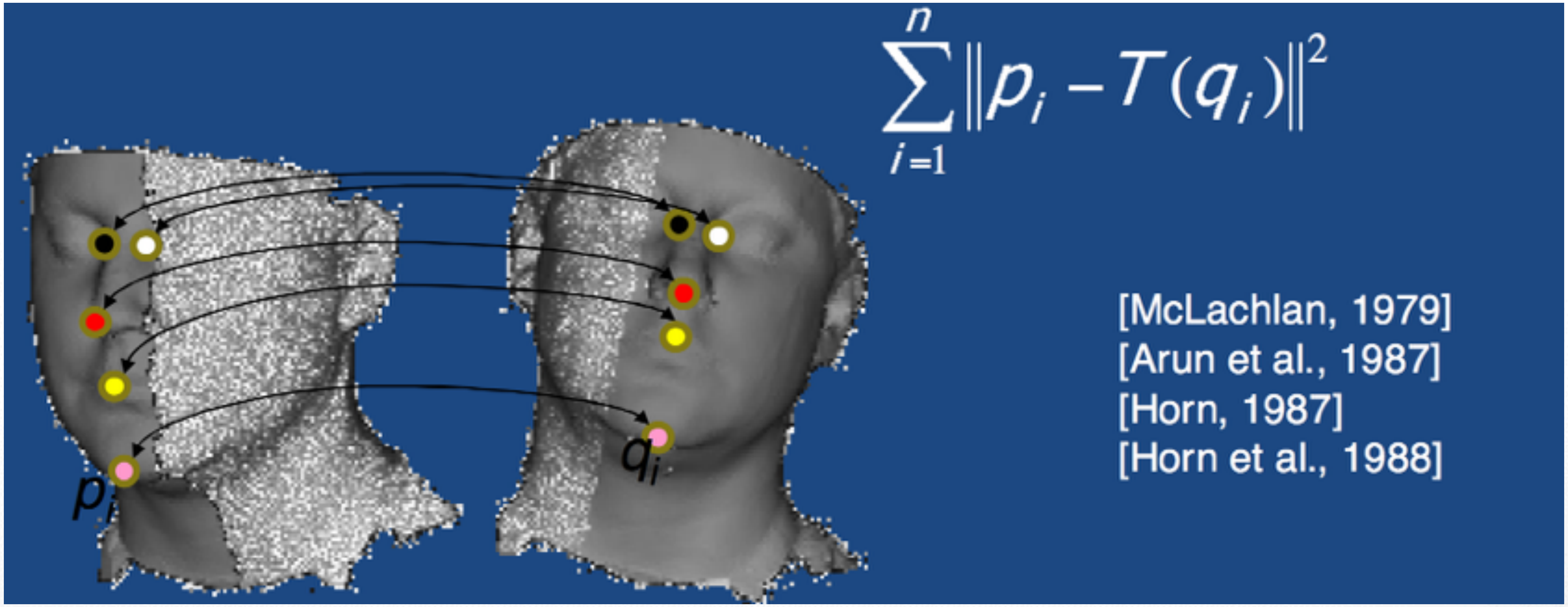
Outline

- **Global Shape Correspondence**
  - Shape Descriptors
  - Alignment
- **Partial Shape Correspondence**
  - From Global to Local
  - Pose Normalization
  - Partial Shape Descriptors
- **Registration**
  - Closed Form Solutions
  - Branch & Bound
  - Random Sample Consensus (RANSAC)

Registration

Ideal Case

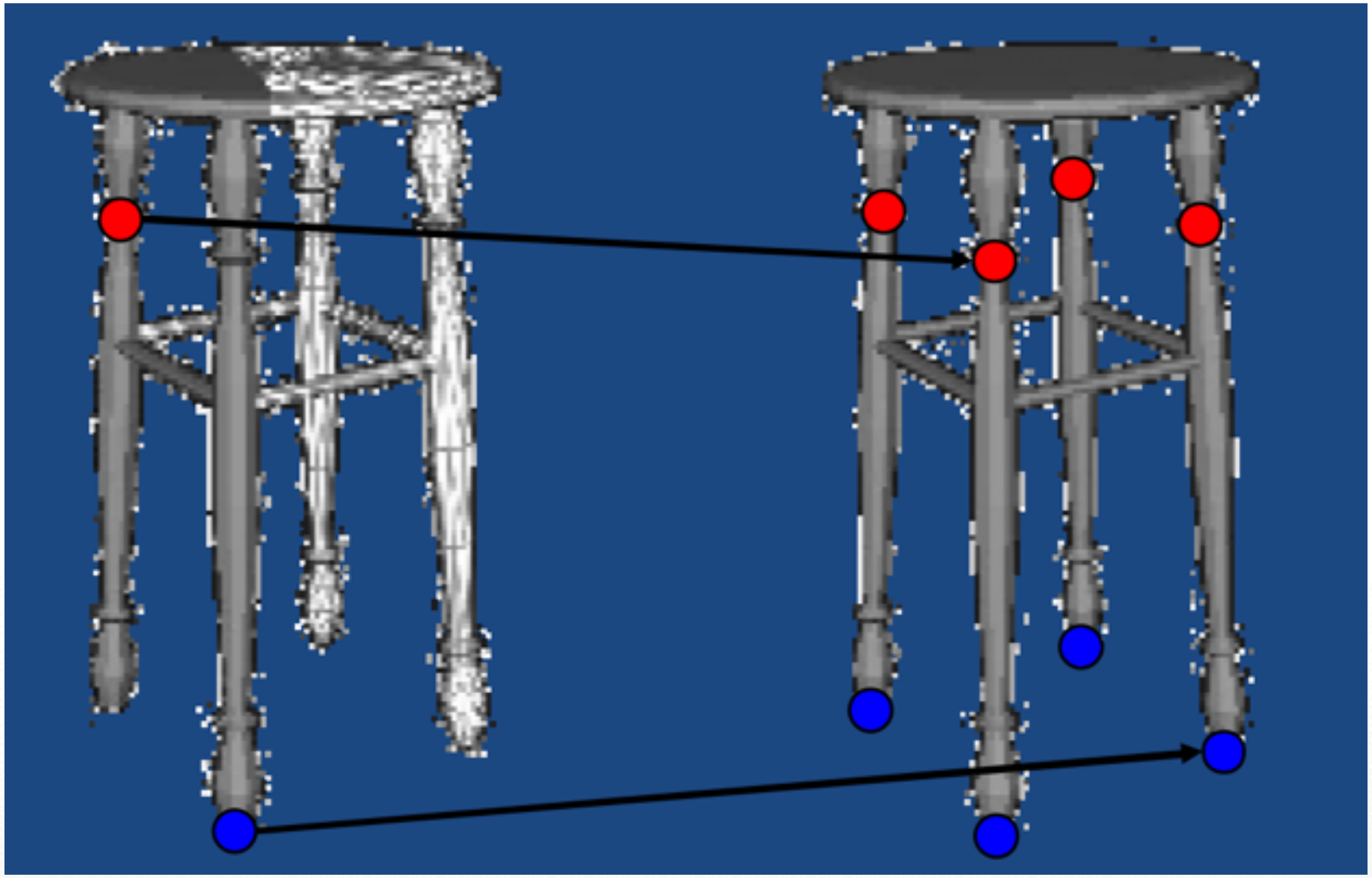
- Every feature point on one scan has a **single** corresponding feature on the other.
- Solve for optimal transformation T



Registration

Challenge:

- Even with good descriptors, symmetries in the model and the locality of descriptors can result in multiple and incorrect correspondences



Registration

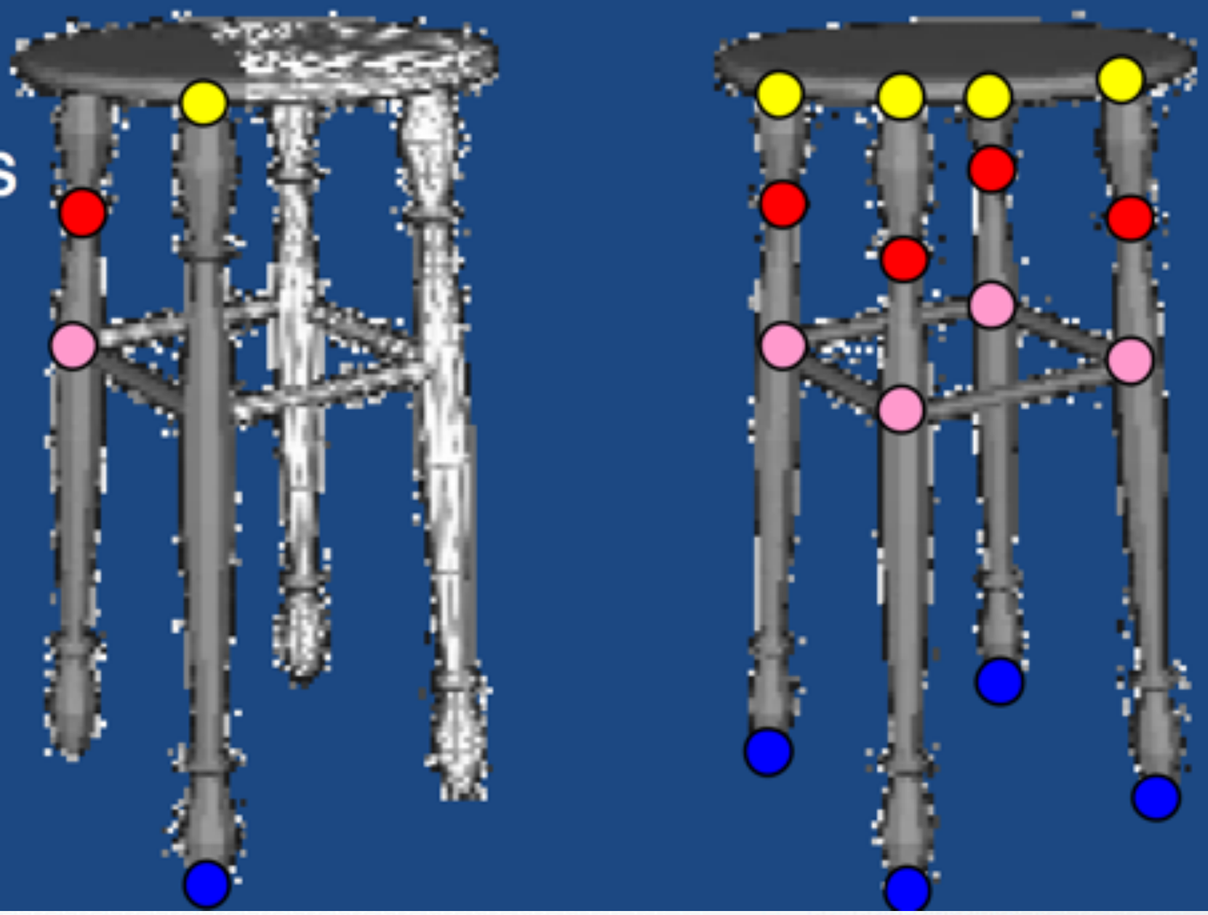
Exhaustive Search

- Compute alignment error at each permutation of correspondences and use the optimal one

$$\text{Error} = \underset{\pi \in \Psi}{\operatorname{argmin}} \left(\underset{T \in E^3}{\operatorname{argmin}} \sum_{i=1}^{n} \left\| p_i - T(\pi(p_i)) \right\|^2 \right)$$

Ψ = Set of possible correspondence

E^3 = Group of rigid body transformations



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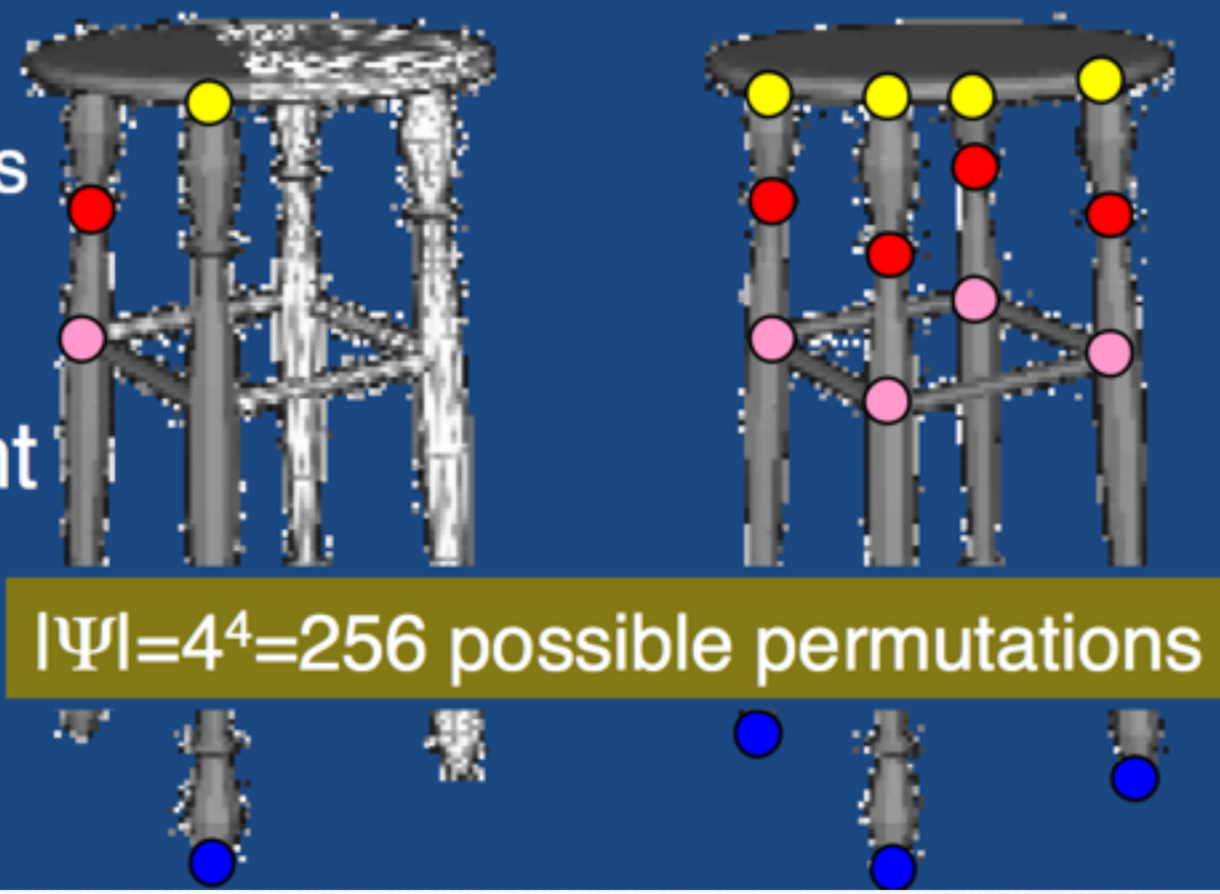
Ψ = Set of possible correspondence

E^3 = Group of rigid body transformations

Given points $\{p_1,\ldots,p_n\}$ on the query, if p_i matches m_i different target points:

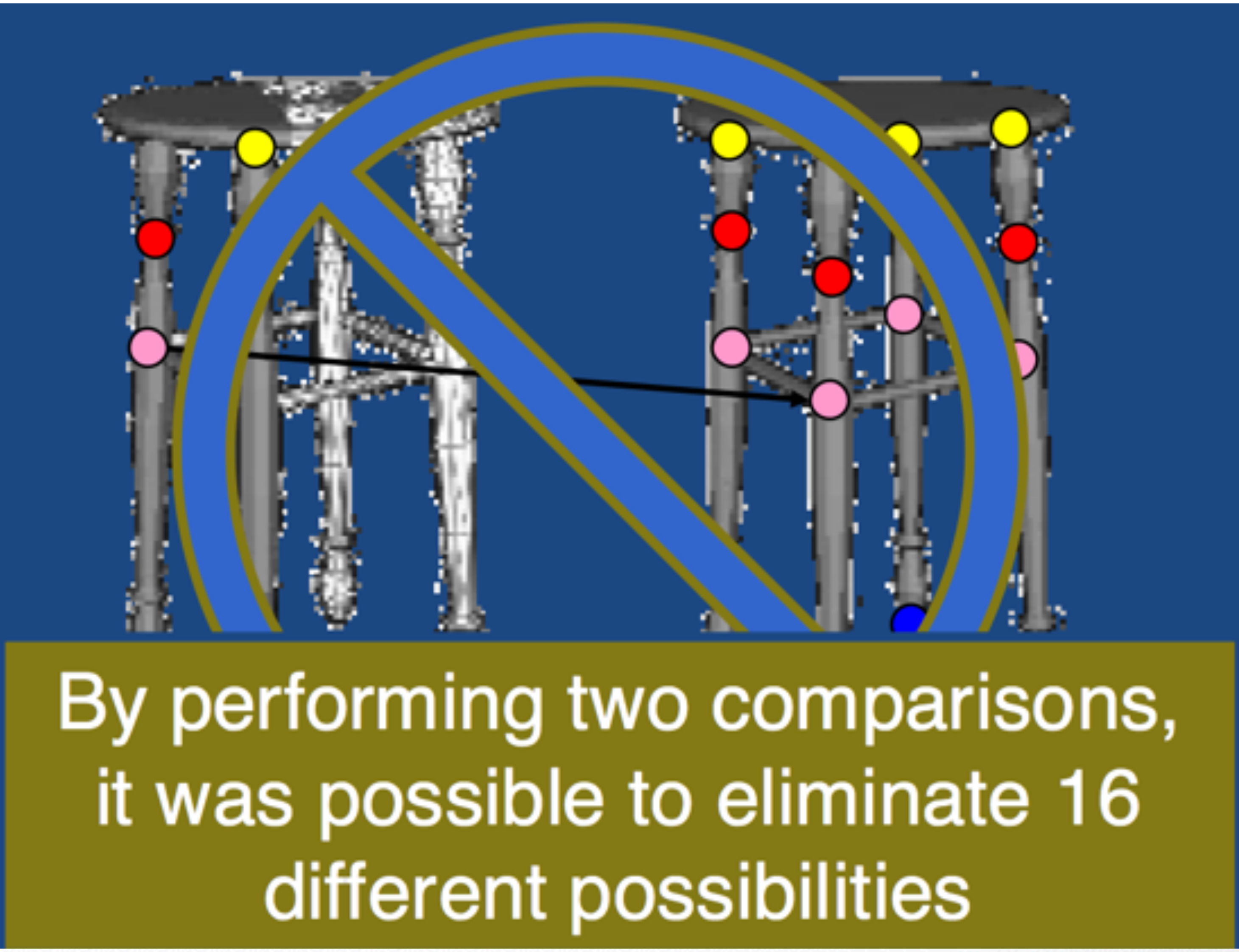
$$|\Psi| = \prod_{i=1}^{n} m_i$$



Registration

Branch & Bound (Decision tree)

- Try all permuations but terminate early if the alignment can be predicted to be bad



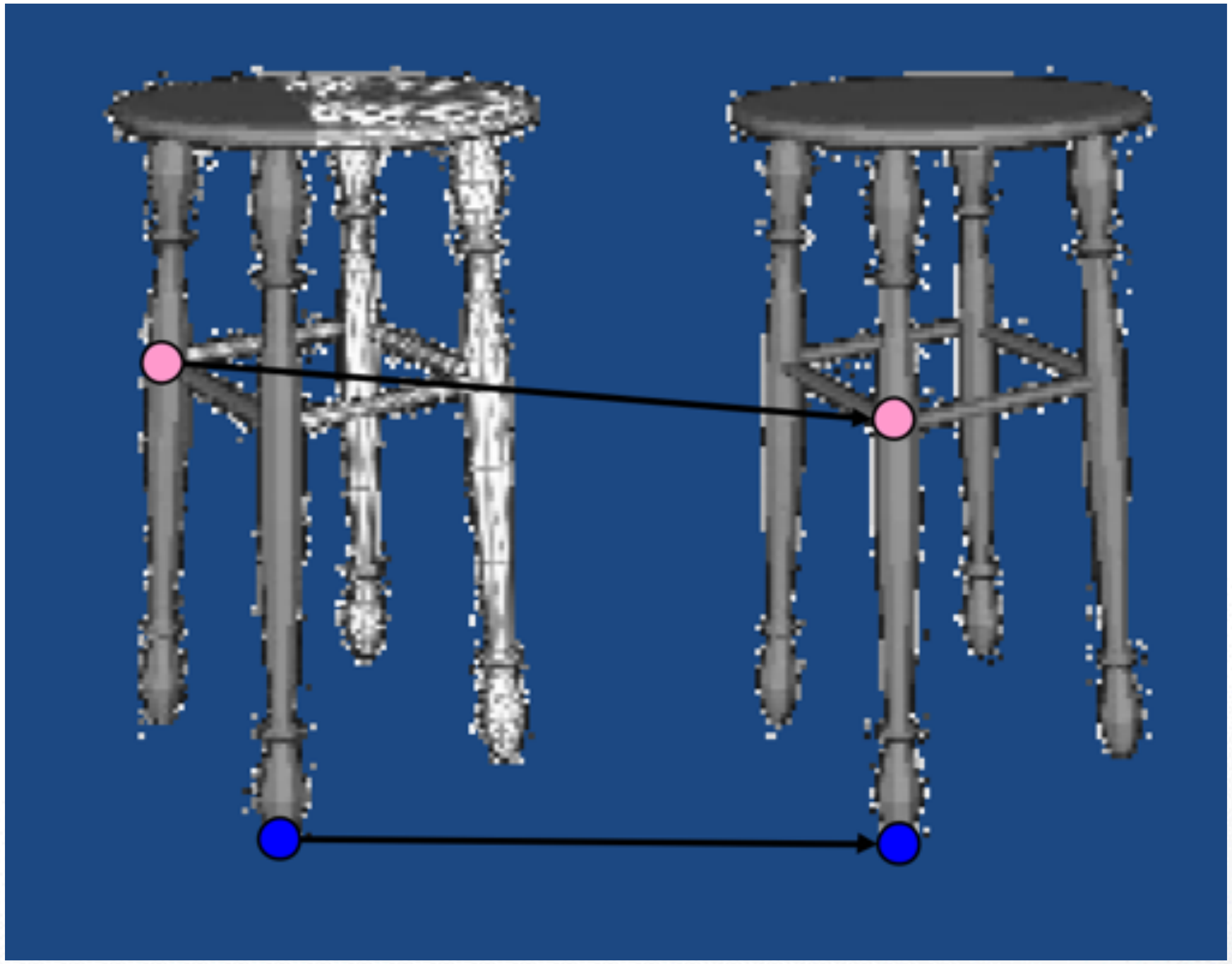
Registration

Goal

- Need to be able to determine if the alignment will be good without knowing all of the correspondences

Observation

- Alignment needs to preserve the lengths between points in a single scan



Registration

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Observation

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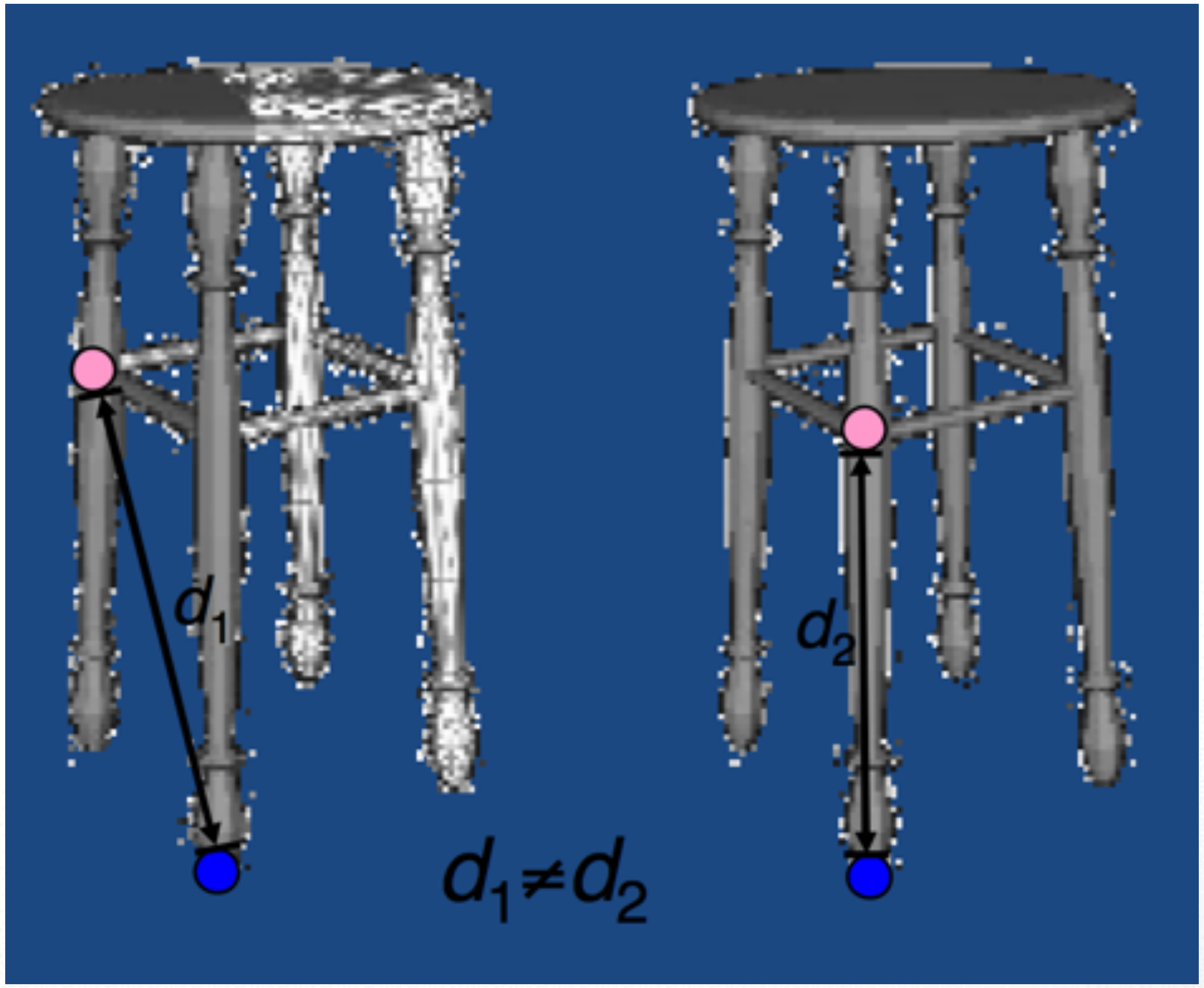
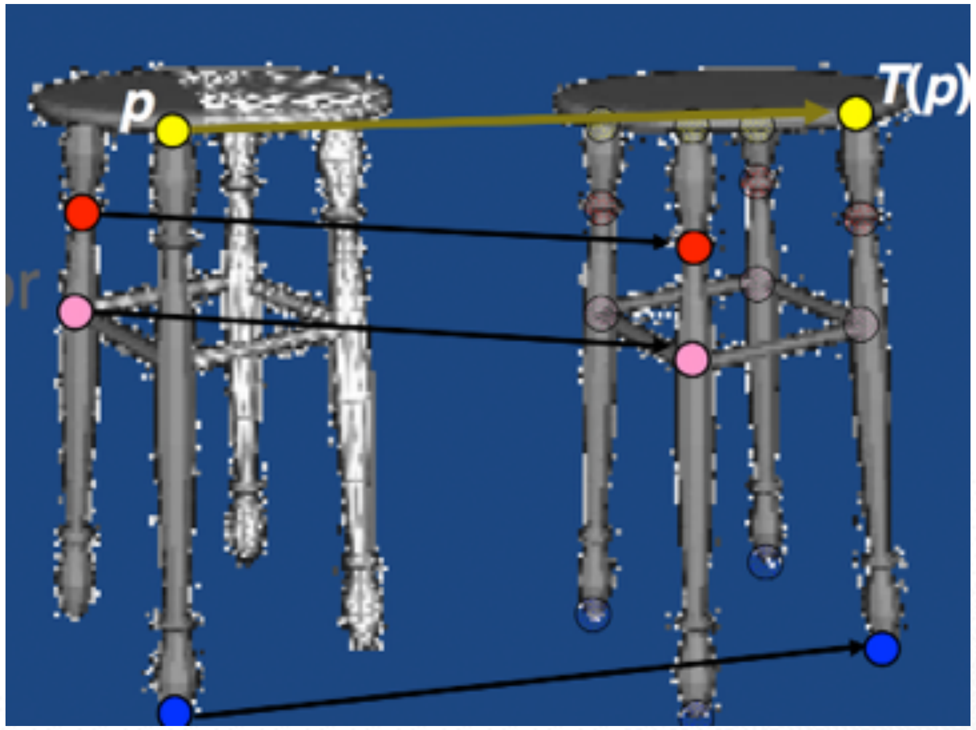


RANdom SAmple Consensus

Algorithm (iterate 100 times)

- Randomly choose 3 points on source
- For all possible correspondences on target:
  - Compute T
  - For every other source p:
    - find closest correspondence T(p)
  - Compute alignment error



Summary

Global Shape Correspondences

- Shape Descriptors
  - Shells (1D)
  - Sectors (2D)
  - Sectors & Shells (3D)
- Alignment
  - Exhaustive Search
  - Normalization
  - Invariance

Summary

Partial-Shape/Point Correspondences

- From Global to Local
  - Center at feature
  - Restrict extent
- Pose Normalization
  - Normal-based alignment
- Partial Shape Descriptors
  - Normalization/invariance
  - Normalization/exhaustive-search

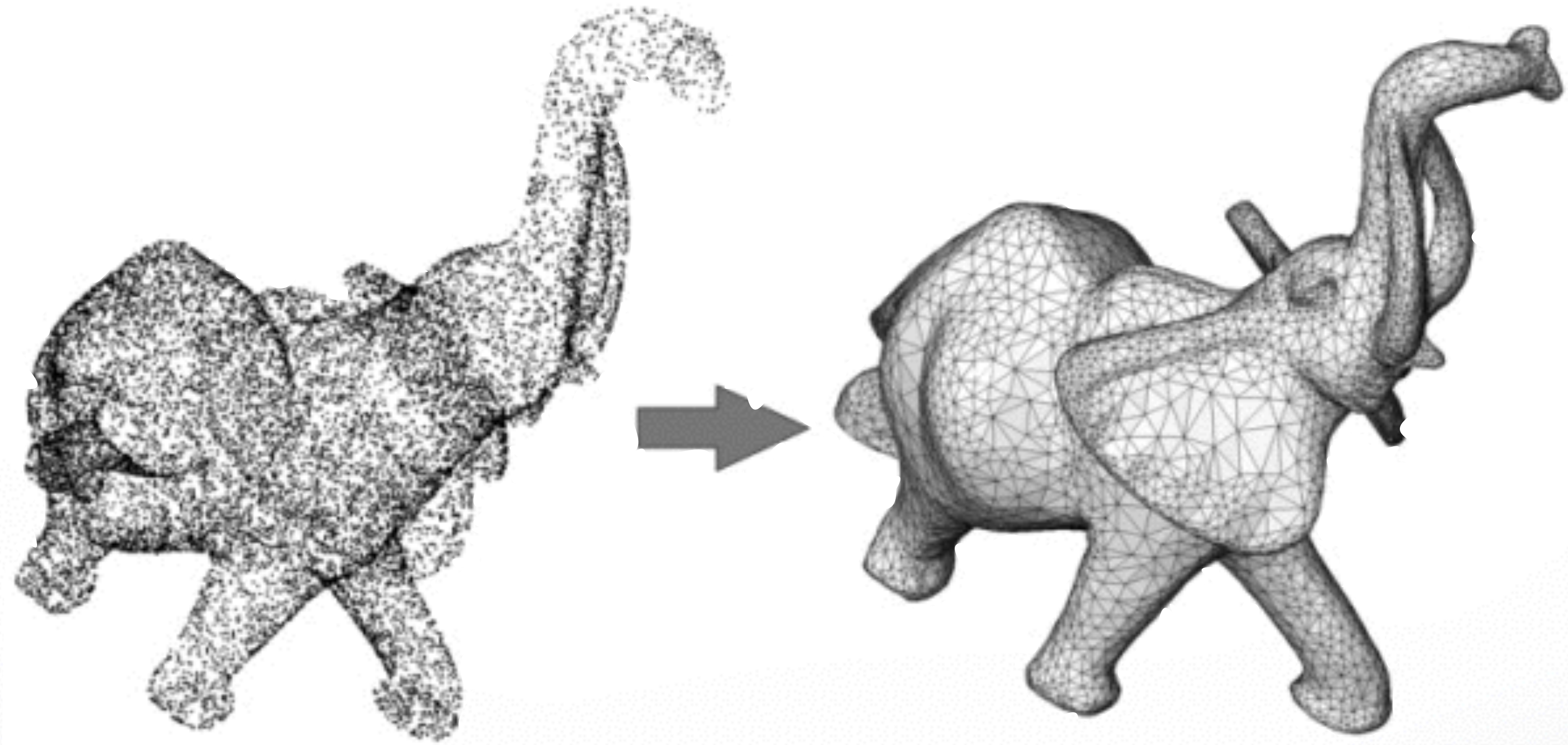
Summary

Registration

- Closed Form Solutions
  - Global symmetry
  - Local self similarity
- Branch & Bound
  - Inter-feature distances for early termination
- RANdom SAmple Consensus
  - Efficient transformation computation

Next Time

Surface Reconstruction



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Thanks!