Exercise 2. Registration
Rigid Registration

• Selecting source points
• Matching points to the target mesh
• Weighting the correspondences
• Rejecting bad pairs
• Compute error metric
• Minimize error metric
Exercise 2

- Perform rigid registration between 10 scans of the Stanford bunny
Exercise 2

• Demo

• ‘SHIFT’ + mouse controls: manual alignment for an initial transformation

• ‘r’: perform single registration step with point to point distance minimization

• ‘SPACE’: perform single registration step with point to plane distance minimization

• ‘n’: load next scan
Exercise 2

- Getting it compiled
- Subsampling
- Bad pairs rejection
- Point to point optimization
- Point to plane optimization
Getting It Compiled

- CMake, OpenGL, OpenMesh
- ANN (Approximate Nearest Neighbor)
  - efficient closest point lookup using kd-tree
Subsampling

- Uniform subsampling within a given radius
  `subsampleRadius`
- `RegistrationViewer::subsample()` in `RegistrationViewer.cc`
Bad Pairs Rejection

- Closest points are computed using ANN
- Prune correspondences based on
  - distance threshold
  - normal compatibility
- RegistrationViewer::calculate_correspondences() in RegistrationViewer.cc
Point to Point Optimization

- Minimize \( E = \sum_{i=1}^{N} \|Rp_i + t - q_i\|_2^2 \)
  by solving a linear system \( Ax = b \)
- Registration::register_point2point() in Registration.cc
Euler Angles

- Three elemental rotations:

\[
\begin{align*}
R_x(\alpha) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}, \\
R_y(\beta) &= \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}, \\
R_z(\gamma) &= \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{align*}
\]

- Any rotation matrix can be decomposed as a product of elemental three rotation matrix

\[
R = R_z(\gamma)R_y(\beta)R_x(\alpha) = \begin{bmatrix}
c_\gamma c_\beta & -c_\alpha s_\gamma + c_\gamma s_\beta s_\alpha & s_\gamma s_\alpha + c_\gamma c_\alpha s_\beta \\
c_\beta s_\gamma & c_\gamma c_\alpha + s_\gamma s_\beta s_\alpha & c_\alpha s_\gamma s_\beta - c_\gamma s_\alpha \\
-s_\beta & c_\beta s_\alpha & c_\beta c_\alpha 
\end{bmatrix}
\]

\[
c_\alpha = \cos \alpha \quad s_\alpha = \sin \alpha
\]
• Linearized Euler angle

(assuming small rotation: $\cos \alpha = 1 \quad \sin \alpha = \alpha$)

$$R = \begin{bmatrix}
    c_\gamma c_\beta & -c_\alpha s_\gamma + c_\gamma s_\beta s_\alpha & s_\gamma s_\alpha + c_\gamma c_\alpha s_\beta \\
    c_\beta s_\gamma & c_\gamma c_\alpha + s_\gamma s_\beta s_\alpha & c_\alpha s_\gamma s_\beta - c_\gamma s_\alpha \\
    -s_\beta & c_\beta s_\alpha & c_\beta c_\alpha
\end{bmatrix} = \begin{bmatrix}
    1 & -\gamma & \beta \\
    \gamma & 1 & -\alpha \\
    -\beta & \alpha & 1
\end{bmatrix}$$

• Linearized transformation

$$x = \begin{bmatrix}
    \alpha & \beta & \gamma & t_x & t_y & t_z
\end{bmatrix}^T$$
Point to Plane Optimization

- Minimize 
  \[ E = \sum_{i=1}^{N} \| n_i^T (R p_i + t - q_i) \|^2 \]
  by solving a linear system \( Ax = b \)

- Registration::register_point2surface() in Registration.cc
Results
Submission

• Deadline: **Monday, Feb 16, 2015 11:59pm**

• Upload a `.zip` compressed file named “Exercise2-YourName.zip” to Blackboard, same as Ex. 1

• Include a “read.txt” file describing how you solve each exercise and the encountered problems
Contact

- email (include “CSCI_599” in title):
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- Highly recommended to post your questions on Blackboard
Thanks!

http://cs599.hao-li.com