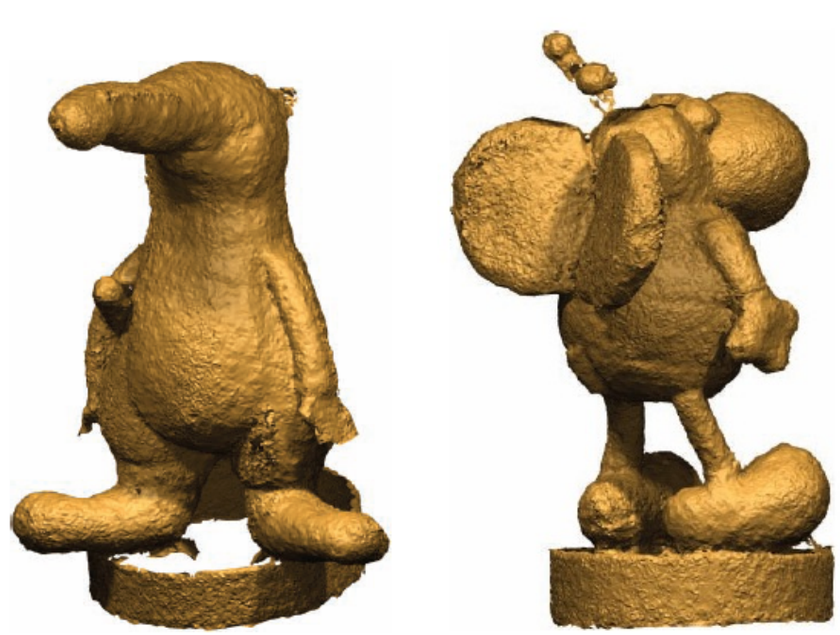
Spring 2014

CSCI 599: **Digital Geometry Processing**

13.1 Dynamic Geometry Processing II



Hao Li

http://cs599.hao-li.com

Prof. Niloy J. Mitra
UCL

Registration without Correspondences

Scan Registration

Bee

Input frames (Selection)

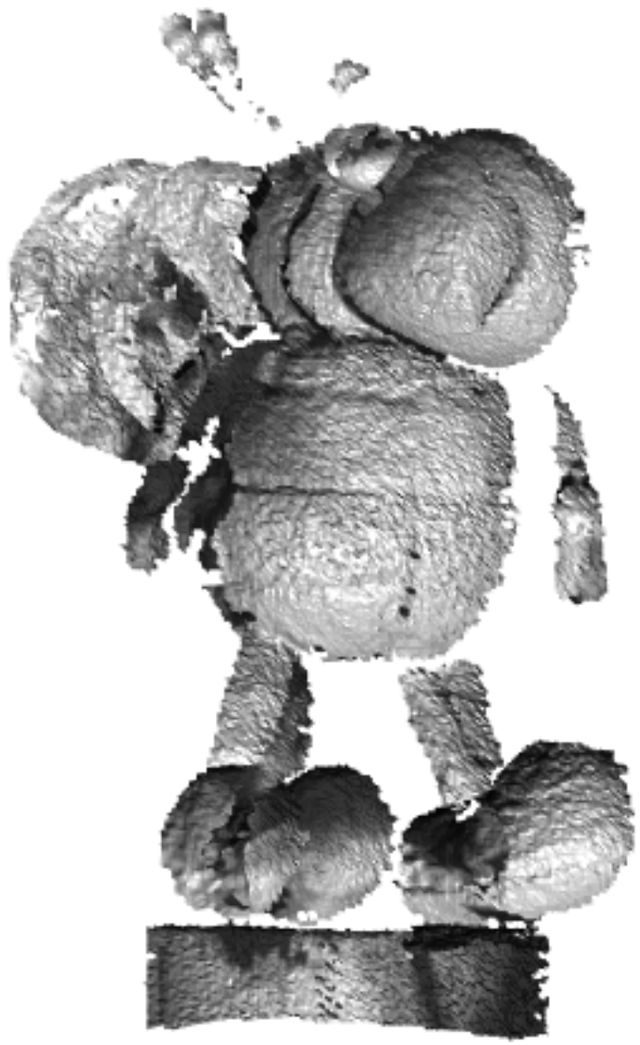
2200 pointclouds scanned at 17 Hz

transformations only for adjacent frames considered

no global error correction

no noise smoothing

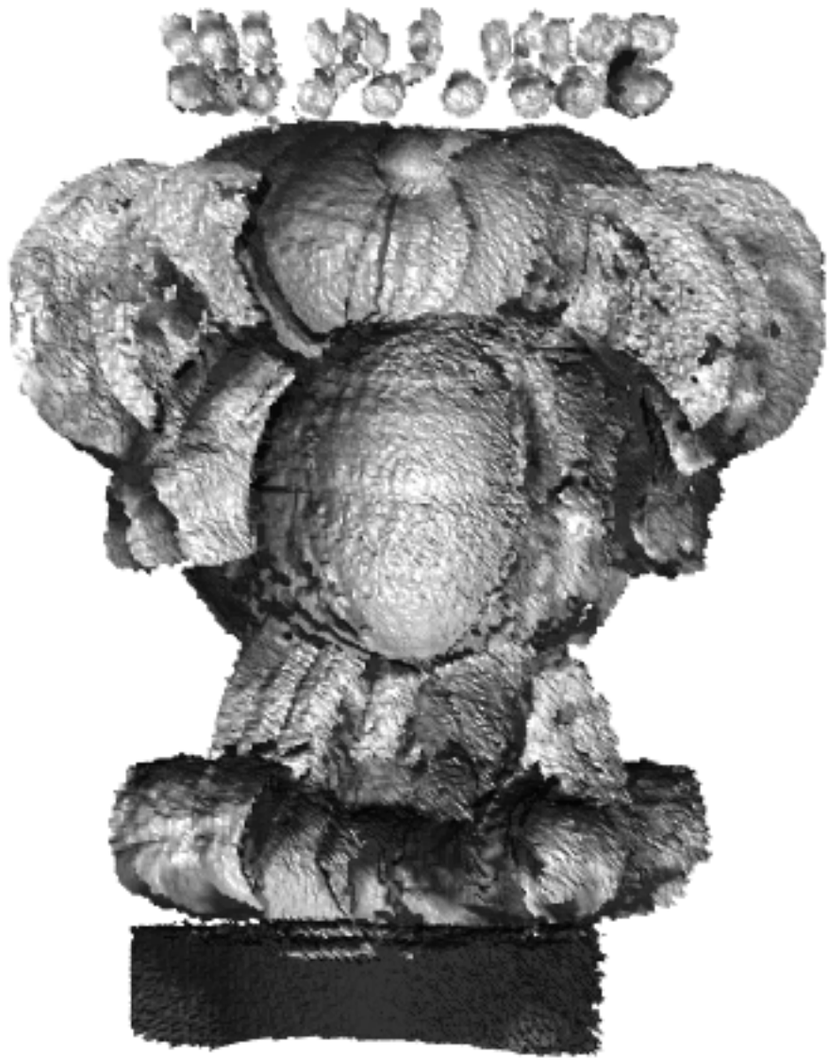
Scan Registration



Solve for inter-frame motion:

$$\alpha := (\mathbf{R}, \mathbf{t})$$

Scan Registration



Solve for inter-frame motion:

$$\alpha_j := (\mathbf{R}_j, \mathbf{t}_j)$$

The Setup

Given:

A set of frames $\{P_0, P_1, \ldots P_n\}$

Goal:

Recover rigid motion $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$ between adjacent frames

The Setup

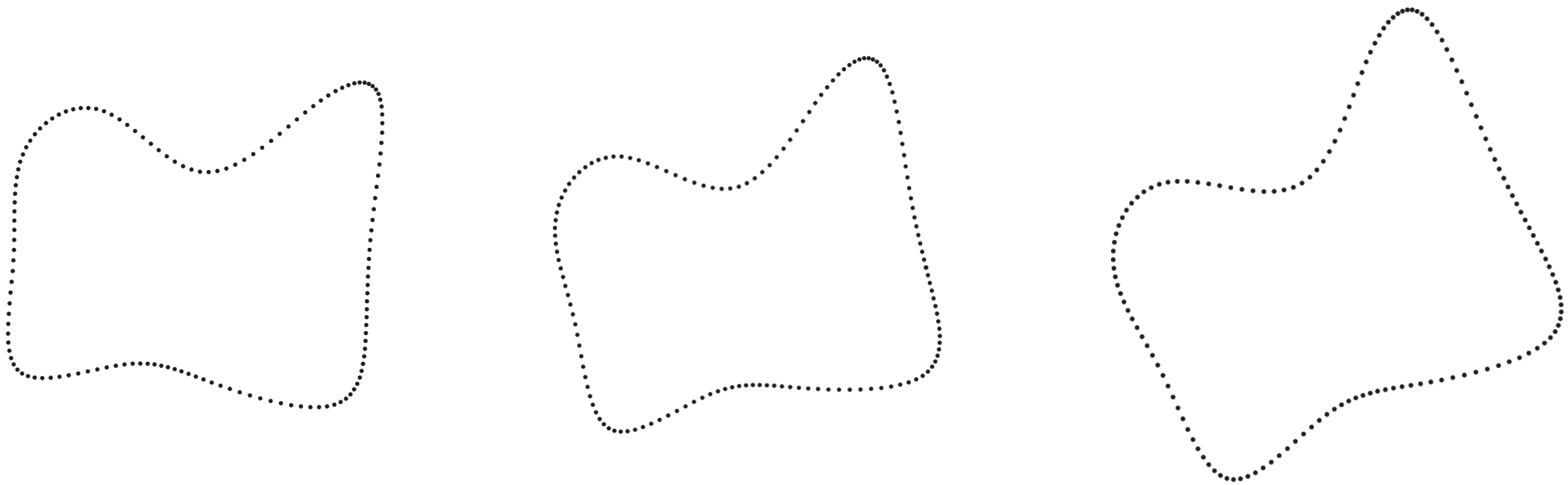
- Smoothly varying object motion
- Unknown correspondence between scans
- Fast acquisition !

motion happens between frames

Contributions

- Rigid registration ! kinematic property of space-time surface (locally exact)
- Registration ! surface normal estimation
- Analysis (see paper for details)
  - Relation to ICP (Iterated Closest Point)
  - Stability analysis
- Extension to deformable/articulated bodies

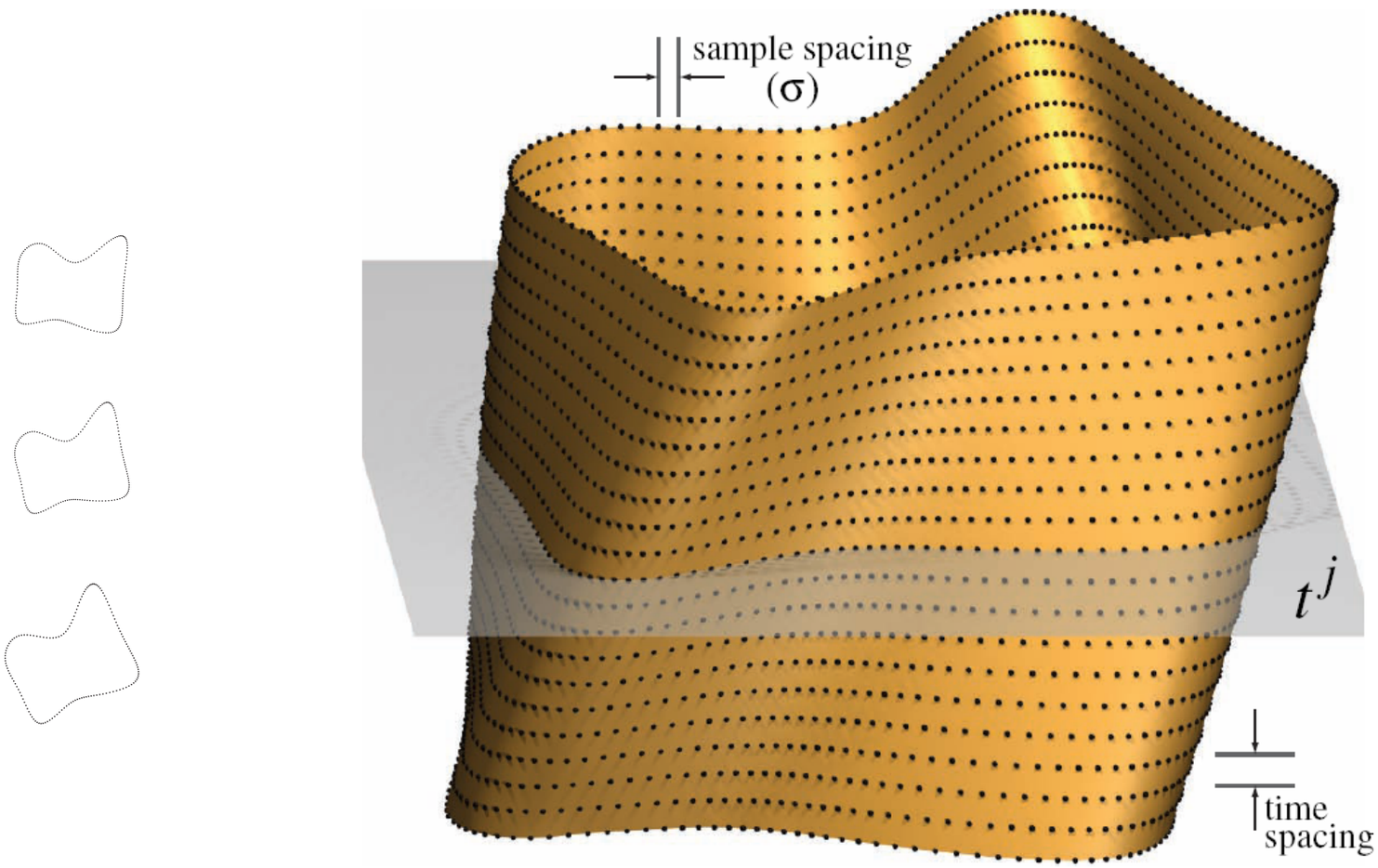
Time Ordered Scans



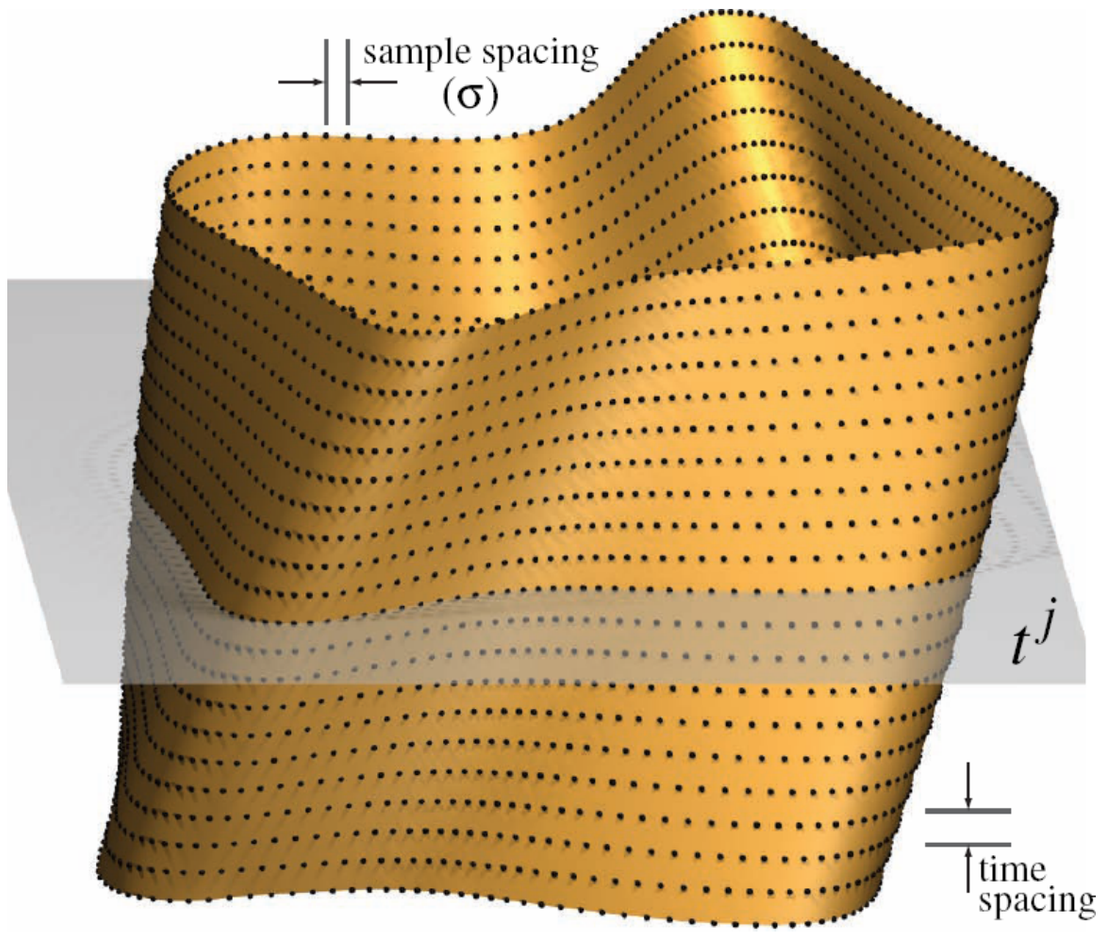
t^j t^{j+1} t^{j+2}

$$\widetilde{P}^j \equiv \{\widetilde{\mathbf{p}}_i^j\} := \{(\mathbf{p}_i^j, t^j), \mathbf{p}_i^j \in \mathbb{R}^d, t^j \in \mathbb{R}\}$$

Space-Time Surface

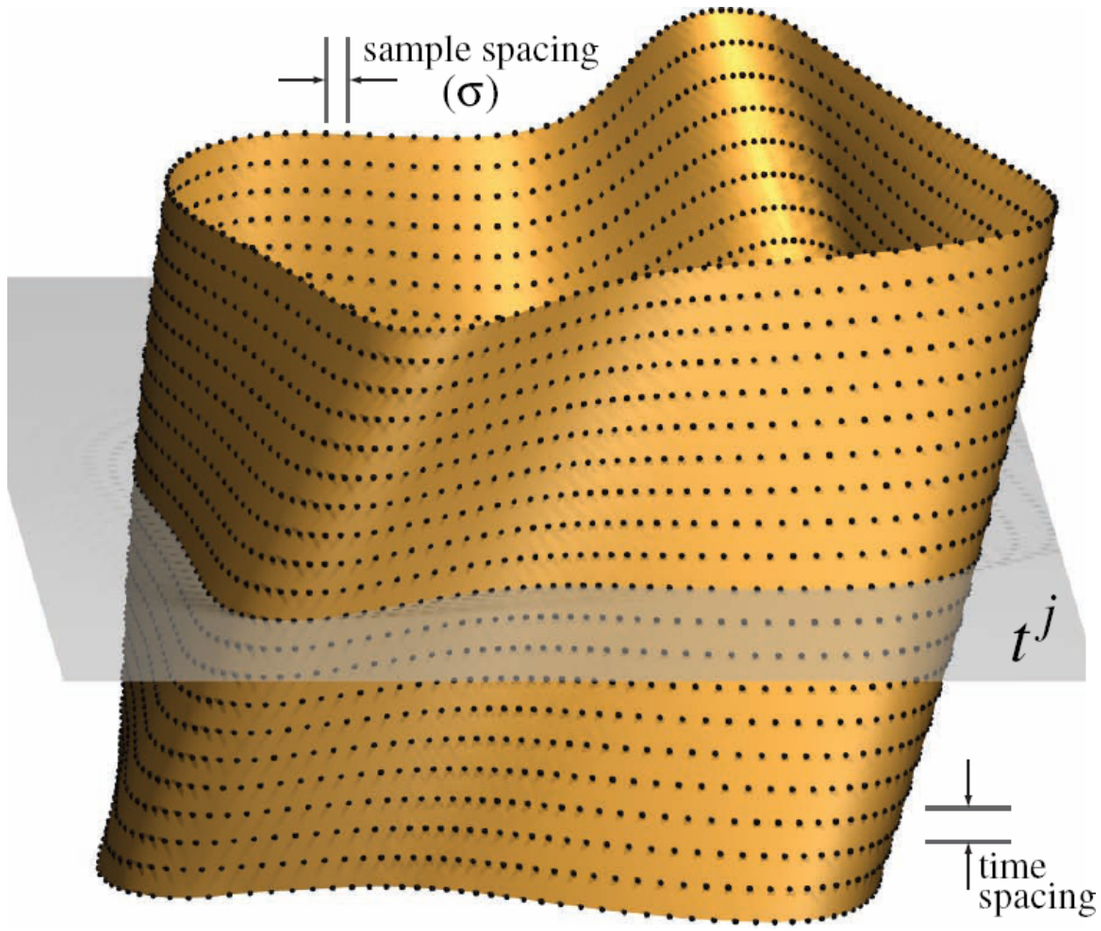


Space-Time Surface



$\widetilde{\mathbf{p}}_i^j$! $\widetilde{\alpha_j}(\widetilde{\mathbf{p}}_i^j) = \left(\mathbf{R}_j\mathbf{p}_i^j + \mathbf{t}_j, t^j + \Delta t^j\right)$

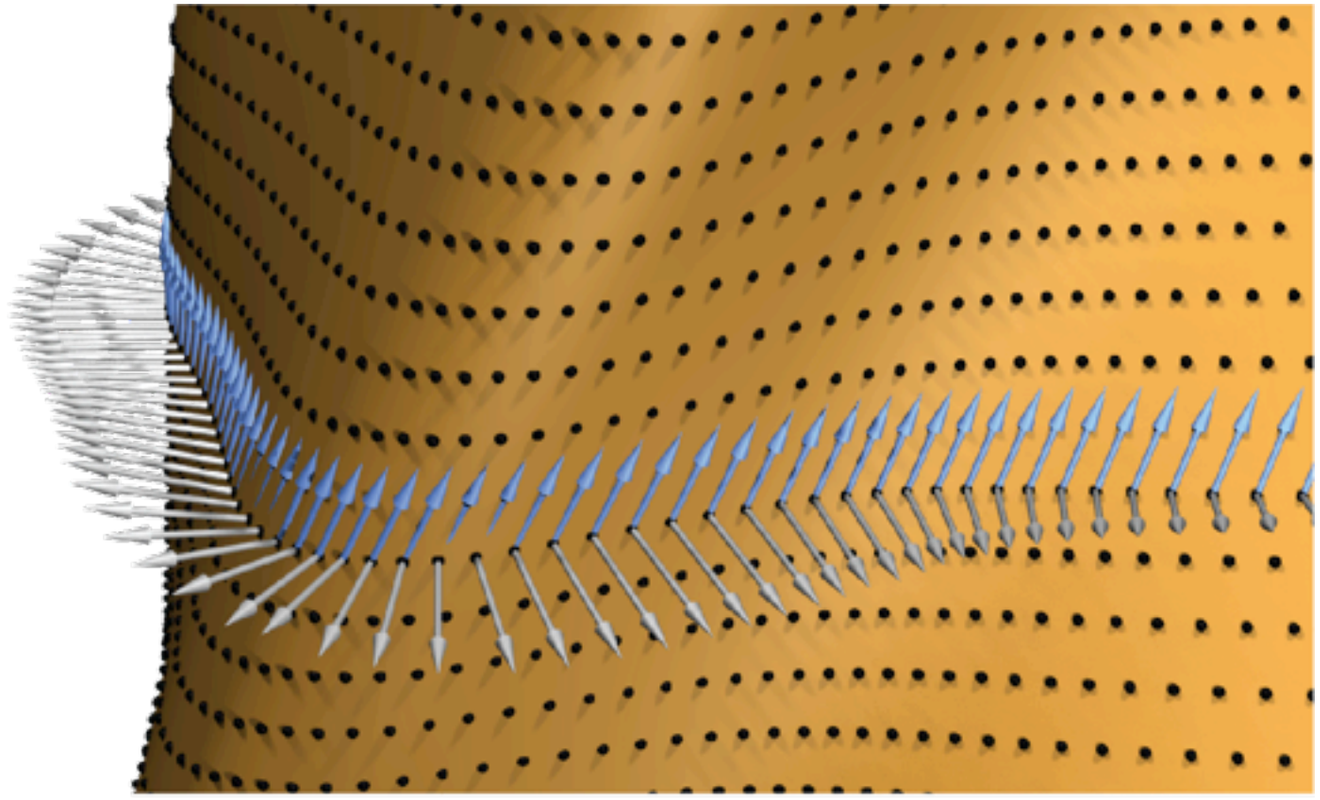
Space-Time Surface



$$\widetilde{\mathbf{p}}_i^j \quad ! \quad \widetilde{\alpha_j}(\widetilde{\mathbf{p}}_i^j) = \left(\mathbf{R}_j \mathbf{p}_i^j + \mathbf{t}_j, t^j + \Delta t^j\right)$$

$$\widetilde{\alpha_j} = \operatorname{argmin} \sum_{i=1}^{|P^j|} d^2(\widetilde{\alpha_j}(\widetilde{\mathbf{p}}_i^j), S)$$

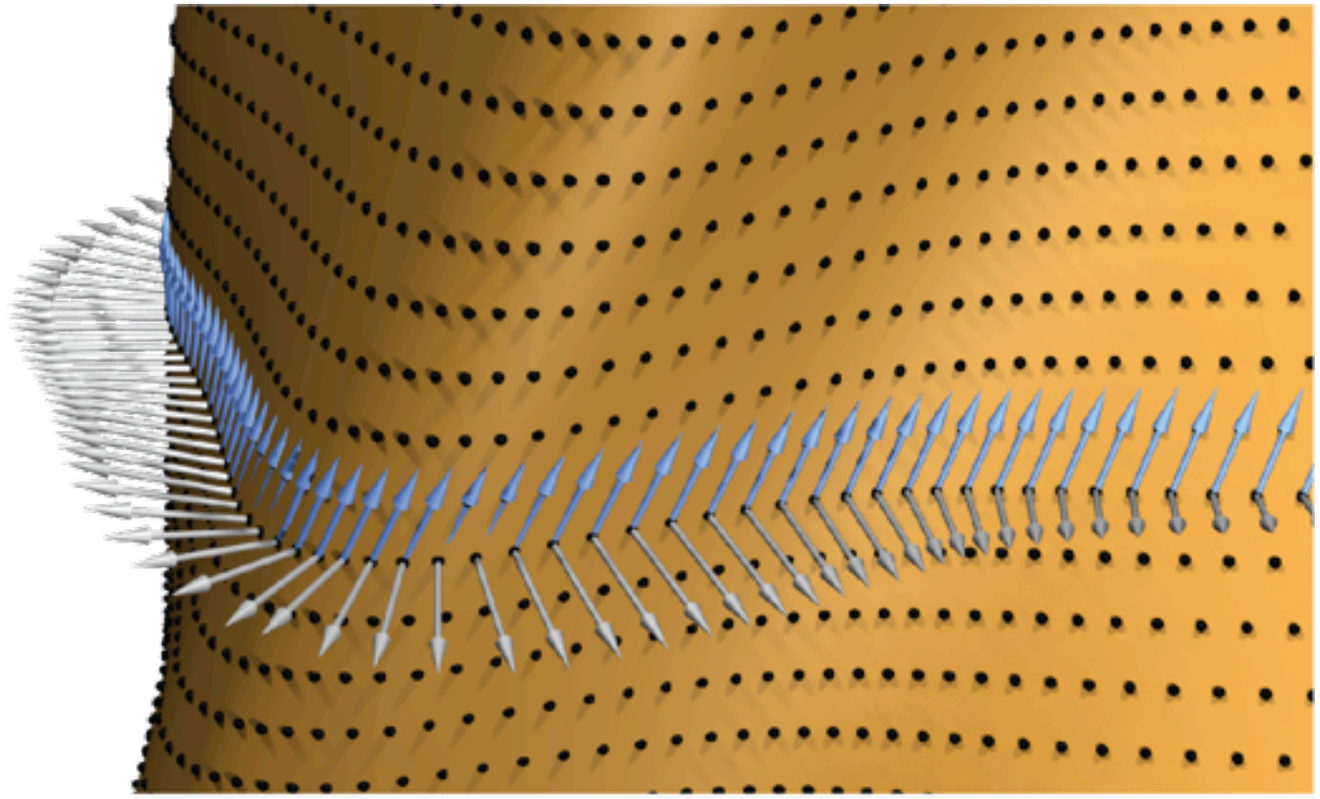
Space-Time Velocity Vectors



Tangential point movement ! velocity vectors orthogonal to surface normals

$$\widetilde{\alpha_j} = \operatorname{argmin} \sum_{i=1}^{|P^j|} d^2(\widetilde{\alpha_j}(\widetilde{\mathbf{p}}_i^j), S)$$

Space-Time Velocity Vectors



Tangential point movement ! velocity vectors orthogonal to surface normals

$$\tilde{v}^{j}(p_i).\tilde{n}^{j}(p_i) = 0$$

Final Steps

(rigid) velocity vectors !

$$\widetilde{\mathbf{v}}(\widetilde{\mathbf{p}}_i^j) = (\mathbf{c}_j \times \mathbf{p}_i^j + \overline{\mathbf{c}}_j, 1)$$

$$\min_{\mathbf{c}_j, \overline{\mathbf{c}}_j} \sum_{i=1}^{|P^j|} w_i^j \left[(\mathbf{c}_j \times \mathbf{p}_i^j + \overline{\mathbf{c}}_j, 1) \cdot \widetilde{\mathbf{n}}_i^j \right]^2$$

Final Steps

(rigid) velocity vectors !

$$\widetilde{\mathbf{v}}(\widetilde{\mathbf{p}}_i^j) = (\mathbf{c}_j \times \mathbf{p}_i^j + \overline{\mathbf{c}}_j, 1)$$

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$$A\mathbf{x} + \mathbf{b} = 0$$

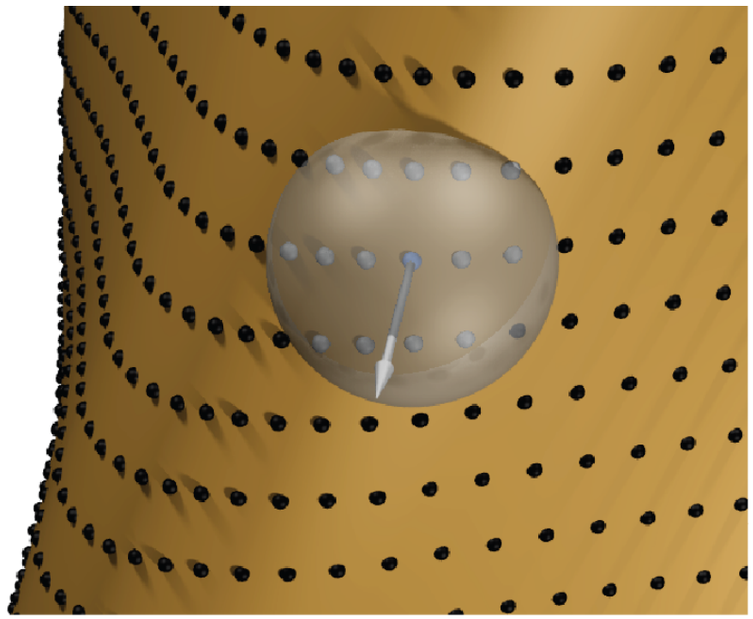
$$A = \sum_{i=1}^{|P^j|} w_i^j \begin{bmatrix} \bar{\mathbf{n}}_i^j \\ \mathbf{p}_i^j \times \bar{\mathbf{n}}_i^j \end{bmatrix} \begin{bmatrix} \bar{\mathbf{n}}_i^{j\,T} & (\mathbf{p}_i^j \times \bar{\mathbf{n}}_i^j)^T \end{bmatrix}$$

$$\mathbf{b} = \sum_{i=1}^{|P^j|} w_i^j n_i^j \begin{bmatrix} \bar{\mathbf{n}}_i^j \\ \mathbf{p}_i^j \times \bar{\mathbf{n}}_i^j \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} \bar{\mathbf{c}}_j \\ \mathbf{c}_j \end{bmatrix}$$

Registration Algorithm

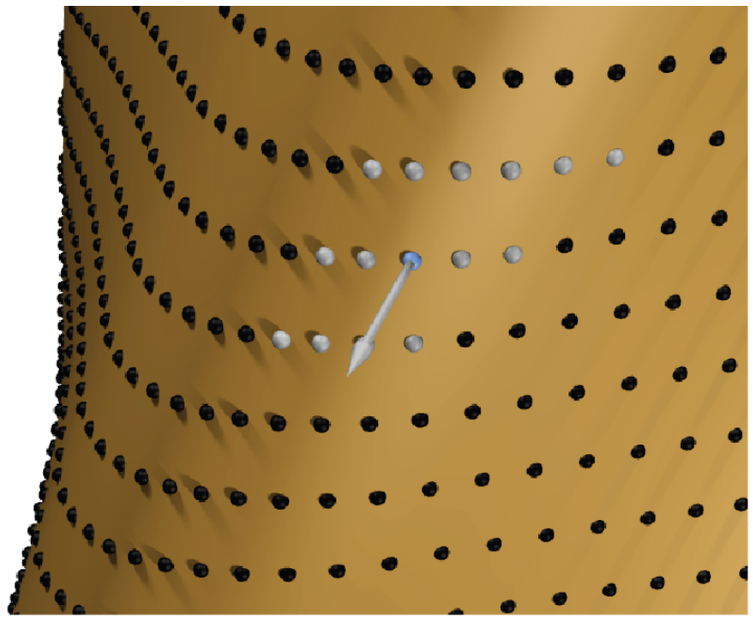
1. Compute time coordinate spacing (σ), and form space-time surface.
2. Compute space time neighborhood using ANN, and locally estimate space-time surface normals.
3. Solve linear system to estimate $(c_j, \bar{c}_j)$.
4. Convert velocity vectors to rotation matrix + translation vector using Plücker coordinates and quarternions.

Normal Estimation: PCA Based



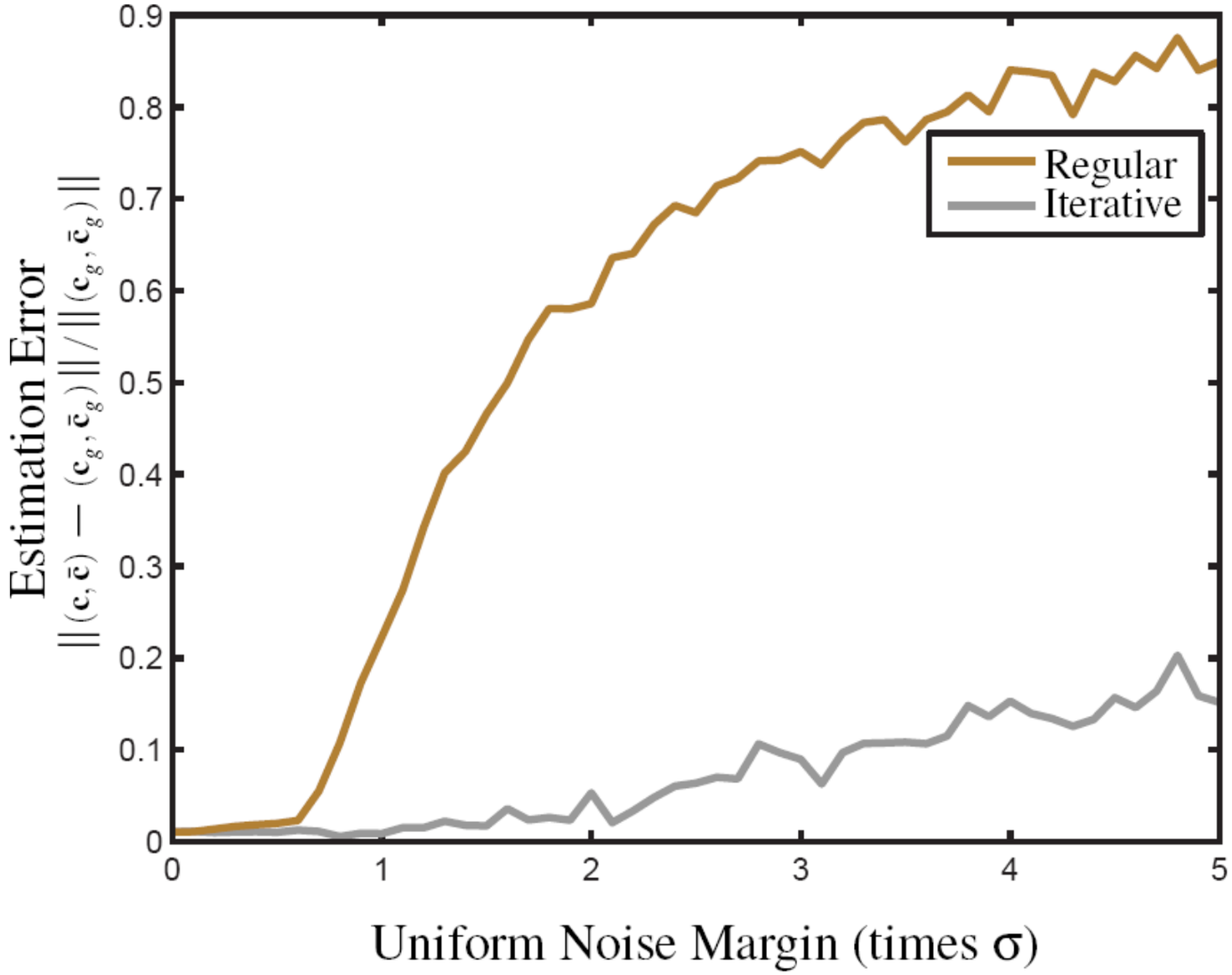
Plane fitting using PCA using chosen neighborhood points.

Normal Estimation: Iterative Refinement



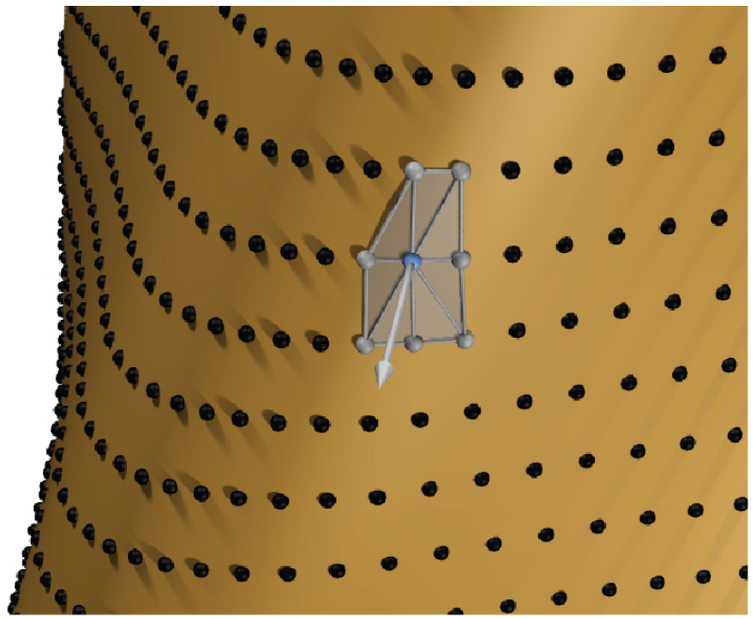
Update neighborhood with current velocity estimate.

Normal Estimation: Effect of Noise



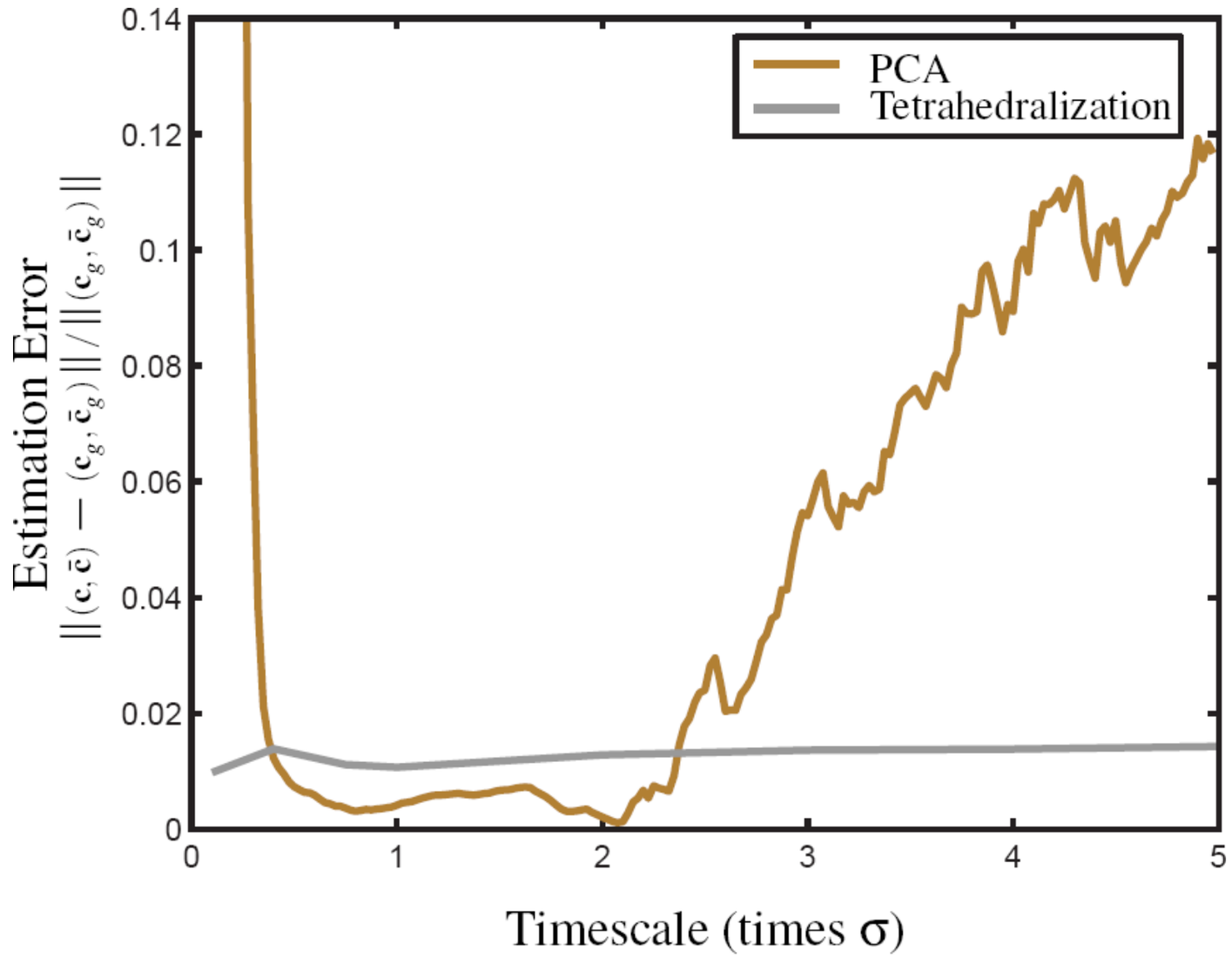
Stable, but more expensive.

Normal Estimation: Local Triangulation



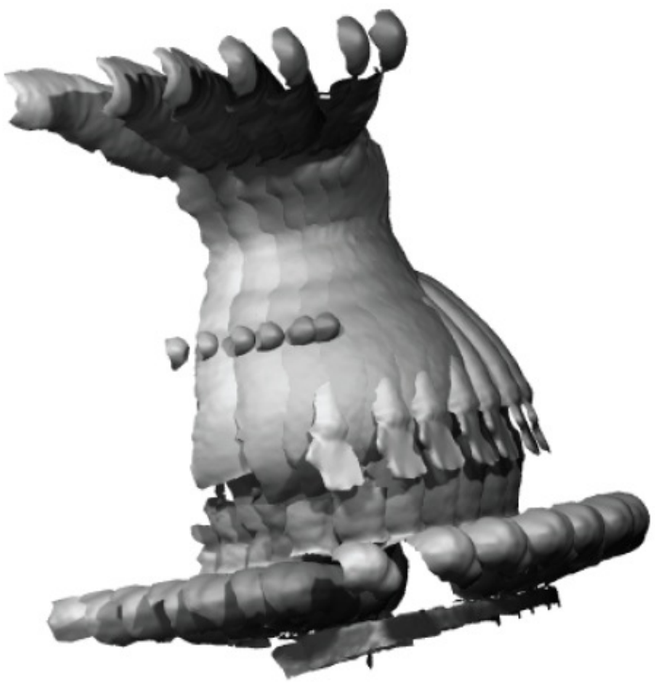
Perform local surface triangulation (tetrahedralization).

Normal Estimation



Stable, but more expensive.

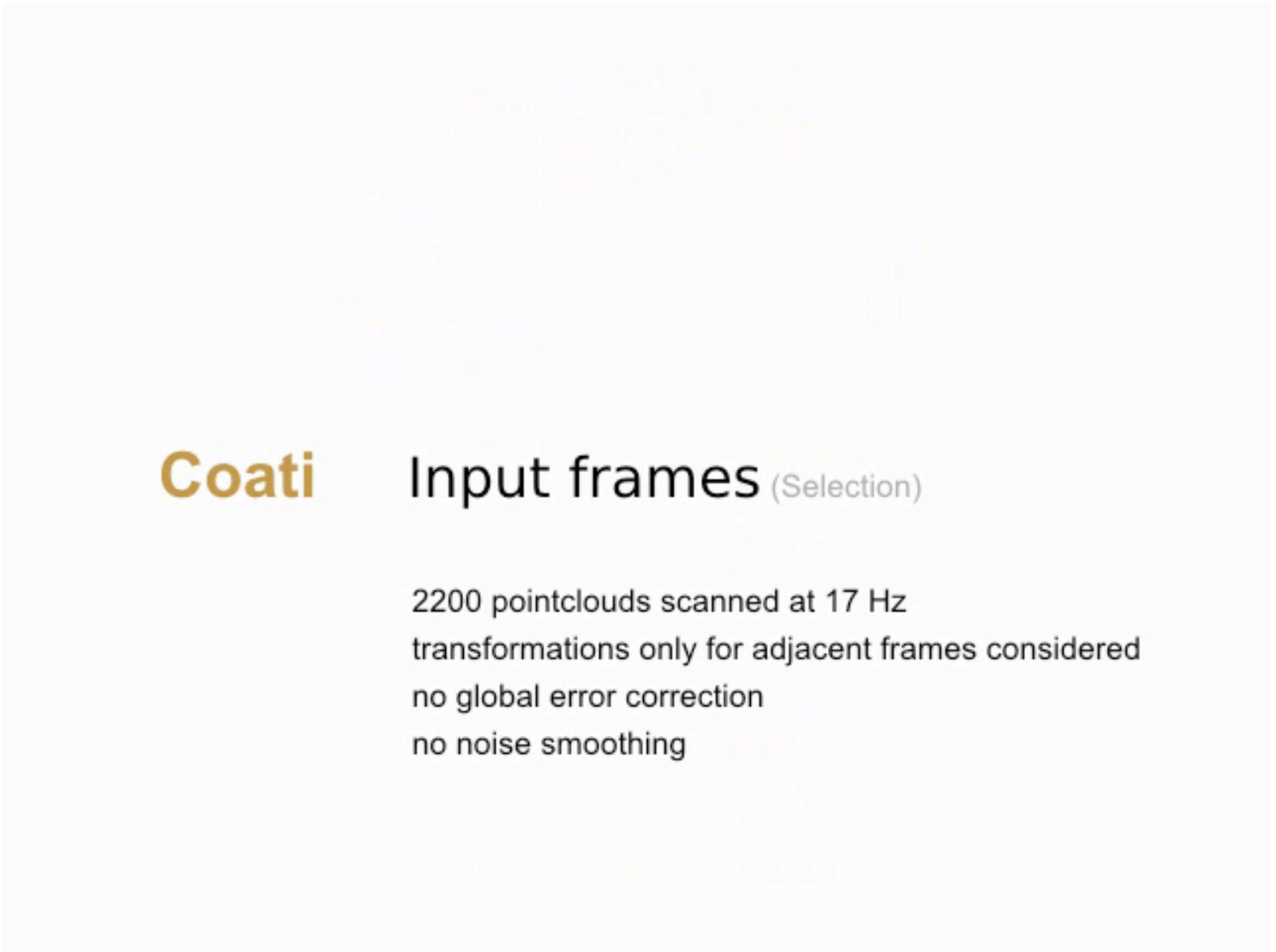
Comparison with ICP



ICP point-plane

Dynamic registration

Rigid/Deformable: Coati Sequence (2.2K frames)

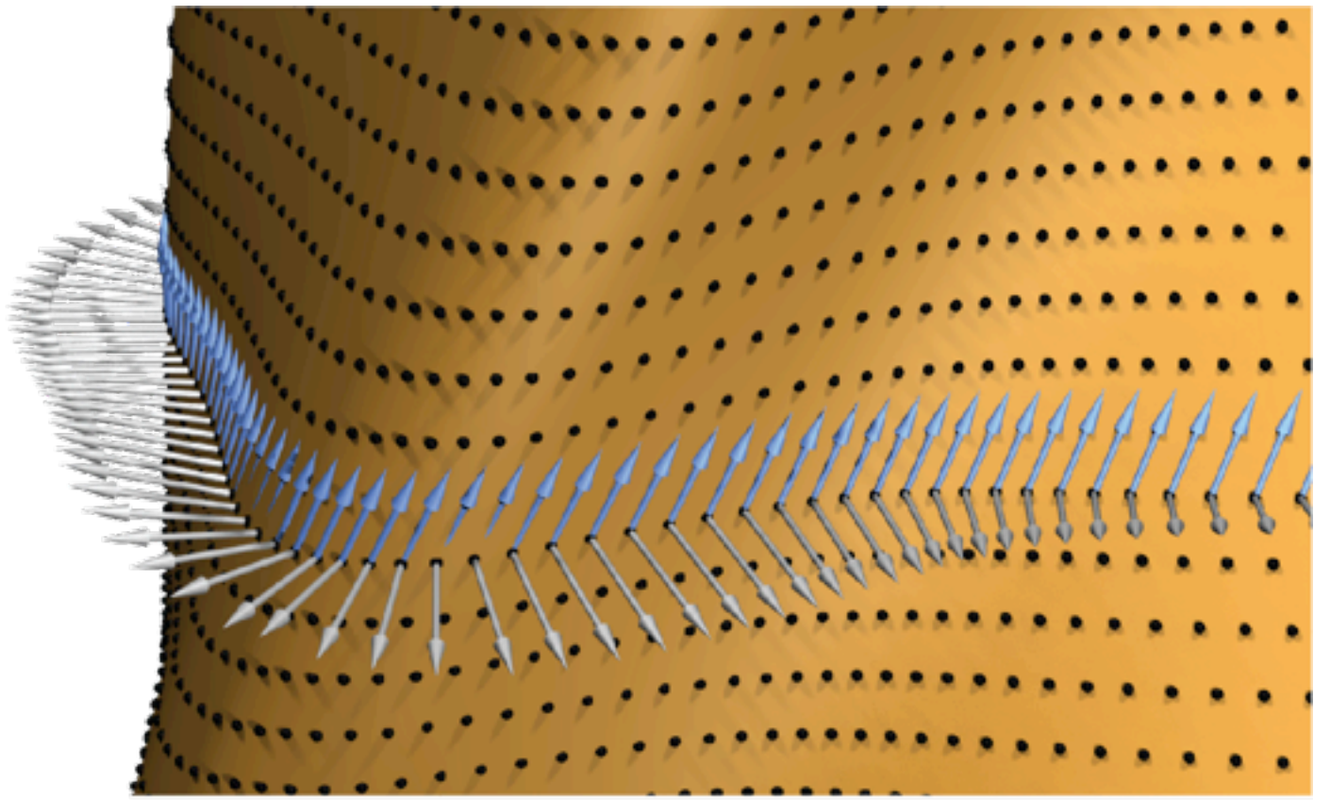


Rigid: Bee Sequence (2,2K frames)

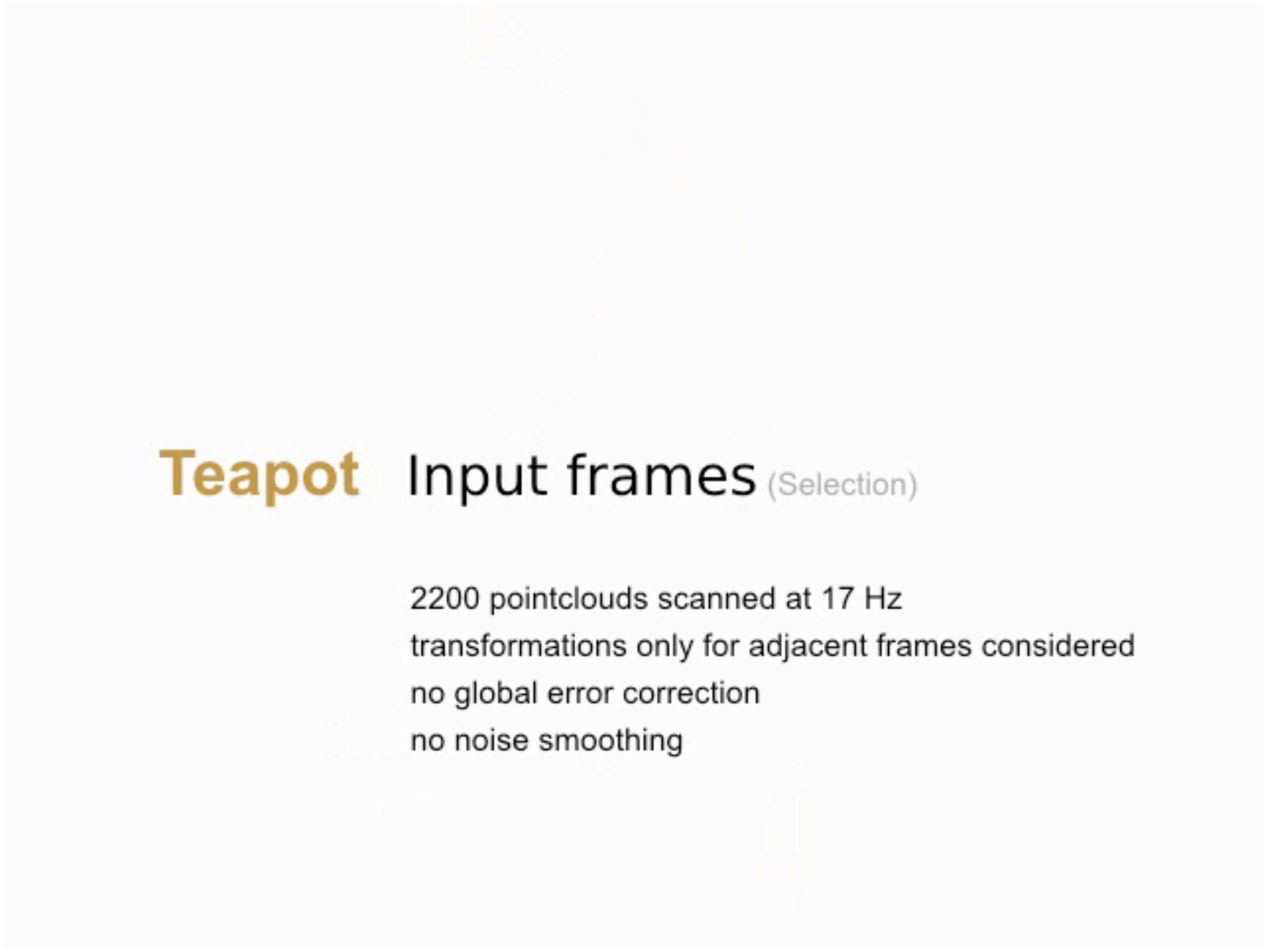
Bee Input frames (Selection)

2200 pointclouds scanned at 17 Hz
transformations only for adjacent frames considered
no global error correction
no noise smoothing

Handling Large Number of Frames



Rigid/Deformable: Teapot Sequence (2.2K frames)



Deformable Bodies

$$\min_{\mathbf{c}_j,\overline{\mathbf{c}}_j} \sum_{i=1}^{|P^j|} w_i^j \left[(\mathbf{c}_j \times \mathbf{p}_i^j + \overline{\mathbf{c}}_j, 1) \cdot \widetilde{\mathbf{n}}_i^j \right]^2$$

- Cluster points, and solve smaller systems.
- Propagate solutions with regularization.

Deformable: Hand (100 frames)

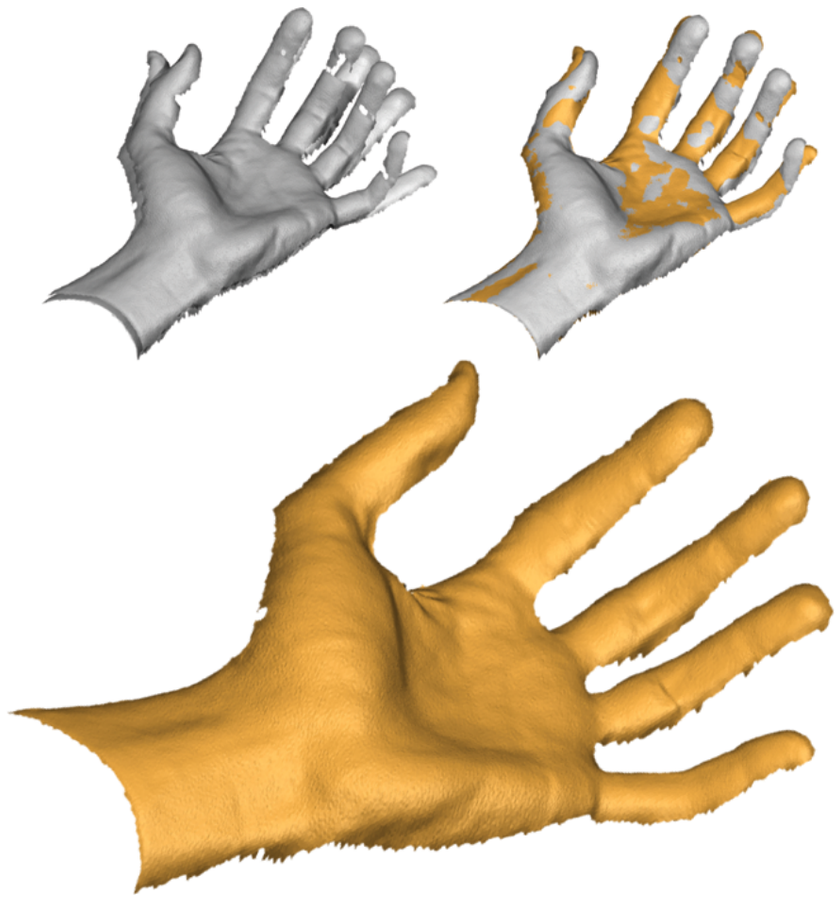
Hand

Input frames & registered result

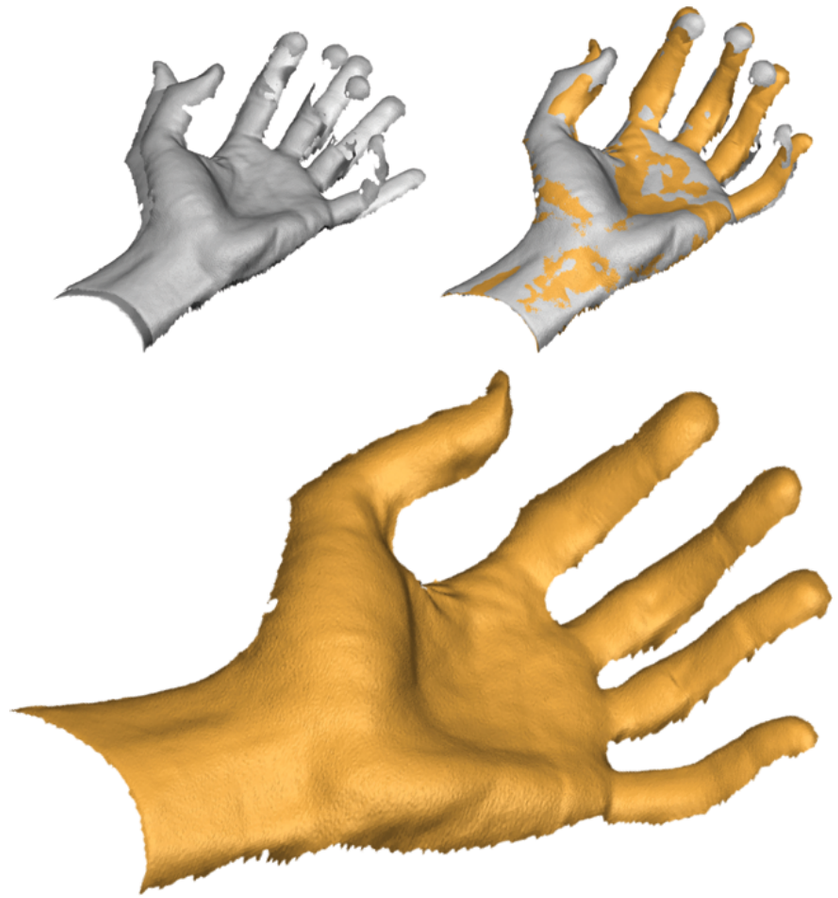
100 pointclouds scanned at 17 Hz
transformations only for adjacent frames considered
no global error correction
no noise smoothing

first frame is tracked
deformation due to severly missing data (e.g. ring finger)

Deformable: Hand (100 frames)



scan #1 ! scan #50



scan #1 ! scan #100

Conclusion

- Simple algorithm using kinematic properties of space-time surface.
- Easy modification for deformable bodies.
- Suitable for use with fast scanners.

Conclusion

- Simple algorithm using kinematic properties of space-time surface.
- Easy modification for deformable bodies.
- Suitable for use with fast scanners.

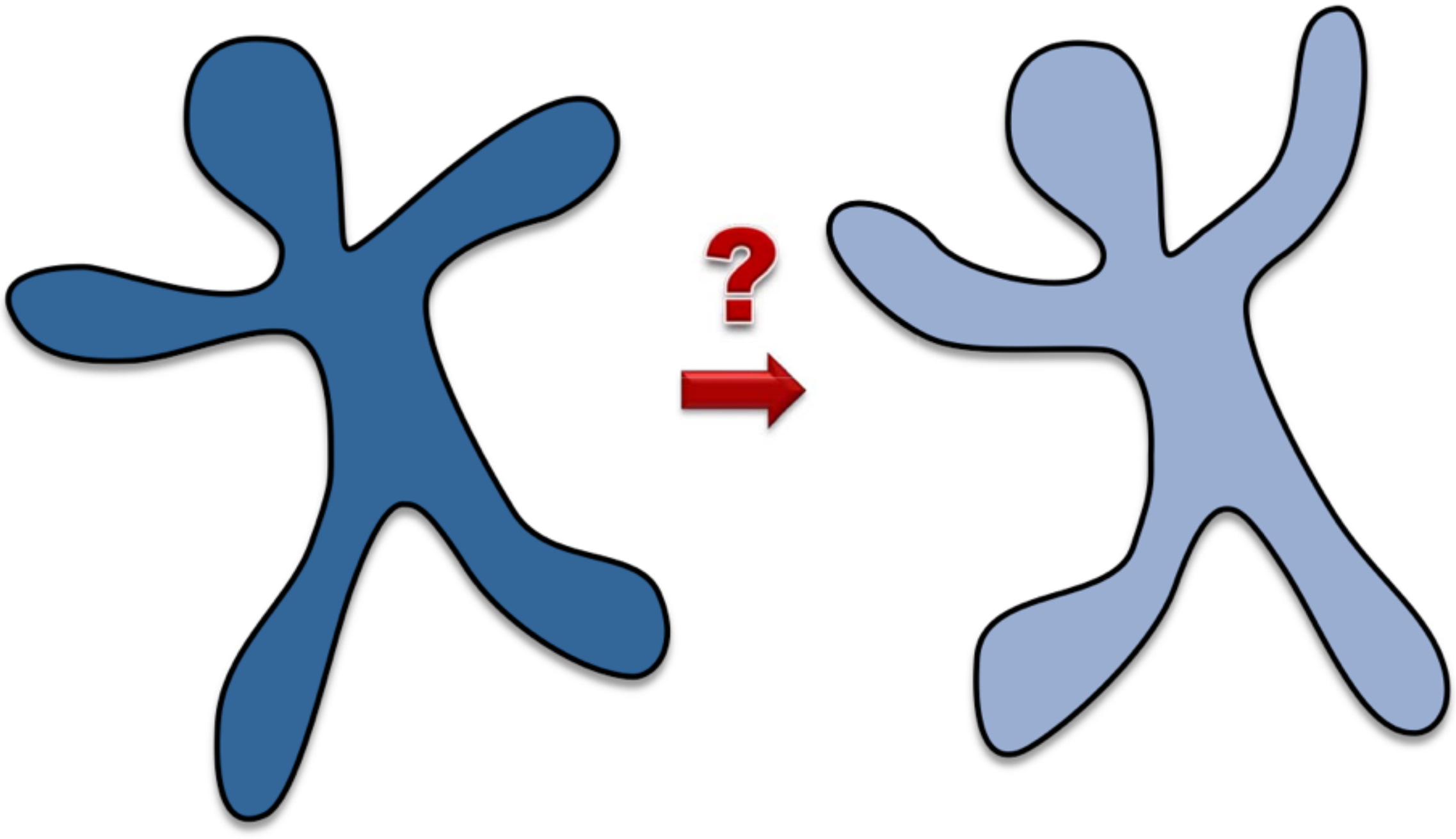
Limitations/Future Work

- Need more scans, dense scans, …
- Sampling condition ! time and space

Prof. Michael Wand
Utrecht University

Animation Reconstruction “Urshapes”!

Correspondences

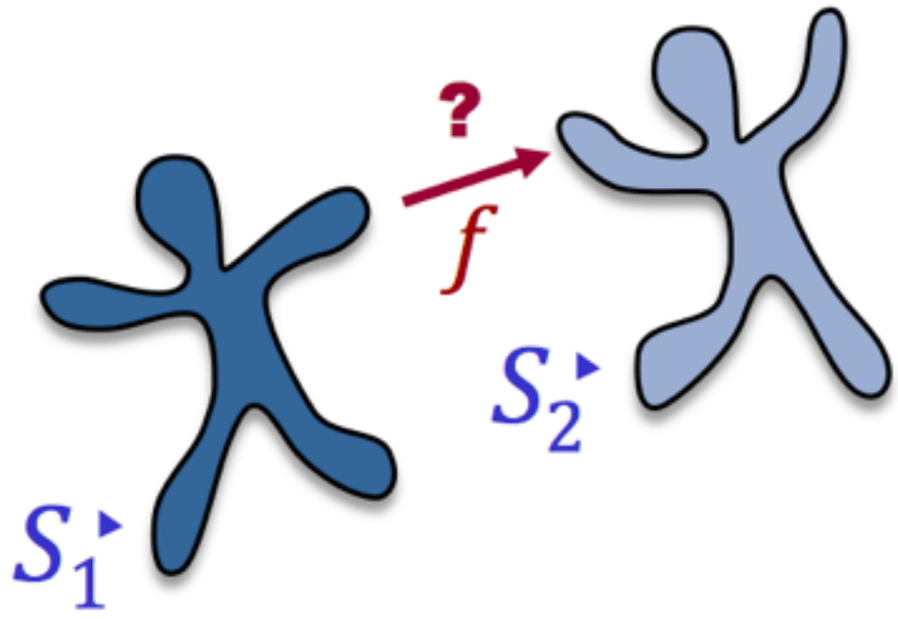


Problem

Problem Statement:

Given:

- Two surfaces $S_1, S_2 \subseteq \mathbb{R}^3$



We are looking for:

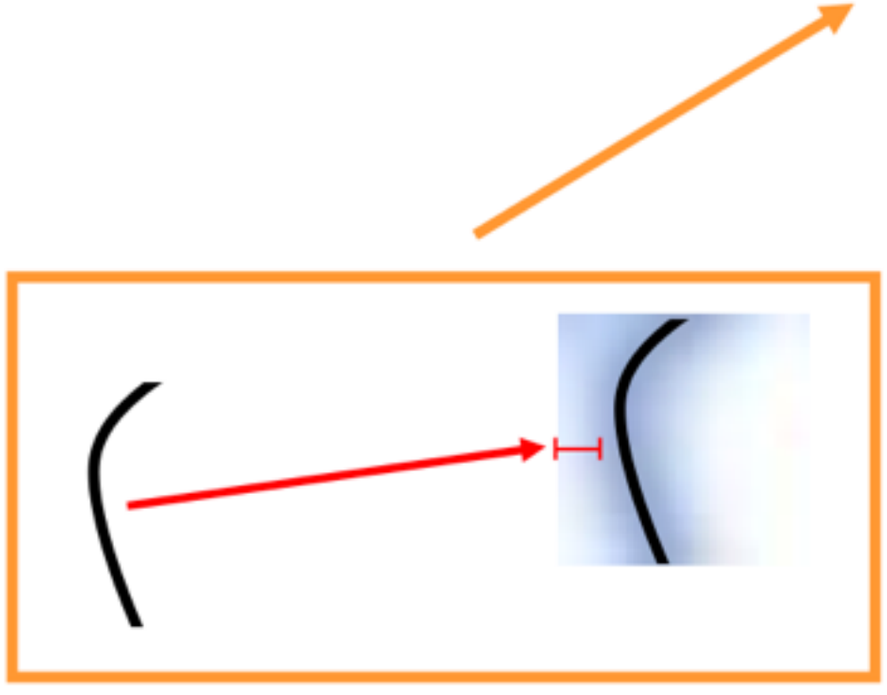
- A *reasonable* deformation function f: $S_1 \to \mathbb{R}^3$ that brings S_1 close to S_2

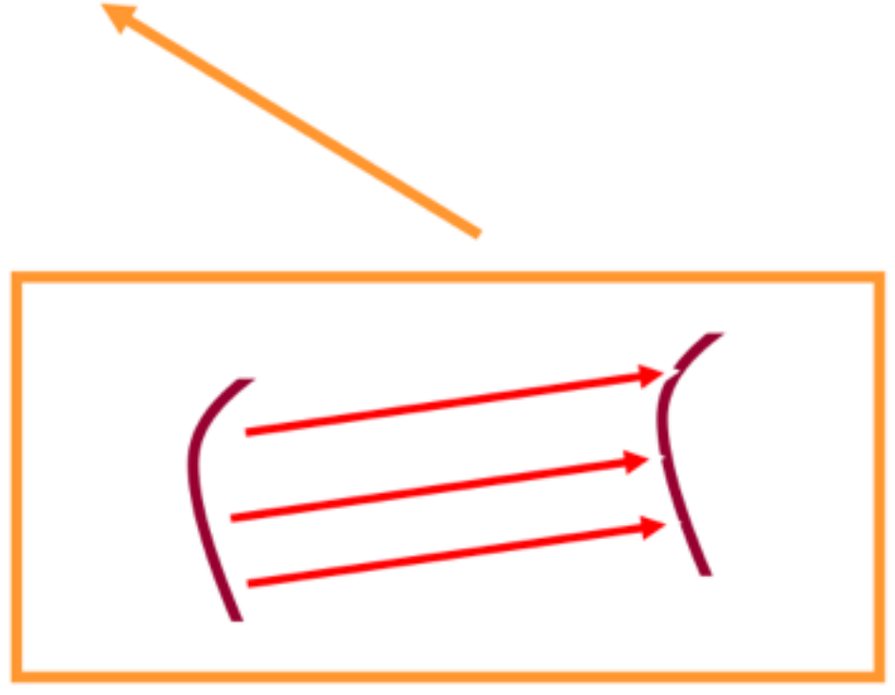
Variational Problem

Variational Problem:

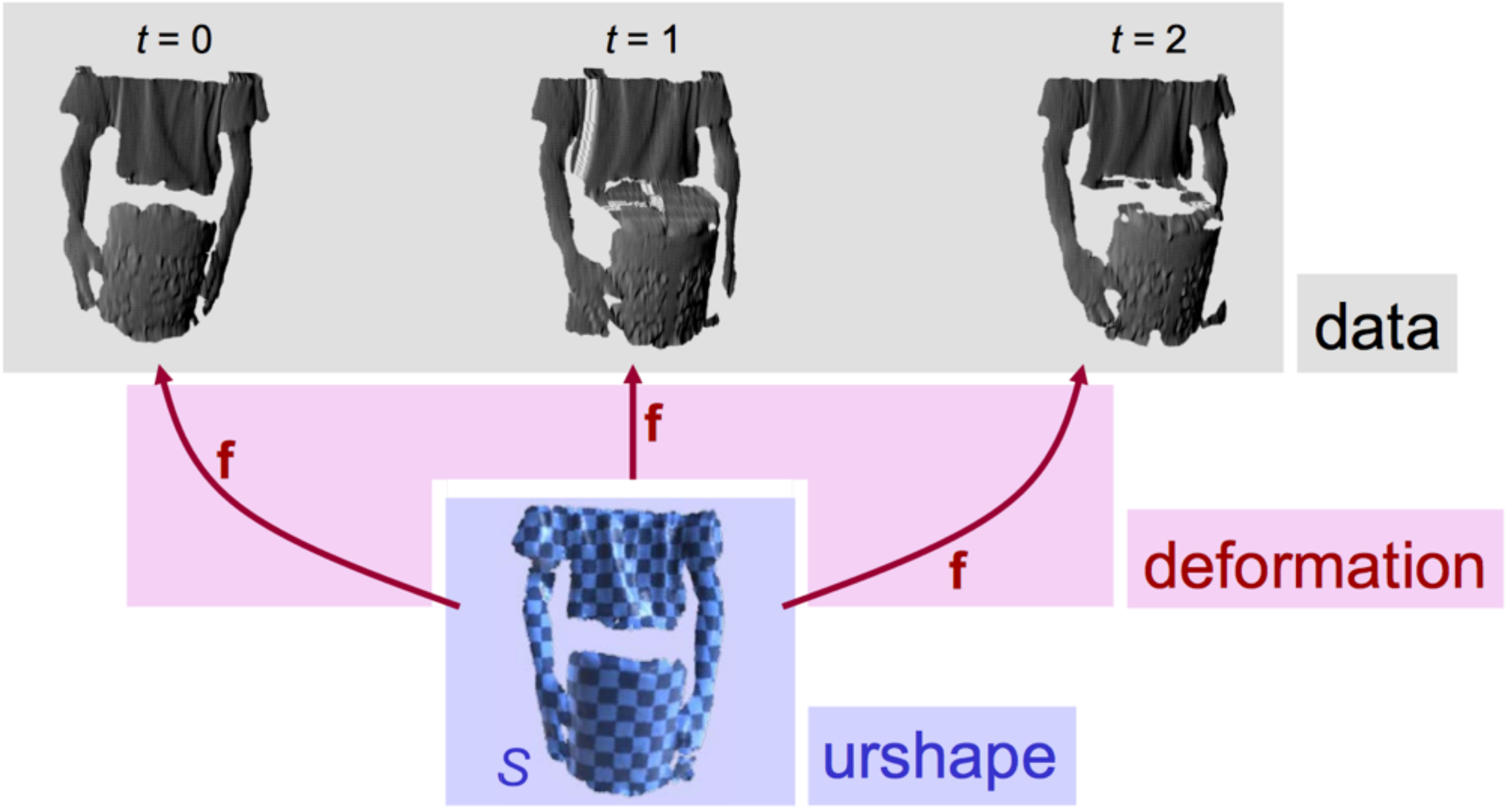
- Formulate as an energy minimization problem:

$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$





Scan Sequence



Hierarchical Merging

data

$\mathbf{f}(S)$

$\mathbf{f}$

S

Hierarchical Merging

data

$\mathbf{f}(S)$

$\mathbf{f}$

S

Initial Urshape

data

f(*S*)

f

S

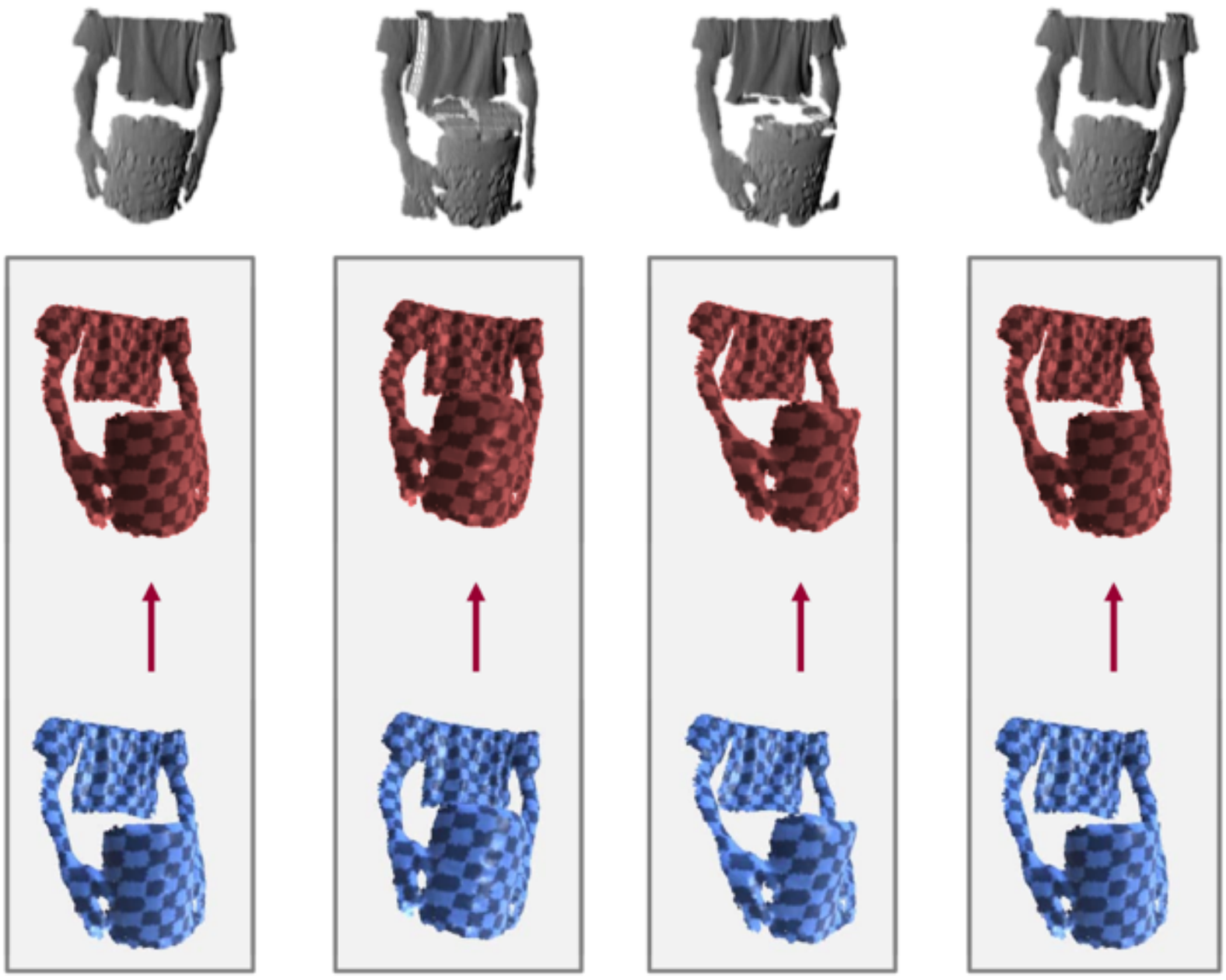
Initial Urshape

data

f(*S*)

f

S



Alignment

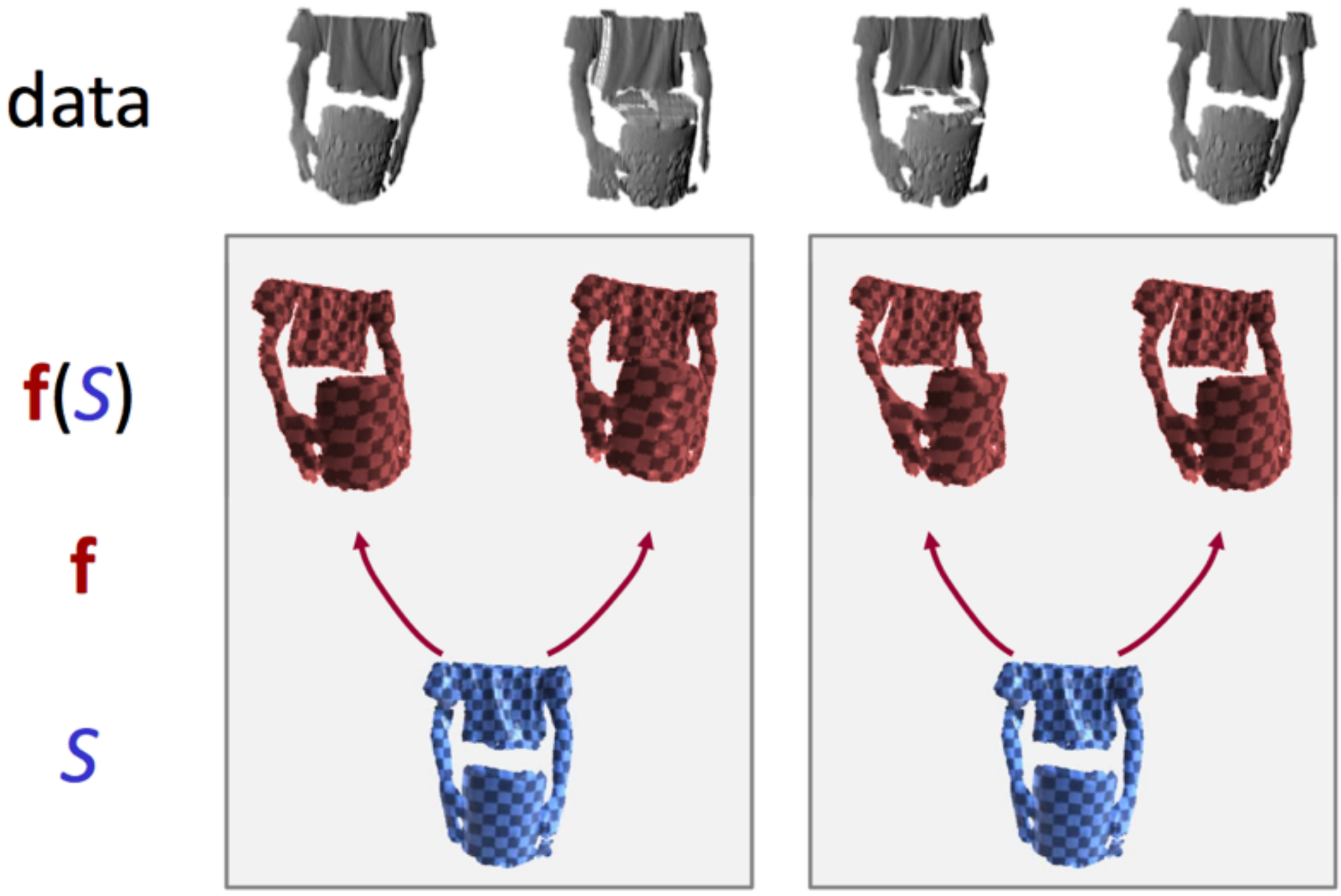
data

f(*S*)

f

S

Alignment



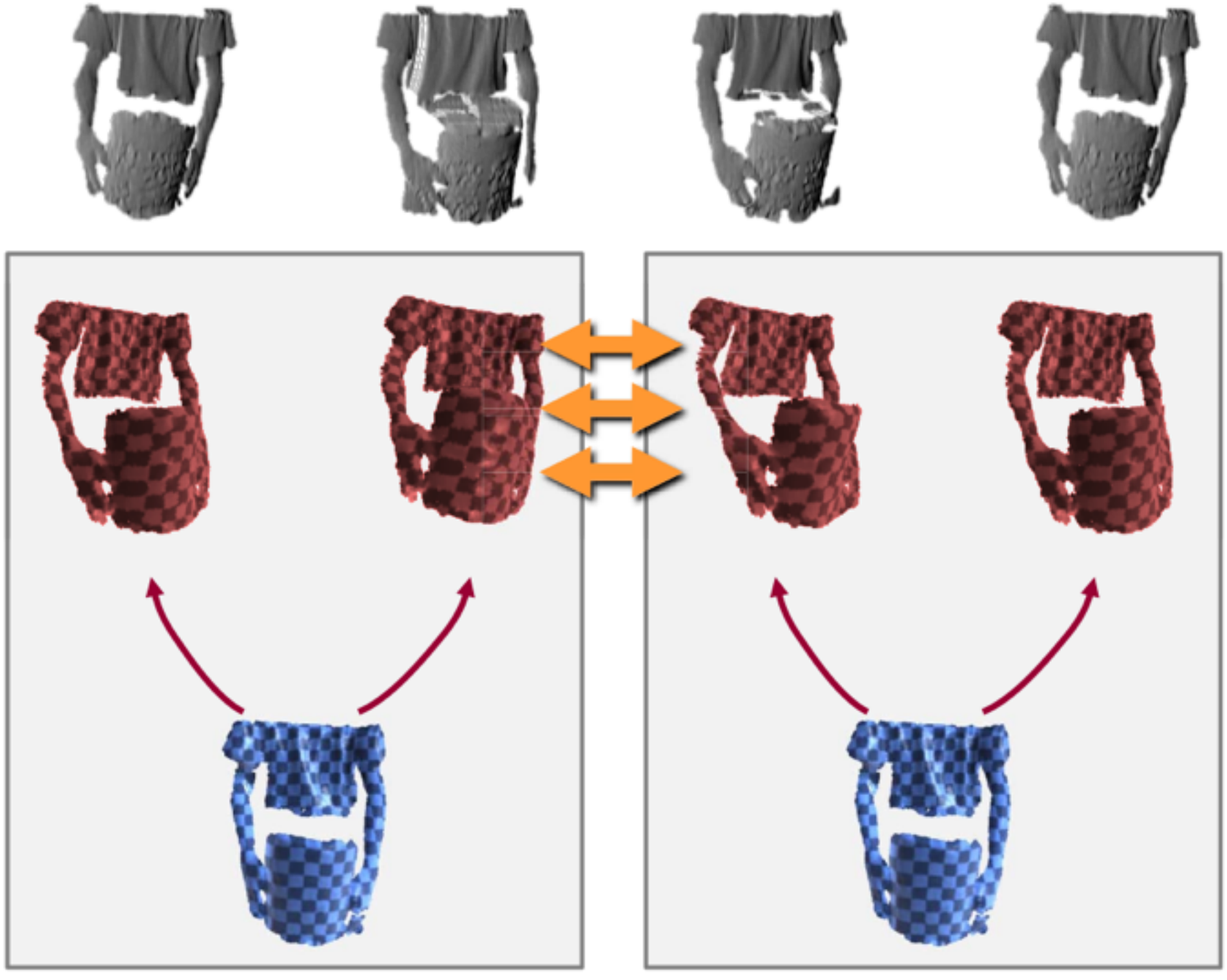
Hierarchical **Alignment**

data

f(*S*)

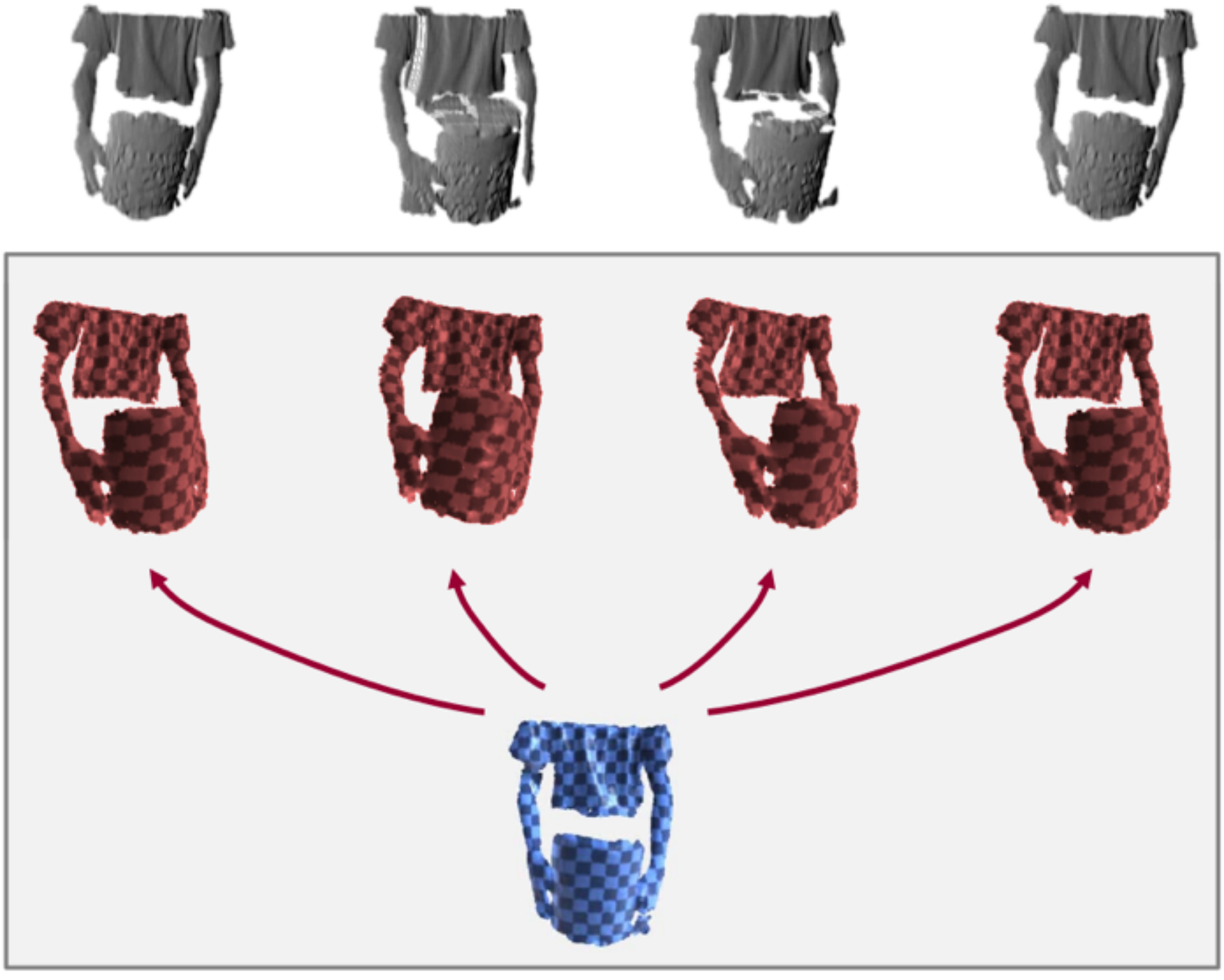
f

S



Hierarchical **Alignment**

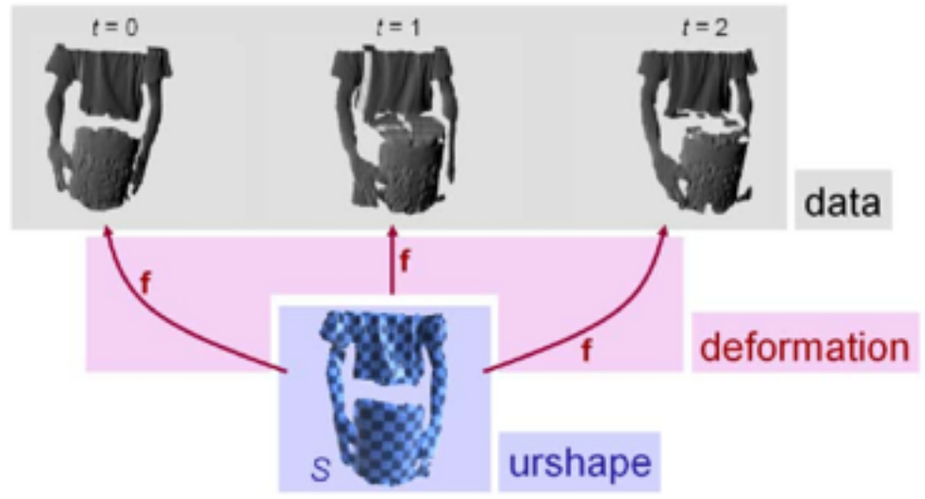
data



Global **Optimization**

Energy Function

$$E(\mathbf{f}, S) = E_{data} + E_{deform} + E_{smooth}$$



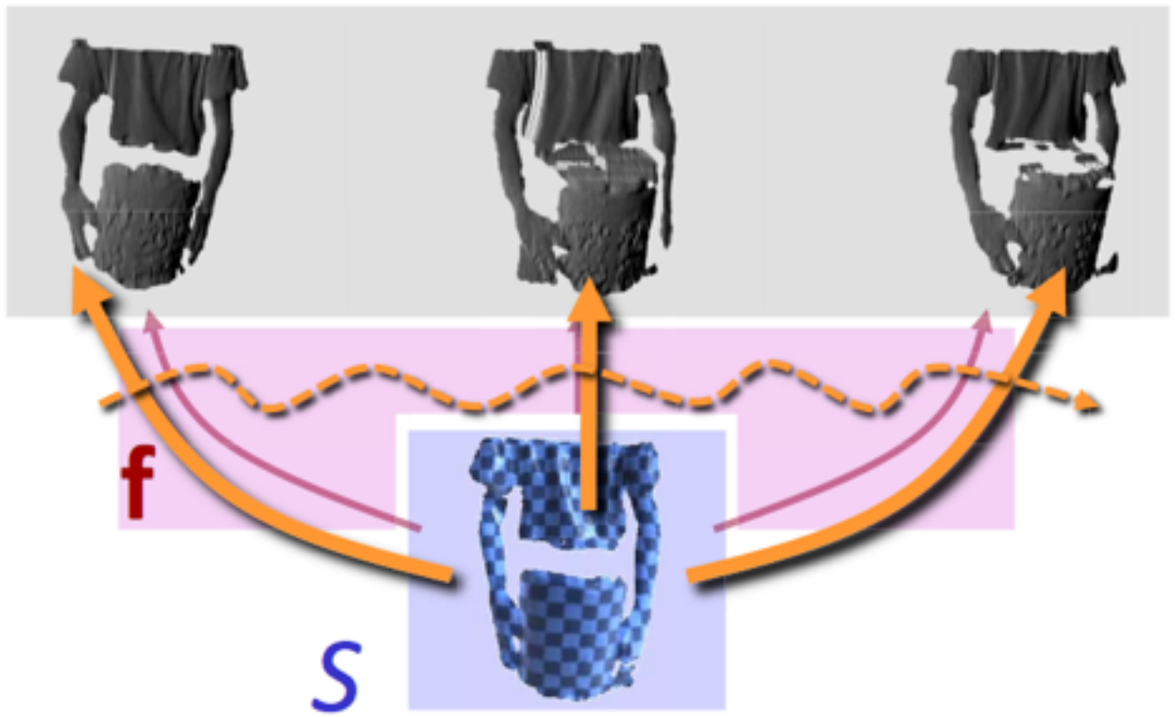
Components

- $E_{data}(\mathbf{f}, S)$ – data fitting
- $E_{deform}(\mathbf{f})$ – elastic deformation, smooth trajectory
- $E_{smooth}(S)$ – smooth surface

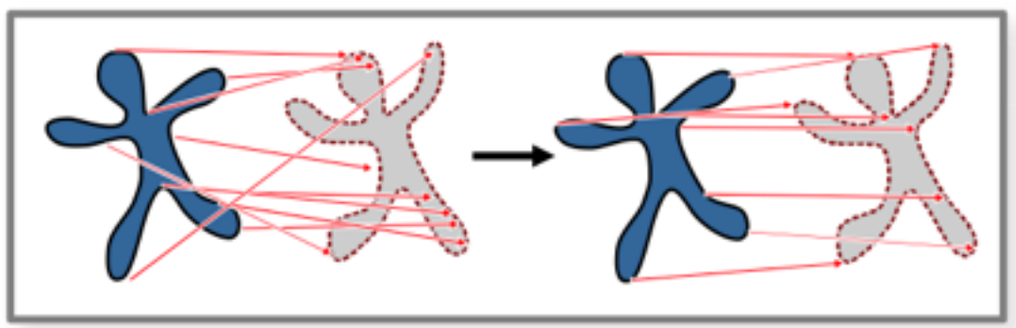
Final Optimization

- Minimize over all frames

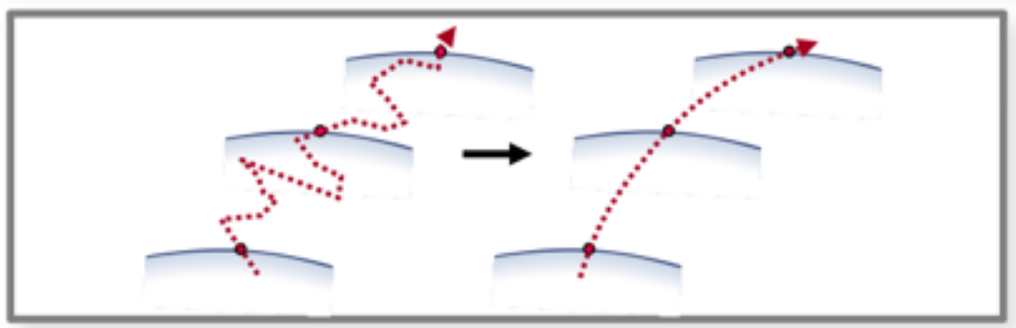
Deformation Field



- Elastic energy

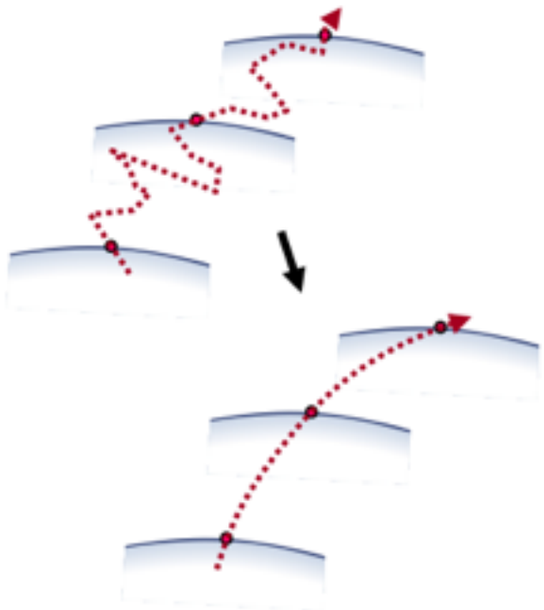


- Smooth trajectories



More Regularization

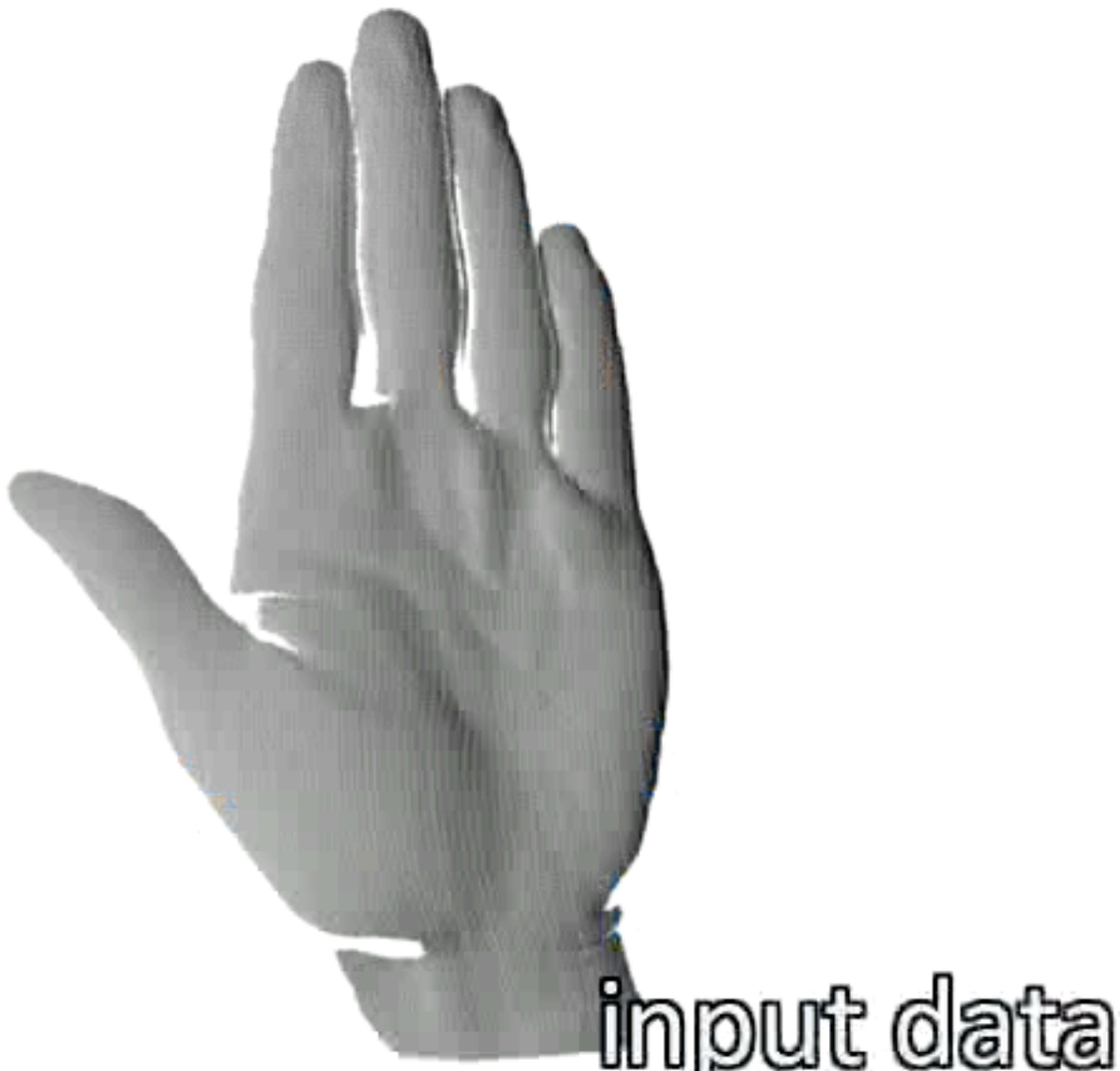
- Acceleration: $E_{acc} = \int_T \int_V |\partial_t^2 \mathbf{f}|^2$
  - Smooth trajectories
- Velocity (weak): $E_{vel} = \int_T \int_V |\partial_t \mathbf{f}|^2$
  - Damping



Results

input data

Results



Extension: Animation Cartography

[Tevs et al. 2012]

http://cs599.hao-li.com

Thanks!