5.1 **Surface Registration**

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Administrative

- Next Thursday, let's capture some stuff, bring yourself or an object of your choice.

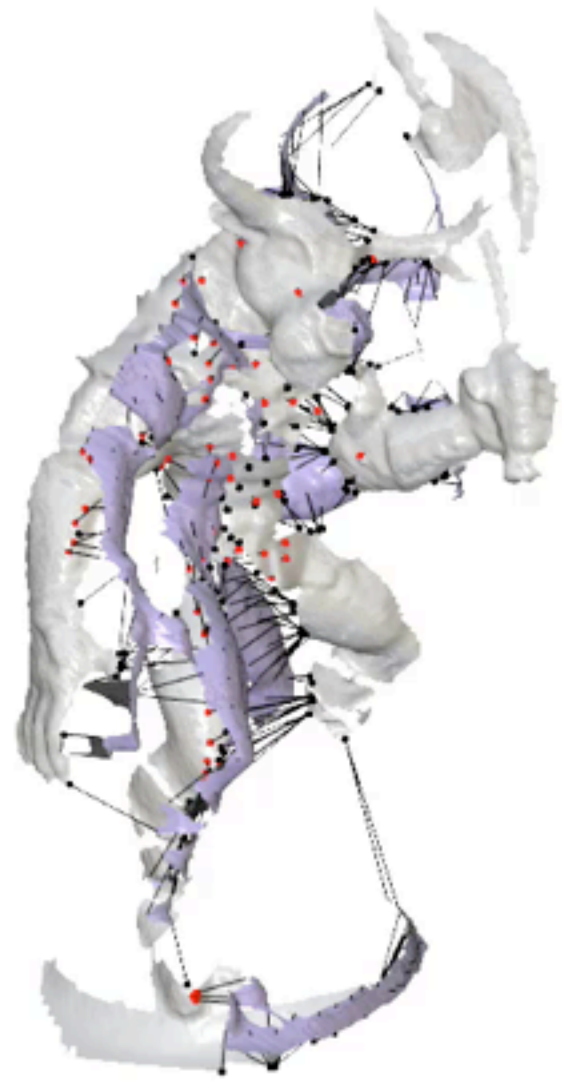
Acknowledgement

Images and Slides are courtesy of

- Prof. Szymon Rusinkiewicz, Princeton University
- ICCV Course 2005: http://www.cis.upenn.edu/~bjbrown/iccv05_course/

Surface Registration

Align two partially-overlapping meshes given initial guess for relative transform

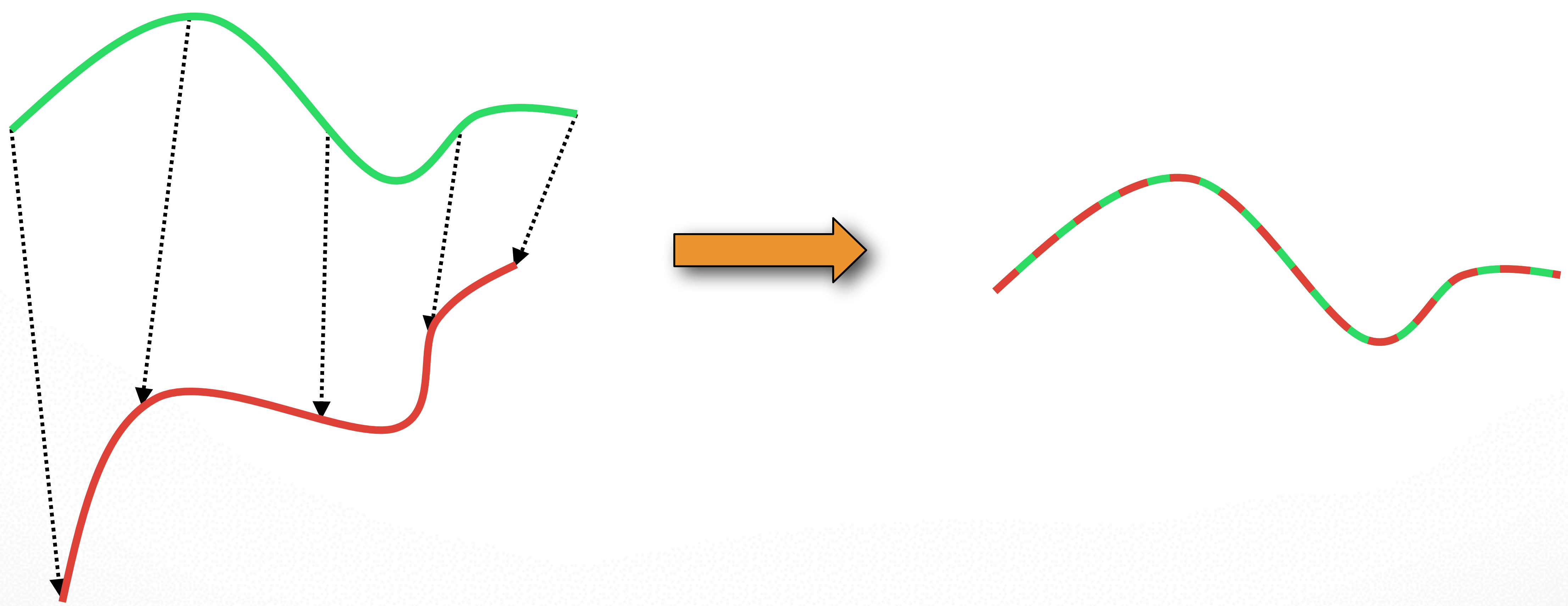


Outline

- ICP: Iterative Closest Points
- Classification of ICP variants
  - Faster alignment
  - Better robustness
- ICP as function minimization

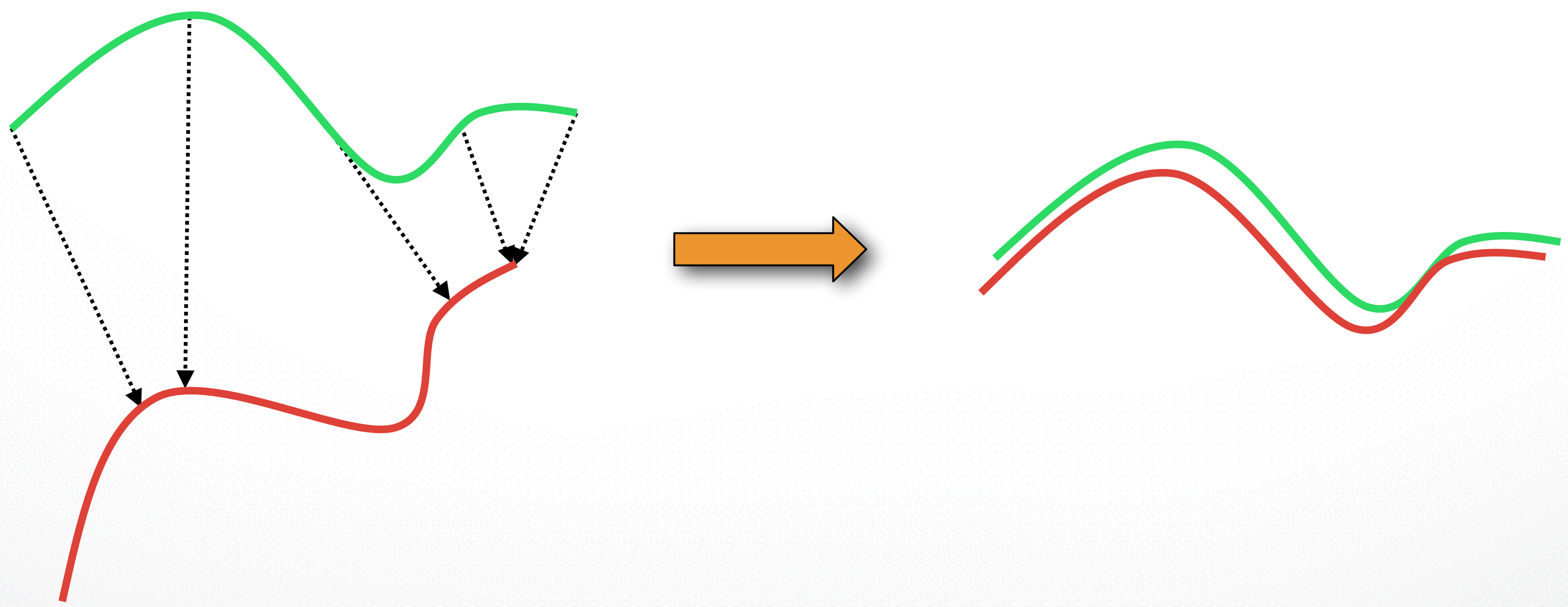
Aligning 3D Data

If correct correpondences are known, can find correct relative rotation/translation



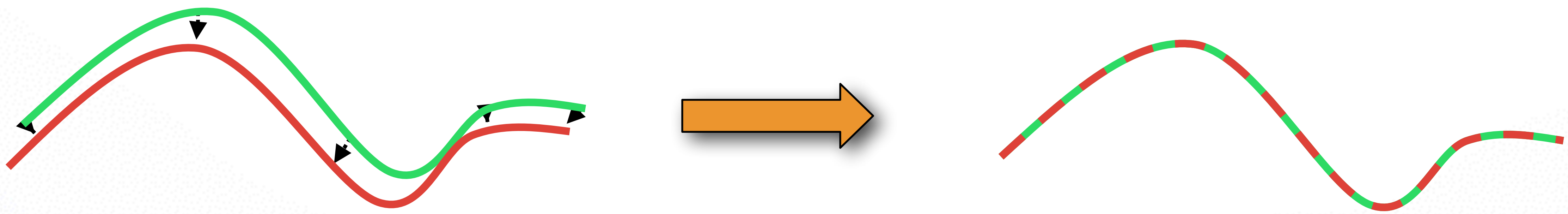
Aligning 3D Data

- How to find correspondences: User input? Feature detection? Signatures?
- Alternatives: assume **closest** points correspond



Aligning 3D Data

- … and iterate to find alignment
  - Iterative Closest Points (ICP) [Besl & Mckay]
- Converges if starting position “close enough”



Basic ICP

- **Select** e.g., 1000 random points
- **Match** each to closest point on other scan, using data structure such as *k*-d tree
- **Reject** pairs with distance > *k* times median
- Construct **error function**:

$$E = \sum \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i\|^2$$

- **Minimize** (closed form solution in [Horn 87])

Shape Matching: Translation

- Define bary-centered point sets

$$
\begin{aligned}
\bar{\mathbf{p}} &:= \frac{1}{m}\sum_{i=1}^{m}\mathbf{p}_i & \qquad \bar{\mathbf{q}} &:= \frac{1}{m}\sum_{i=1}^{m}\mathbf{q}_i \\
\hat{\mathbf{p}}_i &:= \mathbf{p}_i - \bar{\mathbf{p}} & \qquad \hat{\mathbf{q}}_i &:= \mathbf{q}_i - \bar{\mathbf{q}}
\end{aligned}
$$

- Optimal translation vector $\mathbf{t}$ maps barycenters onto each other

$$\mathbf{t} = \bar{\mathbf{p}} - \mathbf{R}\bar{\mathbf{q}}$$

Shape Matching: Rotation

- Approximate nonlinear rotation by general matrix

$$\min_{\mathbf{R}} \sum_i \|\hat{\mathbf{p}}_i - \mathbf{R}\hat{\mathbf{q}}_i\|^2 \quad \rightarrow \quad \min_{\mathbf{A}} \sum_i \|\hat{\mathbf{p}}_i - \mathbf{A}\hat{\mathbf{q}}_i\|^2$$

- The least squares linear transformation is

$$\mathbf{A} = \left(\sum_{i=1}^{m} \hat{\mathbf{p}}_i \hat{\mathbf{q}}_i^T\right) \cdot \left(\sum_{i=1}^{m} \hat{\mathbf{q}}_i \hat{\mathbf{q}}_i^T\right)^{-1} \in \mathbb{R}^{3\times 3}$$

- SVD & Polar decomposition extracts rotation from $\mathbf{A}$

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad \rightarrow \quad \mathbf{R} = \mathbf{U}\mathbf{V}^T$$

ICP Variants

Variants on the following stages of ICP have been proposed

1. **Selecting** source points (from one or both meshes)
2. **Matching** to points in the other mesh
3. **Weighting** the correspondences
4. **Rejecting** certain (outliers) point pairs
5. Assigning an **error metric** to the current transform
6. **Minimizing** the error metric w.r.t. transformation

ICP Variants

Can analyze various aspects of performance:

- Speed
- Stability
- Tolerance of noise and/or outliers
- Maximum initial misalignment

Comparisons of many variants in

- [Rusinkiewicz & Levoy, 3DIM 2001]

ICP Variants

1. Selecting source points (from one or both meshes)
2. Matching to points in the other mesh
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5. **Assigning an error metric to the current transform**
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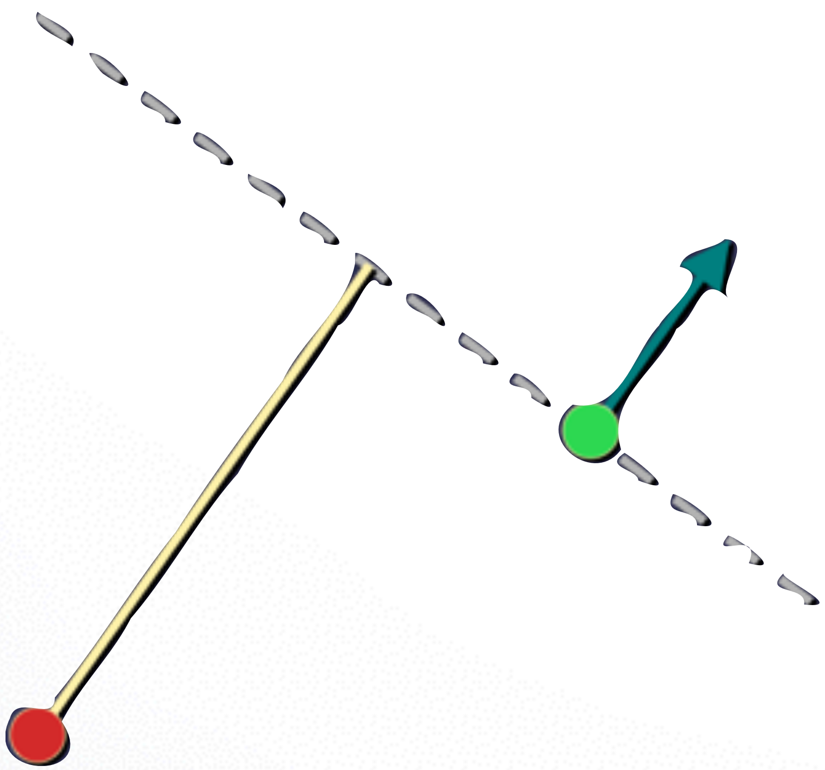
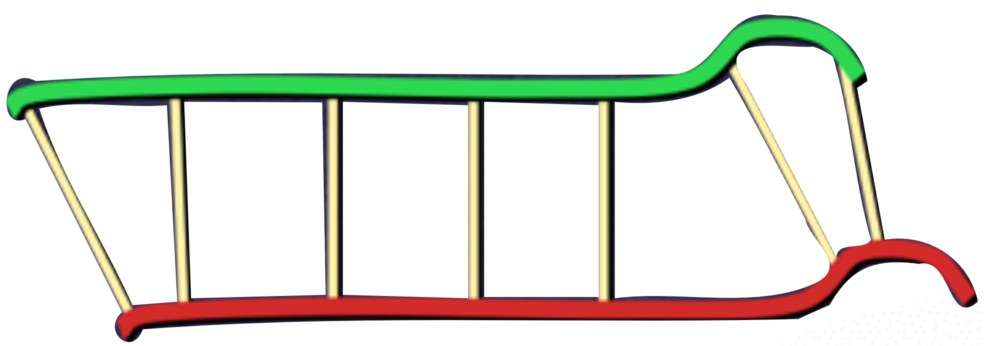
Point-to-Plane Error Metric

Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]





Point-to-Plane Error Metric

- Error function:

$$E = \sum \left((\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i)^\top \mathbf{n_i} \right)^2$$

where $\mathbf{R}$ is a rotation matrix, $\mathbf{t}$ is a translation vector

- Linearize (i.e. assume that $\sin\theta \approx \theta$, $\cos\theta \approx 1$):

$$E \approx \sum \left((\mathbf{p}_i - \mathbf{q}_i)^\top \mathbf{n}_i \right) + \mathbf{r}^\top (\mathbf{p}_i \times \mathbf{n}_i) + \mathbf{t}^\top \mathbf{n}_i)^2 \qquad \mathbf{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

- Result: overconstrained linear system

Point-to-Plane Error Metric

- Overconstrained linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} \leftarrow & \mathbf{p}_1 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_1 & \rightarrow \\ \leftarrow & \mathbf{p}_2 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_2 & \rightarrow \\ & \vdots & & & \vdots & \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} r_x \\ r_y \\ r_z \\ t_x \\ t_y \\ t_z \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -(\mathbf{p}_1 - \mathbf{q}_1)^\top \mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2)^\top \mathbf{n}_2 \\ \vdots \end{bmatrix}$$

- Solve using least squares

$$\mathbf{A}^\top \mathbf{A}\mathbf{x} = \mathbf{A}^\top \mathbf{b}$$

$$\mathbf{x} = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \mathbf{b}$$

Improving ICP Stabilitiy

- Closest **compatible** point
- Stable sampling

ICP Variants

1. Selecting source points (from one or both meshes)
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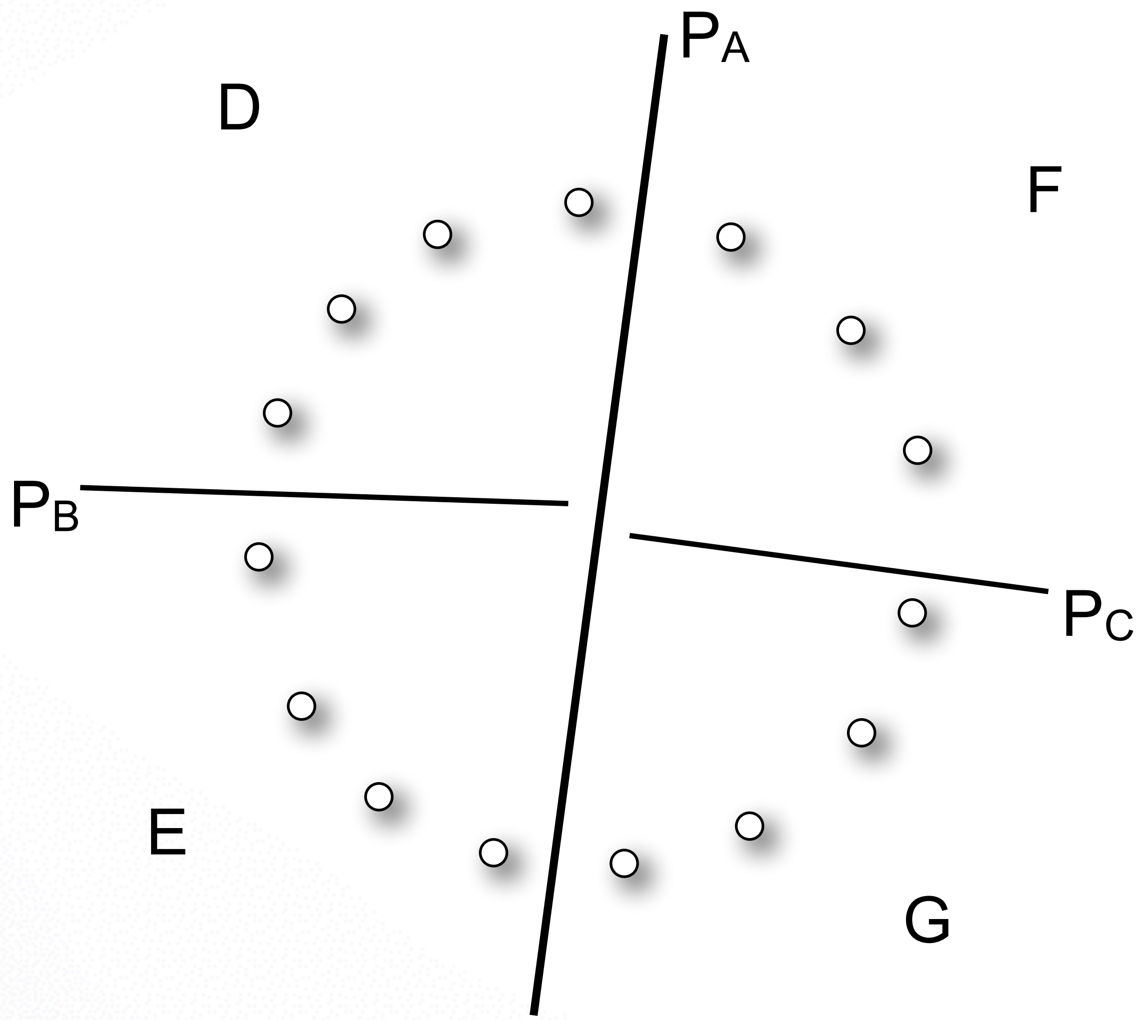
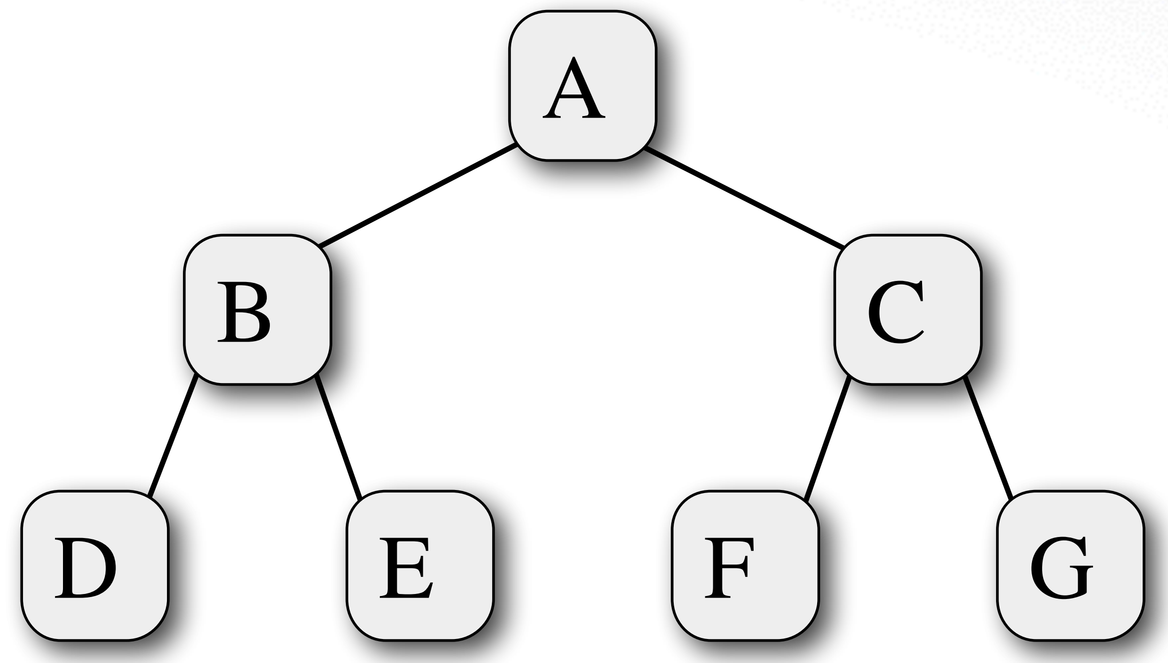
Closest Point Search

- Find closest point of a query point
  - Brute force: $O(n)$ complexity

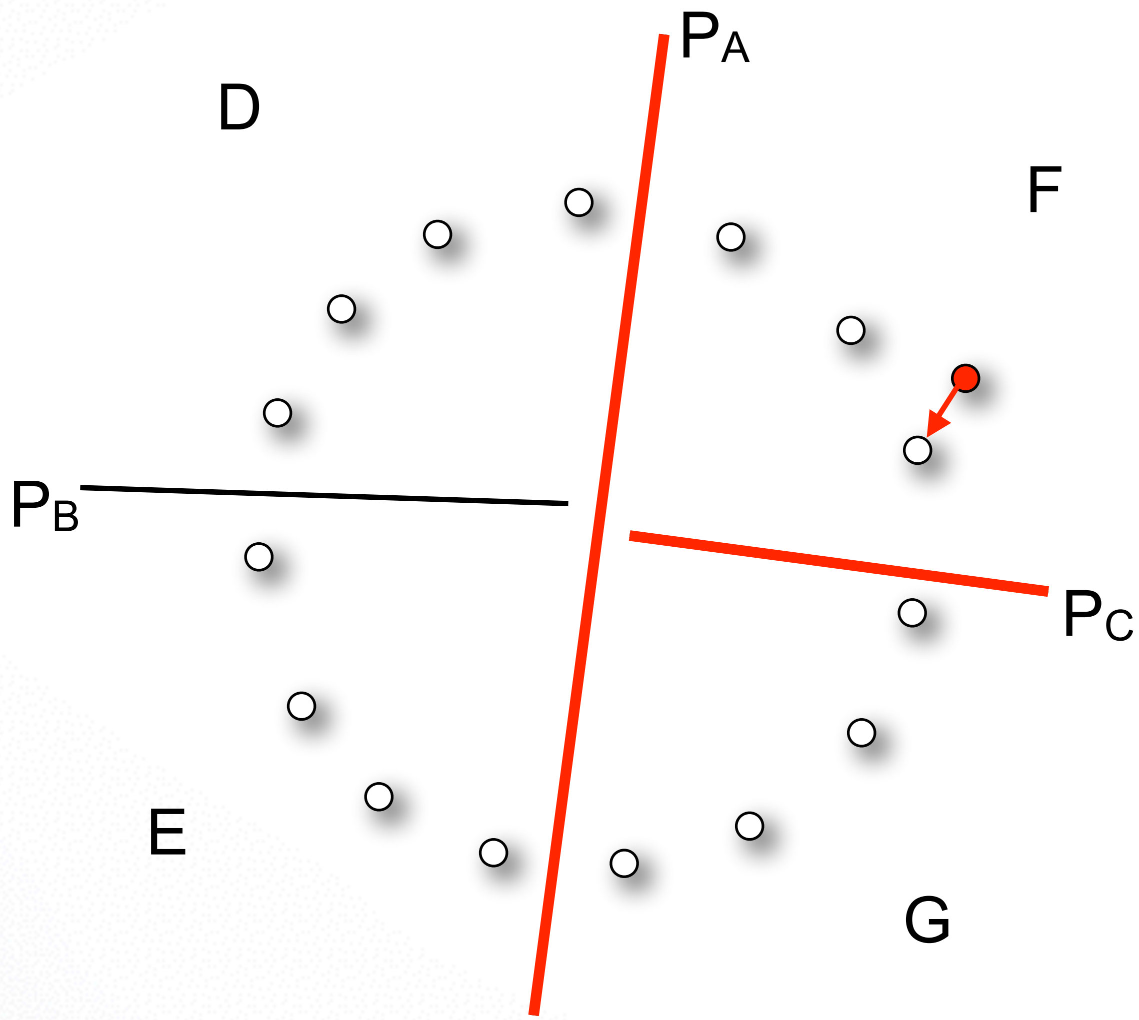
- Use Hierarchical BSP tree
  - Binary space partitioning tree (general kD-tree)
  - Recursively partition 3D space by planes
  - Tree should be balanced, put plane at median
  - $\log(n)$ tree levels, complexity $O(n \log n)$

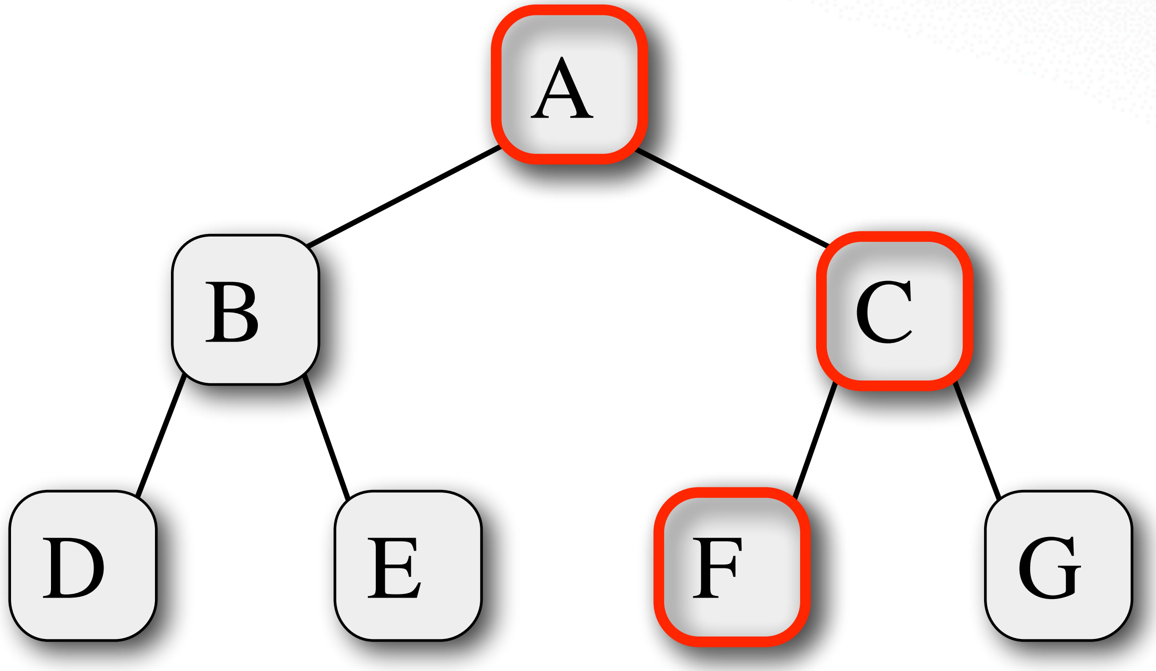
BSP Closest Point Search



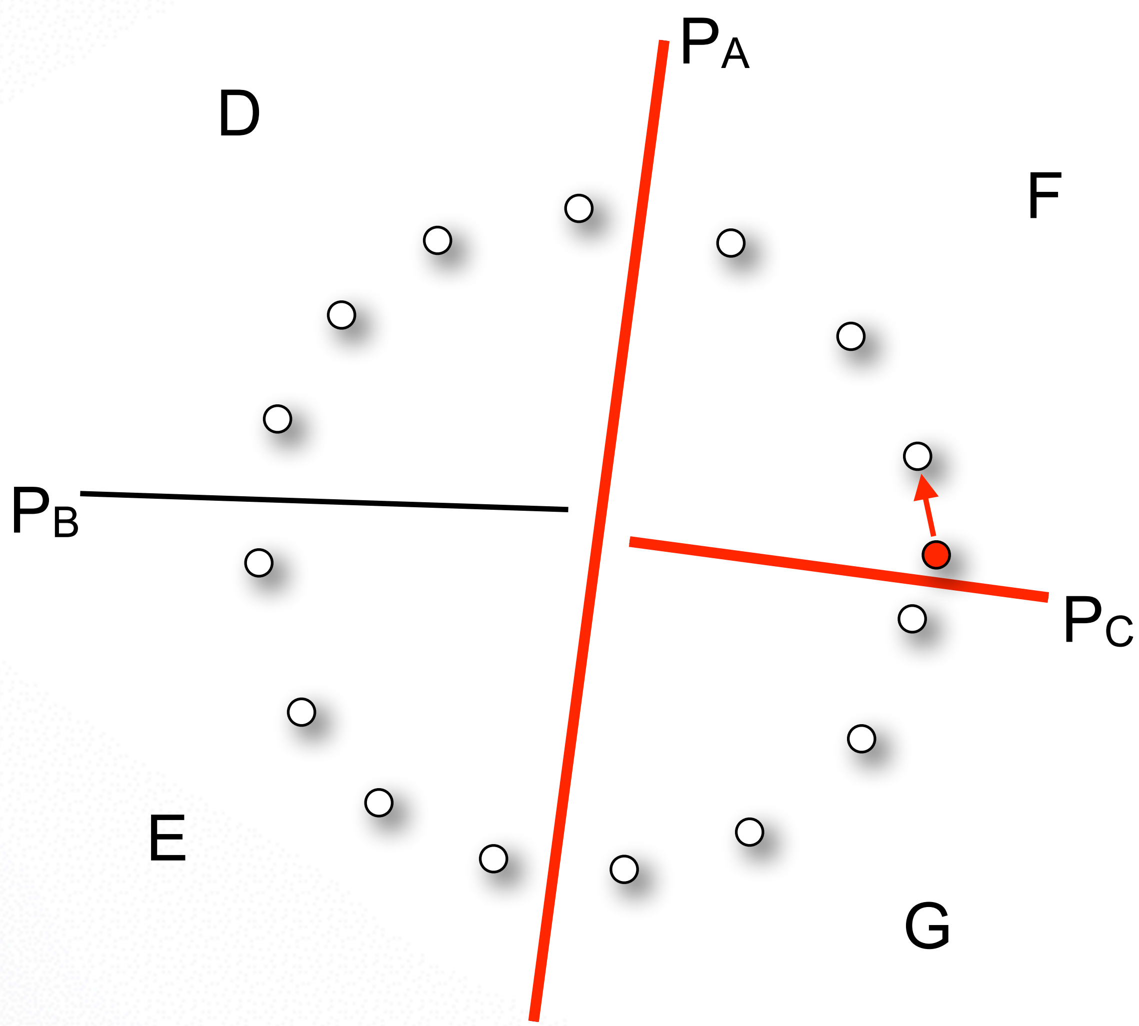


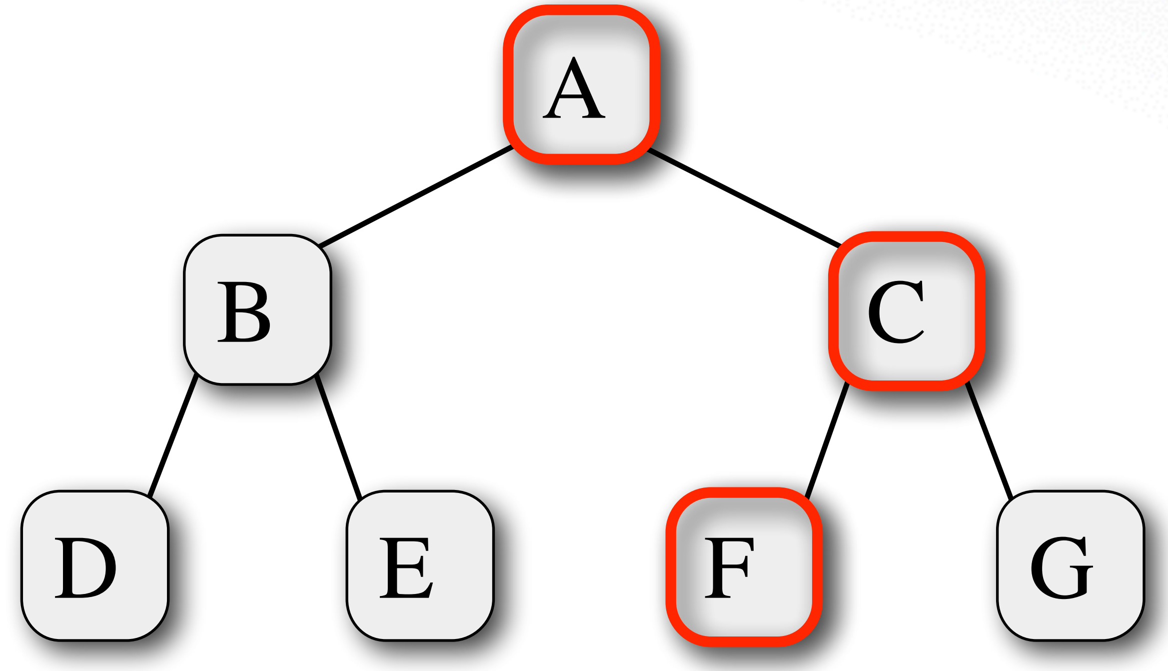
BSP Closest Point Search



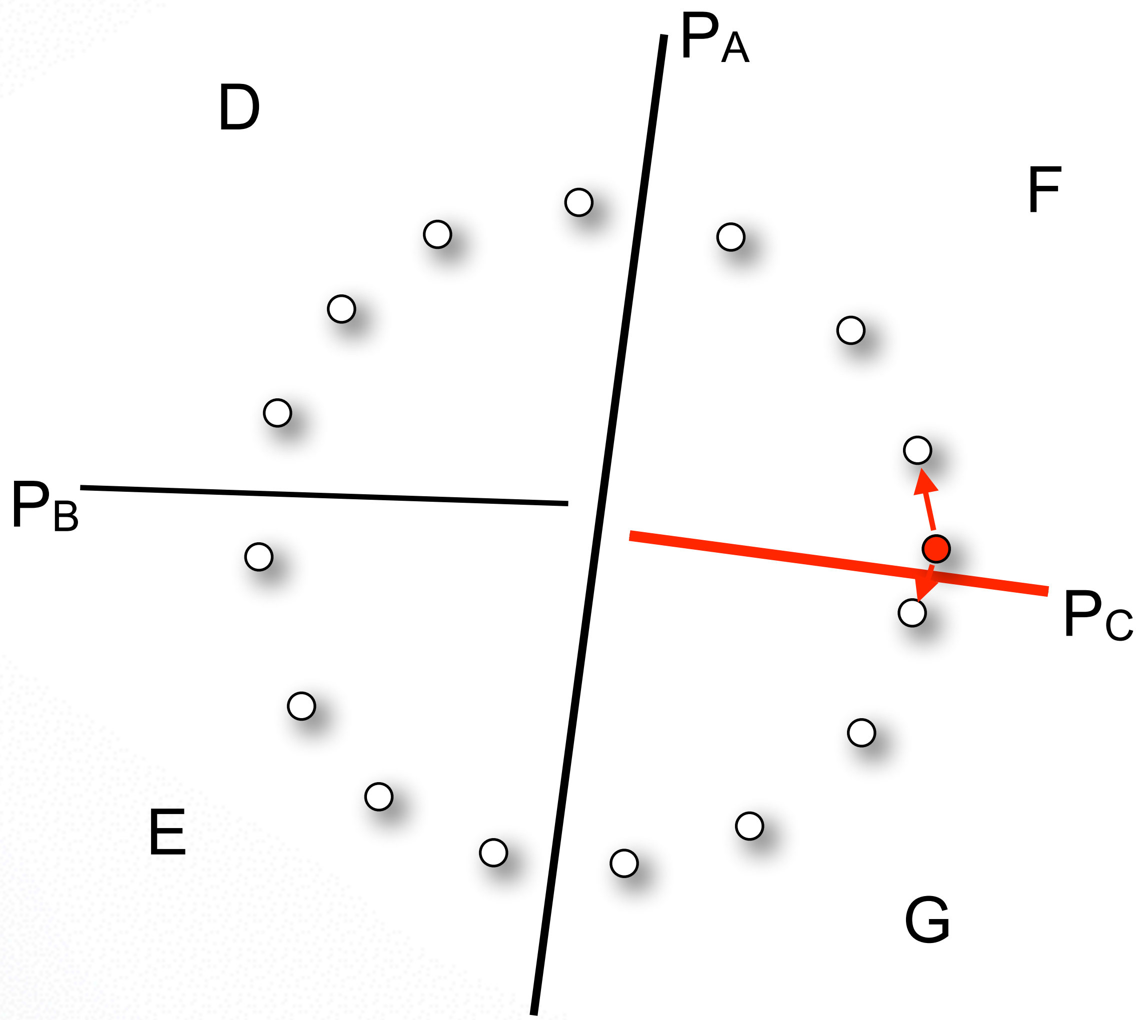


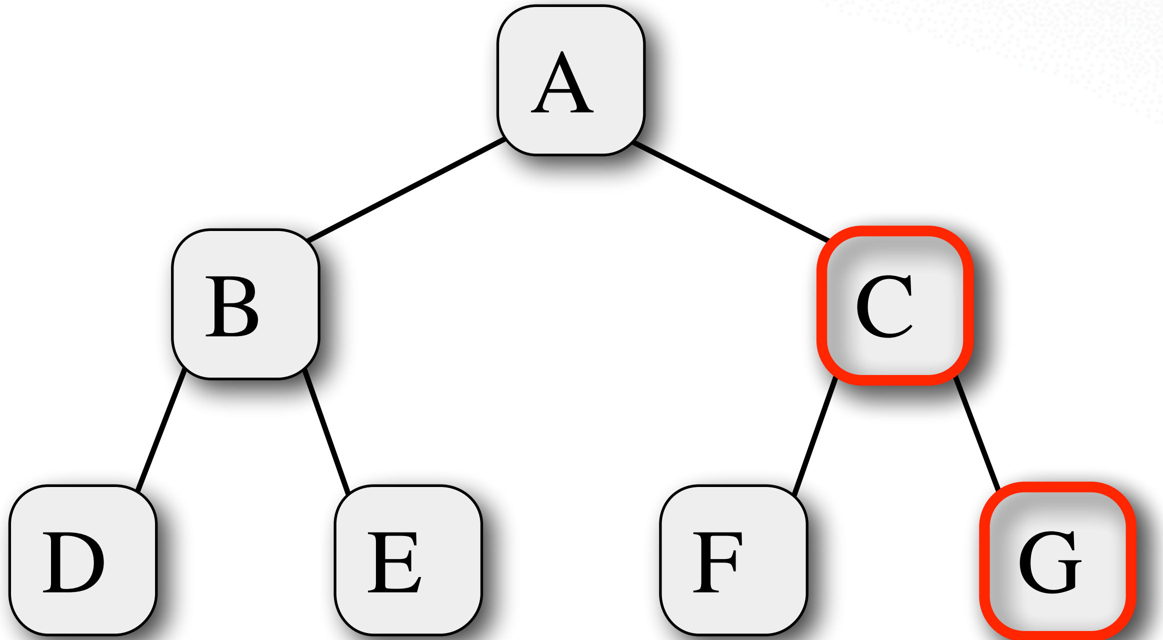
BSP Closest Point Search



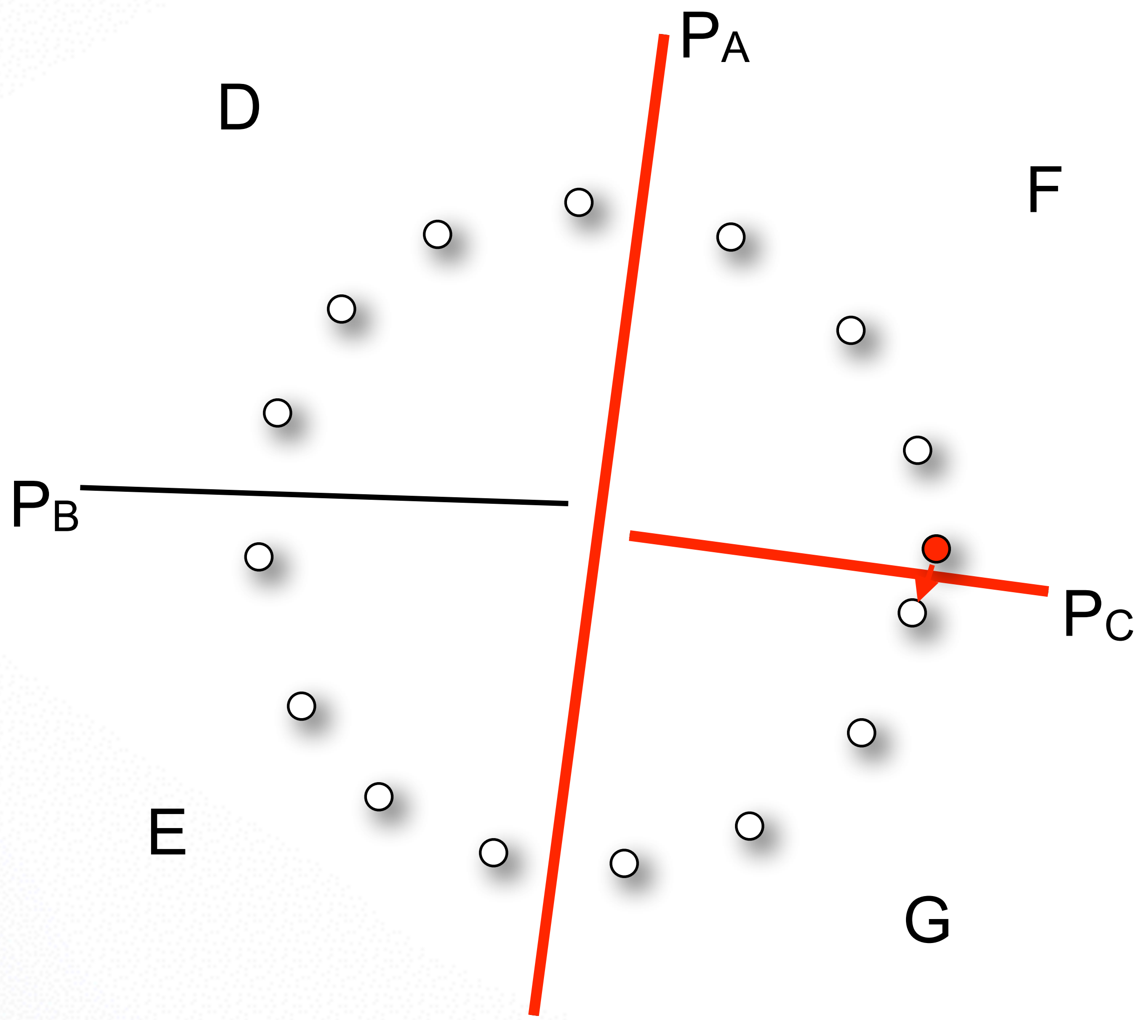


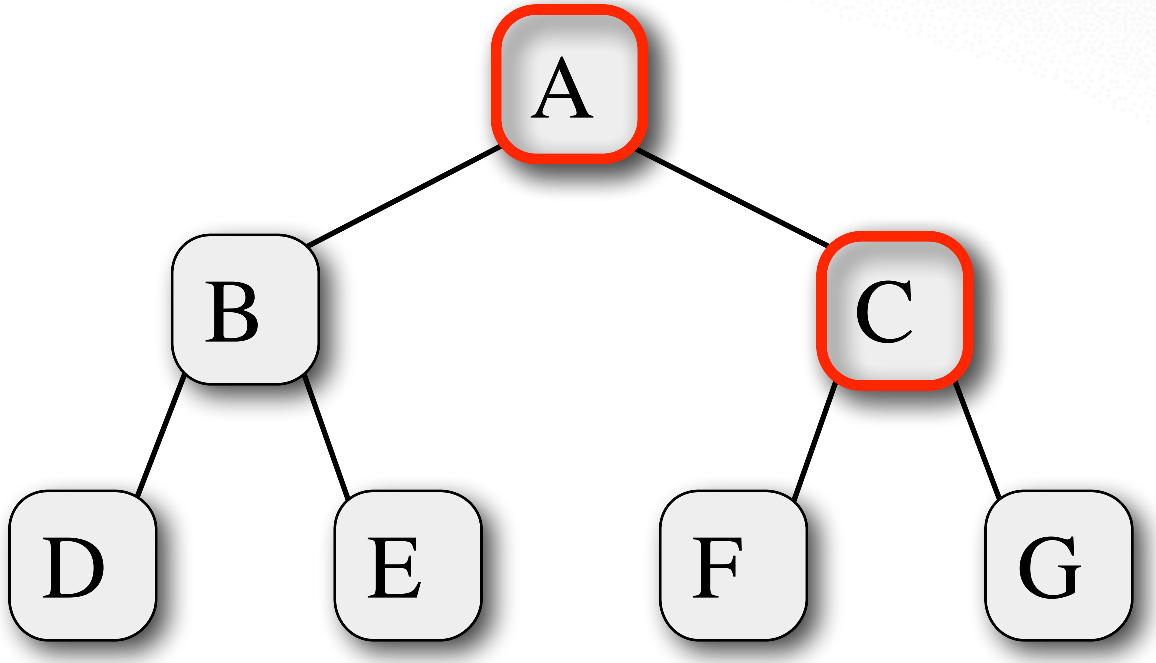
BSP Closest Point Search





BSP Closest Point Search





BSP Closest Point Search

```
BSPNode::dist(Point x, Scalar& dmin)
{
  if (leaf_node())
    for each sample point p[i]
      dmin = min(dmin, dist(x, p[i]));

  else
  {
    d = dist_to_plane(x);
    if (d < 0)
    {
      left_child->dist(x, dmin);
      if (|d| < dmin) right_child->dist(x, dmin);
    }
    else
    {
      right_child->dist(x, dmin);
      if (|d| < dmin) left_child->dist(x, dmin);
    }
  }
}
```

Closest Compatible Point

- Closest point often a bad approximation to corresponding point
- Can improve matching effectiveness by restricting match to **compatible** points
  - Compatibility of colors [Godin et al. '94]
  - Compatibility of normals [Pulli '99]
  - Other possibilities: curvature, higher-order derivatives, and other local features (remember: data is noisy)

ICP Variants

1. **Selecting source points (from one or both meshes)**
2. Matching to points in the other mesh
3. Weighting the correspondences
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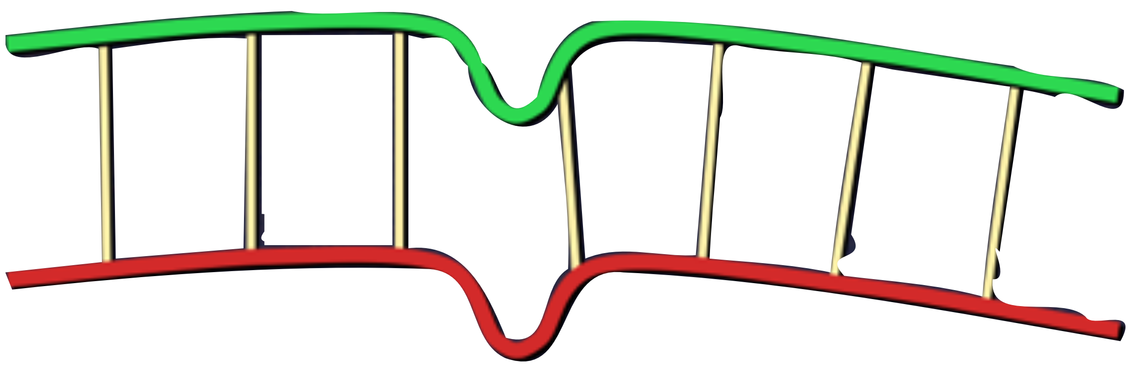
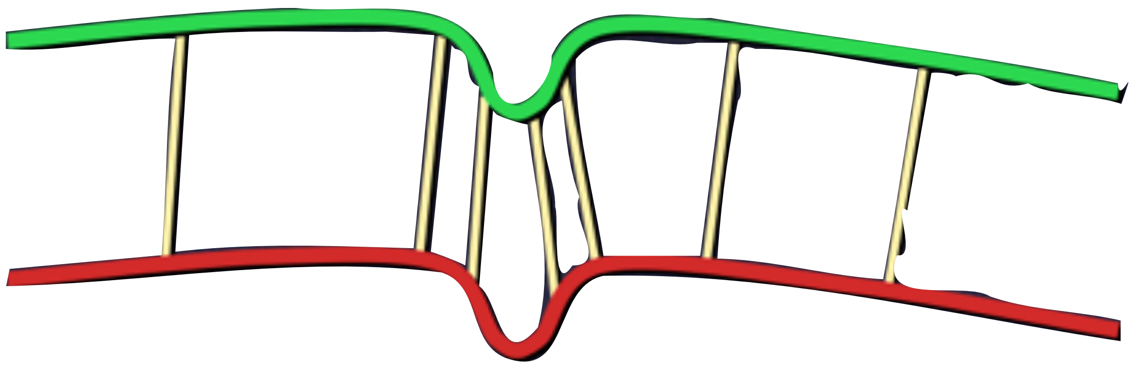
Selecting Source Points

- Use all points
- Uniform subsampling
- Random sampling
- **Stable sampling** [Gelfand et al. 2003]
    - Select samples that constrain all degrees of freedom of the rigid-body transformation

Stable Sampling



Uniform Sampling



Stable Sampling

Covariance Matrix

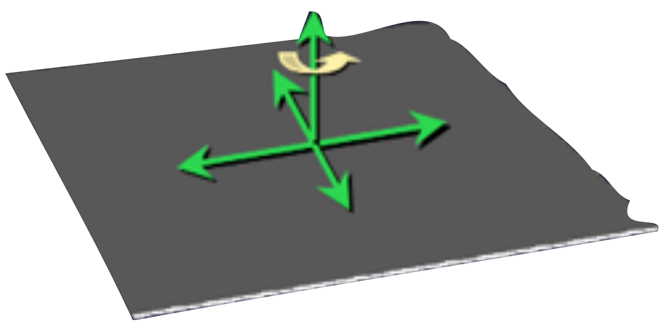
- Aligning transform is given by $\mathbf{A}^\top\mathbf{A}\mathbf{x} = \mathbf{A}^\top\mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} \leftarrow & \mathbf{p}_1 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_1 & \rightarrow \\ \leftarrow & \mathbf{p}_2 \times \mathbf{n}_1 & \rightarrow & \leftarrow & \mathbf{n}_2 & \rightarrow \\ & \vdots & & & \vdots & \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} r_x \\ r_y \\ r_z \\ t_x \\ t_y \\ t_z \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -(\mathbf{p}_1 - \mathbf{q}_1)^\top\mathbf{n}_1 \\ -(\mathbf{p}_2 - \mathbf{q}_2)^\top\mathbf{n}_2 \\ \vdots \end{bmatrix}$$

- Covariance matrix $\mathbf{C} = \mathbf{A}^\top\mathbf{A}$ determines the change in error when surfaces are moved from optimal alignment

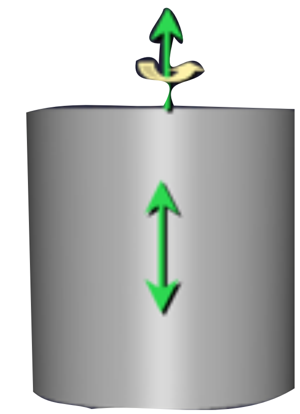
Sliding Directions

- Eigenvectors of $\mathbf{C}$ with small eigenvalues correspond to sliding transformations

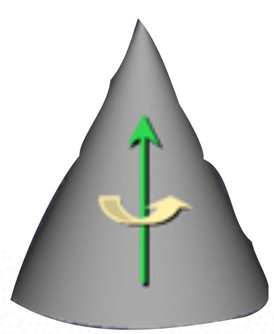


3 small eigenvalues
2 translation
1 rotation

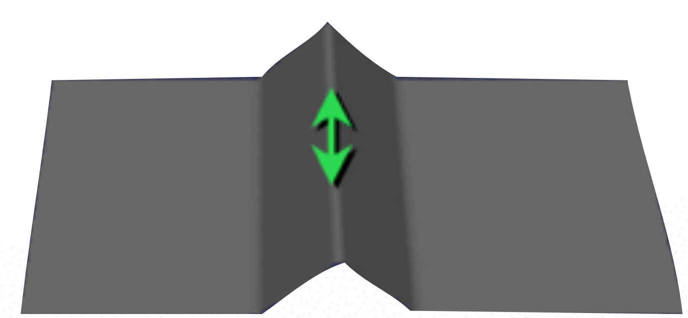
3 small eigenvalues
3 rotation



2 small eigenvalues
1 translation
1 rotation



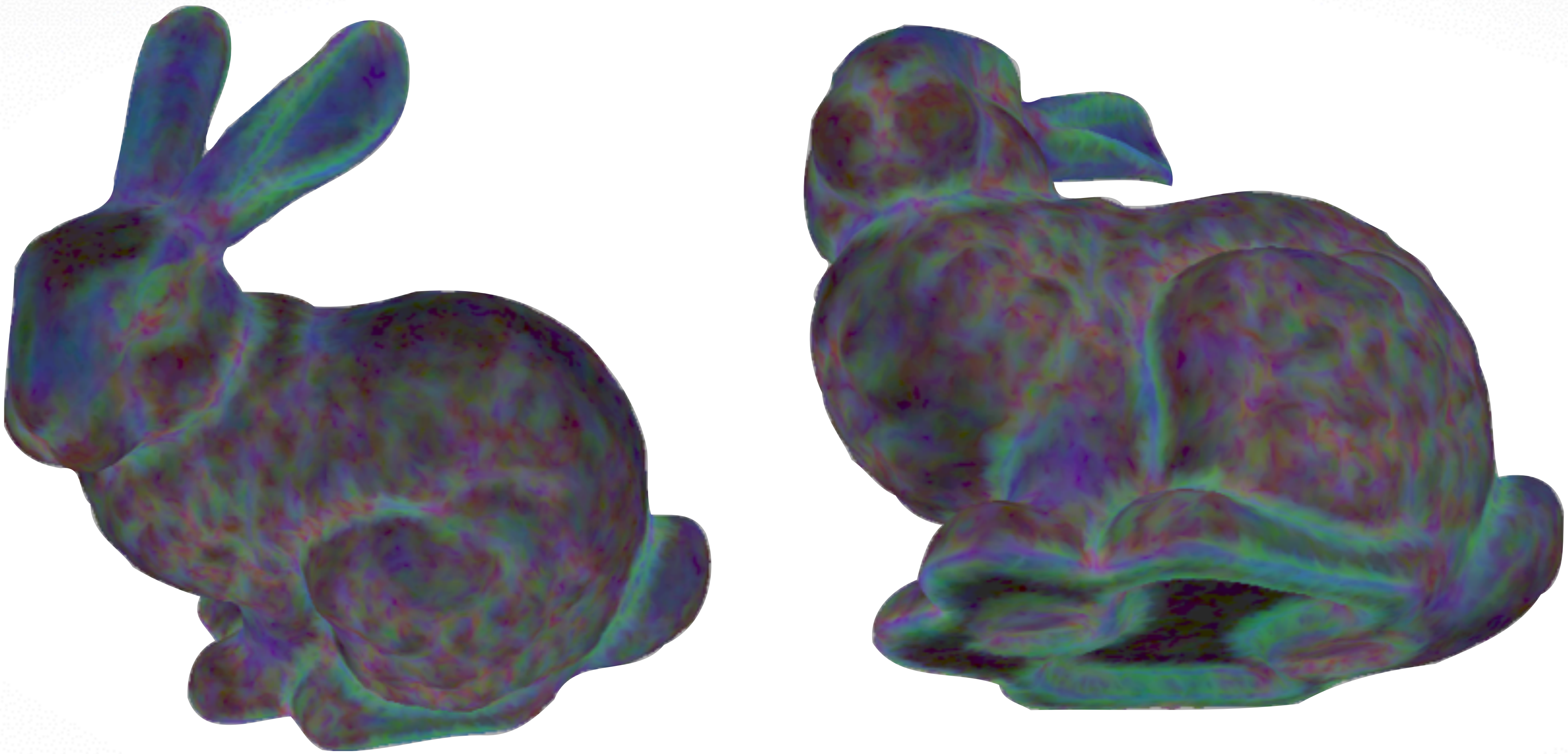
1 small eigenvalue
1 rotation



1 small eigenvalue
1 translation

[Gelfand]

Stability Analysis



Key:

Black	3 DOFs stable	Green	5 DOFs stable
Blue	4 DOFs stable	Red	6 DOFs stable

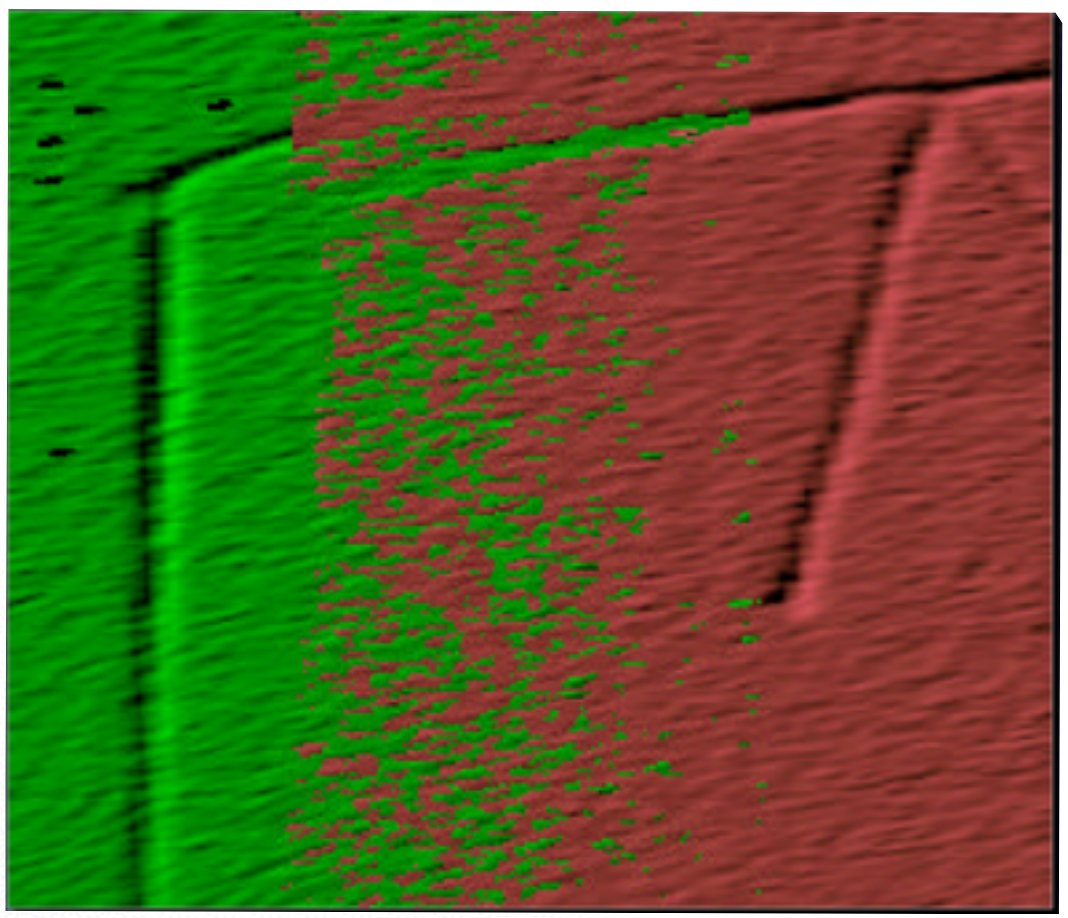
Sample Selection

- Select points to prevent small eigenvalues
  - Based on $\mathbf{C}$ obtained from sparse sampling

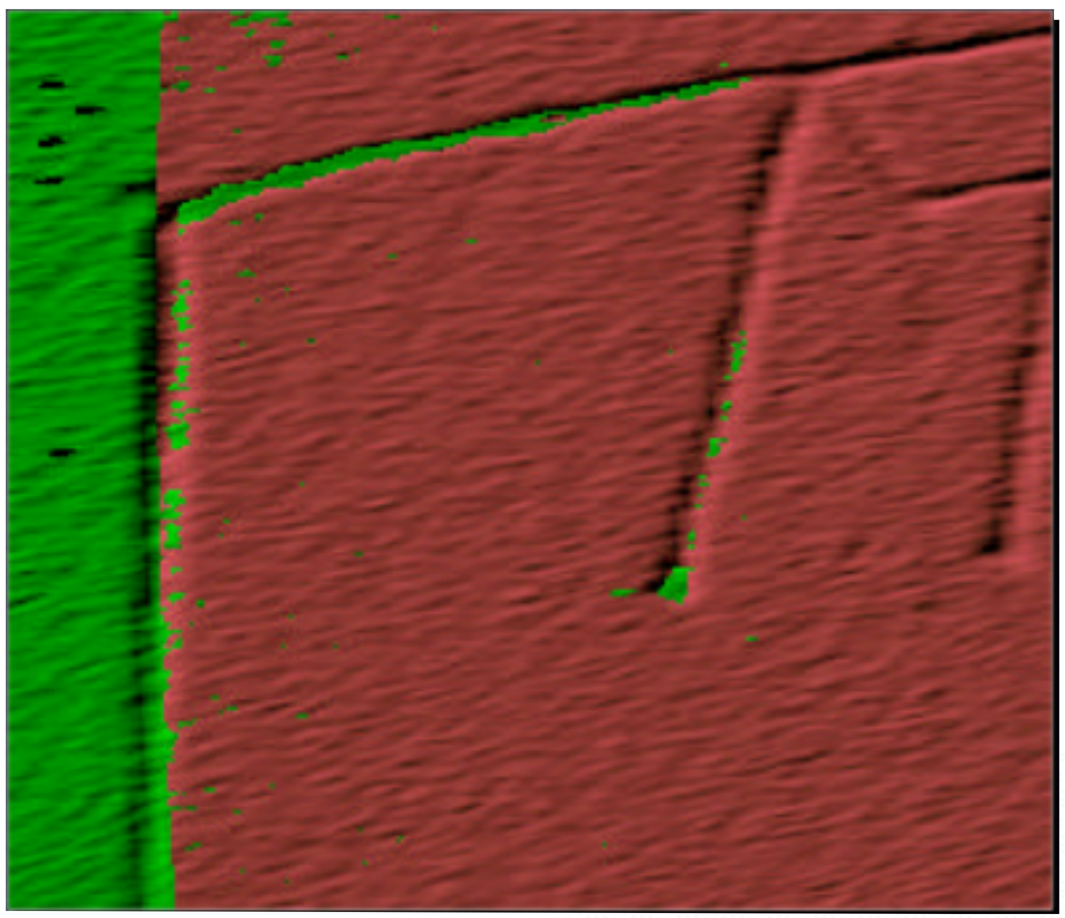
- Simpler variant: normal-space sampling
  - select points with uniform distribution of normals
  - **Pro**: faster, does not require eigenanalysis
  - **Con**: only constrains translation

Result

Stability-based or normal-space sampling important for smooth areas with small features



Random Sampling



Normal-space Sampling

Selection vs. Weighting

- Could achieve same effect with weighting
- Hard to ensure enough samples in features except at high sampling rates
- However, have to build special data structure
- Preprocessing / run-time cost tradeoff

Improving ICP Speed

Projection-based matching

1. Selecting source points (from one or both meshes)
2. **Matching to points in the other mesh**
3. Weighting the correspondences
4. Rejecting certain (outliers) point pairs
5. Assining an error metric to the current transform
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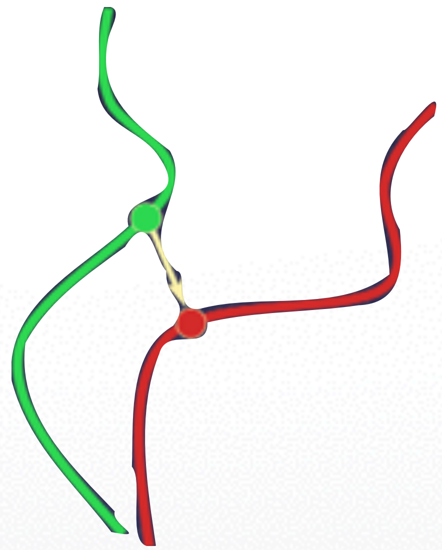
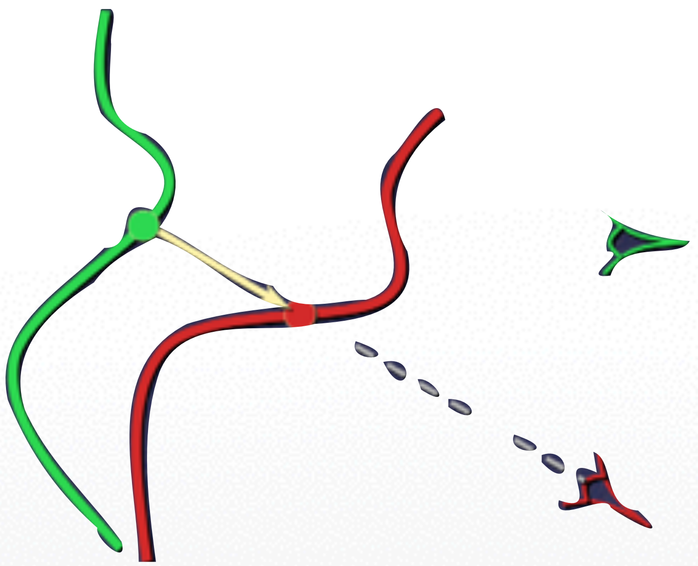
Finding Corresponding Points

- Finding Closest point is most expensive stage of the ICP algorithm
  - Brute force search – O(n)
  - Spatial data structure (e.g., k-d tree) – O(log n)



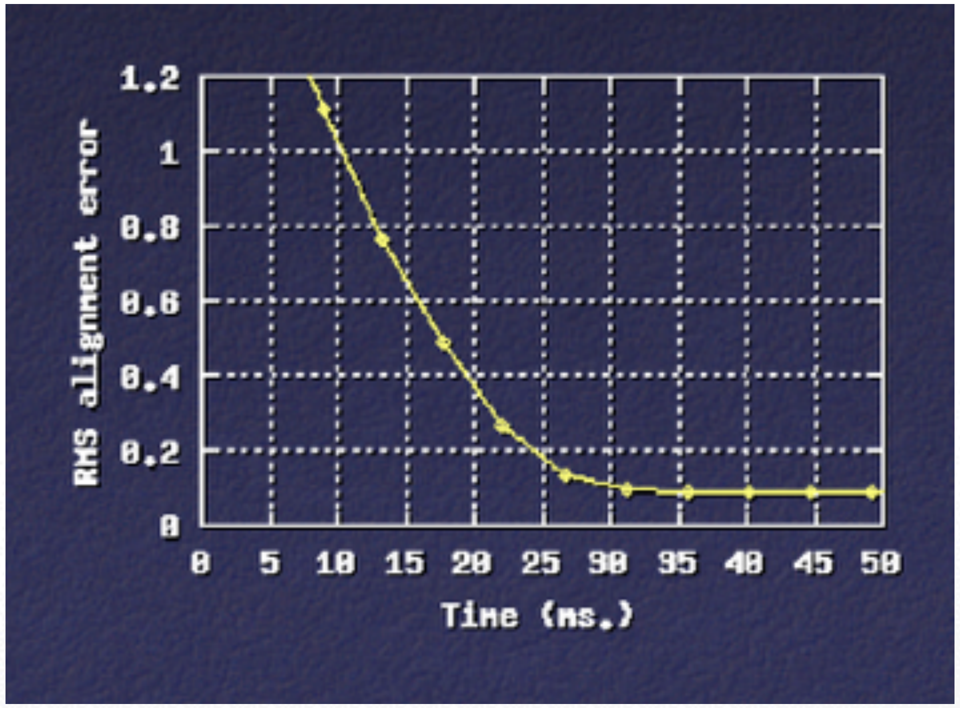
Projection to Find Correspondence

- Idea: use a simpler algorithm to find correspondences
- For range images, can simply project point [Blais 95]
  - Constant-time
  - Does not require precomputing a spatial data structure



Projection-Based Matching

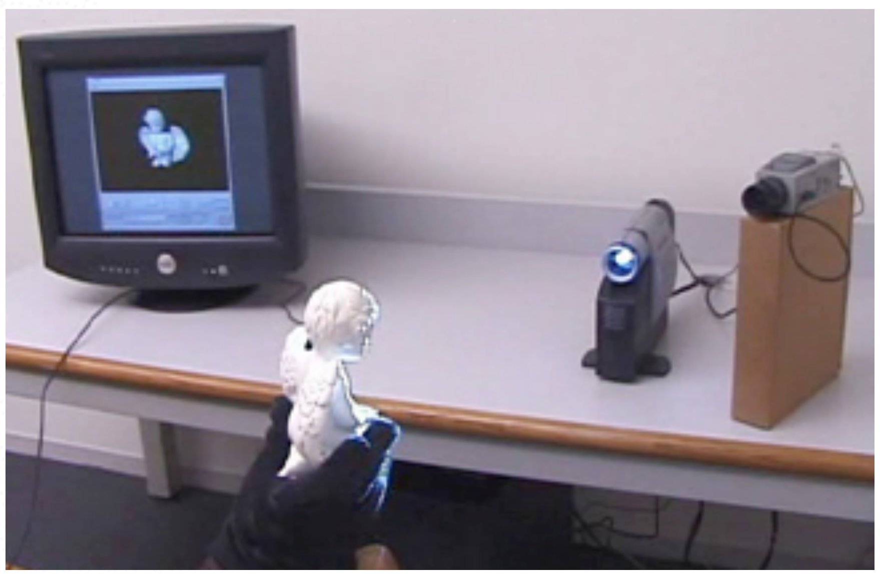
- Slightly worse performance per iteration
- Each iteration is on to two orders of magnitude faster than closest point
- Result: can align two range images in a few milliseconds, vs. a few seconds



Application

- Given:
  - A scanner that returns range images in real time
  - Fast ICP
  - Real-time merging and rendering
- Result: 3D model acquisition
  - Tight feedback loop with user
  - Can see and fill holes while scanning

Examples



[Rusinkiewicz et al. ‘02]

Artec Group

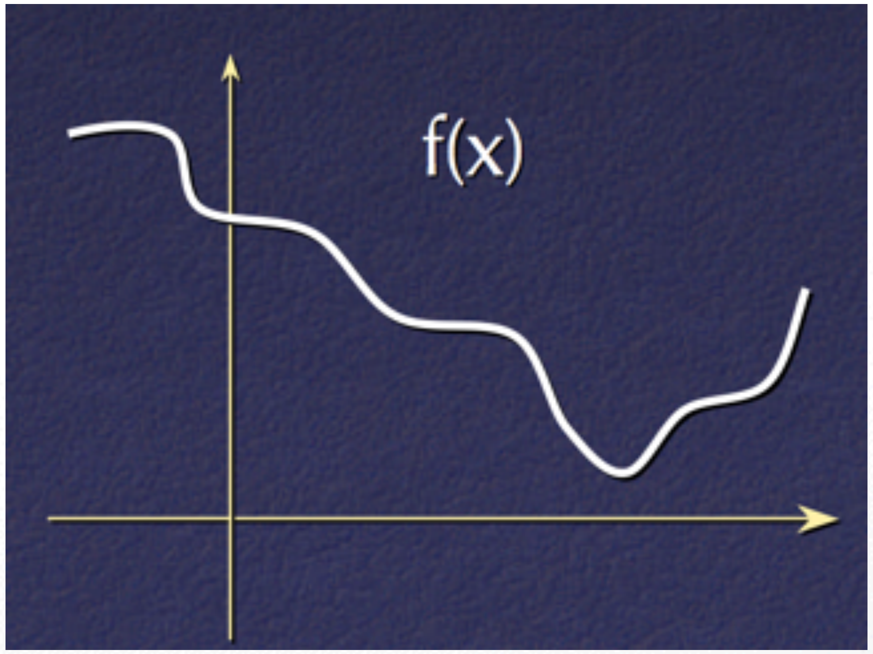
[Newcombe et al. ’11]
KinectFusion

Theoretical Analysis of ICP Variants

- One way of studying performance is via empirical tests on various scenes
- How to analyze performance analytically?
- For example, when does point-to-plane help? Under what conditions does projection-based matching work?

What does ICP do?

- Two ways of thinking about ICP:
  - Solving correspondence problem
  - **Minimizing point-to-surface squared distance**
- ICP is like Newton's method on an approximation of the distance function



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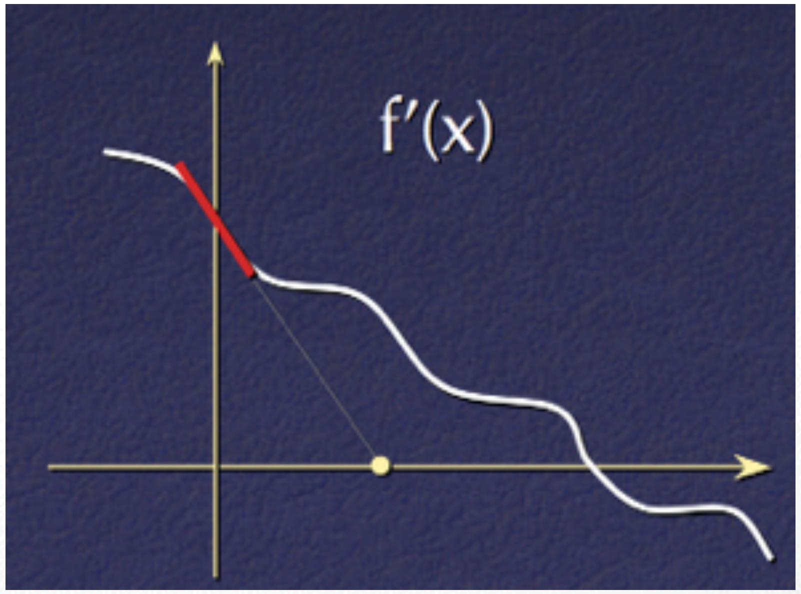
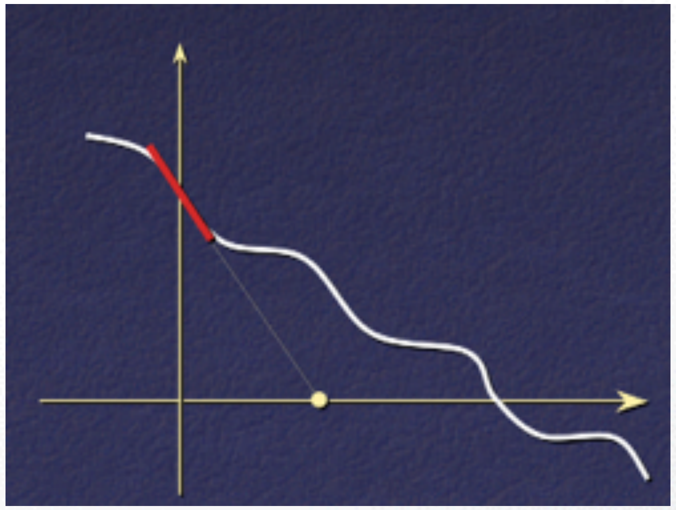
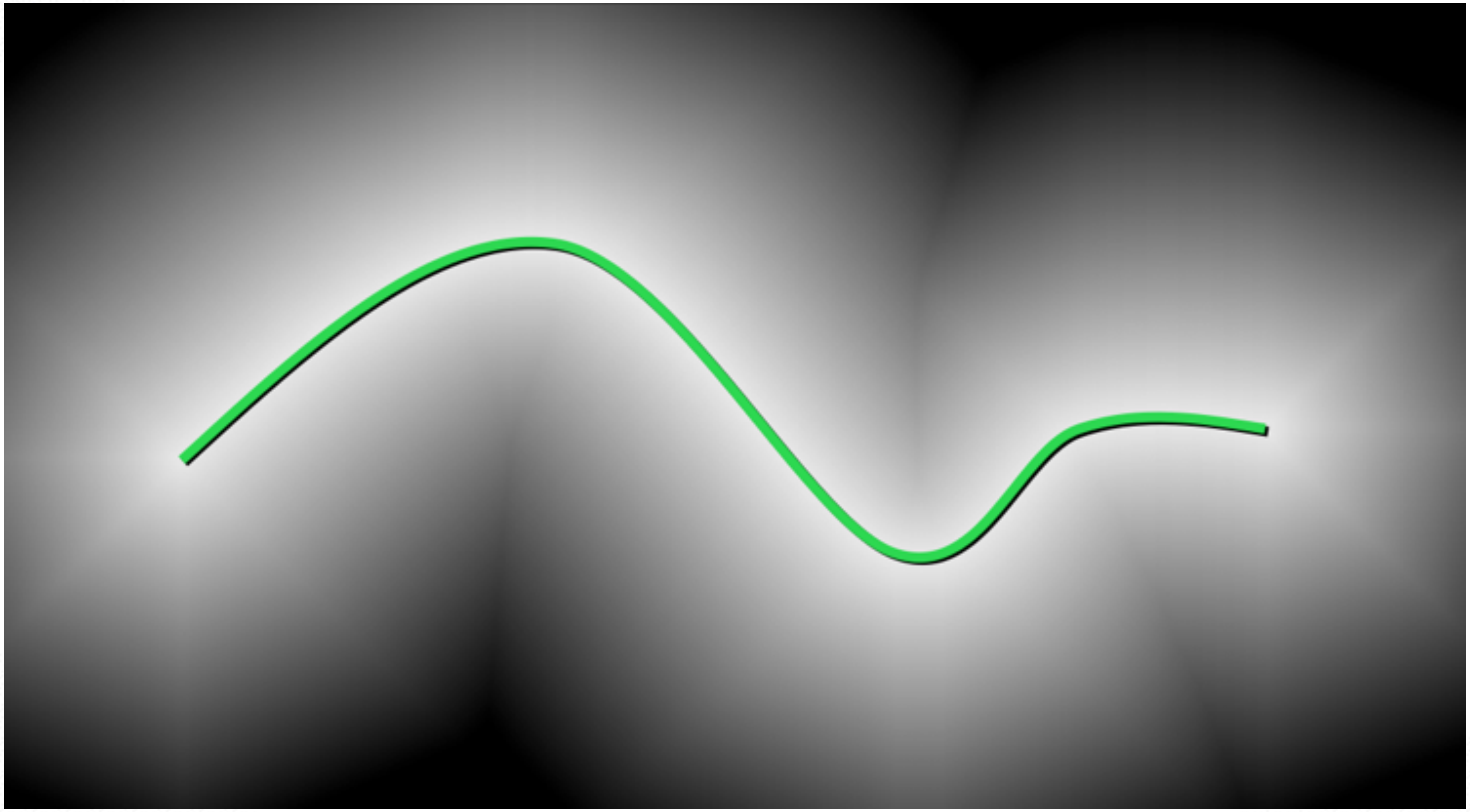


What does ICP do?

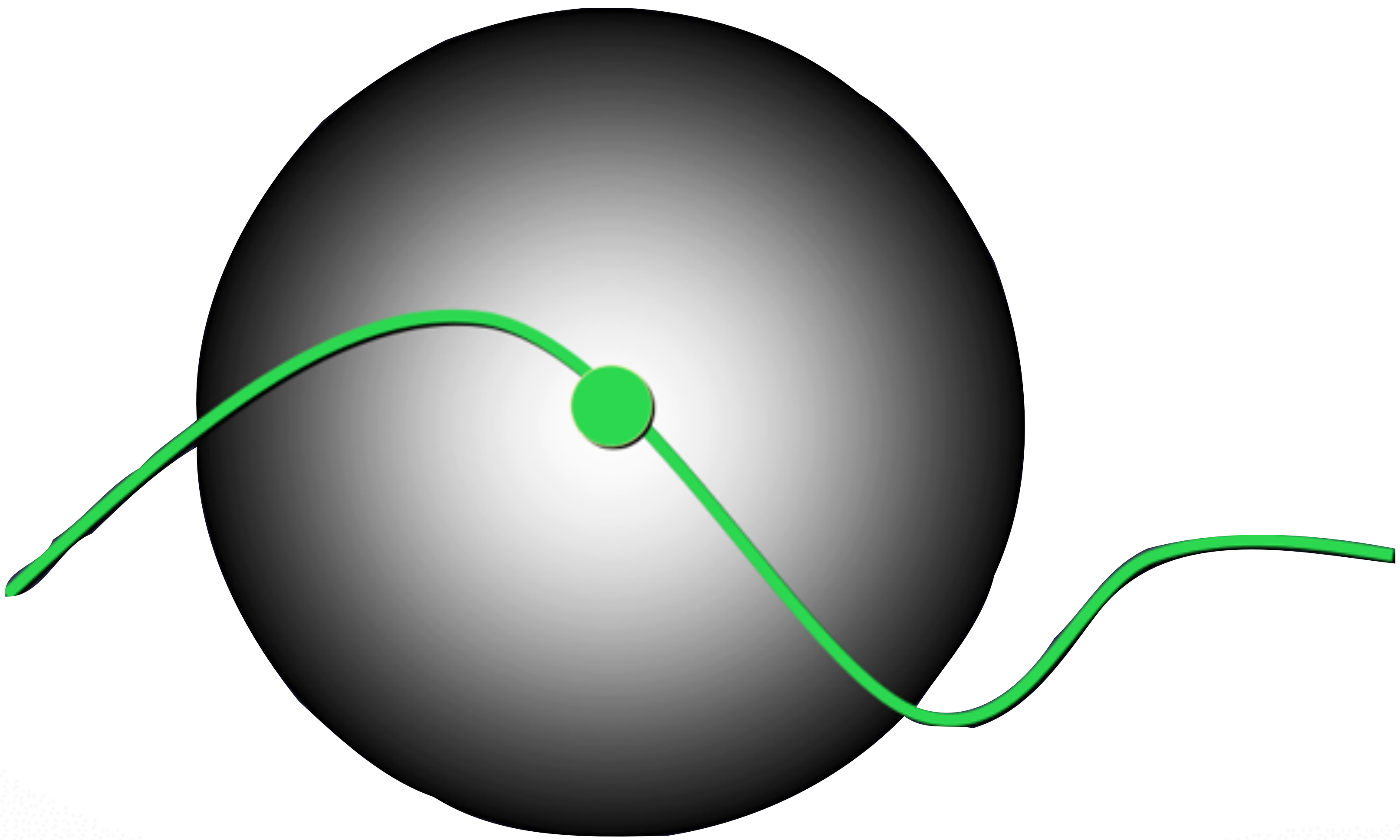
- Two ways of thinking about ICP:
  - Solving correspondence problem
  - **Minimizing point-to-surface squared distance'**
- ICP is like Newton's method on an approximation of the distance function
  - ICP variants affect shape of the global error function or local approximation



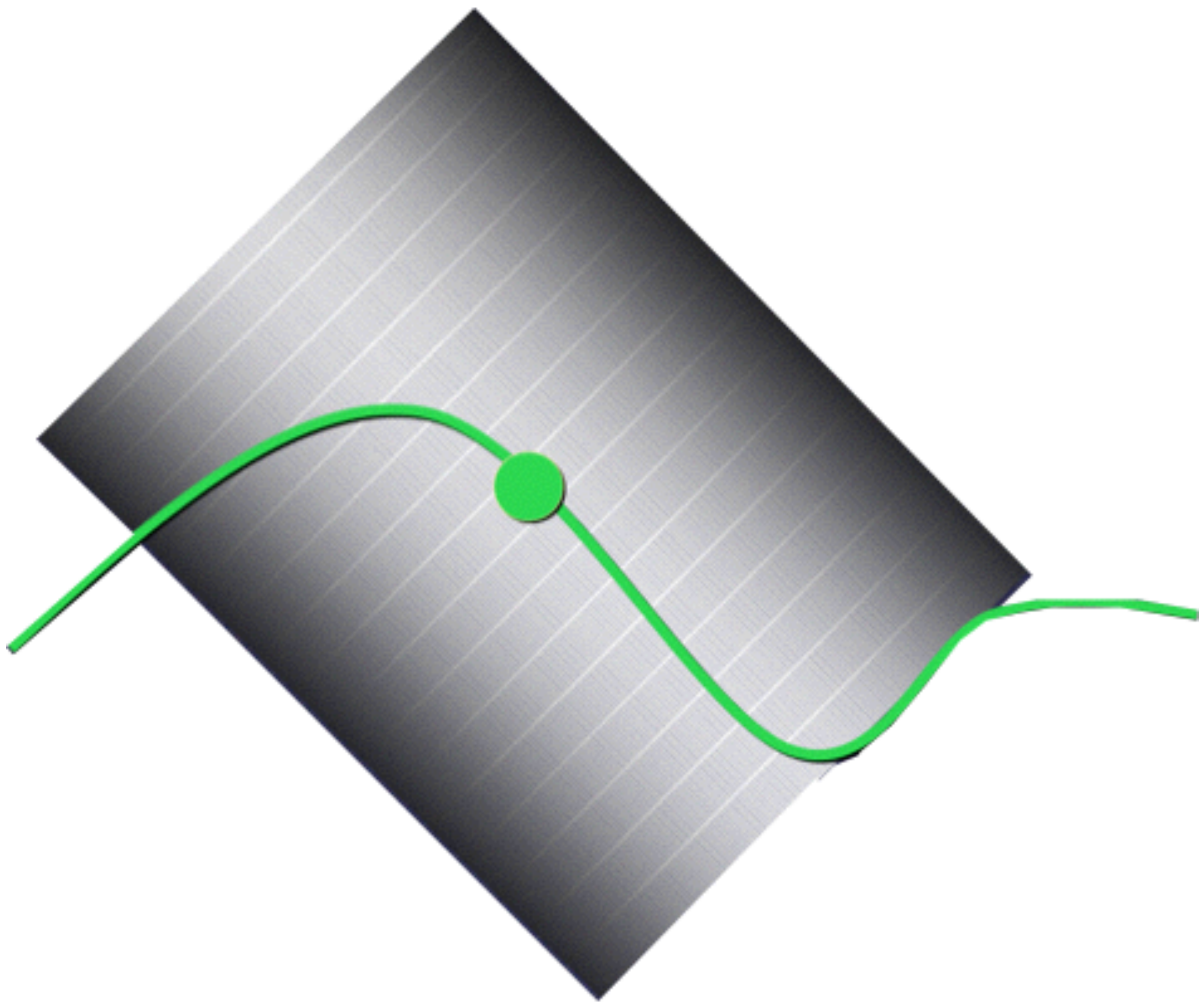
Point-to-Surface Distance



Point-to-Point Distance

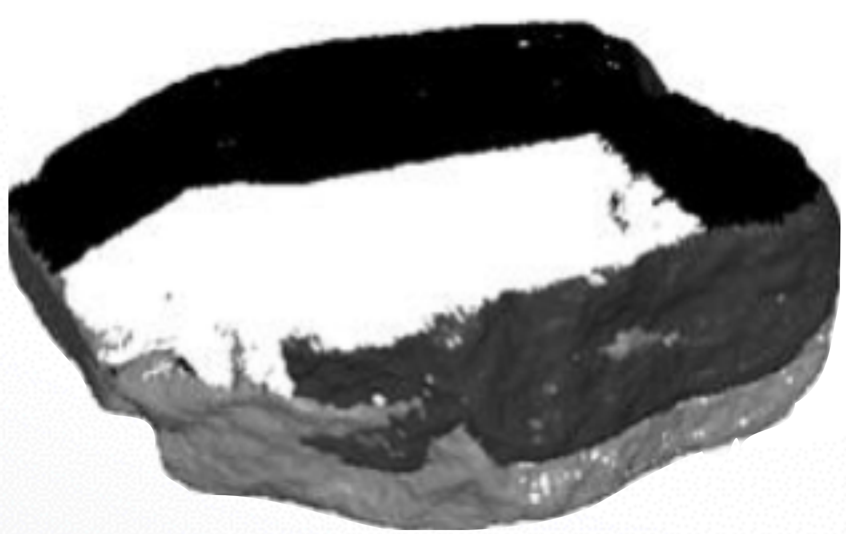


Point-to-Plane Distance



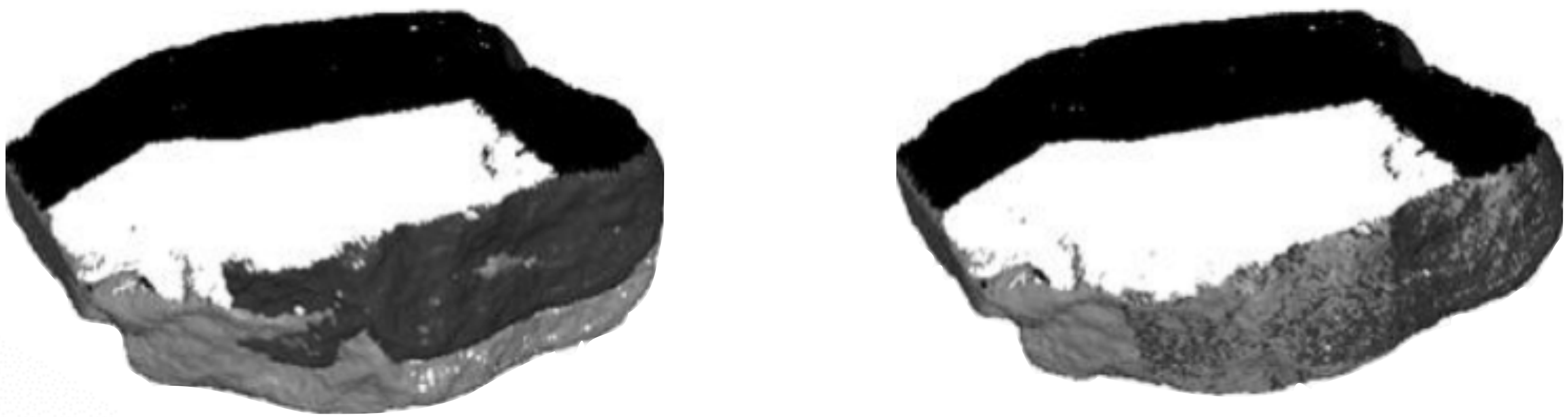
Global Registration Goal

- Given: n scans around an object
- Goal: align them all
- First attempt: apply ICP to each scan to one other



Global Registration Goal

- Want method for distributing accumulated error among all scans



Approach #1: Avoid the Problem

- In some cases, have 1 (possibly low-resolution) scan that covers most surface
- Align all other scans to this “anchor” [Turk 94]
- Disadvantage: not always practical to obtain anchor scan

Approach #2: The Greedy Solution

- Align each new scan to all previous scans [Masuda ’96]
- Disadvantages:
  - Order dependent
  - Doesn’t spread out error

Approach #3: The Brute-Force Solution

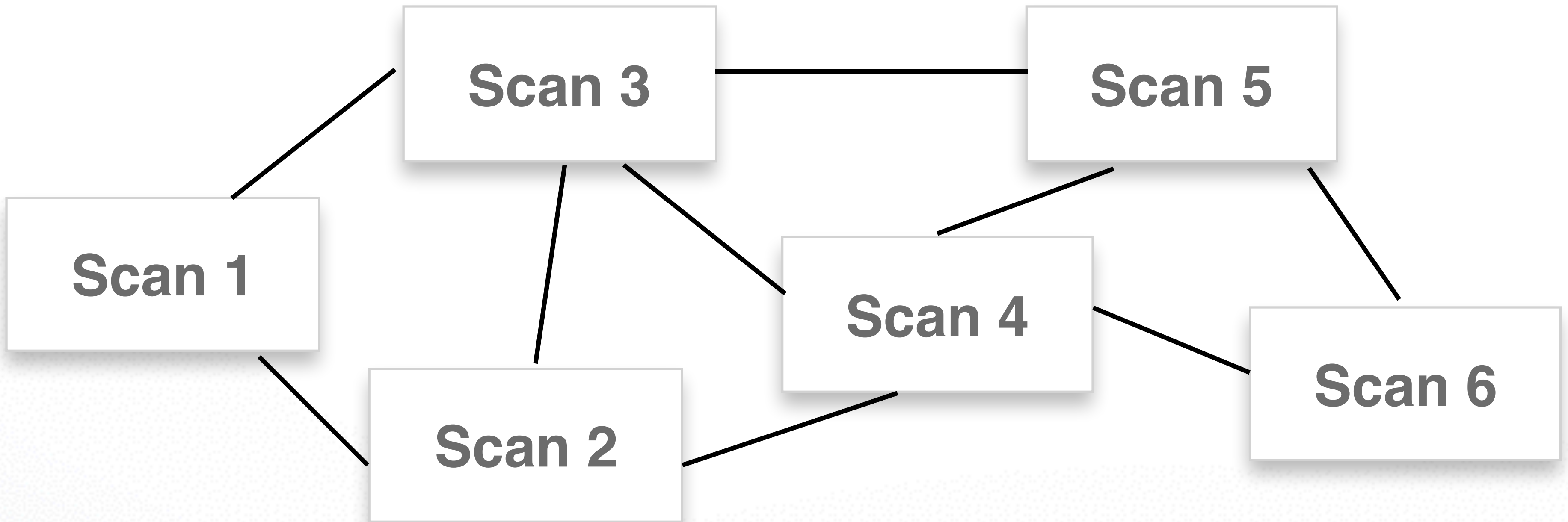
- While not converged:
  - For each scan:
    - For each point:
      - For every other scan
        - Find closest point
  - Minimize error w.r.t. transforms of **all** scans
- Disadvantage:
  - Solve $(6n)\times(6n)$ matrix equation, where n is number of scans

Approach #3a: Slightly Less Brute-Force Solution

- While not converged:
  - For each scan:
    - For each point:
      - For every other scan
        - Find closest point
  - Minimize error w.r.t. transforms of **this** scans
- Faster than previous method (matrices are 6x6) [Bergevin '96, Benjemaa '97]

Graph Methods

- Many global registration algorithms create a graph of **pairwise alignments** between scans



Sharp et al. Algorithm

- Perform pairwise ICPs, record sample (e.g., 200) of corresponding points
- For each scan, starting w most connected
  - Align scan to existing set
  - While (change in error) > threshold
    - Align each scan to others
- All alignments during global reg phase use precomputed correspondending points.

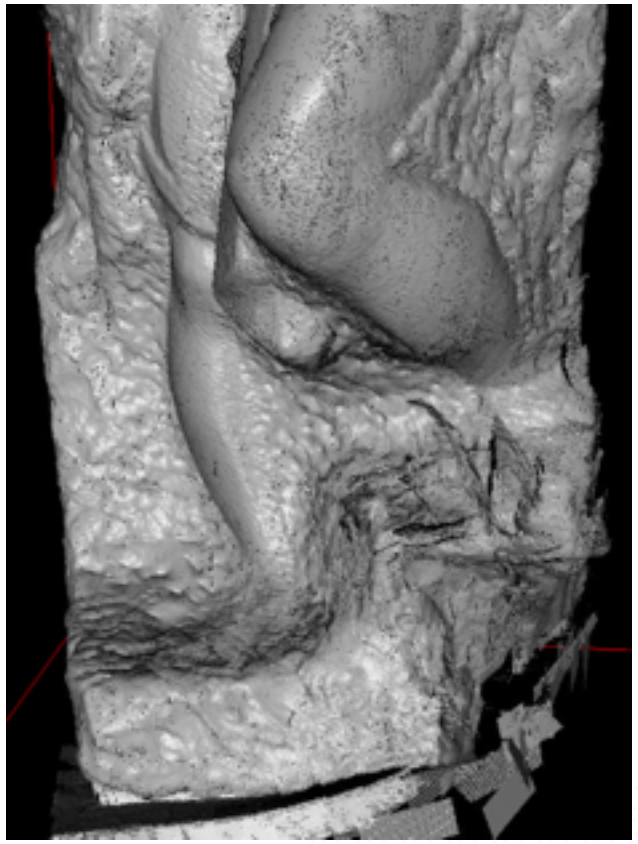
Lu and Milios Algorithm

- Perform pairwise ICPs, record optimal rotation/translation and covariance for each
- Least squares simultaneous minimization of all errors (covariance-weighted)
- Requires linearization of rotations
    - Worse than the ICP case, since don't converge to (incremental rotation) = 0

Bad ICP in Global Registration

One bad ICP can throw off the entire model!

Correct Global Registration



Global Registration Including Bad ICP

Literature

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- Chen & Medioni, “Object modeling by registration of multiple range images”, ICRA1991
- Besl & McKay: A method for registration of 3D shapes, PAMI 1992
- Horn: Closed-form solution of absolute orientation using unit quaternions, Journal Opt. Soc. Amer. 4(4), 1987
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Next Time

3D Capture Session

http://cs599.hao-li.com

Thanks!