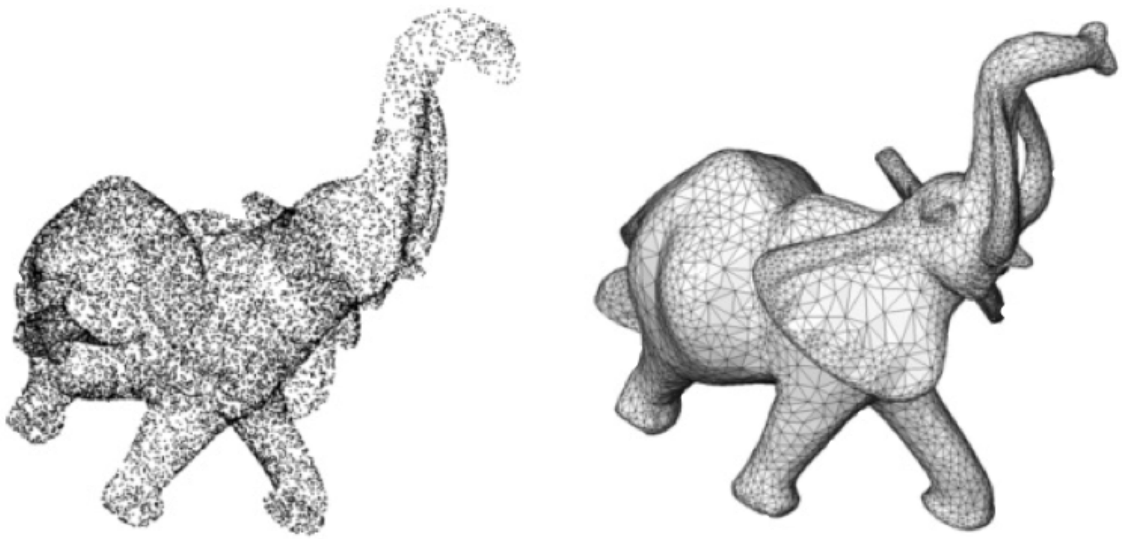
CSCI 599: **Digital Geometry Processing**



6.2 **Surface Reconstruction**

Hao Li

http://cs599.hao-li.com

Surface Reconstruction

physical
model

captured
point cloud

reconstructed
model

Input Data

Set of irregular sample points

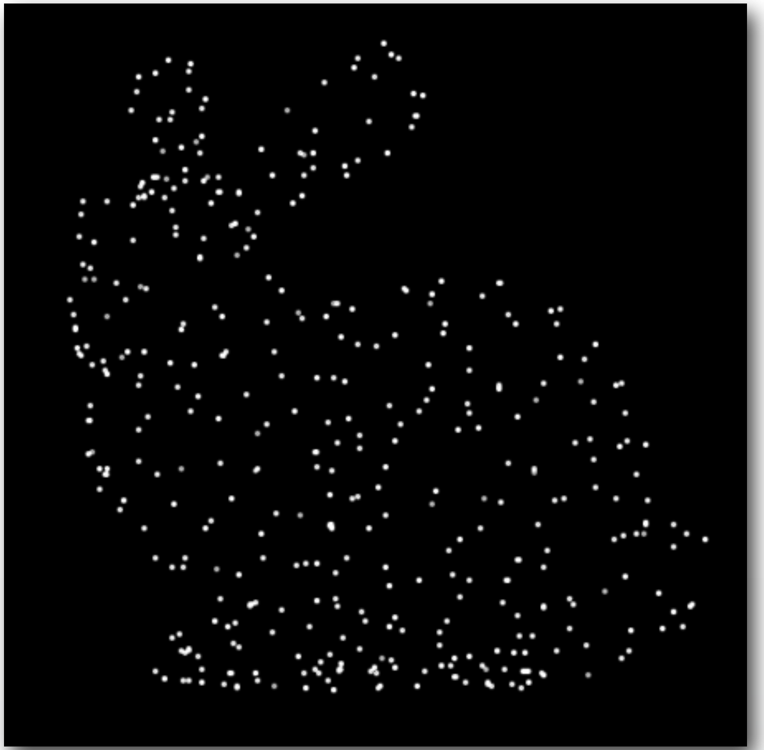
- with or without normals
- examples: multi-view stereo, union of range scan vertices

Set of range scans

- each scan is a regular quad or tri-mesh
- normal vectors can be obtained through local connectivity

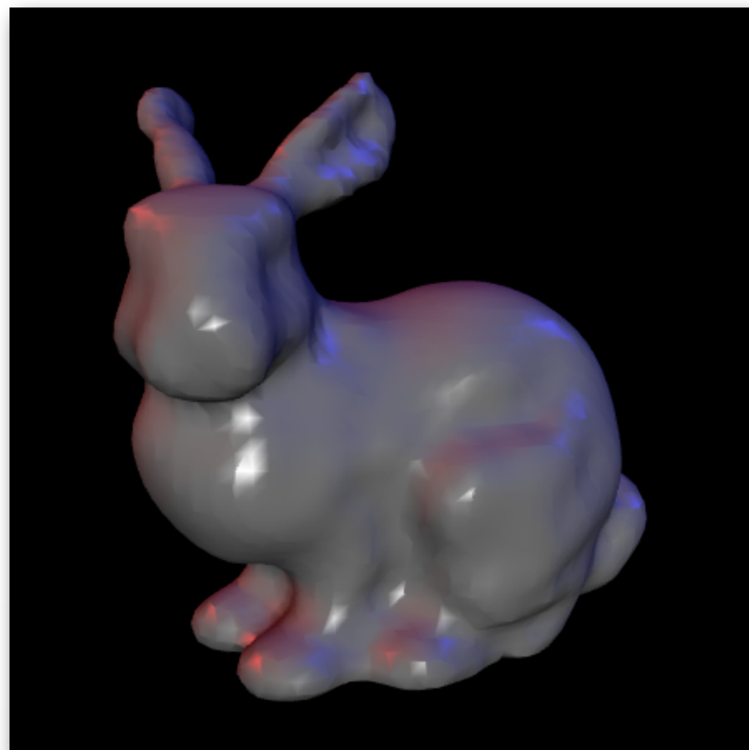
Problem

Given a set of points $\mathcal{P} = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$ with $\mathbf{p}_i \in \mathbb{R}^3$



Problem

Find a manifold surface $\mathcal{S} \subset \mathbb{R}^3$ which approximates $\mathcal{P}$



Two Approaches

Explicit

Local surface connectivity estimation

Point interpolation

Implicit

Signed distance function estimation

Mesh approximation

Two Approaches

Explicit

– Ball pivoting algorithm
– Delaunay triangulation
– Alpha shapes
– Zippering...

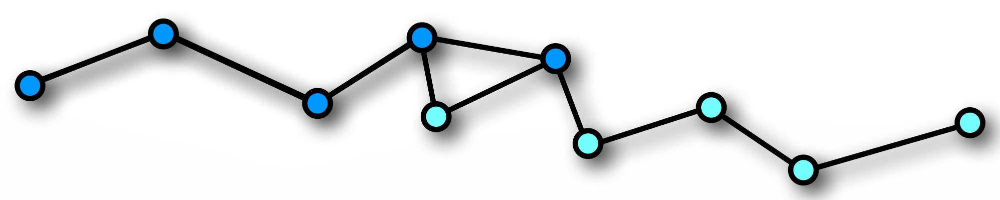
– Image space triangulation

Implicit

– Distance from tangent planes
– SDF estimation via RBF
– ...

Explicit Reconstruction

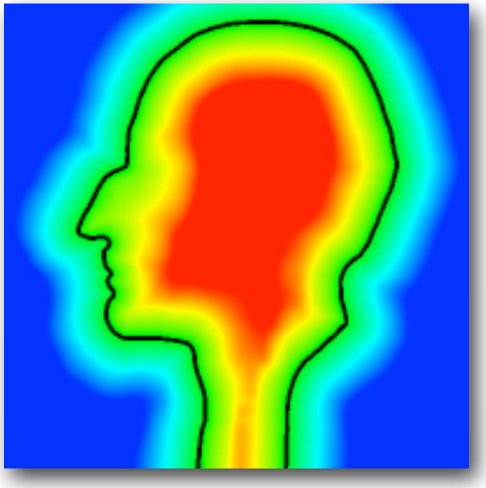
- Connect sample points by triangles
- Exact interpolation of sample points
- Bad for noisy or misaligned data
- Can lead to holes or non-manifold situations



Implicit Reconstruction

Given a set of points $\mathcal{P} = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$ with $\mathbf{p}_i \in \mathbb{R}^3$
Find a manifold surface $\mathcal{S} \subset \mathbb{R}^3$ which approximates $\mathcal{P}$

where $\mathcal{S} = \{\mathbf{x} \,|\, d(\mathbf{x}) = 0\}$ with $d(\mathbf{x})$ a signed distance function



Data Flow

Point cloud

Signed distance function estimation

$d(\mathbf{x}) \downarrow$

Evaluation of distances on uniform grid

$d(\mathbf{i}), \mathbf{i} = [i, j, k] \in \mathbb{Z}^3 \downarrow$

Mesh extraction via marching cubes

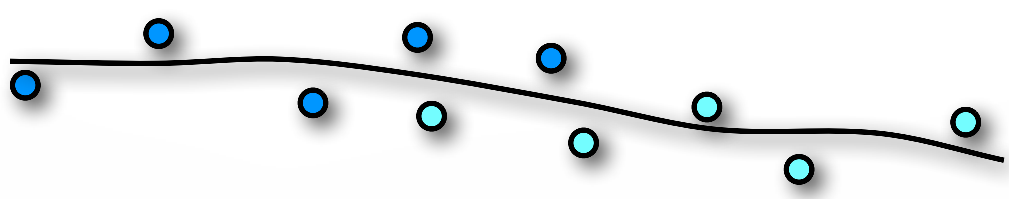
Mesh

Implicit Surface Reconstruction Methods

Mainly differ in their signed distance function

Implicit Reconstruction

- Estimate signed distance function (SDF)
- Extract Zero isosurface by Marching Cubes
- Approximation of input points
- Result is closed two-manifold surface

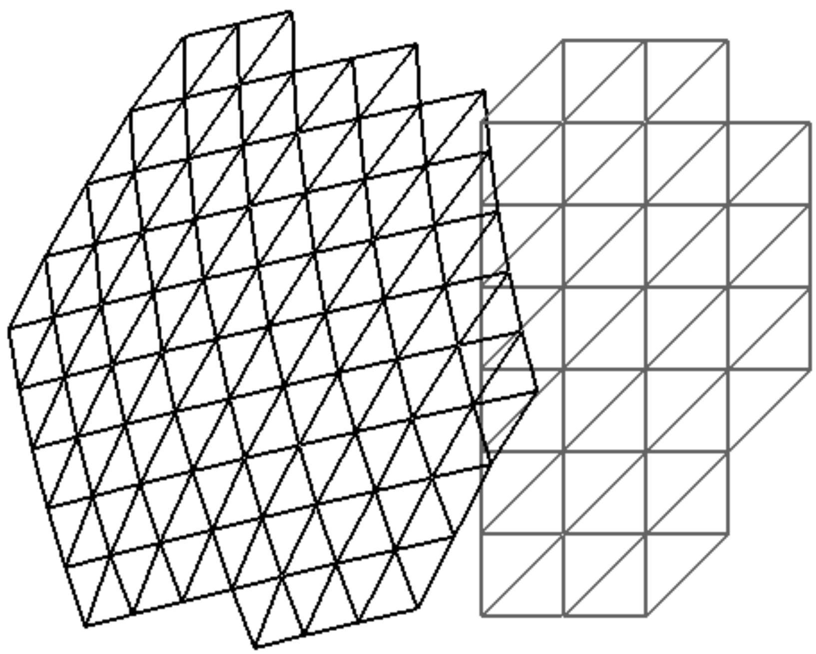


Outline

- **Explicit Reconstruction**
  - Zippering range scans
- **Implicit Reconstruction**
  - SDF from point clouds
  - SDF from range scans
  - Poisson surface reconstruction

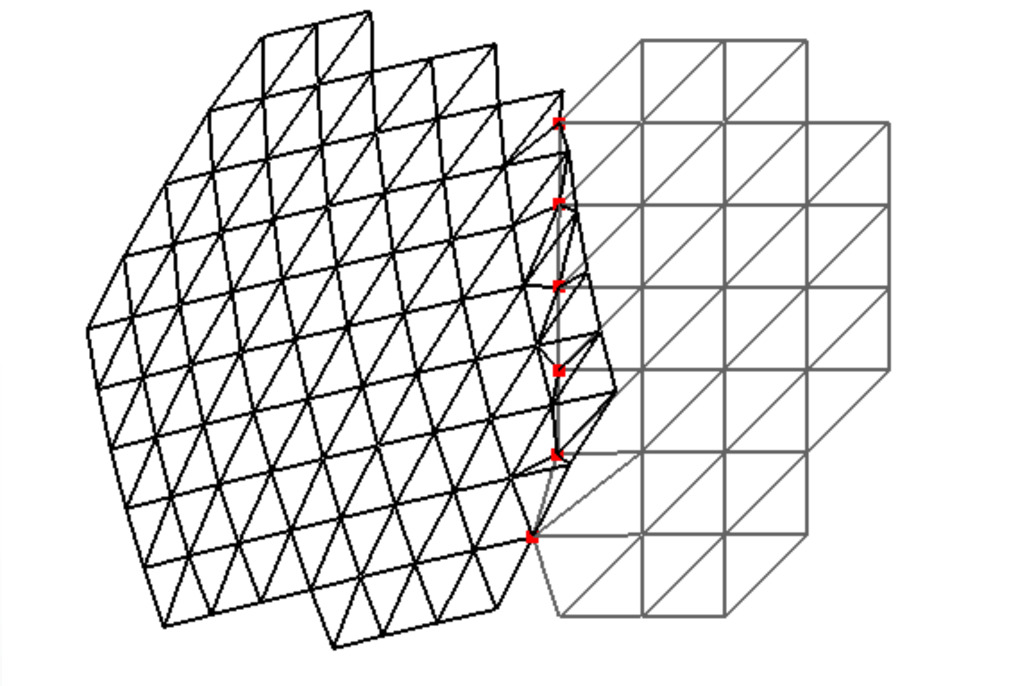
Explicit Reconstruction

“Zipper” several scans to one single model



Explicit Reconstruction

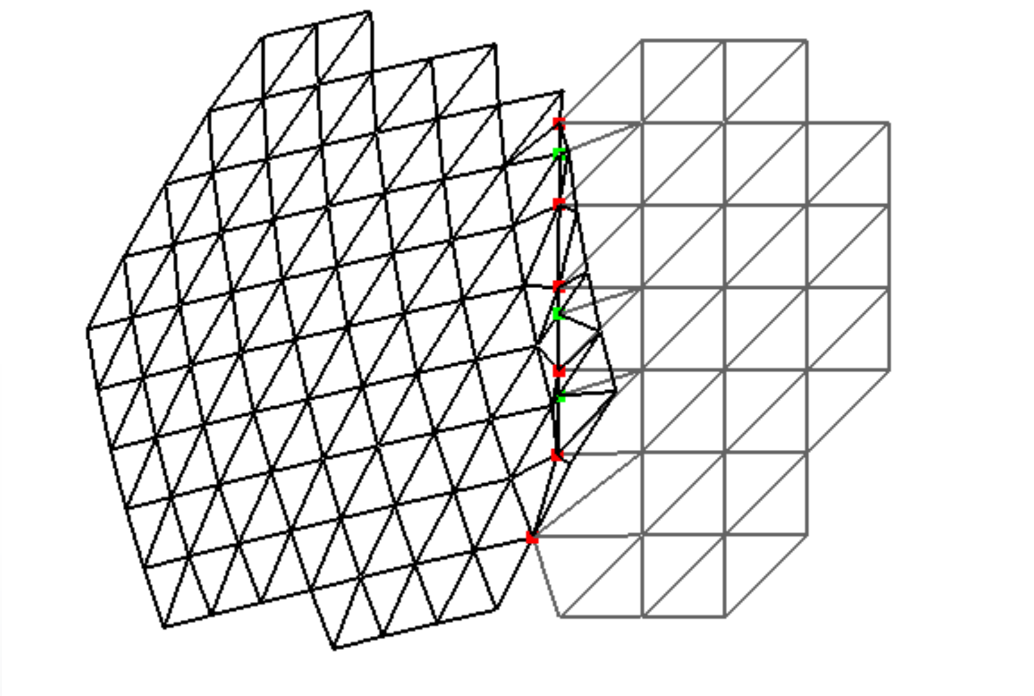
“Zipper” several scans to one single model



Project & insert boundary vertices

Explicit Reconstruction

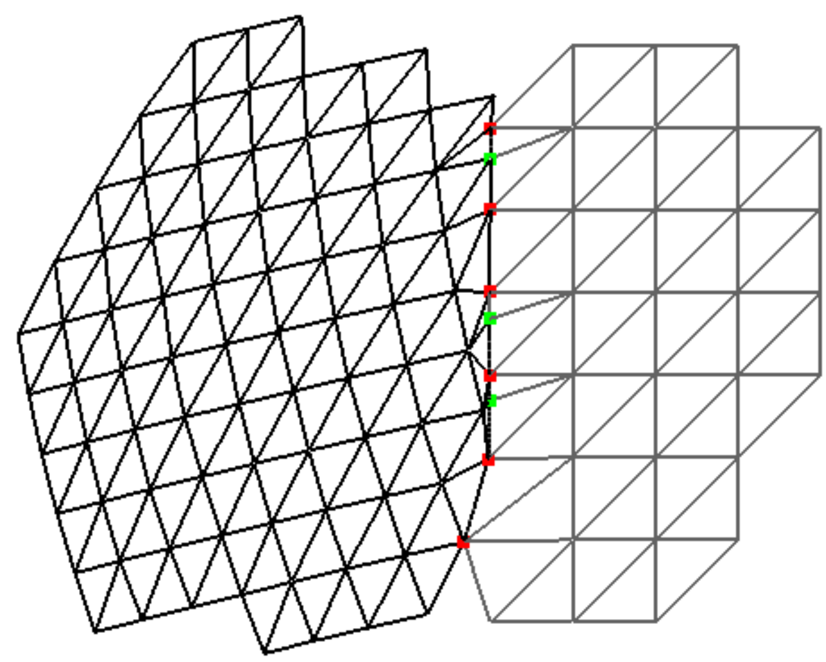
“Zipper” several scans to one single model



Intersect boundary edges

Explicit Reconstruction

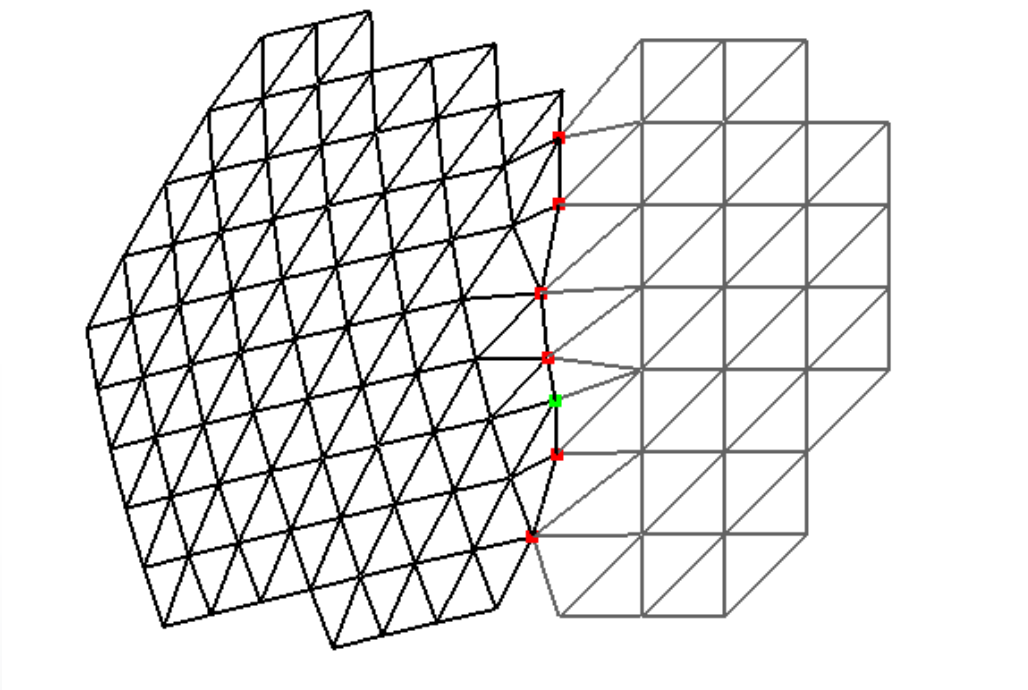
“Zipper” several scans to one single model



Discard overlap region

Explicit Reconstruction

“Zipper” several scans to one single model

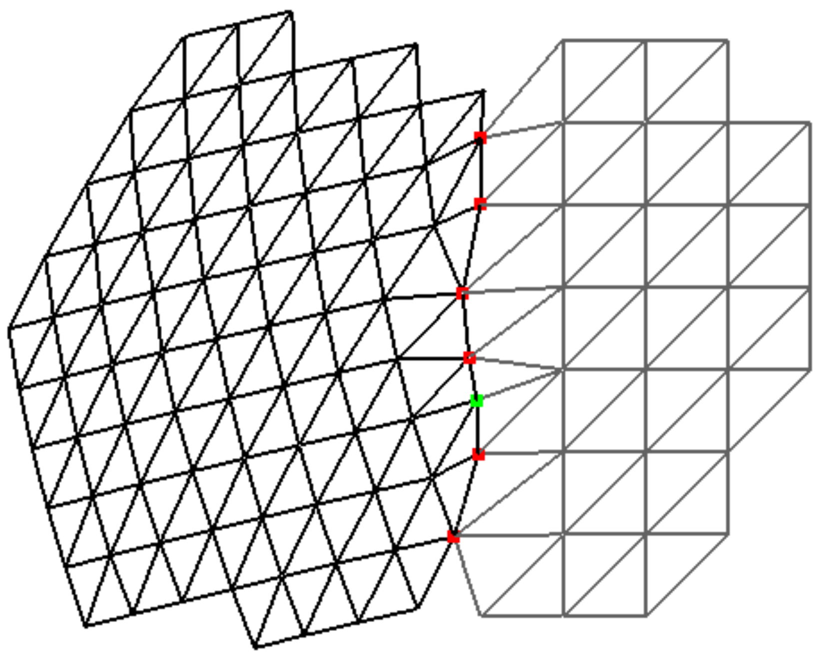


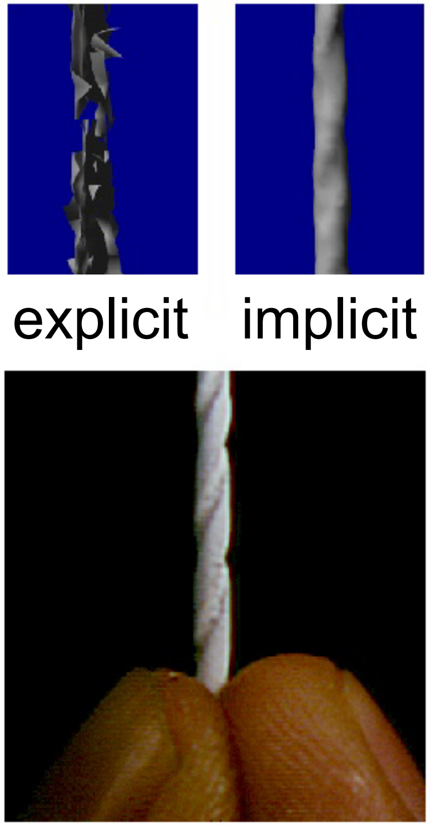
Locally optimize triangulation

Explicit Reconstruction

"Zipper" several scans to one single model

Problems for intricate geometries…





explicit implicit

input model

Mesh Zippering Summary

Pros:

- Preserves regular structure of each scan
- No additional data structures

Cons:

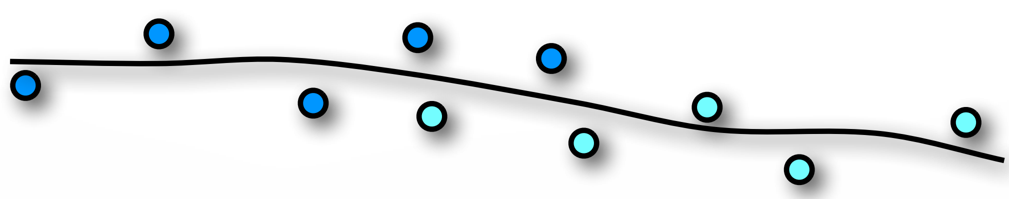
- Zippering can be numerically difficult
- Problems with complex, noisy, incomplete data

Outline

- Explicit Reconstruction
  - Zippering range scans
- **Implicit Reconstruction**
  - **SDF from point clouds**
  - SDF from range scans
  - Poisson surface reconstruction

Implicit Reconstruction

- Estimate signed distance function (SDF)
- Extract Zero isosurface by Marching Cubes
- Approximation of input points
- Watertight manifold by construction



Signed Distance Function

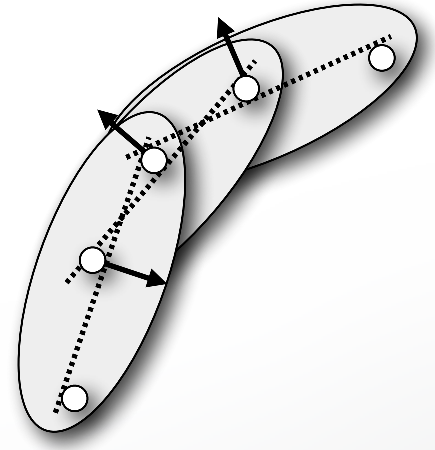
Construct SDF from point samples

- Distance to points is not enough
- Need inside/outside information
- Requires normal vectors

Normal Estimation

Find normal $\mathbf{n}_i$ for each sample point $\mathbf{p}_i$

- Examine local neighborhood for each point
  - Set of k nearest neighbors
- Compute best approximating tangent plane
  - Covariance analysis
- Determine normal orientation
  - Minimal Spanning Tree propagation



Normal Estimation

Find normal $\mathbf{n}_i$ for each sample point $\mathbf{p}_i$

- **Examine local neighborhood for each point**
  - Set of k nearest neighbors
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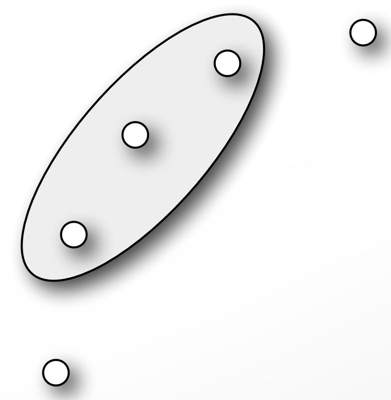


Normal Estimation

Find closest point of a query point

- Find closest point of a query point
  - Brute force: $O(n)$ complexity

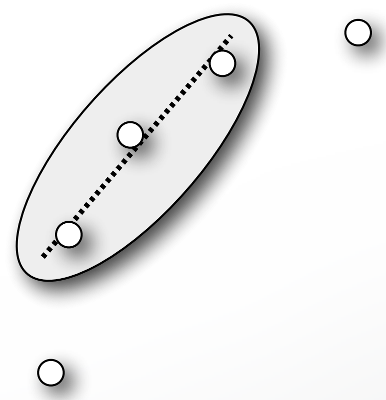
Use Hierarchical BSP tree

- Binary space partitioning tree (general version of kD-tree)
- Recursively partition 3D space by planes
- Tree should be balanced, put plane at median
- $\log(n)$ tree levels, complexity $\log(n)$

Normal Estimation

Find normal $\mathbf{n}_i$ for each sample point $\mathbf{p}_i$

- Examine local neighborhood for each point
    - Set of k nearest neighbors
- **Compute best approximating tangent plane**
    - Covariance analysis
- Determine normal orientation
    - Minimal Spanning Tree propagation



Plane Fitting

Fit a plane with center $\mathbf{c}$ and normal $\mathbf{n}$ to a set of points $\{\mathbf{p}_1, \ldots, \mathbf{p}_m\}$

Minimize least squares error

$$E(\mathbf{c}, \mathbf{n}) = \sum_{i=1}^{m} \left(\mathbf{n}^T (\mathbf{p}_i - \mathbf{c})\right)^2$$

Subject to non-linear constraint

$$\|\mathbf{n}\| = 1$$

Plane Fitting

Reformulate error function

$$
\begin{aligned}
E(\mathbf{c}, \mathbf{n}) &= \sum_{i=1}^{m} \left(\mathbf{n}^T (\mathbf{p}_i - \mathbf{c})\right)^2 \\
&= \sum_{i=1}^{m} \left(\mathbf{n}^T \hat{\mathbf{p}}_i\right)^2 \qquad (\text{with } \hat{\mathbf{p}}_i := \mathbf{p}_i - \mathbf{c}) \\
&= \sum_{i=1}^{m} \hat{\mathbf{p}}_i^T \, \mathbf{n}\mathbf{n}^T \, \hat{\mathbf{p}}_i \qquad (\text{version 1}) \\
&= \sum_{i=1}^{m} \mathbf{n}^T \, \hat{\mathbf{p}}_i \hat{\mathbf{p}}_i^T \, \mathbf{n} \qquad (\text{version 2})
\end{aligned}
$$

Determine c from version 1

Derivative of $E(\mathbf{c}, \mathbf{n})$ w.r.t. $\mathbf{c}$ has to vanish

$$\frac{\partial E(\mathbf{c}, \mathbf{n})}{\partial \mathbf{c}} = \sum_{i=1}^{m} -2\,\mathbf{n}\mathbf{n}^T \hat{\mathbf{p}}_i = -2\,\mathbf{n}\mathbf{n}^T \sum_{i=1}^{m} \hat{\mathbf{p}}_i \stackrel{!}{=} 0$$

This is only possible for

$$\sum_{i=1}^{m} \hat{\mathbf{p}}_i = 0 \quad \Rightarrow \quad \mathbf{c} = \frac{1}{m} \sum_{i=1}^{m} \mathbf{p}_i$$

Plane center is barycenter of points $\mathbf{p}_i$

Determine n from version 2

Represent $\mathbf{n}$ in basis $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$

$$\mathbf{n} = \alpha_1 \mathbf{e}_1 + \alpha_2 \mathbf{e}_2 + \alpha_3 \mathbf{e}_3$$

Since $\mathbf{n}$ has unit length we get

$$1 = \mathbf{n}^\top \mathbf{n} = \alpha_1^2 + \alpha_2^2 + \alpha_3^2$$

Insert into energy formulation

$$\mathbf{n}^T \mathbf{C} \mathbf{n} \; = \; \alpha_1^2 \lambda_1 + \alpha_2^2 \lambda_2 + \alpha_3^2 \lambda_3 \; \geq \; \alpha_1^2 \lambda_3 + \alpha_2^2 \lambda_3 + \alpha_3^2 \lambda_3 \; = \; \lambda_3$$

Minimum is achieved for $\alpha_1 = \alpha_2 = 0, \alpha_3 = 1 \quad \Rightarrow \quad \mathbf{n} = \mathbf{e}_3$

Principal Component Analysis

Plane center is barycenter of points

$$\mathbf{c} = \frac{1}{m} \sum_{i=1}^{m} \mathbf{p}_i$$

Normal is eigenvector w.r.t. smallest eigenvalue of covariance matrix

$$\mathbf{C} = \sum_{i=1}^{m} (\mathbf{p}_i - \mathbf{c})(\mathbf{p}_i - \mathbf{c})^T$$

Normal Estimation

Find normal $\mathbf{n}_i$ for each sample point $\mathbf{p}_i$

- Examine local neighborhood for each point
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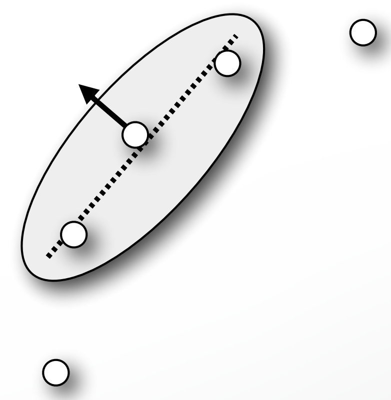
- Compute best approximating tangent plane
  - Covariance analysis
- **Determine normal orientation**
  - Minimal Spanning Tree propagation



Normal Orientation

Riemannian graph connects neighboring points

- Edge (ij) exists if $\mathbf{p}_i \in k\mathrm{NN}(\mathbf{p}_j)$ or $\mathbf{p}_j \in k\mathrm{NN}(\mathbf{p}_i)$

Propagate normal orientation through graph

- For neighbors $\mathbf{p}_i, \mathbf{p}_j$ Flip $\mathbf{n}_j$ if $\mathbf{n}_i^\top \mathbf{n}_j < 0$
- Fails at sharp edges/corners

Propagate along “save” paths (parallel normals)

- Minimum spanning tree with angle-based edge weights

$$w_{ij} = 1 - |\mathbf{n}_i^\top \mathbf{n}_j|$$

Normal Estimation

Find normal $\mathbf{n}_i$ for each sample point $\mathbf{p}_i$

- Examine local neighborhood for each point
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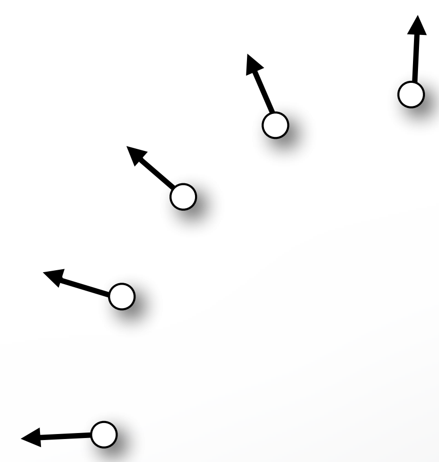
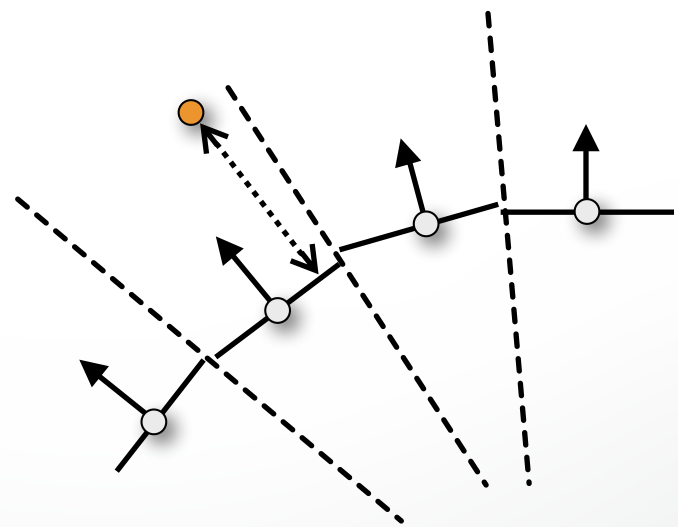


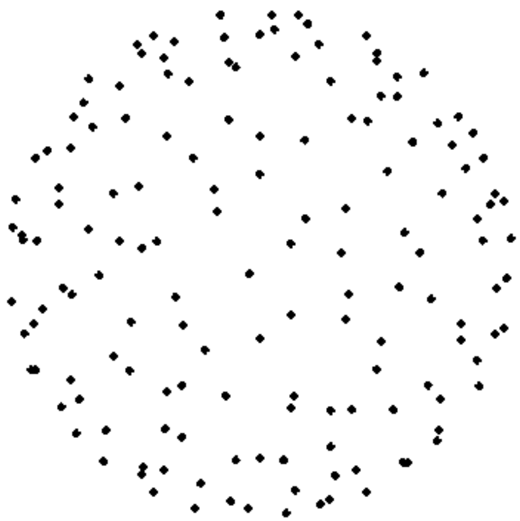
Normal Estimation

Distance from tangent planes [Hoppe 92]

- Points + normals determine local tangent planes
- Use distance from closest point's tangent plane
- Linear approximation in Voronoi cell
- Simple and efficient, but SDF is only $\mathcal{C}^{-1}$



Hoppe ’92 Reconstruction



150 samples

reconstruction
on 50^3 grid

Smooth SDF Approximation

Scattered data interpolation problem

- On-surface constraints $\text{dist}(\mathbf{p}_i) = 0$
- Avoid trivial solution $\text{dist} \equiv 0$
- Off-surface constraints $\text{dist}(\mathbf{p}_i + \mathbf{n}_i) = 1$

Radial basis functions (RBFs)

- Well suited for smooth interpolation
- Sum of shifted, weighted kernel functions

$$\text{dist}(\mathbf{x}) \;=\; \sum_i w_i \cdot \varphi(\|\mathbf{x} - \mathbf{c}_i\|)$$

RBF Interpolation

Interpolate on- and off-surface constraints

$$\operatorname{dist}(\mathbf{x}_j) = \sum_{i=1}^{n} w_i \cdot \varphi(\|\mathbf{x}_j - \mathbf{c}_i\|) \stackrel{!}{=} d_j, \quad j = 1, \ldots, n$$

Choose centers $\mathbf{c}_i$ as constrained points $\mathbf{x}_i$

Solve symmetric linear system for weights w_i

$$\begin{pmatrix} \varphi(\|\mathbf{x_1} - \mathbf{x}_1\|) & \cdots & \varphi(\|\mathbf{x_1} - \mathbf{x}_n\|) \\ \vdots & \ddots & \vdots \\ \varphi(\|\mathbf{x_n} - \mathbf{x}_1\|) & \cdots & \varphi(\|\mathbf{x_n} - \mathbf{x}_n\|) \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix}$$

RBF Interpolation

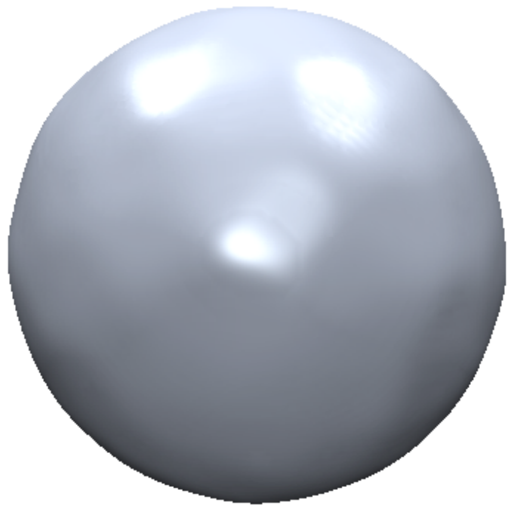
Wendland basis functions

$$\varphi(r) = \left(1 - \frac{r}{\sigma}\right)_{+}^{4} \left(4\frac{r}{\sigma} + 1\right)$$

- Compactly supported in $[0, \sigma]$
- Leads to sparse, symm. pos. def. linear system
- Resulting SDF is $\mathcal{C}^2$ smooth
- But surface is not necessarily fair
- Not suited for highly irregular sampling

Comparison

Hoppe ‘92



Compact RBF
Wendland C^2

RBF Basis Functions

Triharmonic basis functions

$$\phi(r) = r^3$$

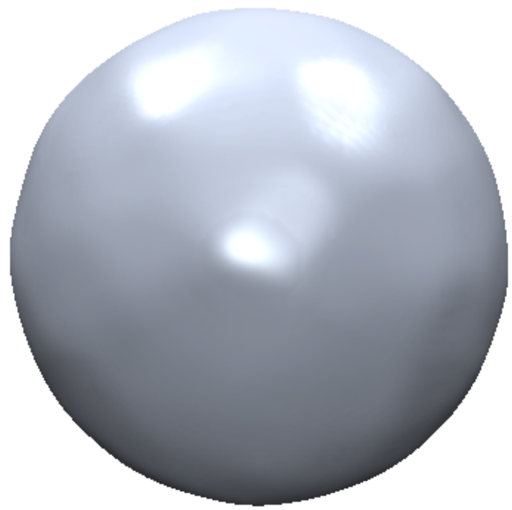
- Globally supported function
- Leads to dense linear system
- SDF is $\mathcal{C}^2$ smooth
- Provably optimal fairness (see smoothing lecture)

$$\int_{\mathbb{R}^3} \left(\frac{\partial^3 \text{dist}}{\partial x\,\partial x\,\partial x}\right)^2 + \left(\frac{\partial^3 \text{dist}}{\partial x\,\partial x\,\partial y}\right)^2 + \cdots + \left(\frac{\partial^3 \text{dist}}{\partial z\,\partial z\,\partial z}\right)^2 \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \;\rightarrow\; \min$$

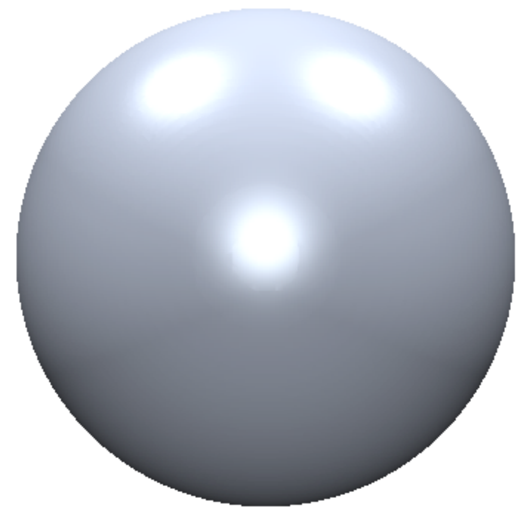
- Works well for irregular sampling

Comparison

Hoppe ‘92



Compact RBF
Wendland C^2



Global RBF
Triharmonic

Complexity Considerations

Solve the linear system for RBF weights

- Hard to solve for large number of samples

Compactly suppoted RBFs

- Sparse linear system
- Efficient CG or sparse Cholesky solver (later…)

Greedy RBF fitting [Carr01]

- Start with a few RBFs only
- Add more RBFs in region of large error

SDF From Points

Pros:

- Result is a closed 2-manifold surface
- Suitable for noisy input data

Cons:

- Solve linear system of RBF weights
- Result is uniformly over-tessellated → mesh decimation
- Can contain poorly shaped triangles → remeshing

Outline

- **Explicit Reconstruction**
  - Zippering range scans
- **Implicit Reconstruction**
  - SDF from point clouds
  - **SDF from range scans**
  - Poisson surface reconstruction

Weighted Average of SDFs

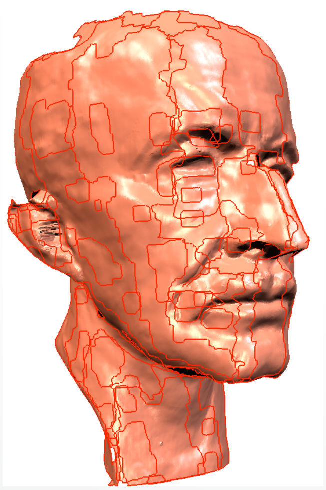
Individual SDFs of each scan: $d_i(\mathbf{x})$

- Distance along scanner's line of sight

Respective weighting functions: $w_i(\mathbf{x})$

- Take scanning angle into account

Global SDF as weighted average

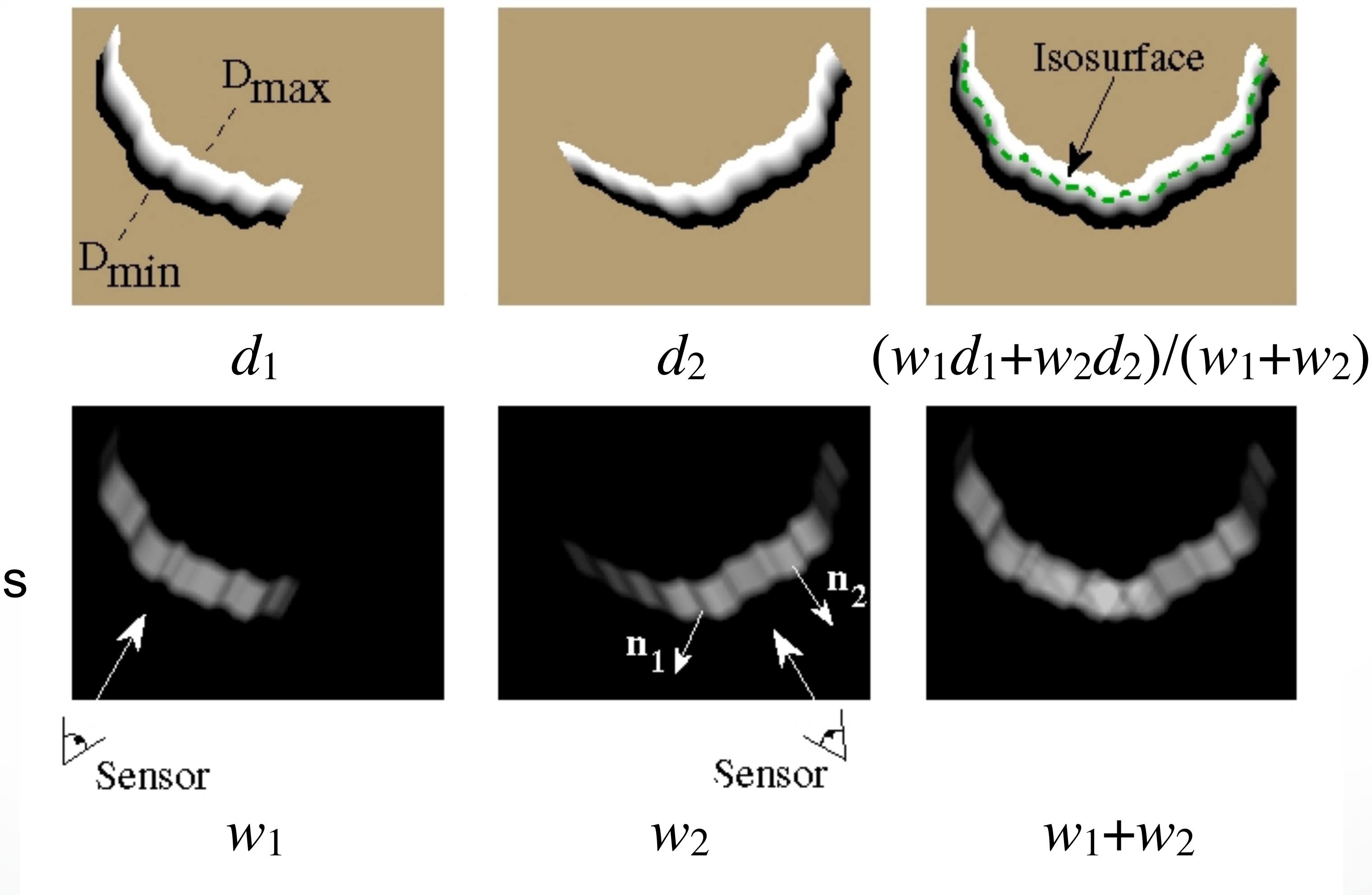


$$D(\mathbf{x}) = \frac{\sum_i w_i(\mathbf{x})\, d_i(\mathbf{x})}{\sum_i w_i(\mathbf{x})}$$

Weighted Average of SDFs

SDFs

Weight Functions



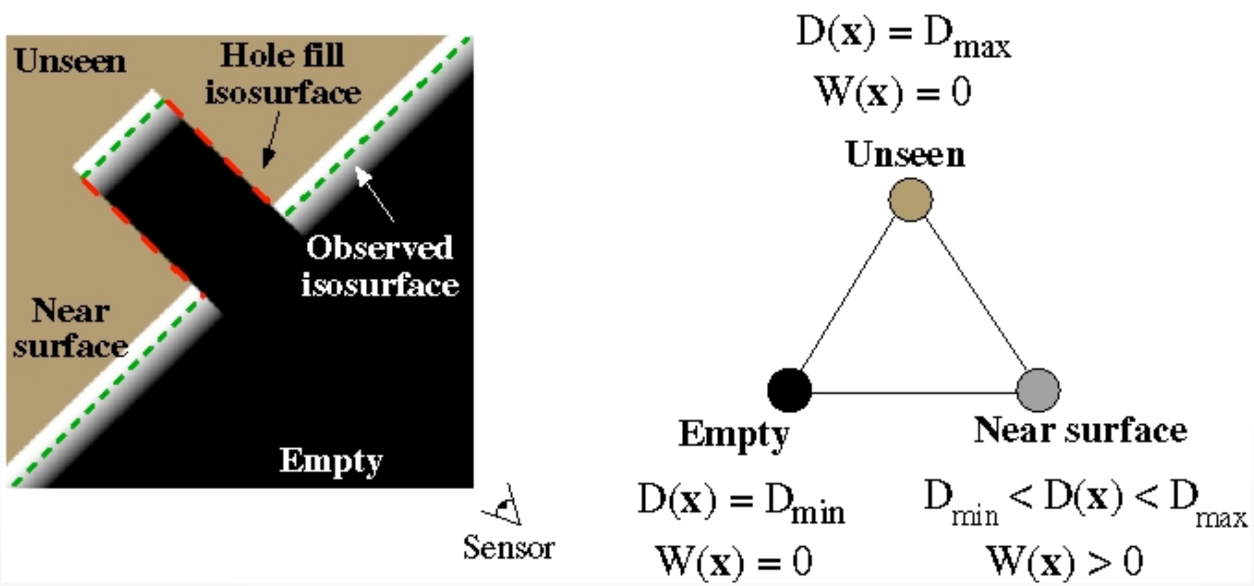
[Curless,Levoy96]

Automatic Hole Filling

Classify grid voxel into three states

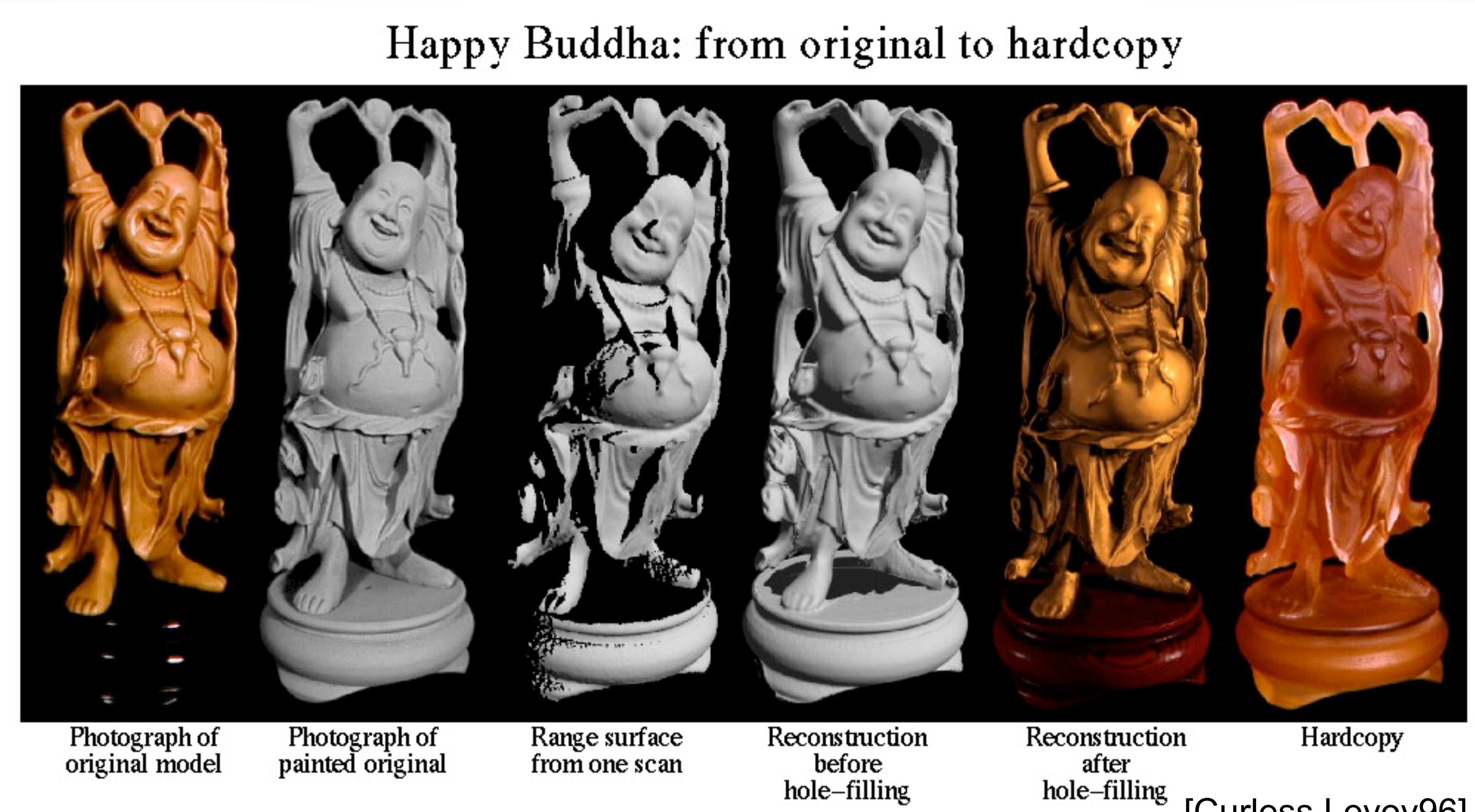
- Empty: Between scanner and surface (space carving)
- Unseen: Behind surface
- Near surface: Close to scanned surface

Marching Cubes automatically fill holes



[Curless,Levoy96]

Volumetric Reconstruction



[Curless,Levoy96]

Digital Michelangelo Project

1G sample points → 8M triangles

4G sample points → 8M triangles

SDF From Range Scans

Pros:

- Result is a closed 2-manifold surface
- Can take scanning information into account

Cons:

- Result is uniformly over-tesselated → mesh decimation
- Can contain poorly shaped triangles → remeshing

References

Reconstruction from point sets

- Hoppe et al.: Surface Reconstruction from Unorganized Points, SIGGRAPH 1992
- Carr etl a.: Reconstruction and representation of 3D objects with radial basis functions, SIGGRAPH 2001

Reconstruction of range scans

- Curless, Levoy: A Volumetric Method for Building Complex Models from Range Images, SIGGRAPH 1996.
- Levoy et al.: Digital Michalangelo Project: 3D Scanning of Large Statues, SIGGRAPH 2000.

Outline

- **Explicit Reconstruction**
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- **Implicit Reconstruction**
  - SDF from point clouds
  - SDF from range scans
  - **Poisson surface reconstruction**

Poisson Surface Reconstruction

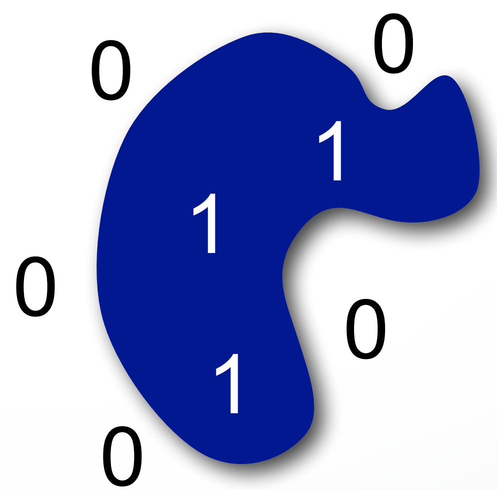
- **Michael Kazhdan**, M. Bolitho, and H. Hoppe, SGP 2006
- Source Code available at:
    - http://www.cs.jhu.edu/~misha/
- Implementation included in Meshlab

Poisson Surface Reconstruction

Indicator Function

- reconstruct the surface by solving for the indicator function of the shape

$$\chi_M(p) = \begin{cases} 1 & \text{if } p \in M \\ 0 & \text{if } p \notin M \end{cases}$$

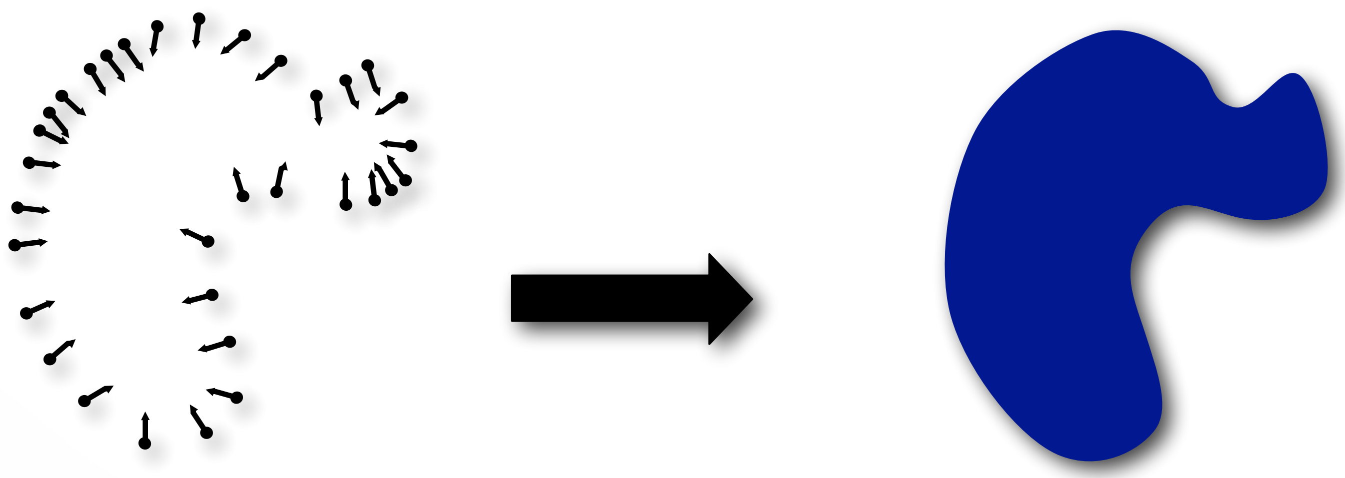


Indicator function

χ_M

Challenge

How to construct the indicator function?



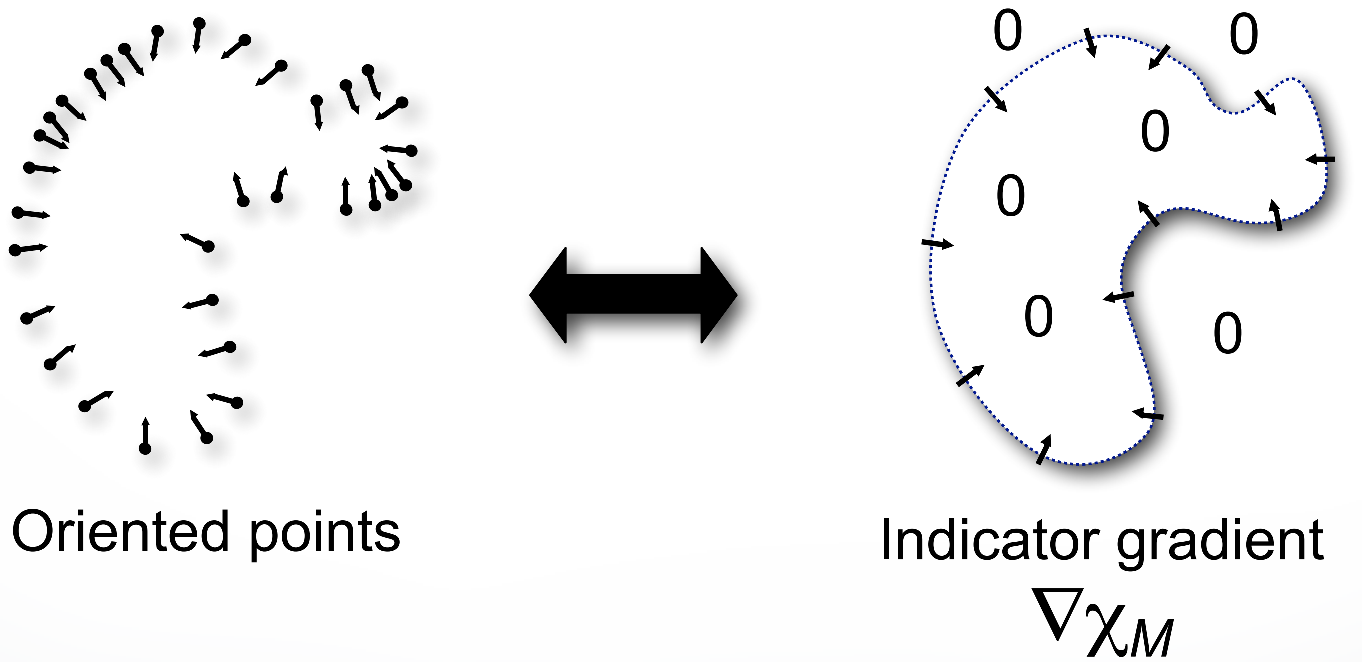
Oriented points

Indicator function

χ_M

Gradient Relationship

There is a relationship between the normal field and gradient of indicator function



Integration

Represent the points by a vector field $\vec{V}$

Find the function χ whose gradient best approximates $\vec{V}$

$$\min_{\chi} \|\nabla \chi - \vec{V}\|$$

Integration as a Poisson Problem

Represent the points by a vector field $\vec{V}$

Find the function χ whose gradient best approximates $\vec{V}$

$$\min_{\chi} \|\nabla\chi - \vec{V}\|$$

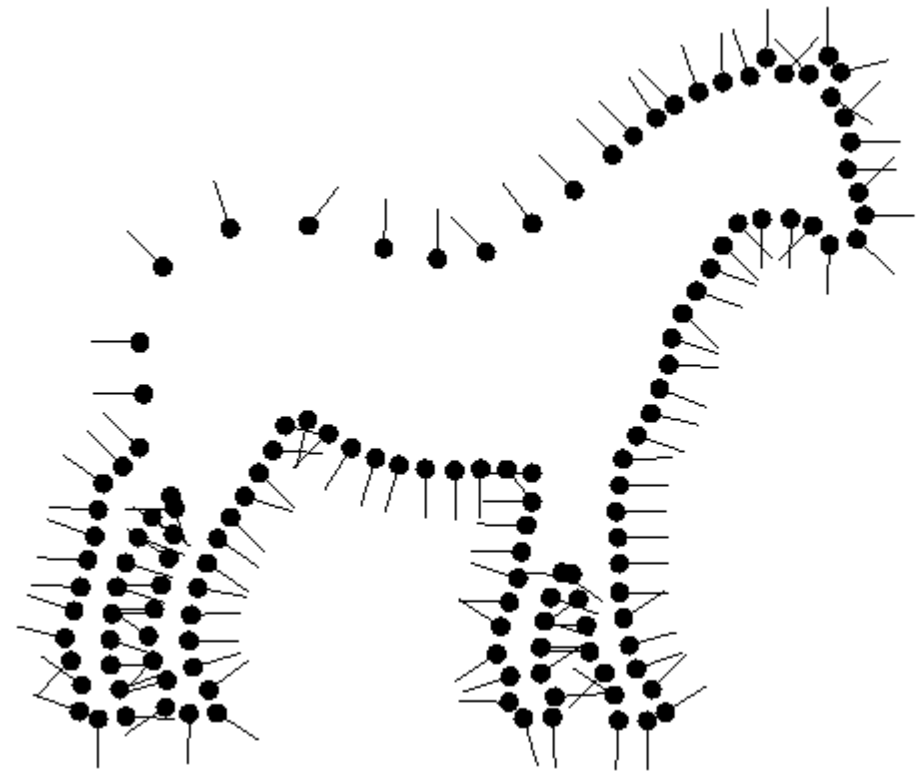
Applying the divergence operator, we can transform this into a Poisson problem:

$$\nabla \times (\nabla\chi) = \nabla \times \vec{V} \quad \Leftrightarrow \quad \Delta\chi = \nabla \times \vec{V}$$

Implementation: Adaptive Octree

Given the Points:

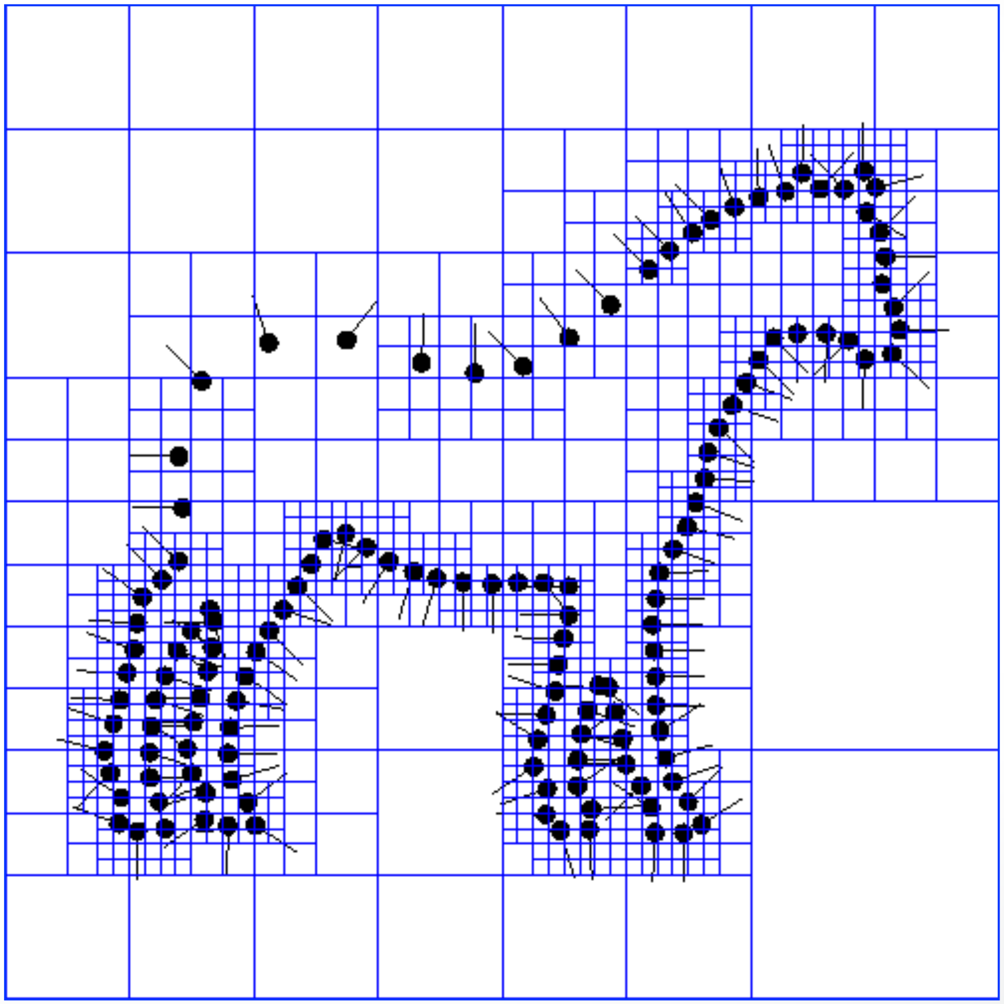
- Set Octree
- Compute vector field
- Compute indicator function
- Extract iso-surface



Implementation: Adaptive Octree

Given the Points:

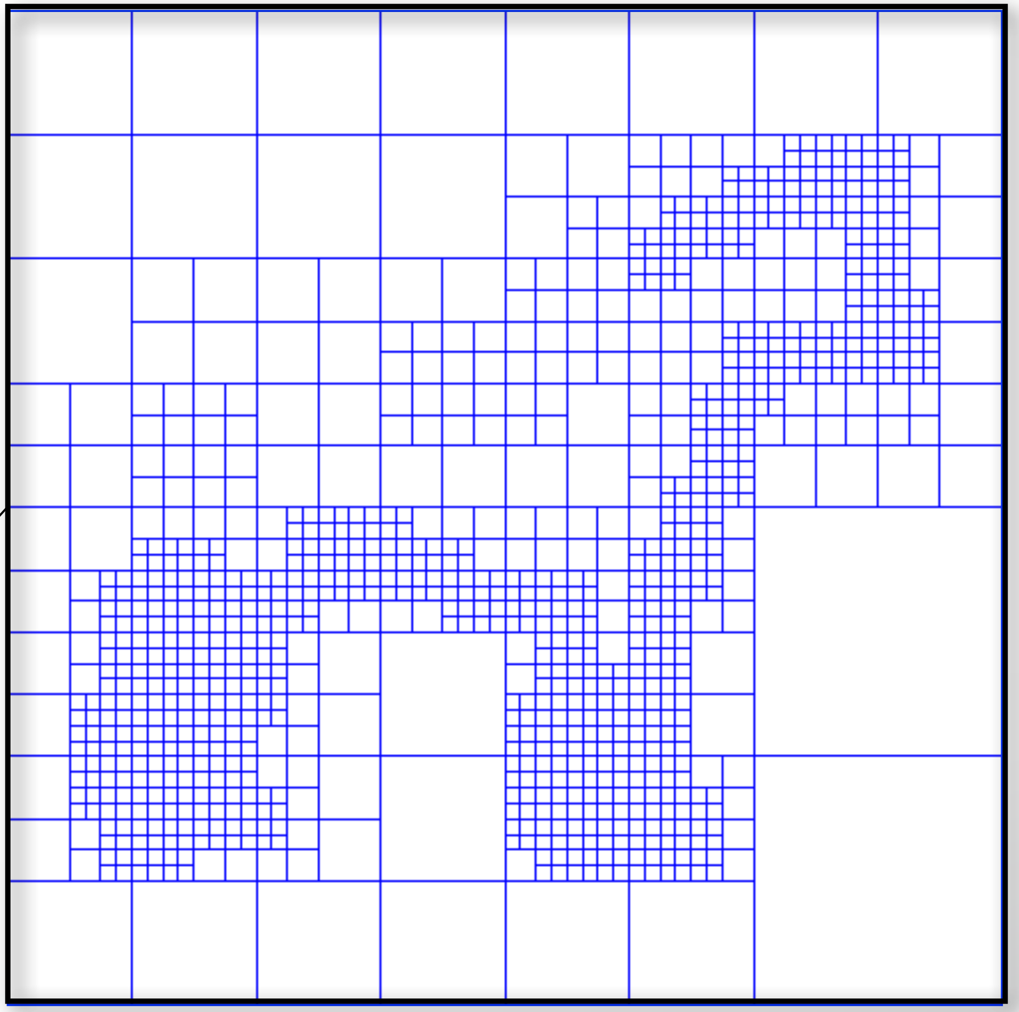
- **Set Octree**
- Compute vector field
- Compute indicator function
- Extract iso-surface

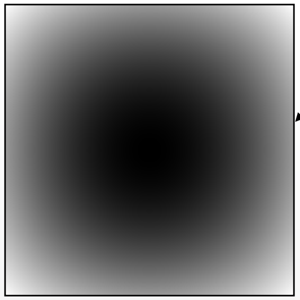


Implementation: Vector Field

Given the Points:

- Set Octree
- **Compute vector field**
  - **Define a function space**
  - Splat the samples
- Compute indicator function
- Extract iso-surface

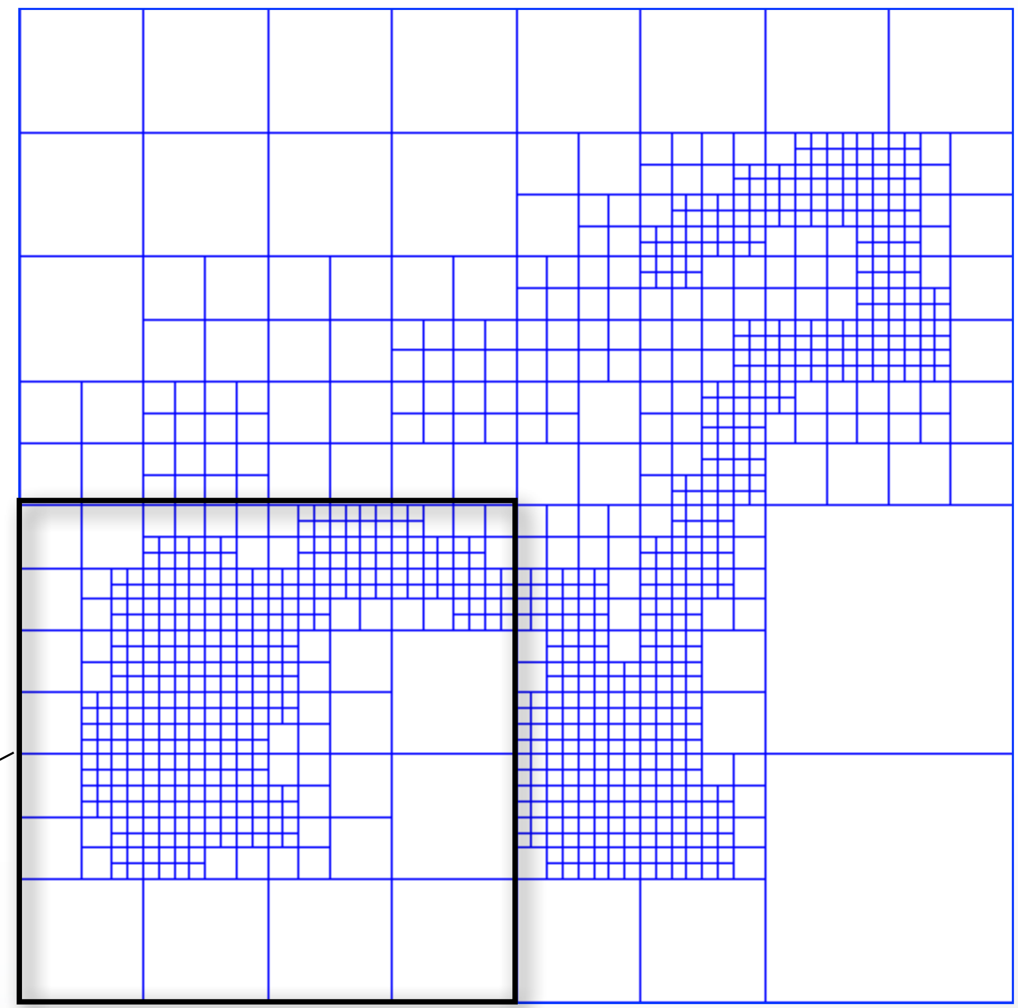


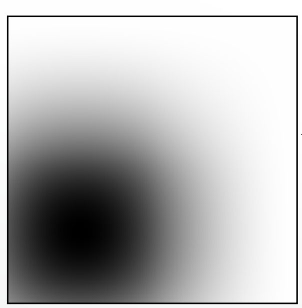


Implementation: Vector Field

Given the Points:

- Set Octree
- **Compute vector field**
  - **Define a function space**
  - Splat the samples
- Compute indicator function
- Extract iso-surface

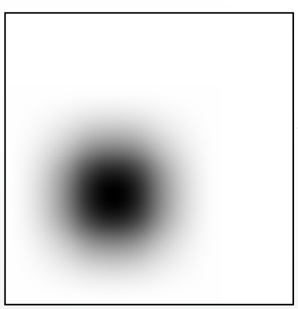


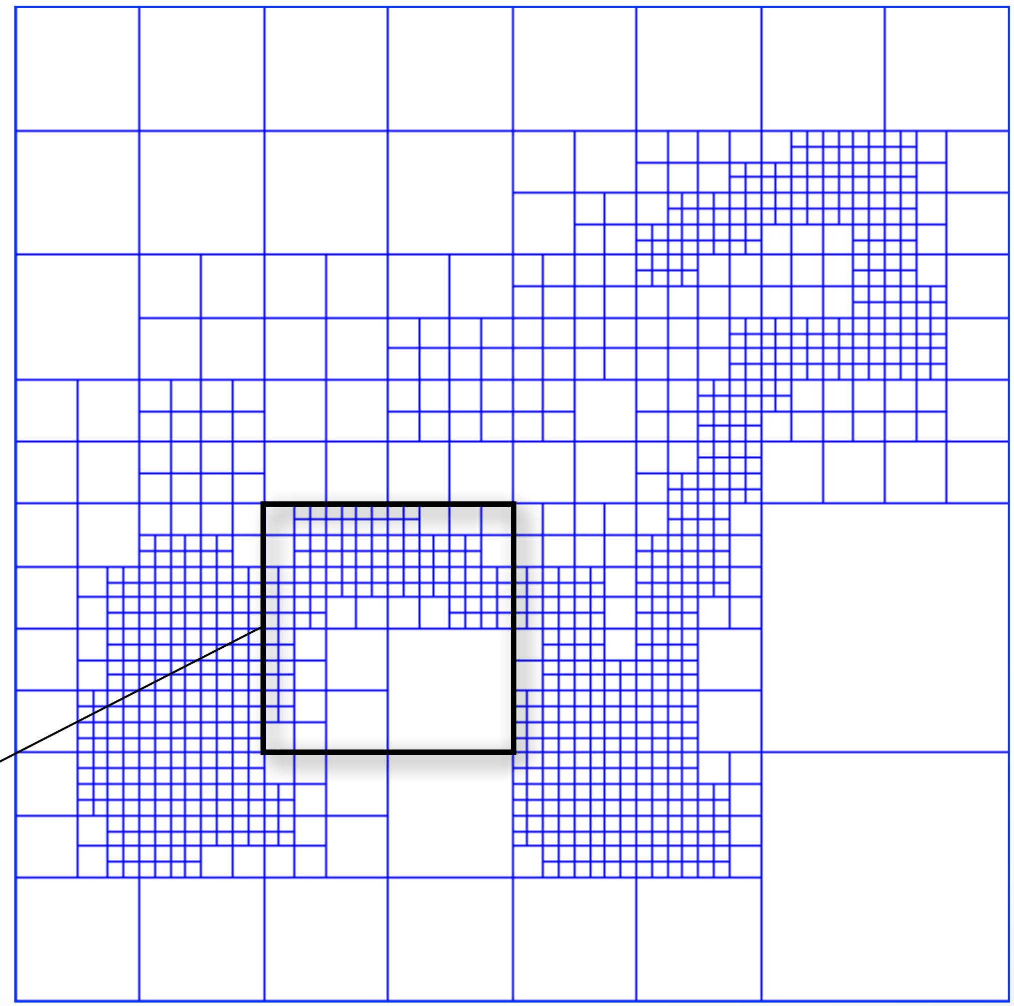


Implementation: Vector Field

Given the Points:

- Set Octree
- **Compute vector field**
  - **Define a function space**
  - Splat the samples
- Compute indicator function
- Extract iso-surface

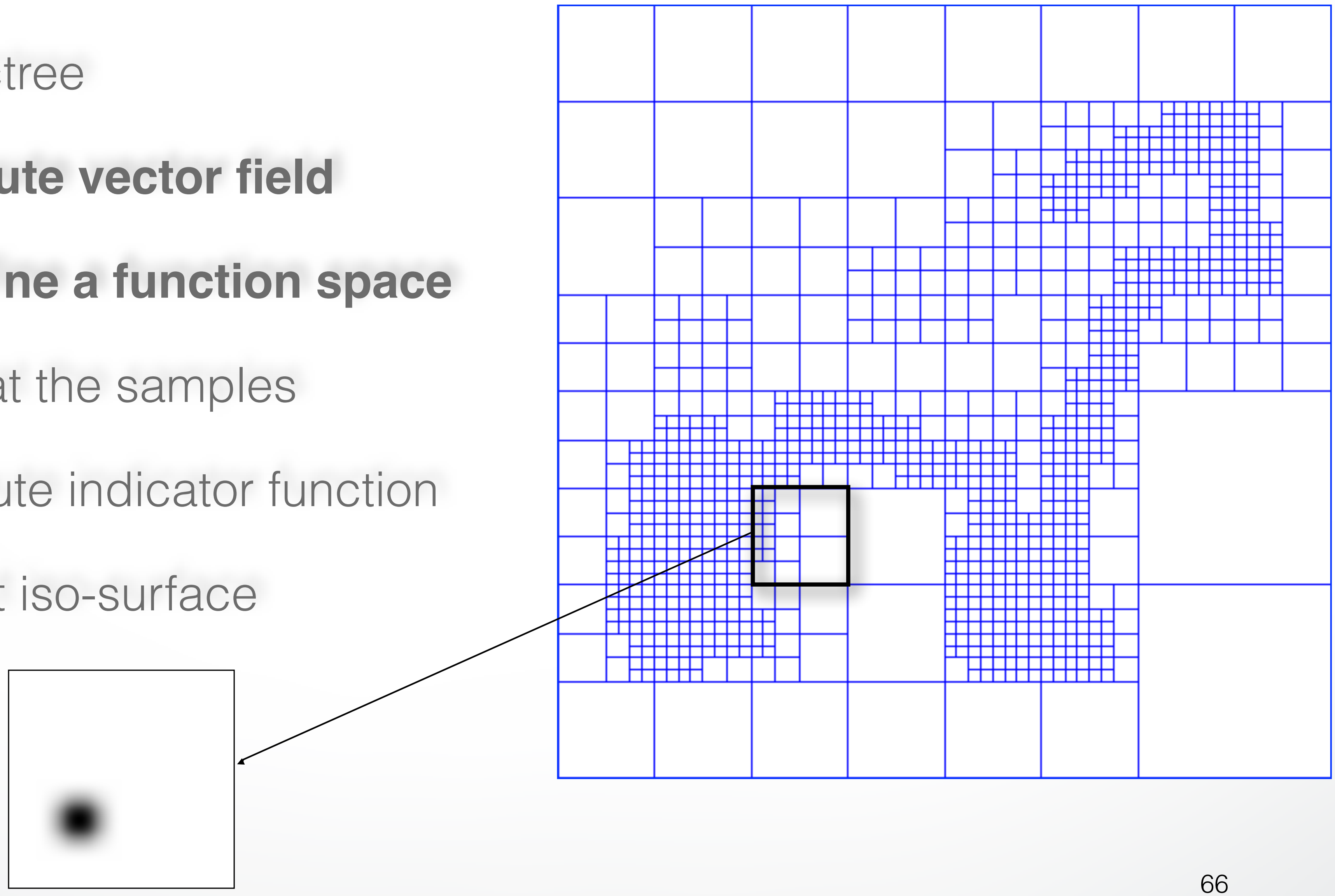




Implementation: Vector Field

Given the Points:

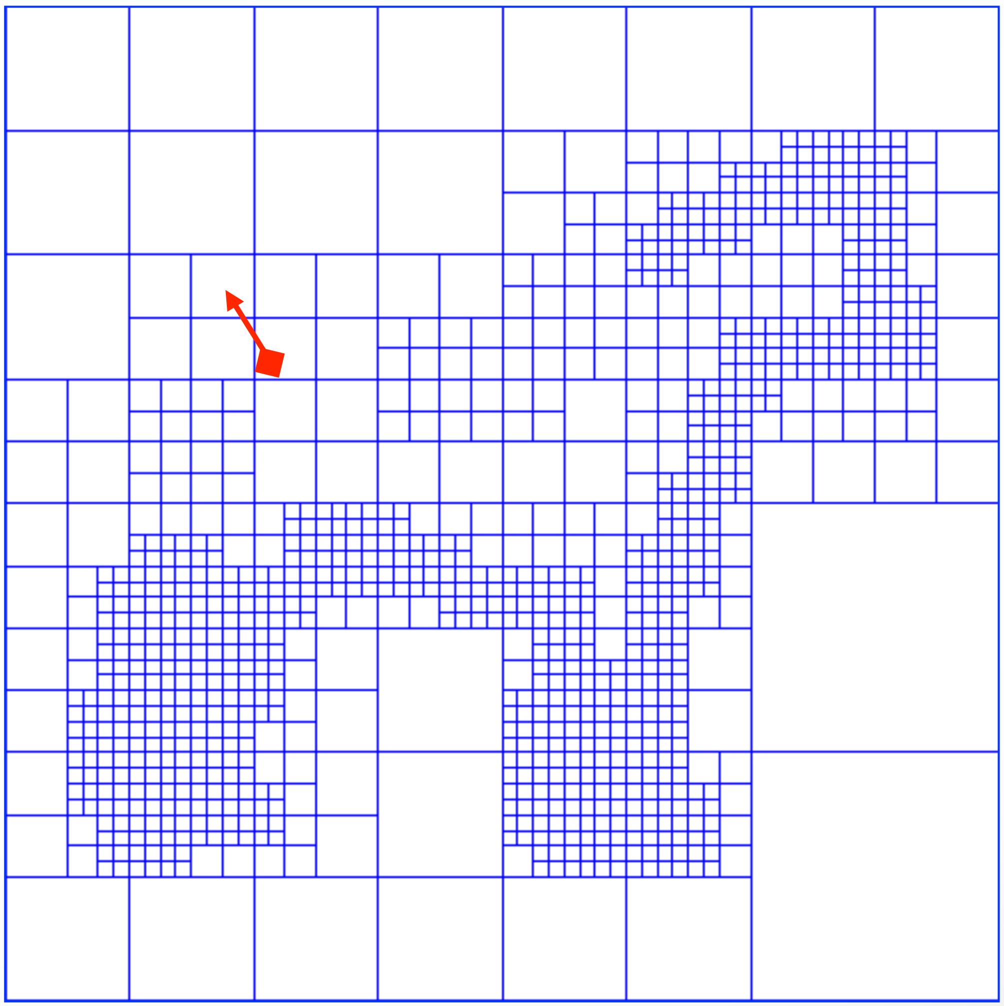
- Set Octree
- **Compute vector field**
  - **Define a function space**
  - Splat the samples
- Compute indicator function
- Extract iso-surface



Implementation: Vector Field

Given the Points:

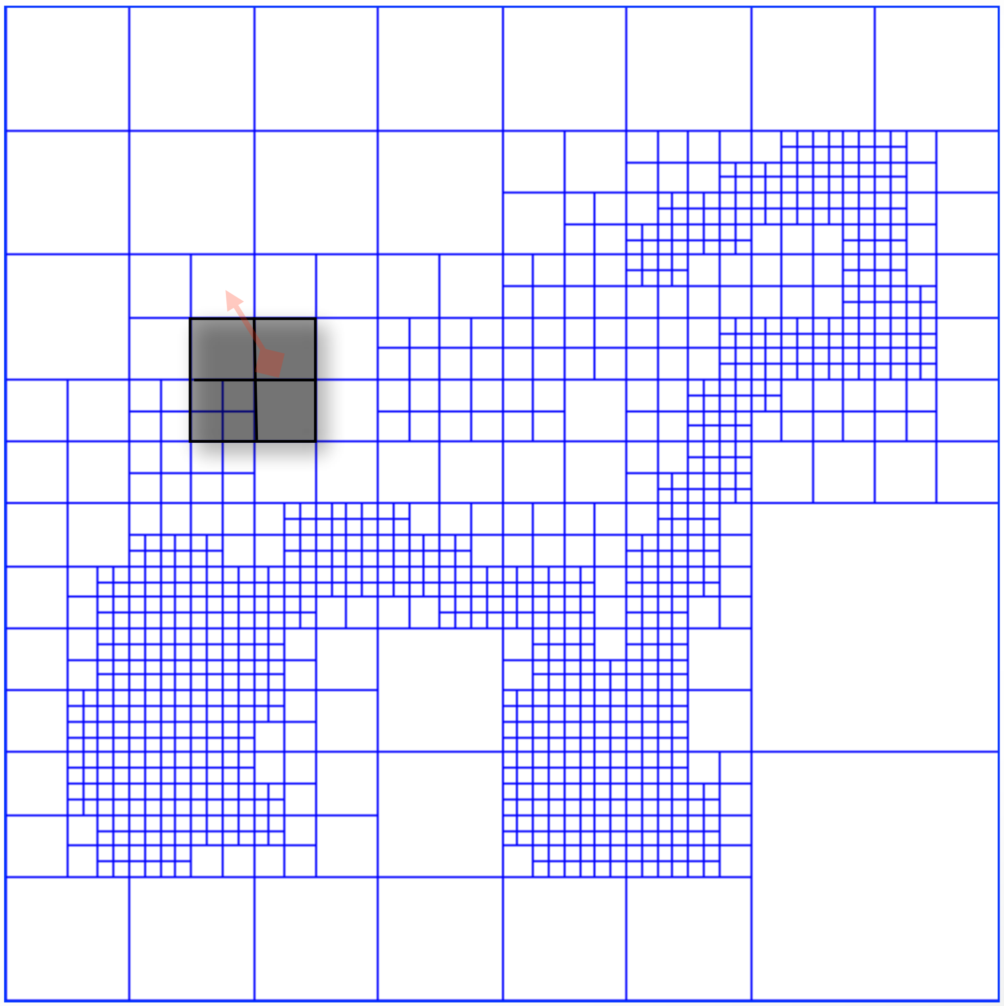
- Set Octree
- **Compute vector field**
  - Define a function space
  - **Splat the samples**
- Compute indicator function
- Extract iso-surface



Implementation: Vector Field

Given the Points:

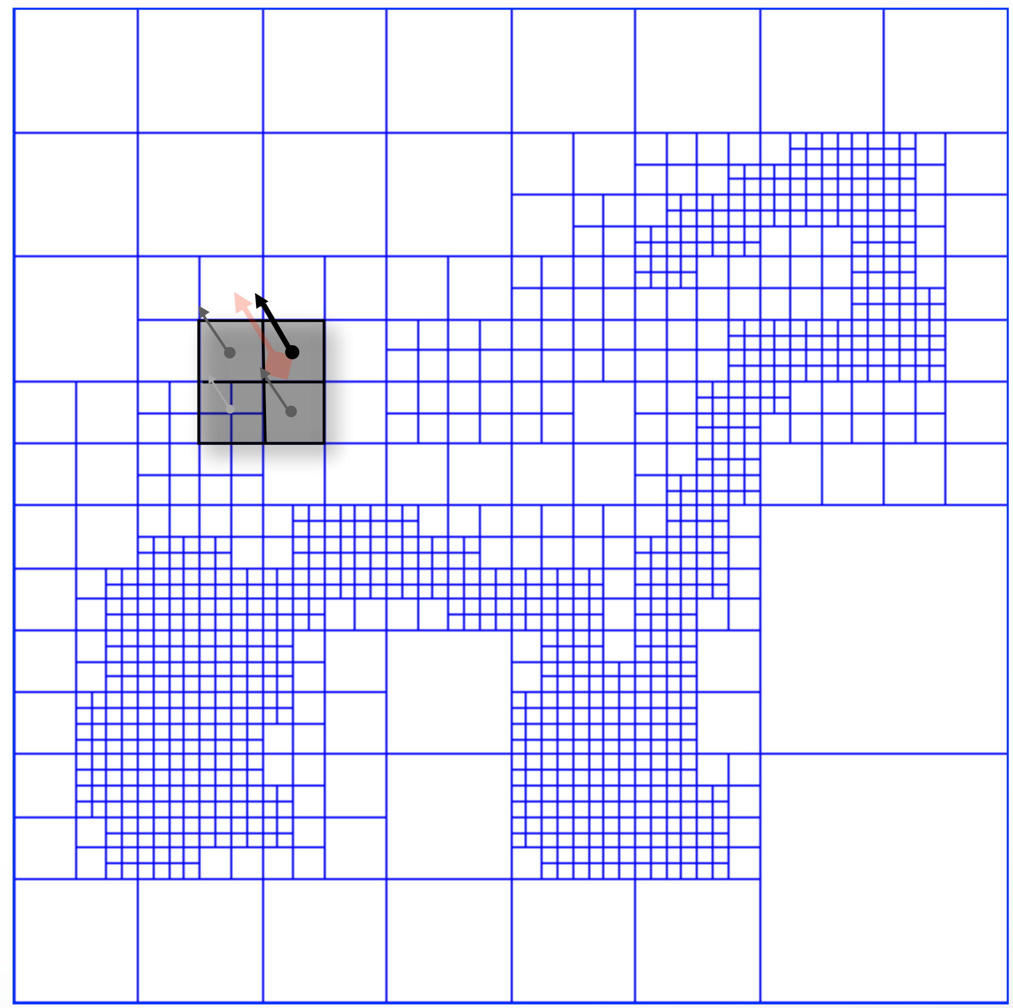
- Set Octree
- **Compute vector field**
  - Define a function space
  - **Splat the samples**
- Compute indicator function
- Extract iso-surface



Implementation: Vector Field

Given the Points:

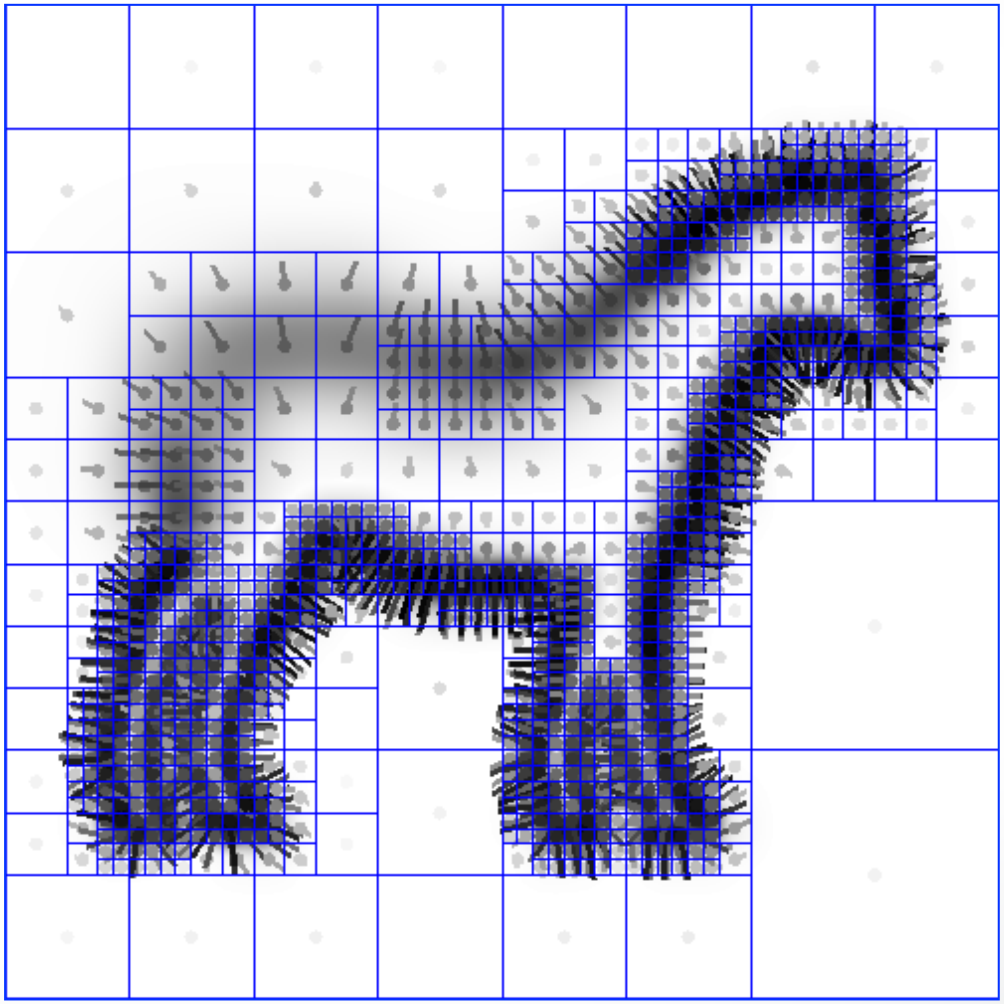
- Set Octree
- **Compute vector field**
  - Define a function space
  - **Splat the samples**
- Compute indicator function
- Extract iso-surface



Implementation: Vector Field

Given the Points:

- Set Octree
- **Compute vector field**
  - Define a function space
  - **Splat the samples**
- Compute indicator function
- Extract iso-surface



Implementation: Indicator Function

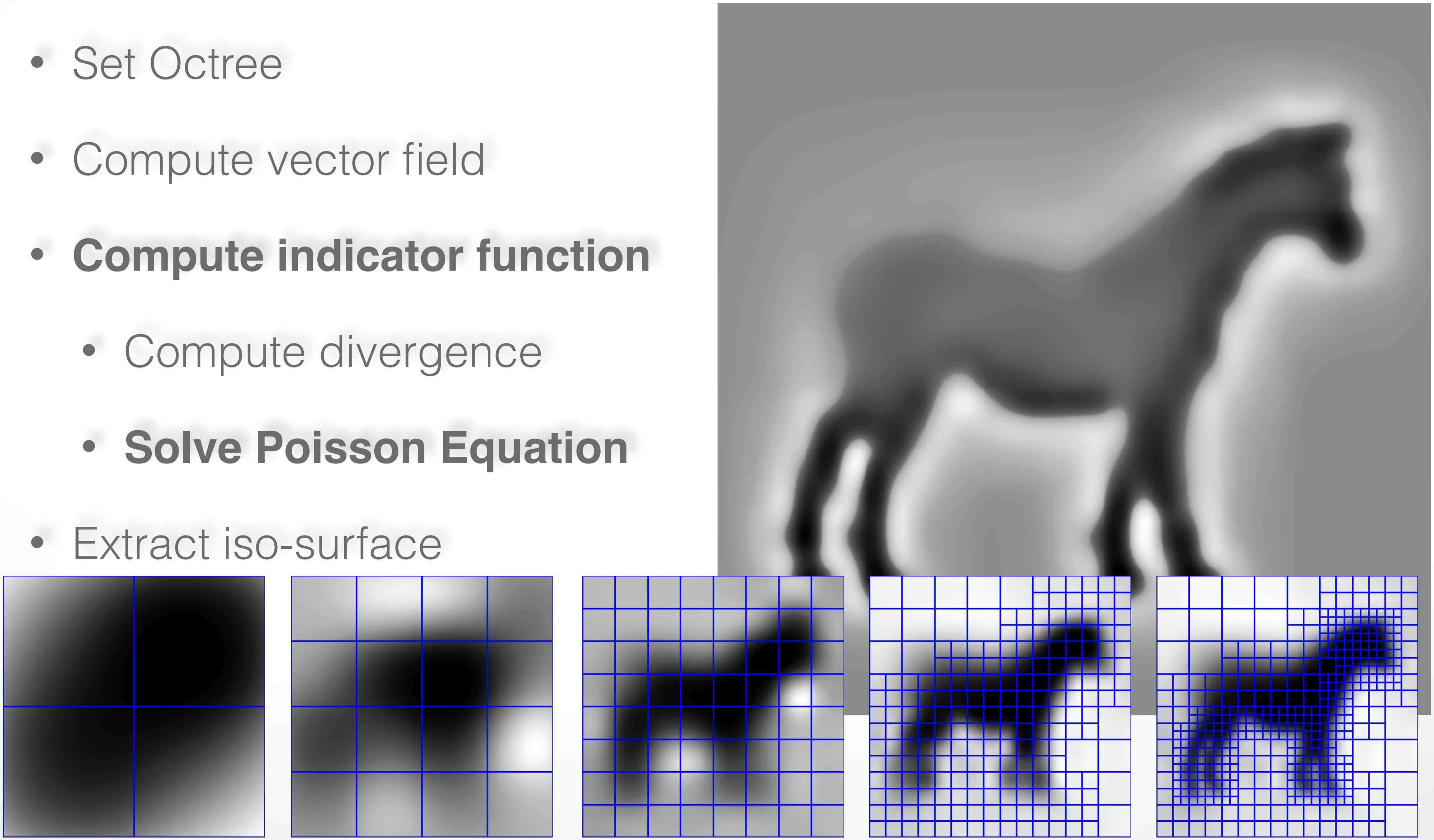
Given the Points:

- Set Octree
- Compute vector field
- **Compute indicator function**
    - **Compute divergence**
    - Solve Poisson Equation
- Extract iso-surface

Implementation: Indicator Function

Given the Points:

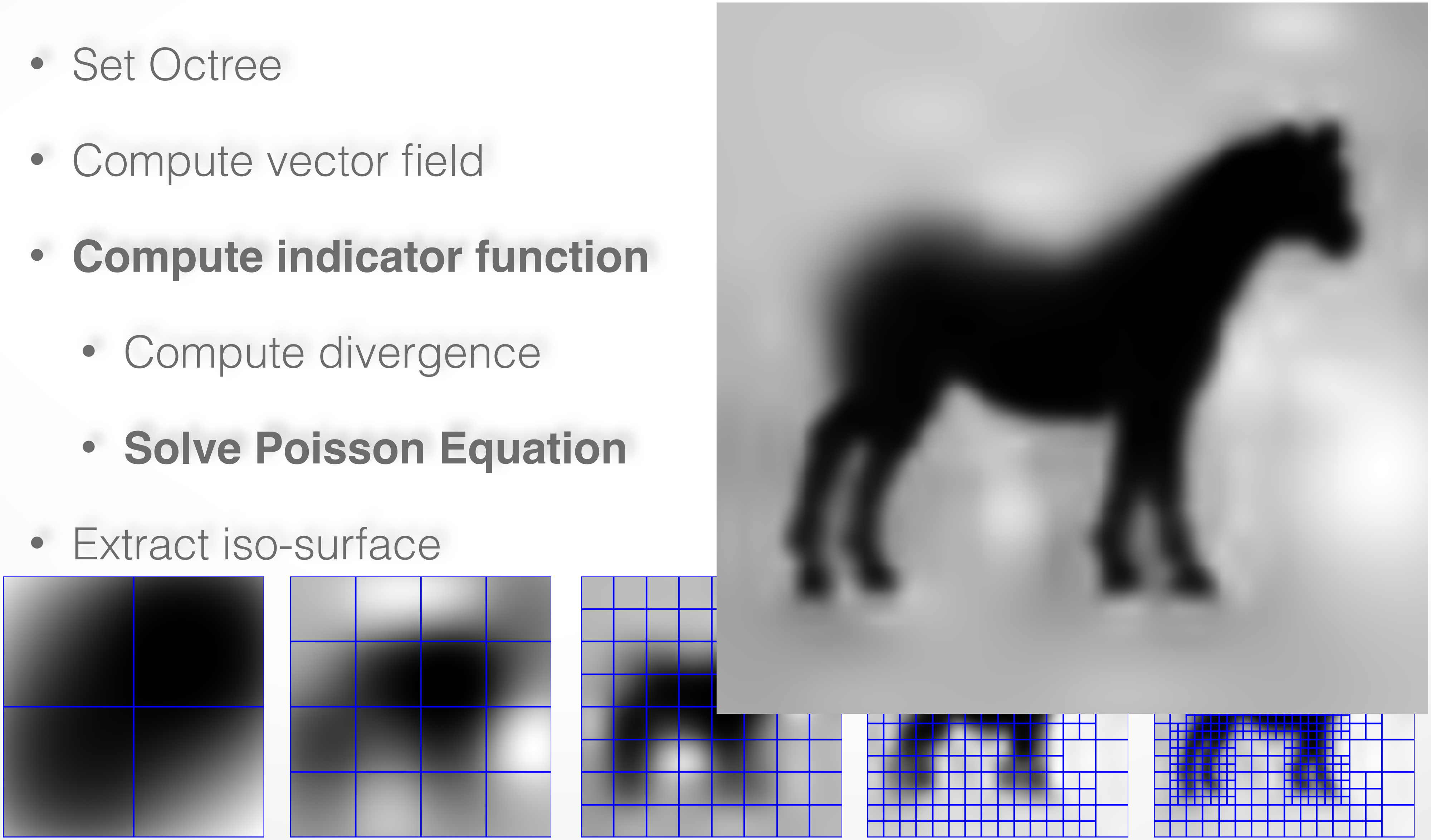
- Set Octree
- Compute vector field
- **Compute indicator function**
  - Compute divergence
  - **Solve Poisson Equation**
- Extract iso-surface



Implementation: Indicator Function

Given the Points:

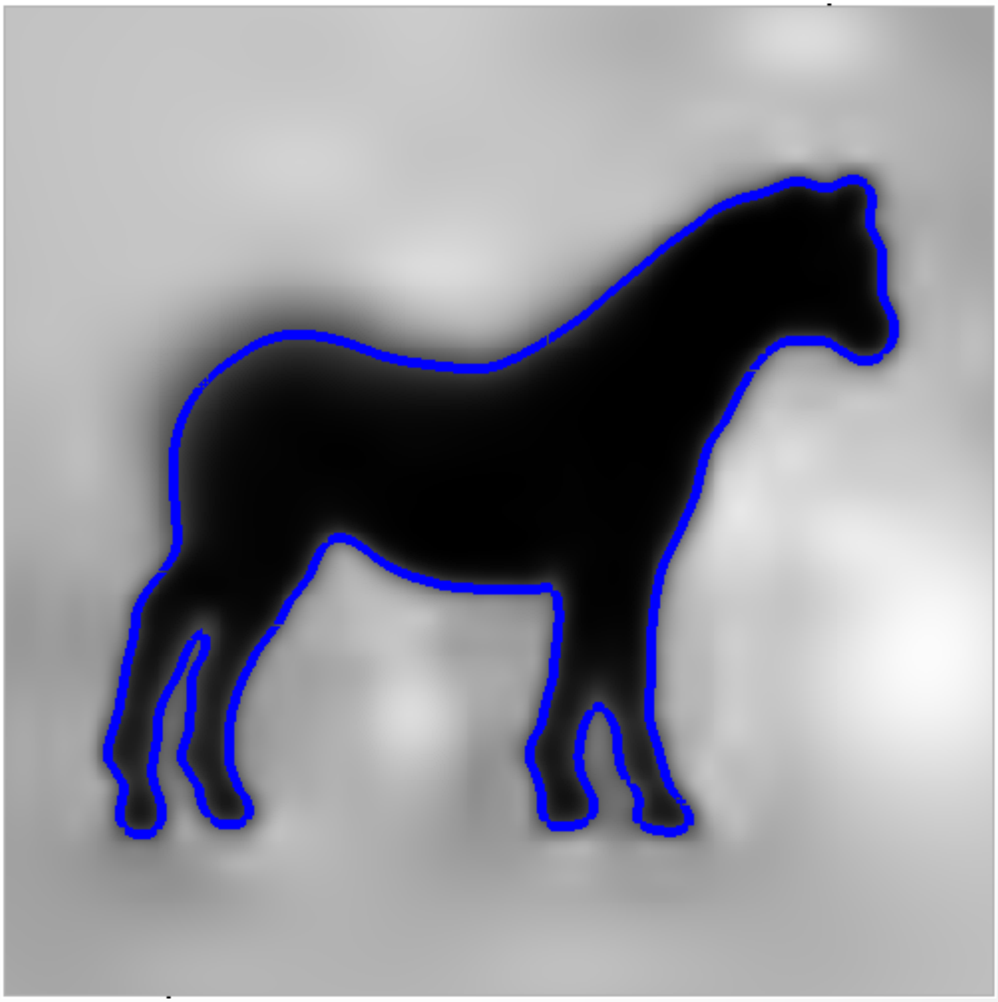
- Set Octree
- Compute vector field
- **Compute indicator function**
  - Compute divergence
  - **Solve Poisson Equation**
- Extract iso-surface



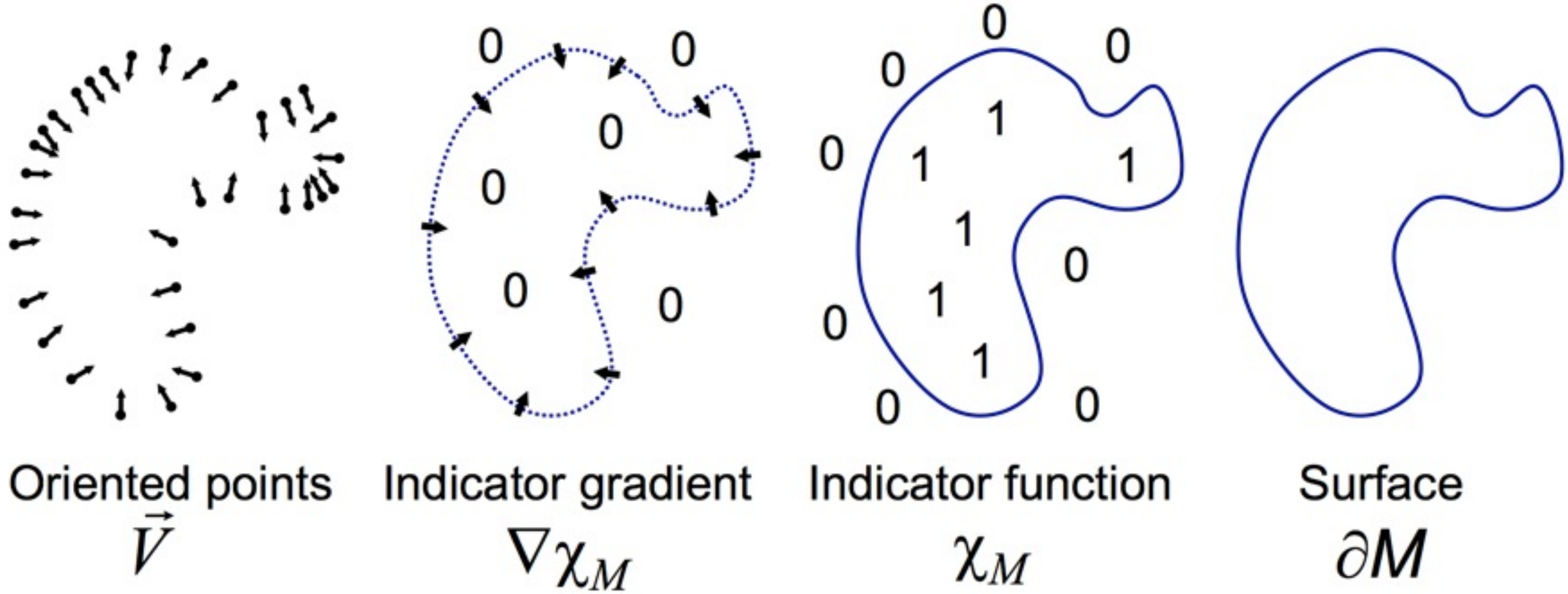
Implementation: Iso-Surface

Given the Points:

- Set Octree
- Compute vector field
- Compute indicator function
- **Extract iso-surface**



Summary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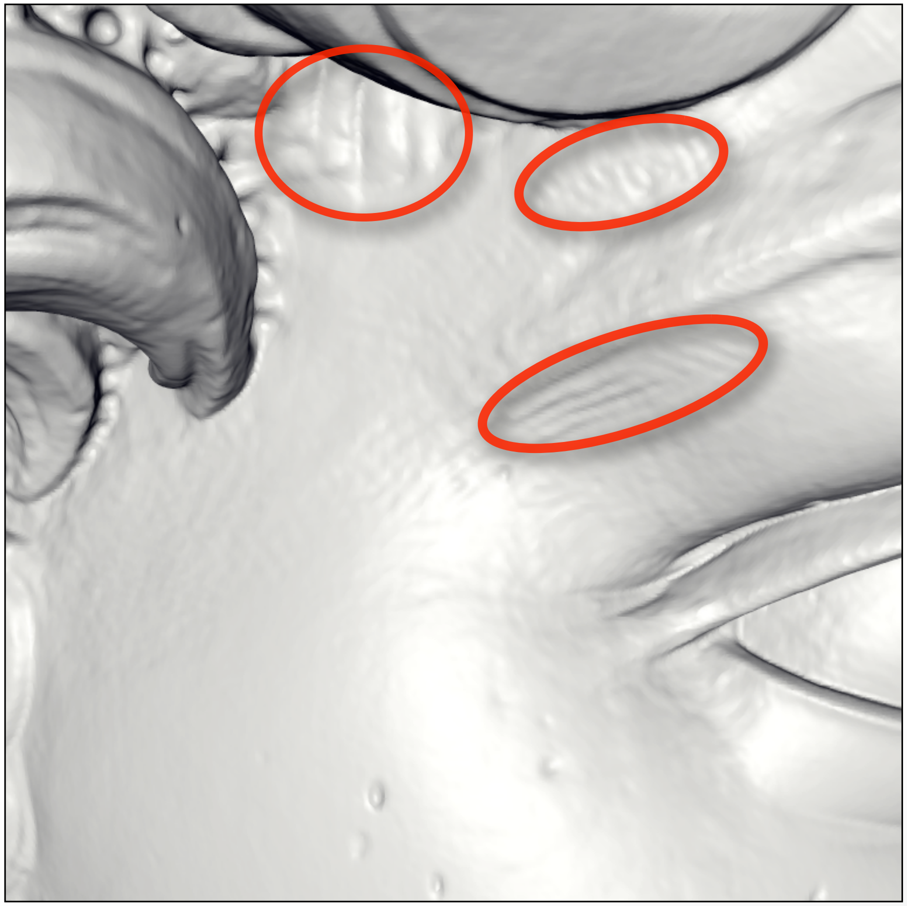
1
1
Oriented points
$\vec{V}$
Indicator gradient
$\nabla\chi_M$
Indicator function
χ_M
Surface
∂M

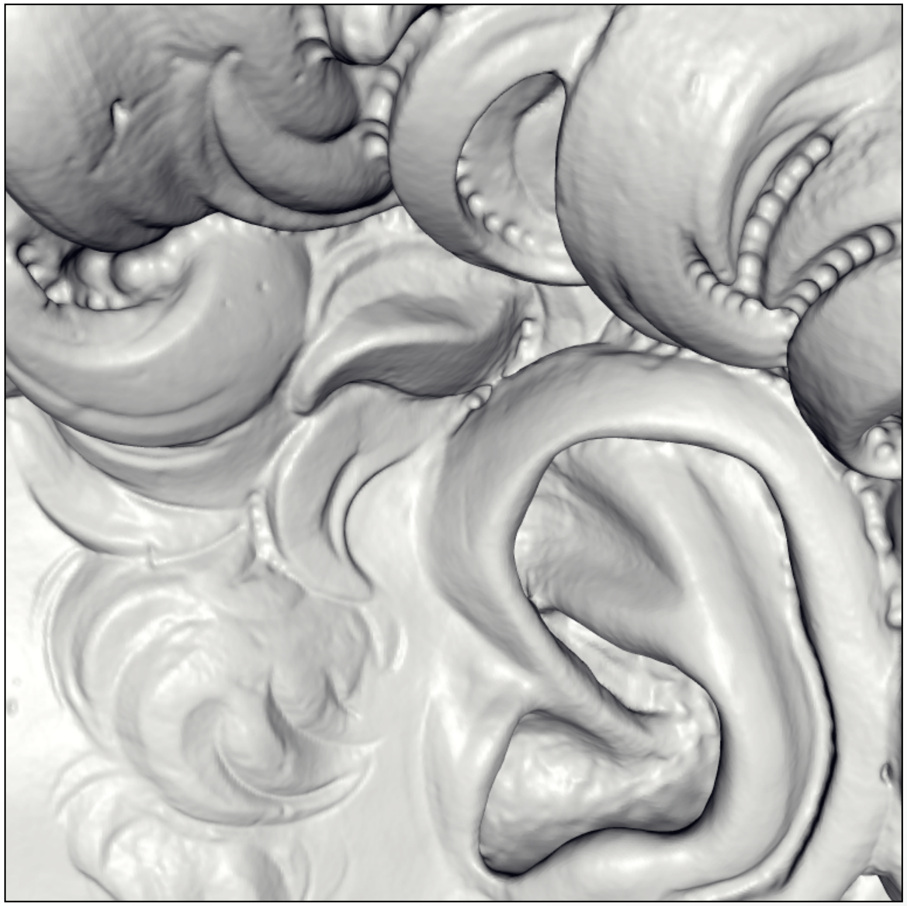
Michelangelo's David

- 215 million data points from 1000 scans
- 22 million triangle reconstruction
- Compute Time: 2.1 hours
- Peak Memory: 6600MB

David – Chisel marks

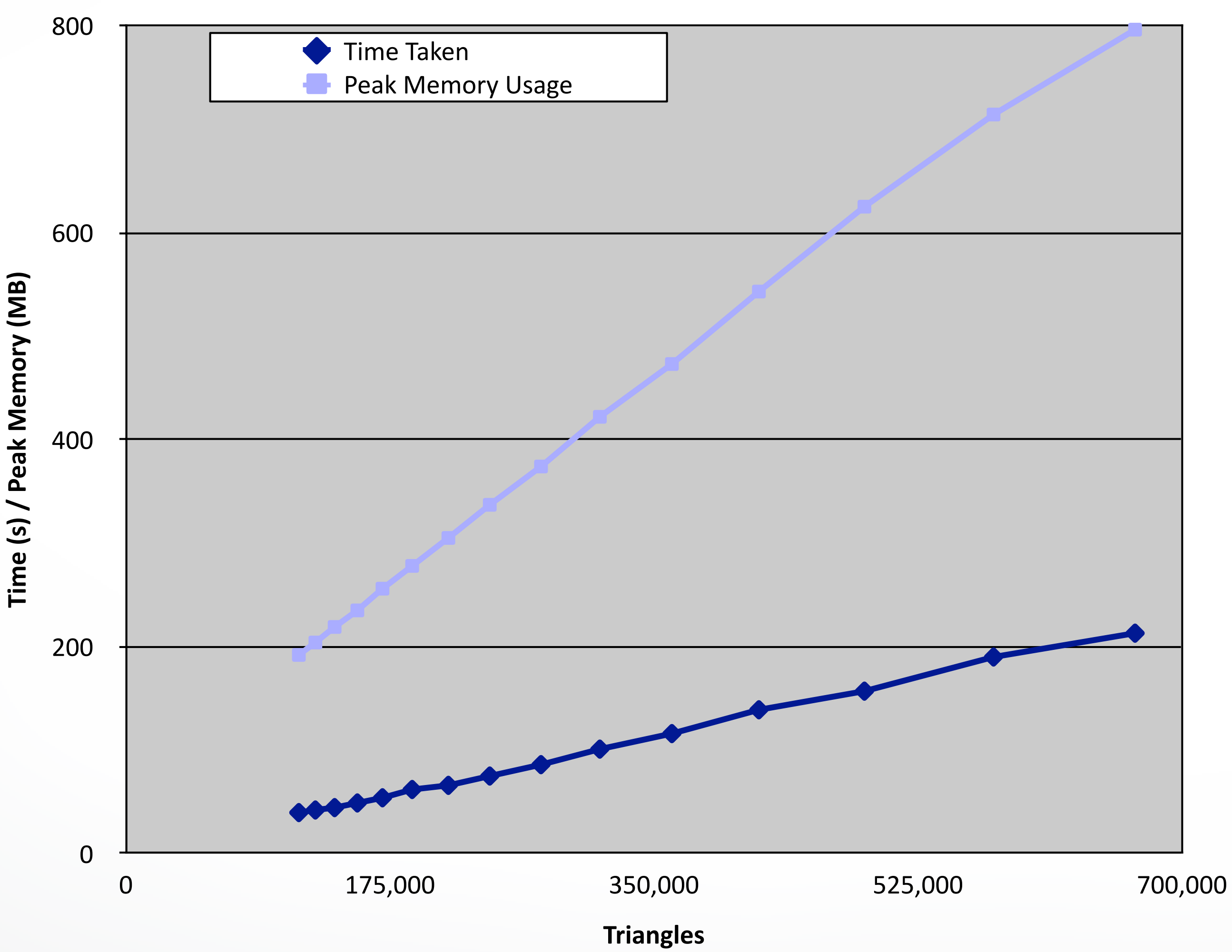


David – Drill marks



David – Drill marks

Scalability – Buddha Model



Stanford Bunny

Power Crust

FastRBF

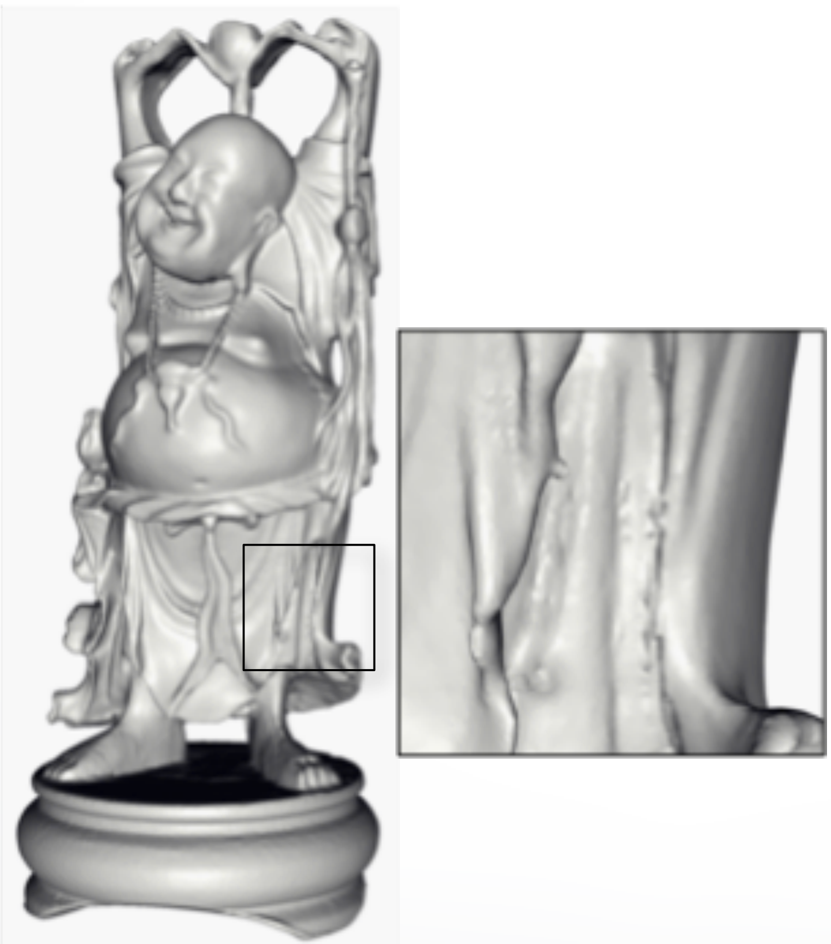
MPU

VRIP

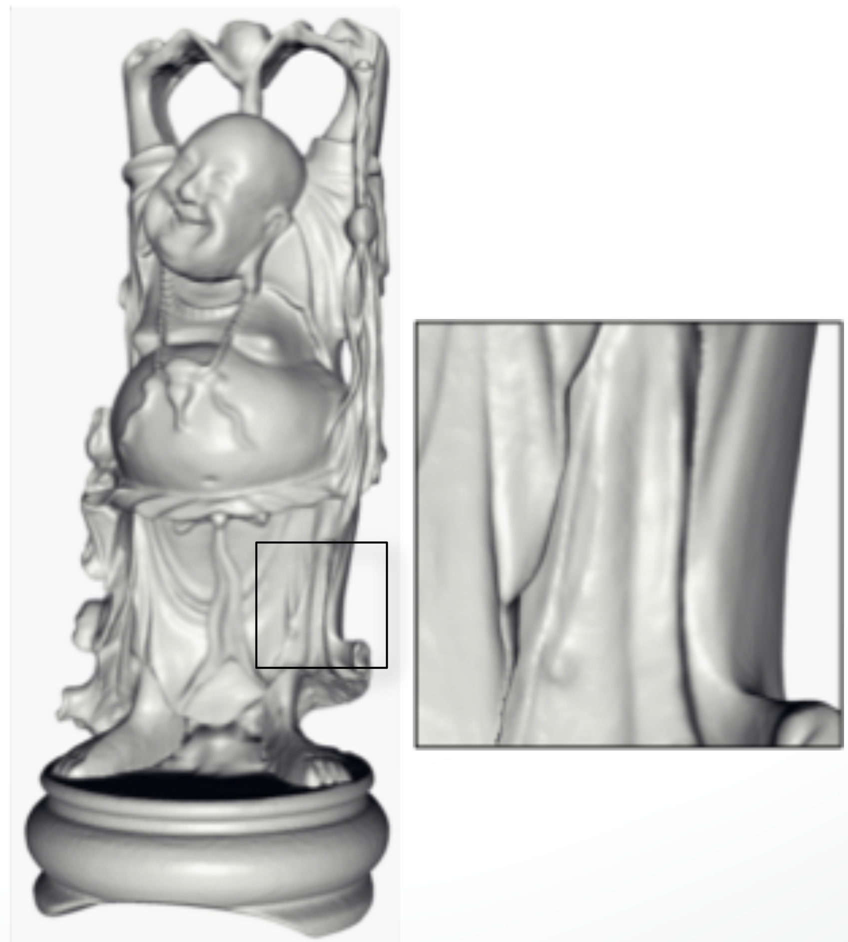
FFT Reconstruction

Possion Reconstruction

VRIP Comparison

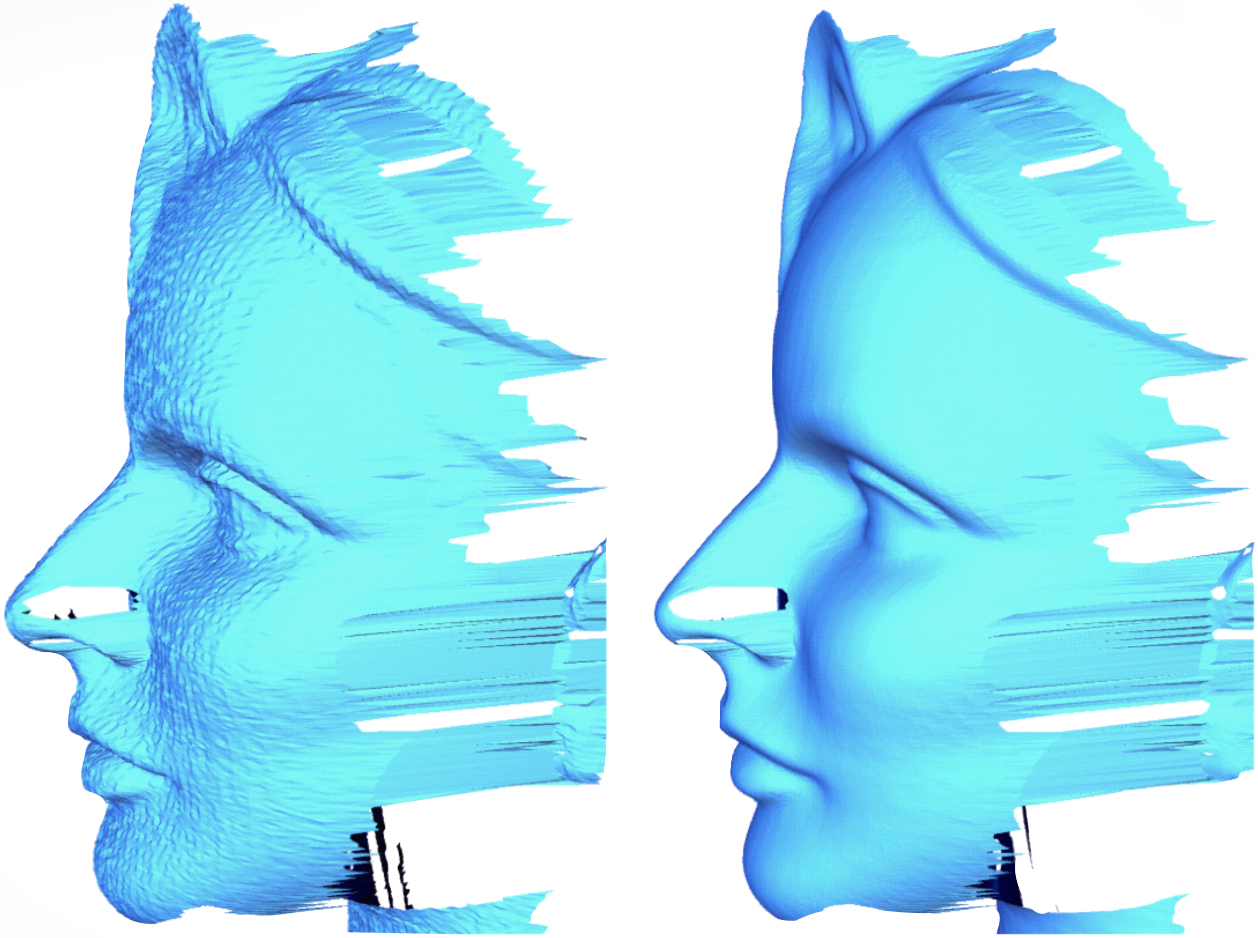


VRIP



Poission Reconstruction

Next Time



Surface Smoothing

http://cs599.hao-li.com

Thanks!