4.1 Polygon Meshes and Implicit Surfaces

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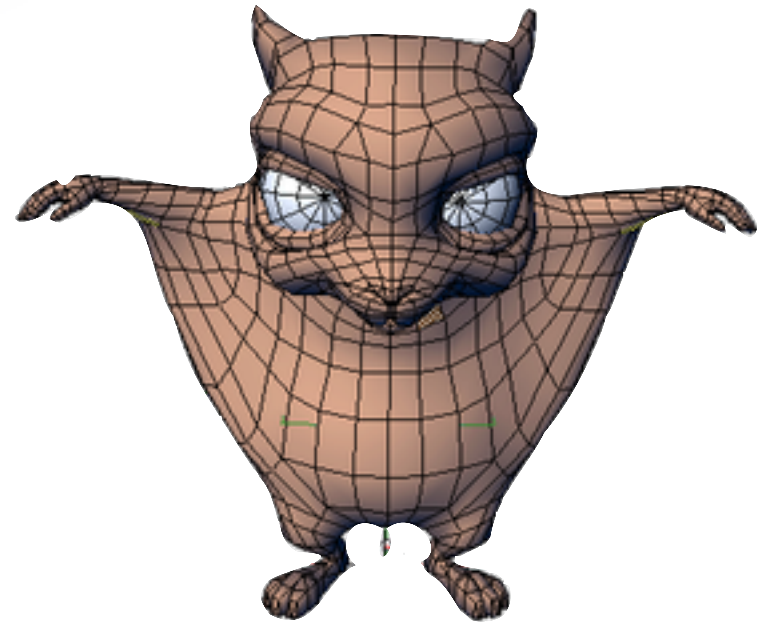
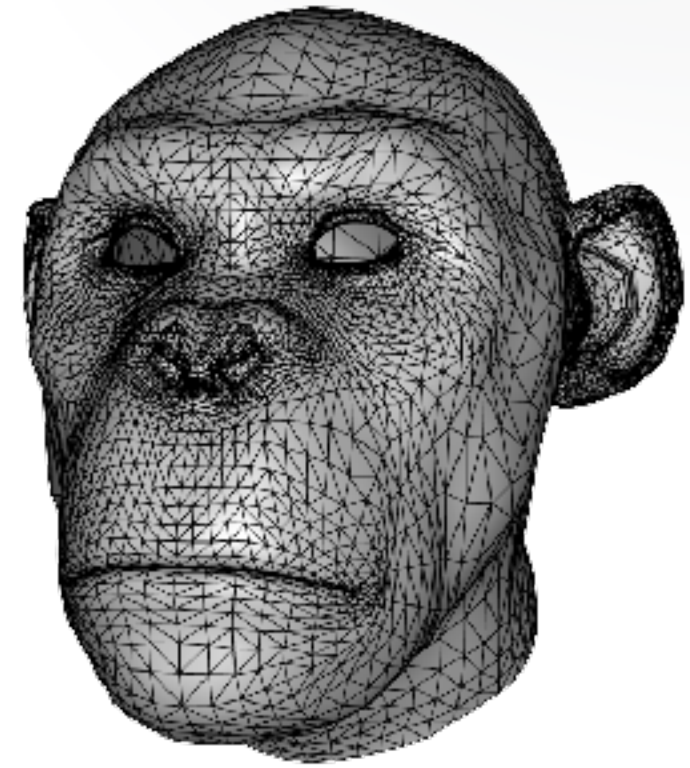
Geometric **Representations**

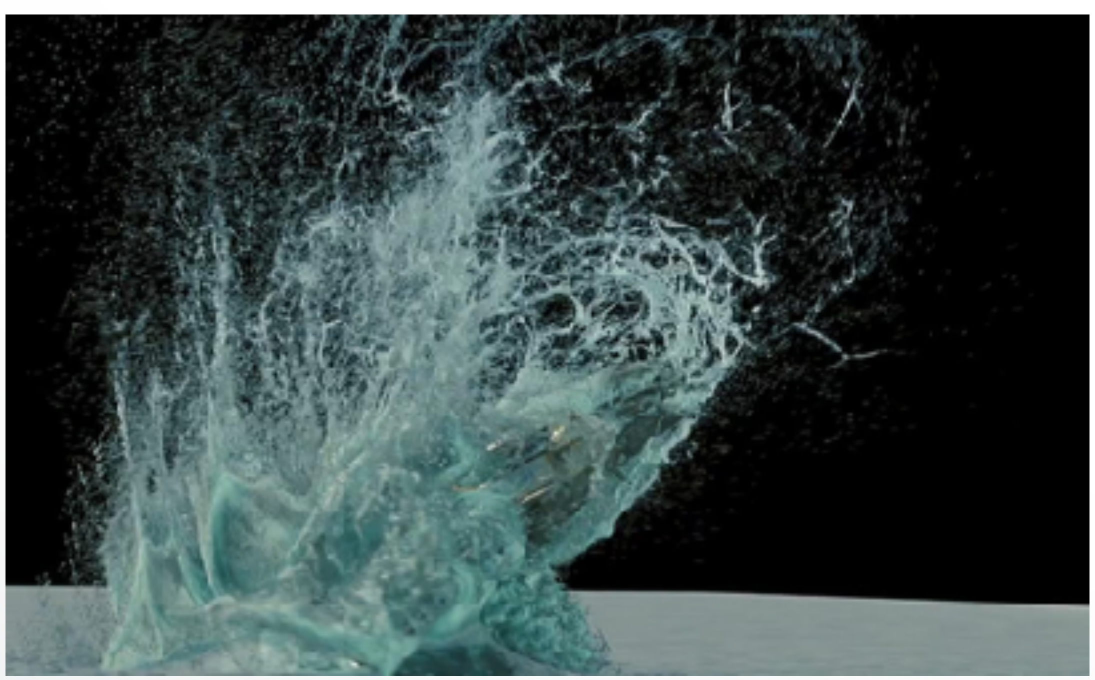
point based



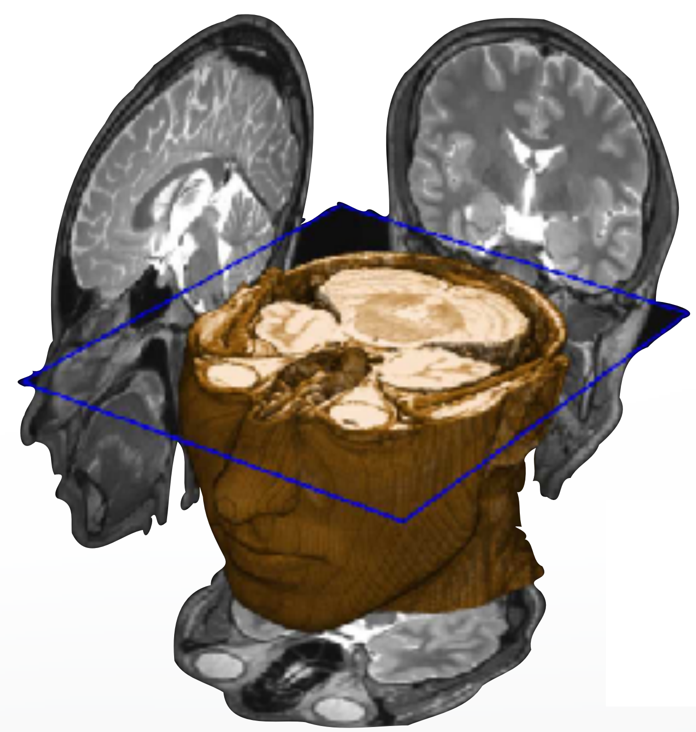
quad mesh



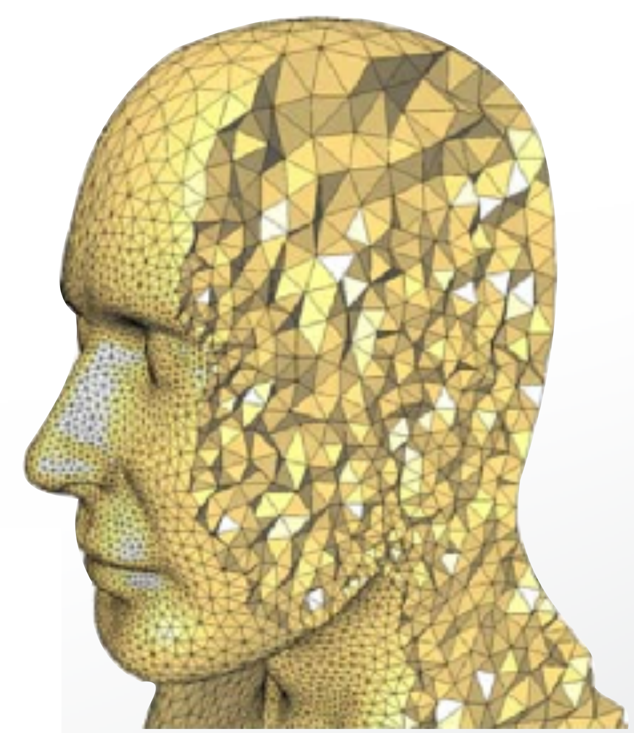
triangle mesh



implicit surfaces / particles



volumetric



tetrahedrons

Modeling Complex Shapes

Source: Wikipedia

- An equation for a sphere is possible, but how about an equation for a telephone, or a face?
- Complexity is achieved using simple pieces
  - polygons, parametric surfaces, or implicit surfaces
- Goals
  - Model anything with arbitrary precision (in principle)
  - Easy to build and modify
  - Efficient computations (for rendering, collisions, etc.)
  - Easy to implement (a minor consideration...)

What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
- Ability to model what we need
- Smoothness and continuity
- Ability to evaluate derivatives
- Ability to do collision detection
- Ease of rendering

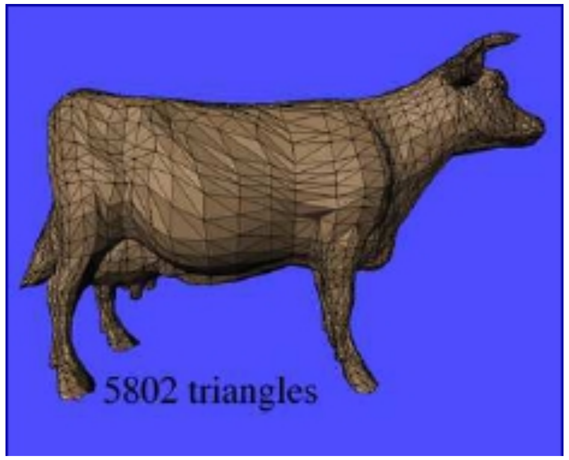
No single technique solves all problems!

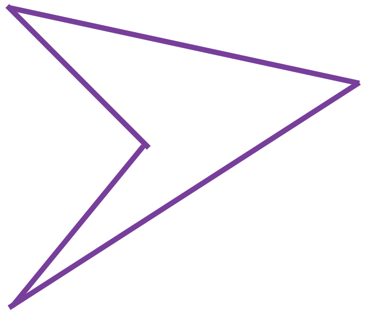
Shape Representations

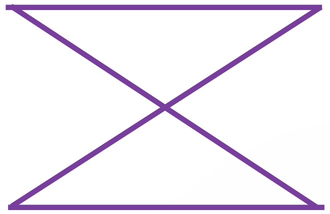
- Polygon Meshes
- Parametric Surfaces
- Implicit Surfaces

Polygon Meshes

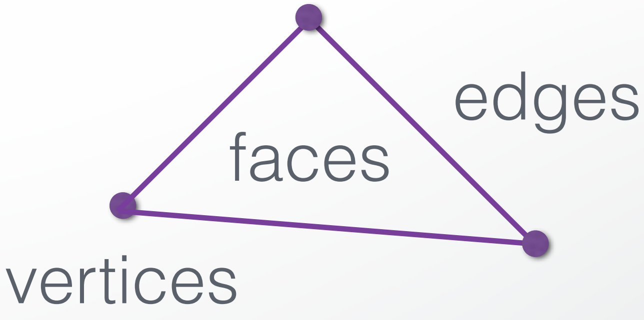
- Any shape can be modeled out of polygons
  - – if you use enough of them…
- Polygons with how many sides?
  - Can use triangles, quadrilaterals, pentagons, … n-gons
  - Triangles are most common
  - When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting







- Polygon meshes are built out of
  - vertices (points)
  - edges (line segments between vertices)
  - faces (polygons bounded by edges)



Polygon Models in OpenGL

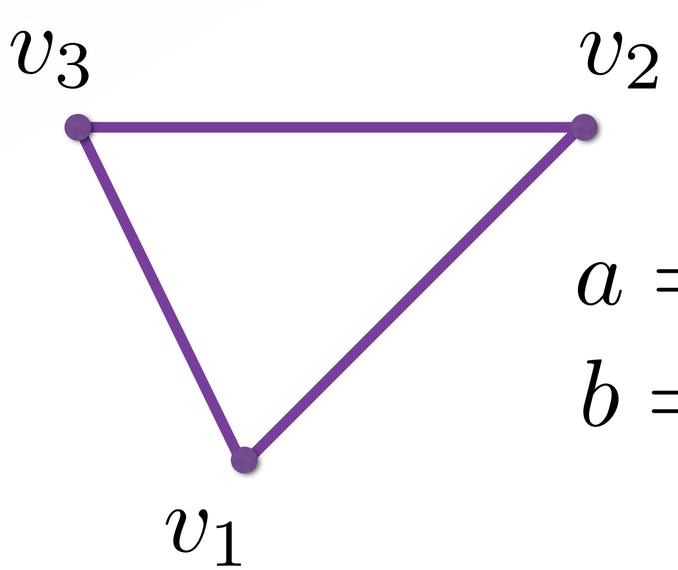
- for faceted shading

```
glNormal3fv(n);
glBegin(GL_POLYGONS);
    glVertex3fv(vert1);
    glVertex3fv(vert2);
    glVertex3fv(vert3);
glEnd();
```

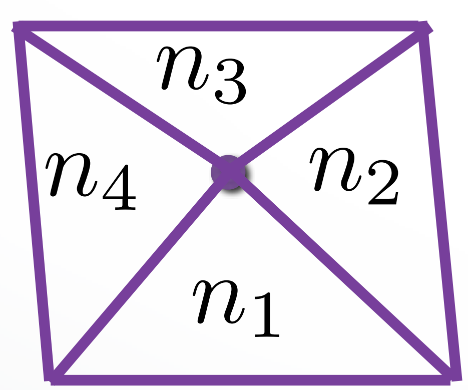
- for smooth shading

```
glBegin(GL_POLYGONS);
    glNormal3fv(normal1);
    glVertex3fv(vert1);
    glNormal3fv(normal2);
    glVertex3fv(vert2);
    glNormal3fv(normal3);
    glVertex3fv(vert3);
glEnd();
```

Normals



$$a = v_2 - v_1$$
$$b = v_3 - v_1$$



- Triangle defines unique plane
  - can easily compute normal

$$n = \frac{a \times b}{\|a \times b\|}$$

  - depends on vertex orientation!
  - clockwise order gives

$$n' = -n$$

- Vertex normals less well defined
  - can average face normals
  - works for smooth surfaces
  - but not at sharp corners (think of a cube)

Where Meshes Come From

- Model manually
  - Write out all polygons
  - Write some code to generate them
  - Interactive editing: move vertices in space
- Acquisition from real objects
  - 3D scanners, vision systems
  - Generate set of points on the surface
  - Need to convert to polygons

Mesh Data Structures

- How to store geometry & connectivity?
- compact storage and file formats
- Efficient algorithms on meshes
  - Time-critical operations
  - All vertices/edges of a face
  - All incident vertices/edges/faces of a vertex

Data Structures

Different Data Structures:

- Different topological data storage
- Most important ones are face and edge-based (since they encode connectivity)
- Design decision ~ memory/speed trade-off

Face Set (STL)

Face:

- 3 vertex positions

Triangles		
x_{11} y_{11} z_{11}	x_{12} y_{12} z_{12}	x_{13} y_{13} z_{13}
x_{21} y_{21} z_{21}	x_{22} y_{22} z_{22}	x_{23} y_{23} z_{23}
...	...	...
x_{F1} y_{F1} z_{F1}	x_{F2} y_{F2} z_{F2}	x_{F3} y_{F3} z_{F3}

9*4 = 36 B/f (single precision)
72 B/v (Euler Poincaré)

No explicit connectivity

Shared Vertex (OBJ,OFF)

Indexed Face List:

- Vertex: position
- Face: Vertex Indices

Vertices
x_1 y_1 z_1
...
x_V y_V z_V

Triangles
i_{11} i_{12} i_{13}
...
...
...
...
i_{F1} i_{F2} i_{F3}

12 B/v + 12 B/f = 36B/v

No explicit adjacency info

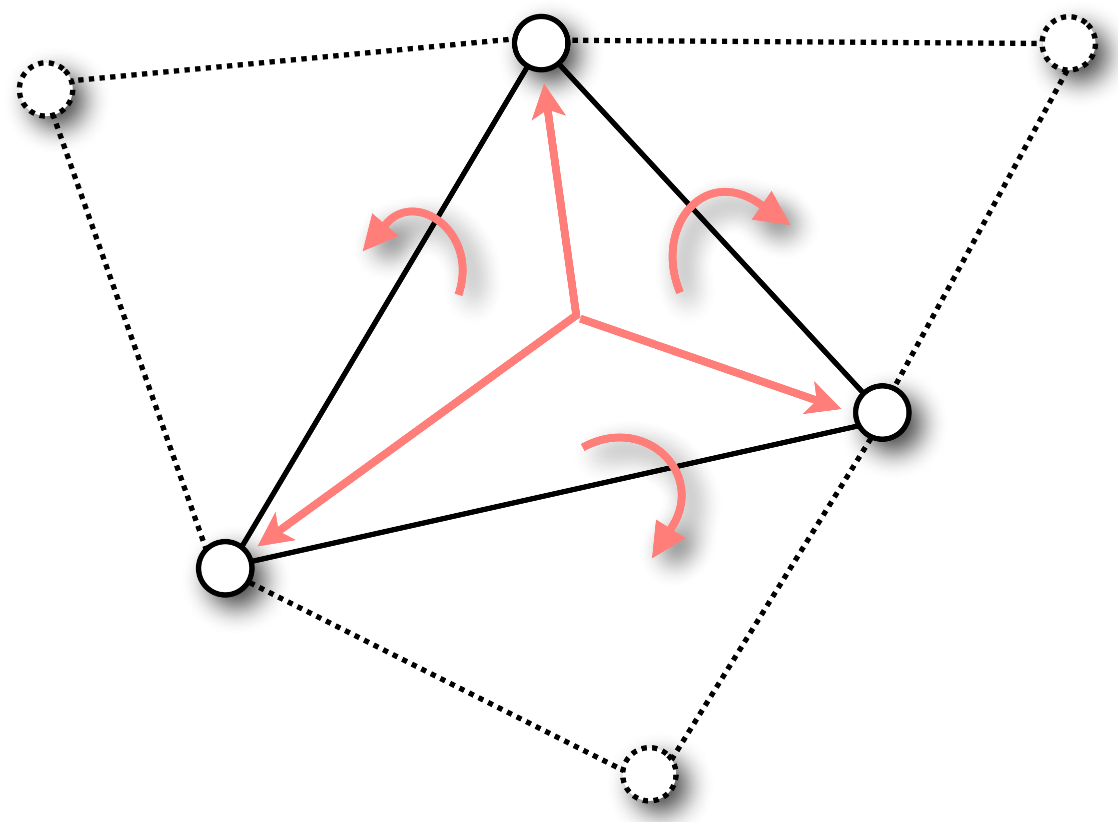
Face-Based Connectivity

Vertex:

- position (12B)
- 1 face (4B)

Face:

- 3 vertices (12B)
- 3 face neighbors (24B)



64 B/v

No edges: Special case handling for arbitrary polygons

Edges always have the same topological structure

↓

Efficient handling of polygons with variable valence

(Winged) **Edge-Based Connectivity**

Vertex:

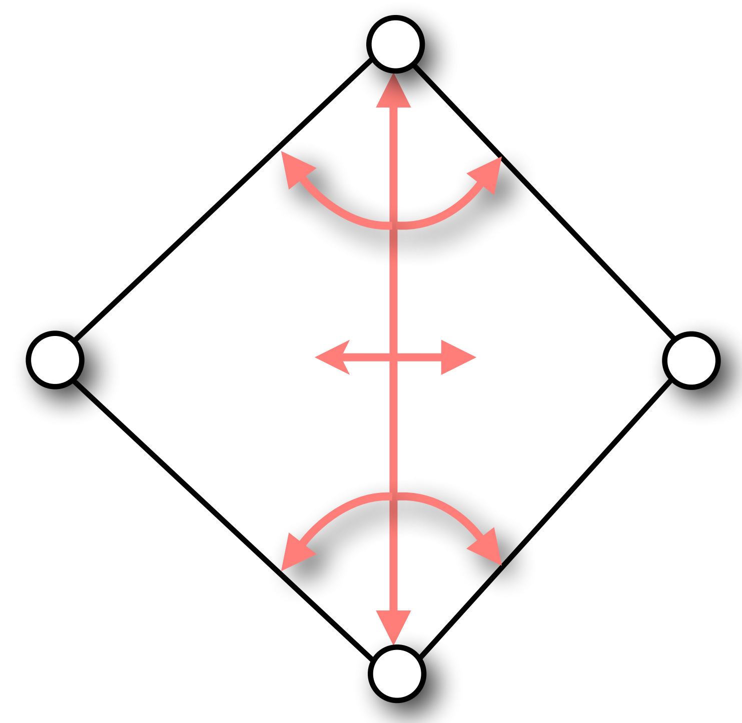
- position
- 1 edge

Edge:

- 2 vertices
- 2 faces
- 4 edges

Face:

- 1 edges



120 B/v

Edges have no orientation: special case handling for neighbors

Halfedge-Based Connectivity

Vertex:

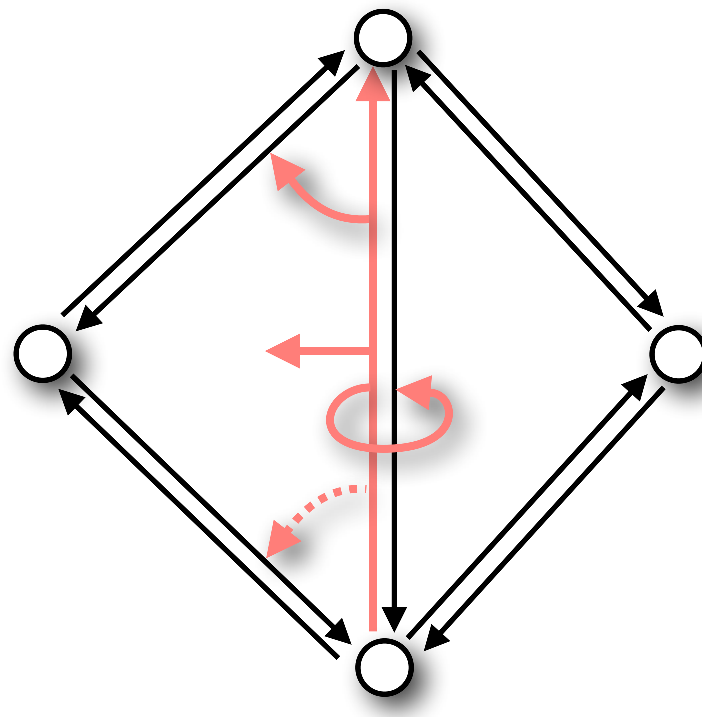
- position
- 1 halfedge

Edge:

- 1 vertex
- 1 face
- 1, 2, or 3 halfedges

Face:

- 1 halfedge



96 to 144 B/v

Edges have orientation: No-runtime overhead due to arbitrary faces

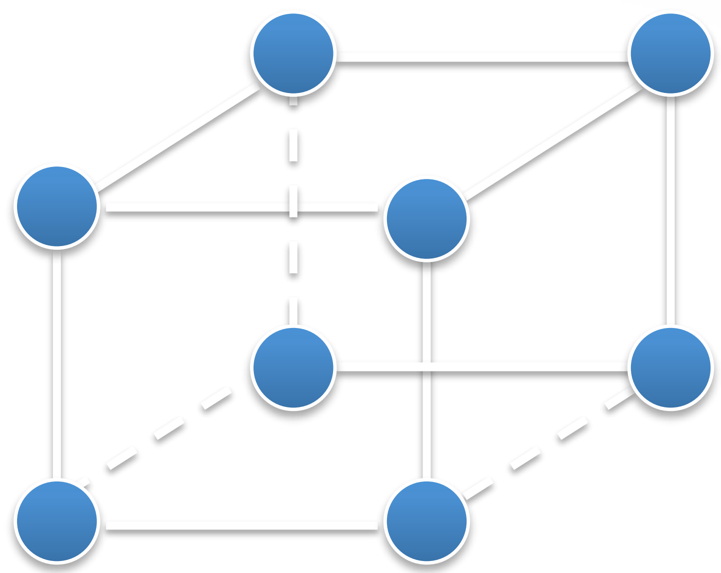
Data Structures for Polygon Meshes

- Simplest (but dumb)
  - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
  - redundant: each vertex stored multiple times
- Vertex List, Face List
  - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
  - List of triangles, each a triple of vertex id's (or pointers) topological (connectivity, adjacency) info only

    Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.
- Fancier schemes:
  - Store more topological info so adjacency queries can be answered in O(1) time.
  - *Winged-edge data structure* – edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).

A File Format for Polygon Models: OBJ

```
# OBJ file for a 2x2x2 cube
v  -1.0  1.0  1.0   - Vertex 1
v  -1.0 -1.0  1.0   - Vertex 2
v   1.0 -1.0  1.0   - Vertex 3
v   1.0  1.0  1.0   - …
v  -1.0  1.0 -1.0
v  -1.0 -1.0 -1.0
v   1.0 -1.0 -1.0
v   1.0  1.0 -1.0
f   1  2  3  4
f   8  7  6  5
f   4  3  7  8
f   5  1  4  8
f   5  6  2  1
f   2  6  7  3
```



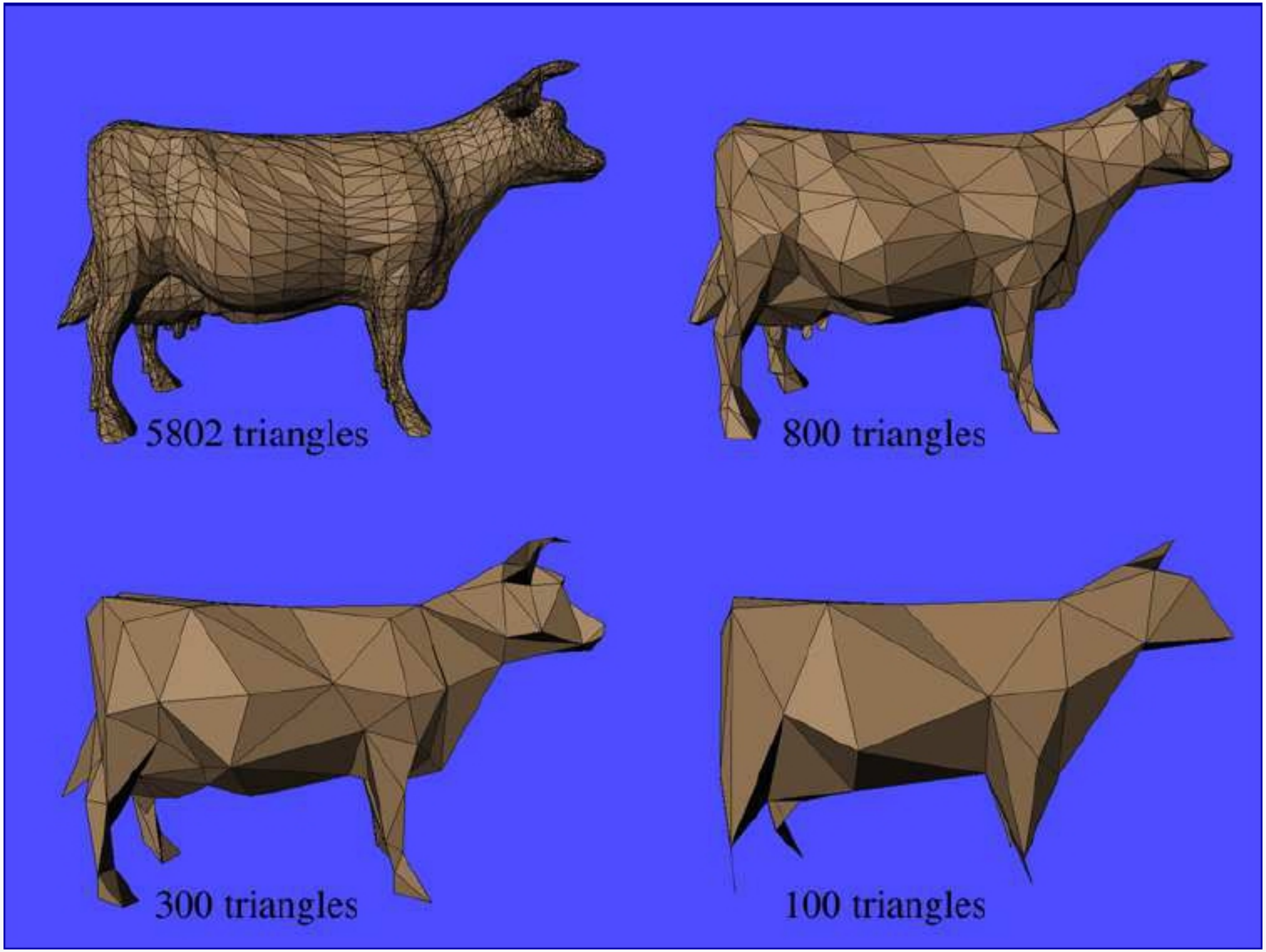
Syntax:

v *x y z* - a vertex a (x,y,z)

f $v_1\ v_2 \ldots v_n$ - a face with vertices $v_1\ v_2 \ldots v_n$

#anything - *comment*

How Many Polygons to Use?



Why Level of Detail?

- Different models for near and far objects
- Different models for rendering and collision detection
- Compression of data recorded from the real world

- We need automatic algorithms for reducing the polygon count without
  - losing key features
  - getting artifacts in the silhouette
  - popping

Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes

- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?

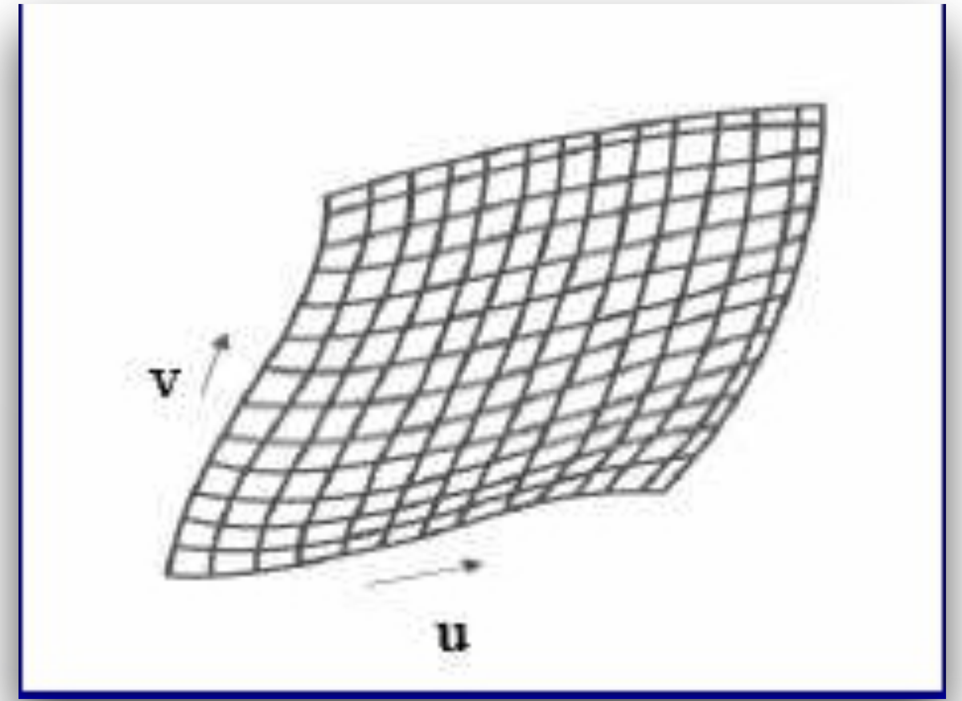
Shape Representations

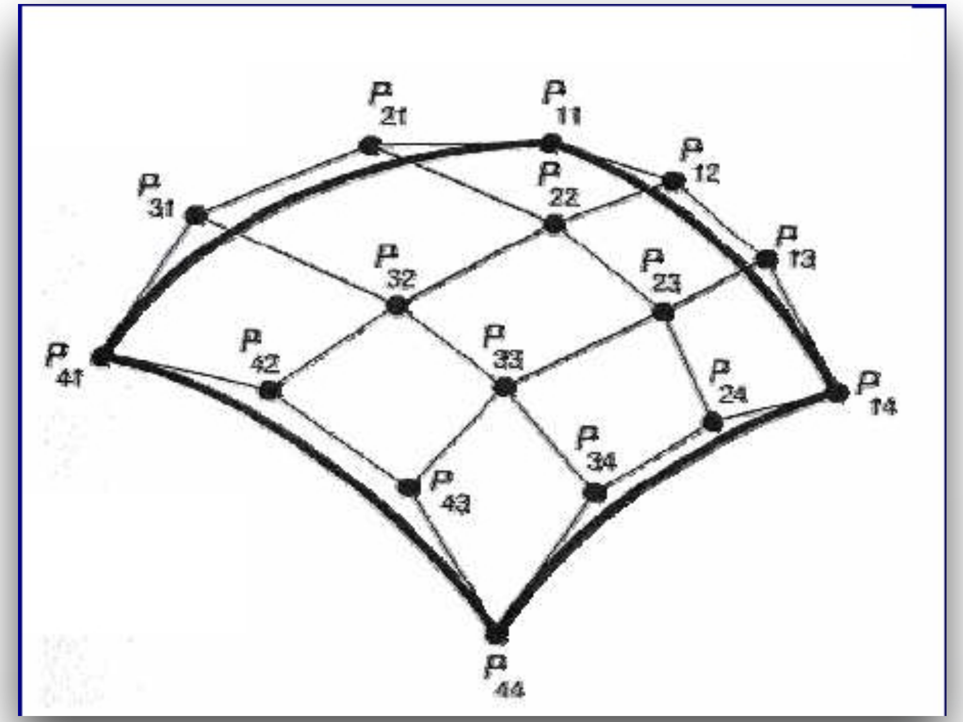
- Polygon Meshes
- Parametric Surfaces
- Implicit Surfaces

Parametric Surfaces

$$p(u, v) = [x(u, v), y(u, v), z(u, v)]$$

- e.g. plane, cylinder, bicubic surface, swept surface



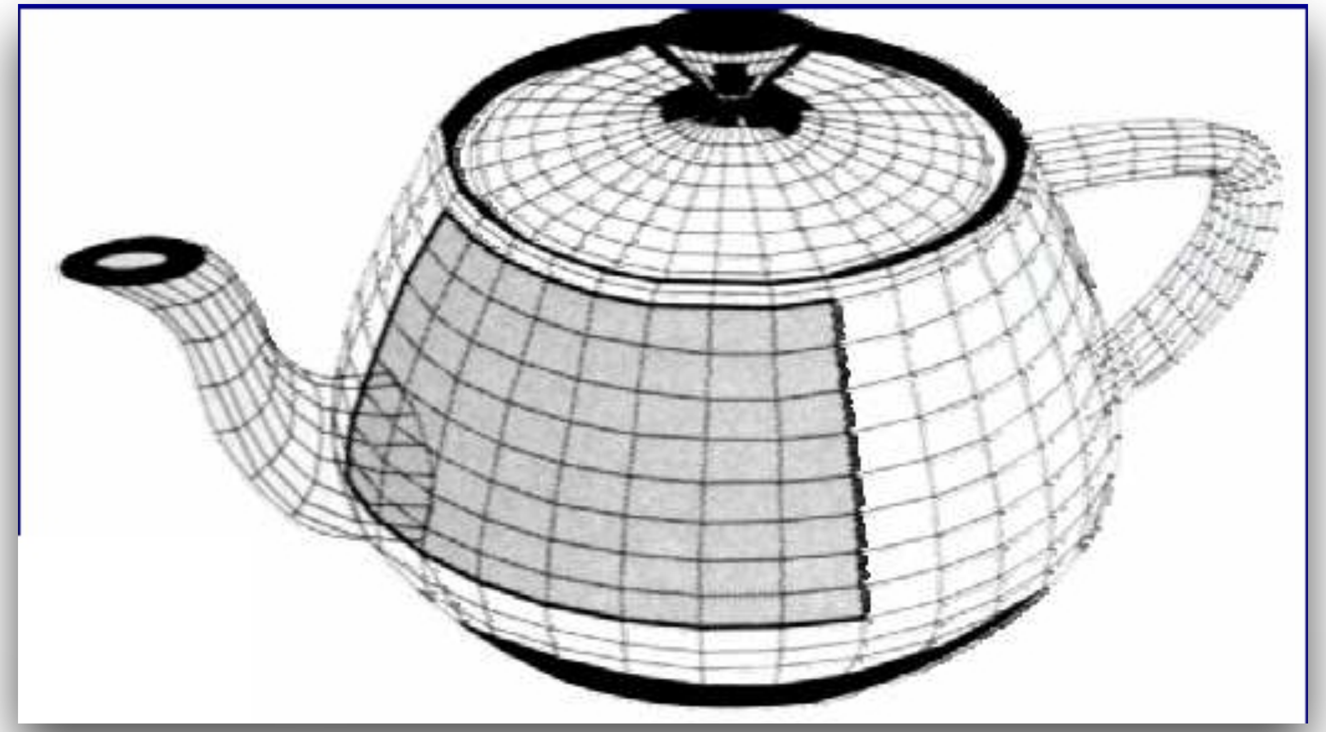


Bezier patch

Parametric Surfaces

$$p(u,v) = [x(u,v), y(u,v), z(u,v)]$$

- e.g. plane, cylinder, bicubic surface, swept surface



Utah teapot

Parametric Representation

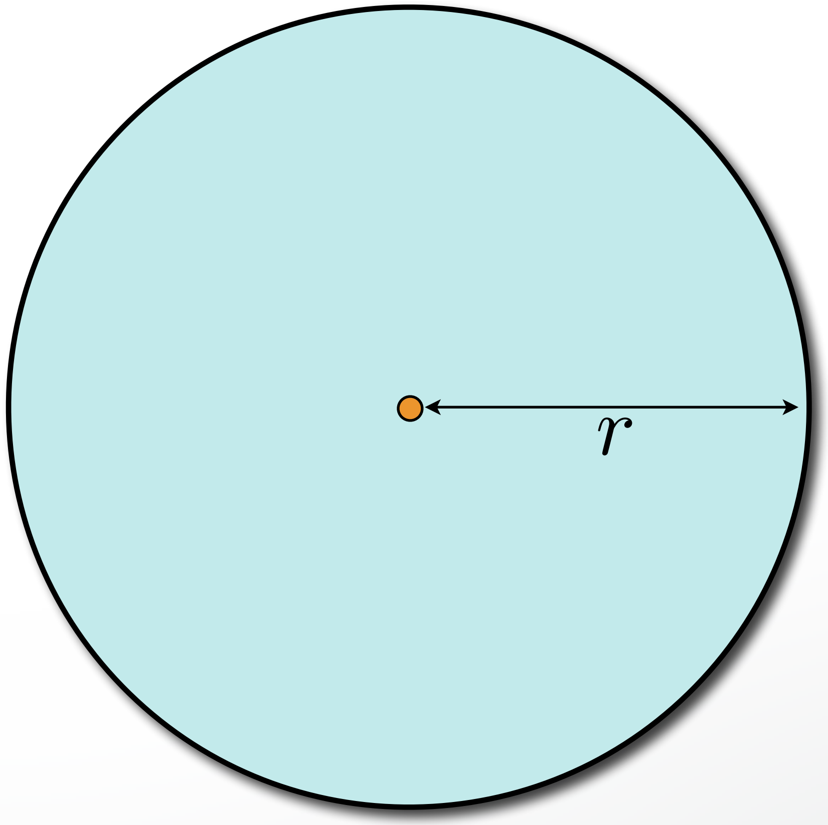
Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

2D example: A Circle

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$



Parametric Representation

Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

2D example: Island coast line

$$\mathbf{f} : [0, 2\pi] \to \mathbb{R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} ? \\ ? \end{pmatrix}$$

Piecewise Approximation

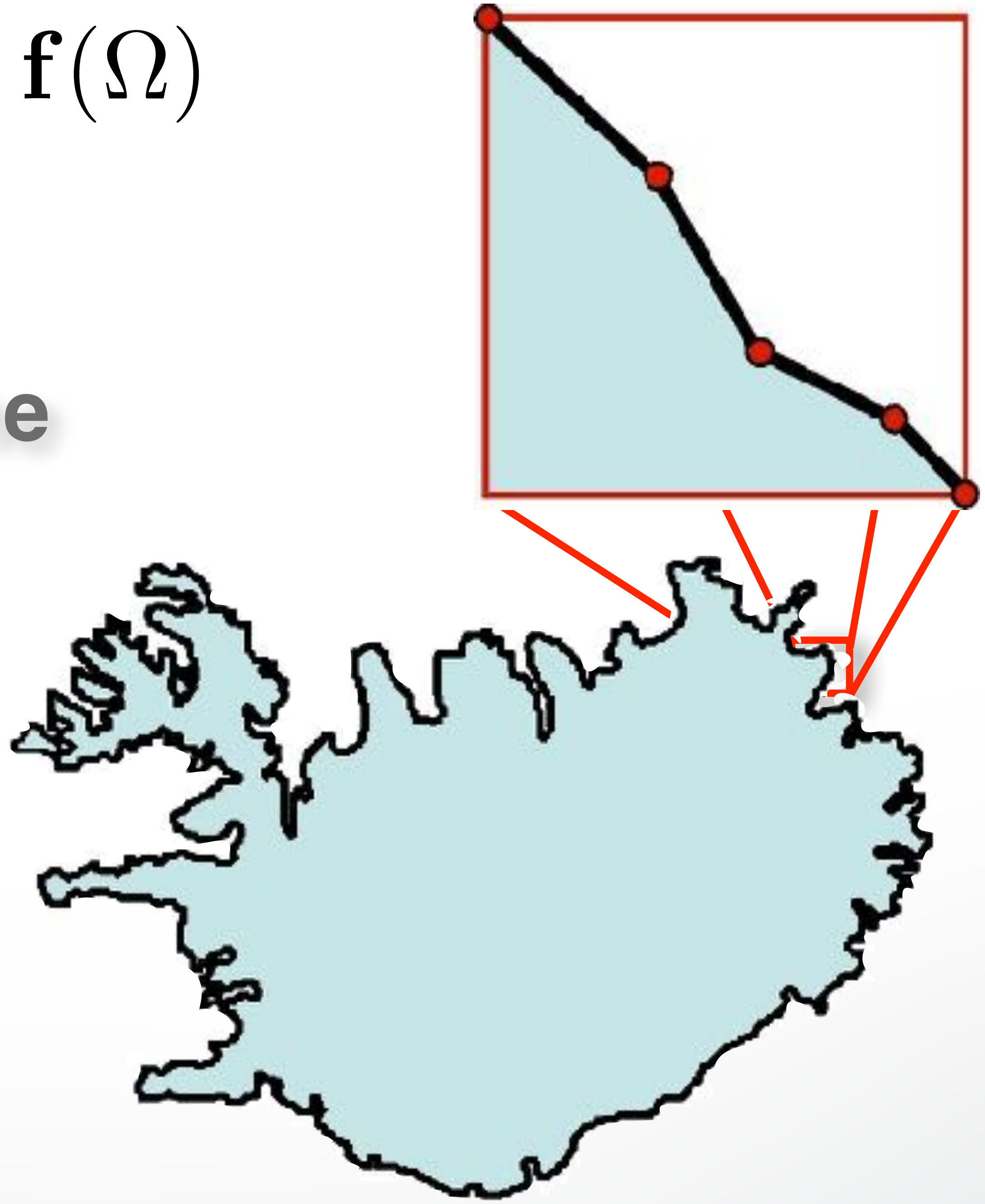
Surface is the range of a function

$$\mathbf{f} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad \mathcal{S}_\Omega = \mathbf{f}(\Omega)$$

2D example: Island coast line

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathbb{R}^2$$

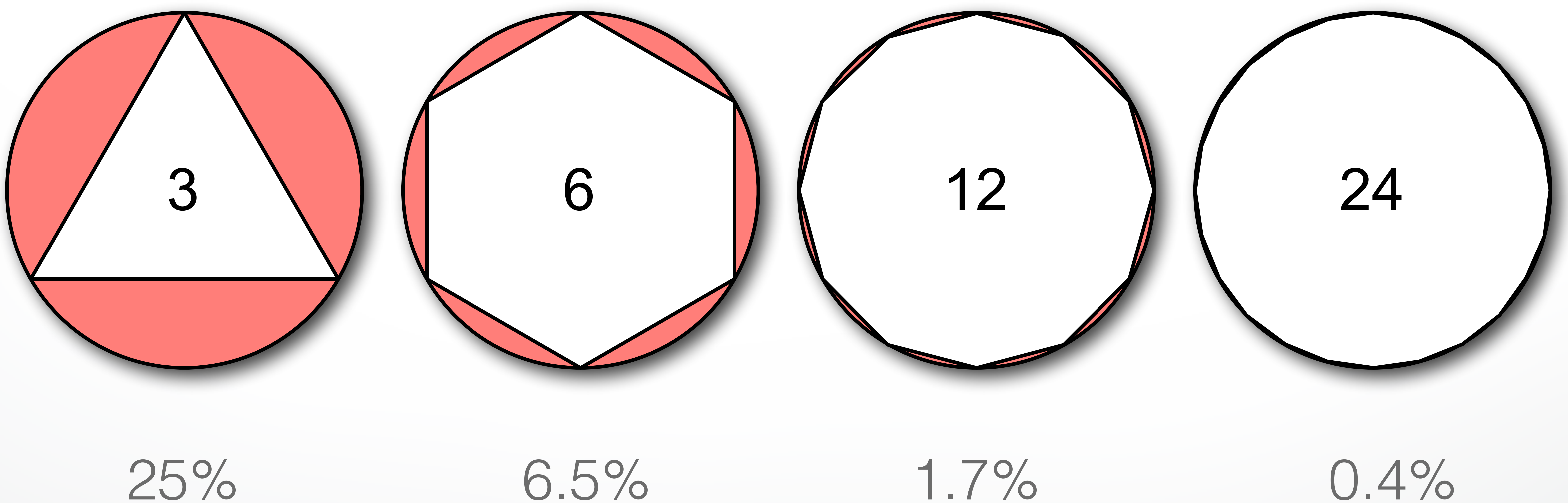
$$\mathbf{f}(t) = \begin{pmatrix} ? \\ ? \end{pmatrix}$$



Polygonal Meshes

Polygonal meshes are a good compromise

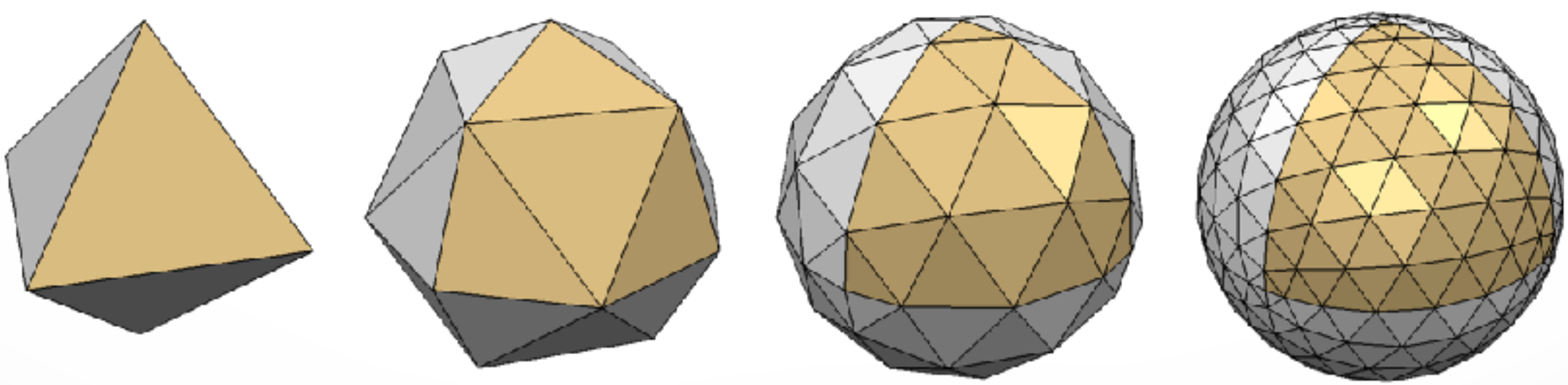
- Piecewise linear approximation → error is $O(h^2)$



Polygonal Meshes

Polygonal meshes are a good compromise

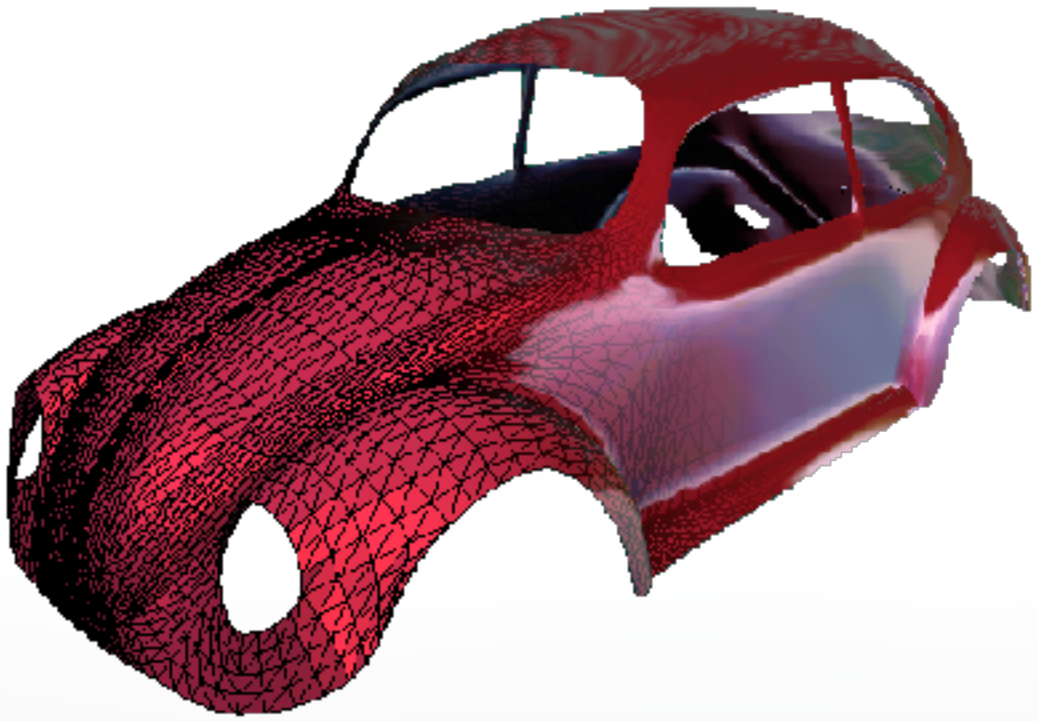
- Piecewise linear approximation → error is $O(h^2)$
- Error inversely proportional to #faces



Polygonal Meshes

Polygonal meshes are a good compromise

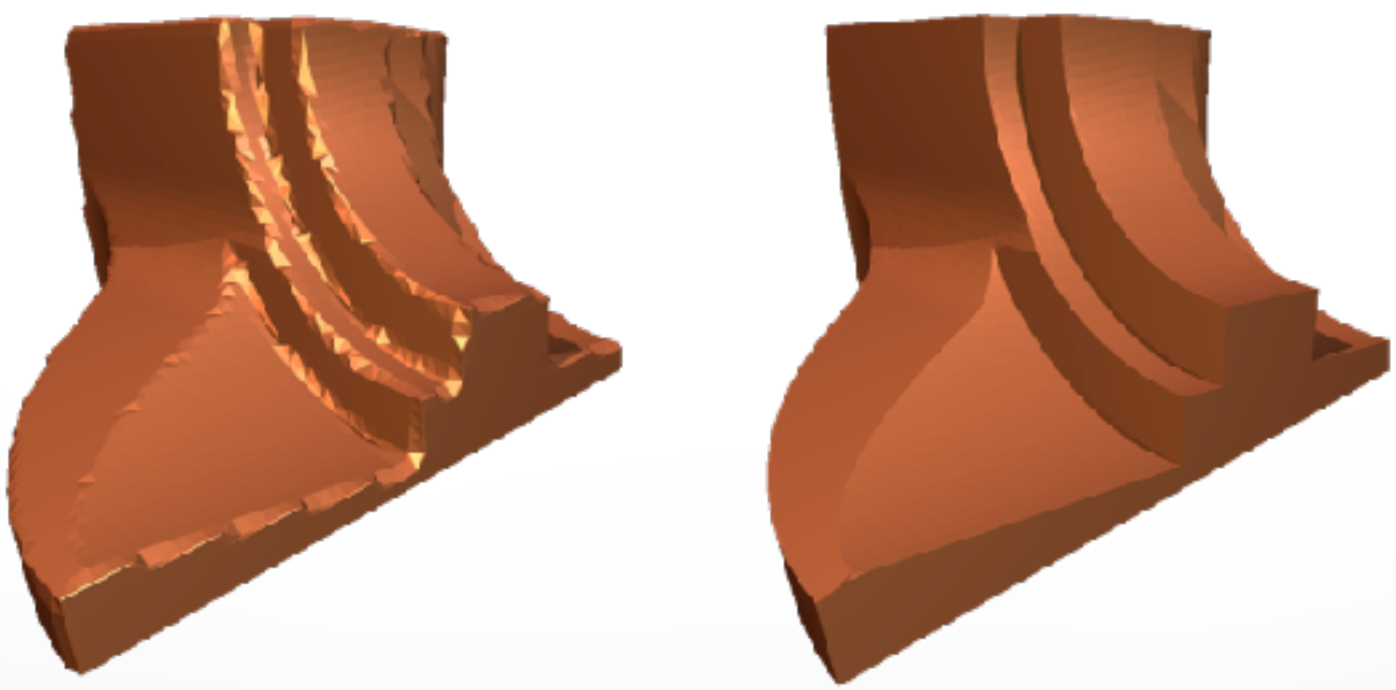
- Piecewise linear approximation → error is $O(h^2)$
- Error inversely proportional to #faces
- Arbitrary topology surfaces



Polygonal Meshes

Polygonal meshes are a good compromise

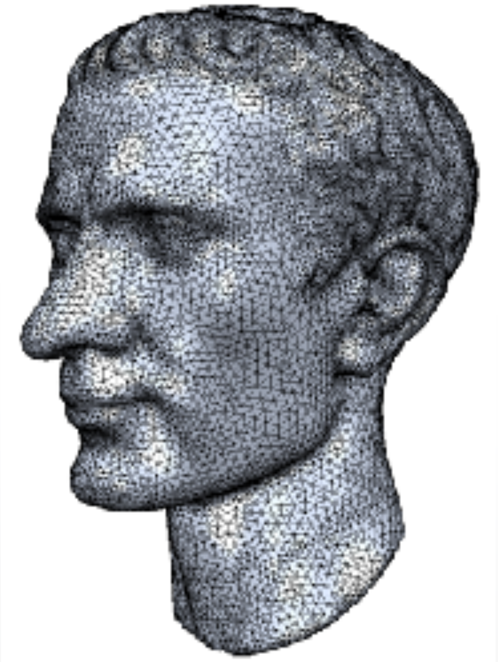
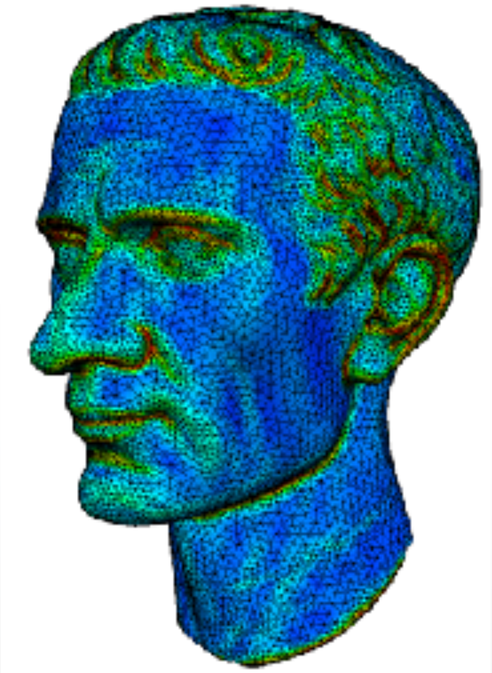
- Piecewise linear approximation → error is $O(h^2)$
- Error inversely proportional to #faces
- Arbitrary topology surfaces
- Piecewise smooth surfaces



Polygonal Meshes

Polygonal meshes are a good compromise

- Piecewise linear approximation → error is $O(h^2)$
- Error inversely proportional to #faces
- Arbitrary topology surfaces
- Piecewise smooth surfaces
- Adaptive sampling

Polygonal Meshes

Polygonal meshes are a good compromise

- Piecewise linear approximation → error is $O(h^2)$
- Error inversely proportional to #faces
- Arbitrary topology surfaces
- Piecewise smooth surfaces
- Adaptive sampling
- Efficient GPU-based rendering/processing

Parametric Surfaces

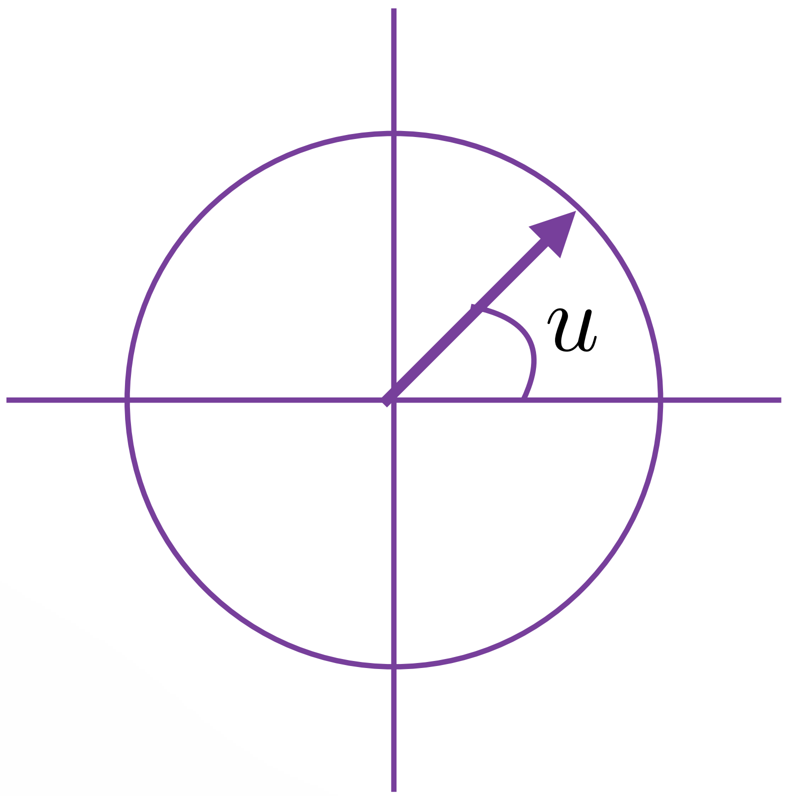
- Why better than polygon meshes?
  - Much more compact
  - More convenient to control --- just edit control points
  - Easy to construct from control points
- What are the problems?
  - Work well for smooth surfaces
  - Must still split surfaces into discrete number of patches
  - Rendering times are higher than for polygons
  - Intersection test? Inside/outside test?

Shape Representations

- Polygon Meshes
- Parametric Surfaces
- Implicit Surfaces

Two Ways to Define a Circle

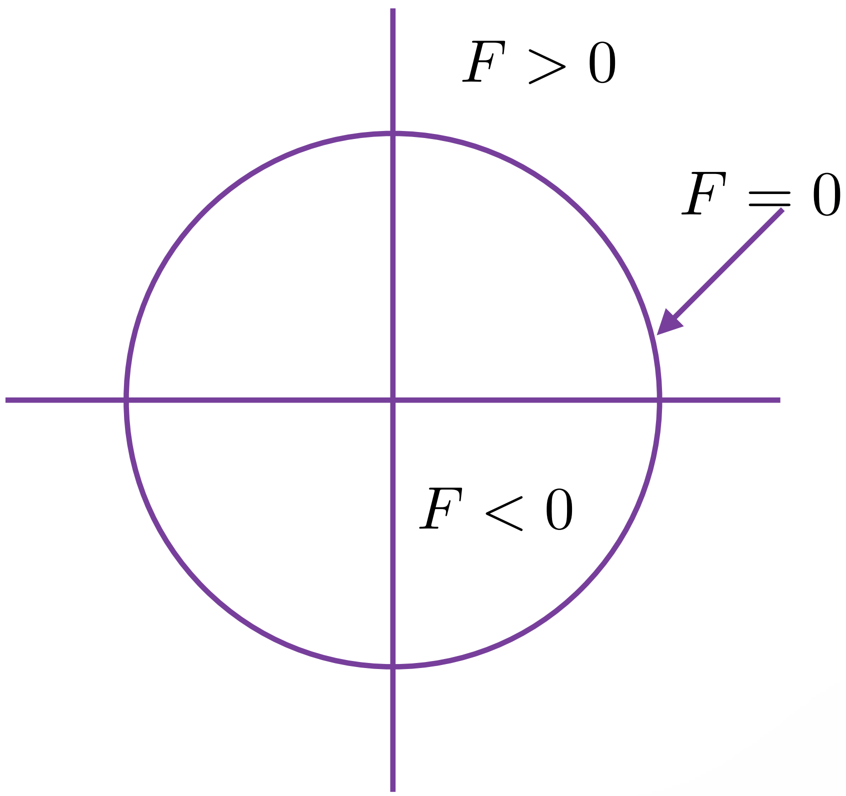
Parametric



$$x = f(u) = r cos(u)$$

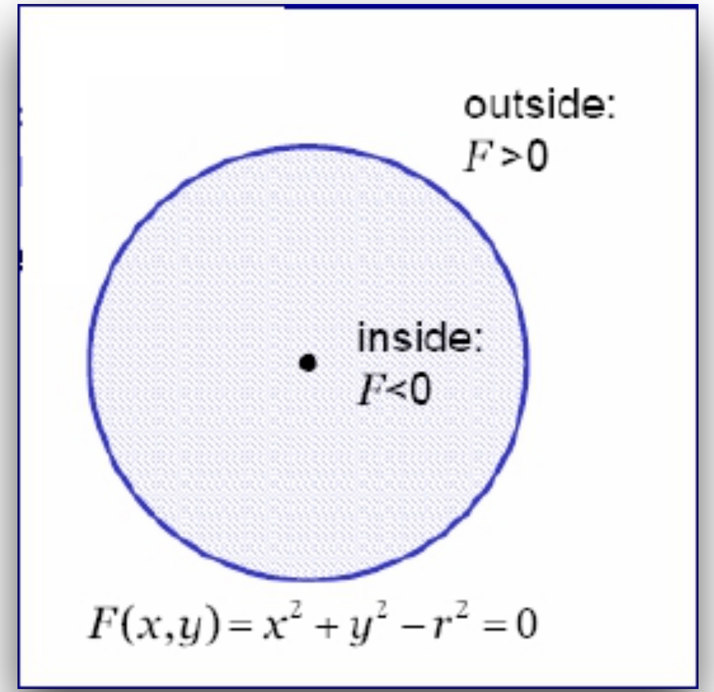
$$y = g(u) = r sin(u)$$

Implicit



$$F(x, y) = x^2 + y^2 - r^2$$

Implicit Surfaces



- well defined inside/outside
- polygons and parametric surfaces do not have this information
- Computing is hard:
  -implicit functions for a cube? telephone?

- Implicit surface: $F(x, y, z) = 0$

  - e.g. plane, sphere, cylinder, quadric, torus, blobby models

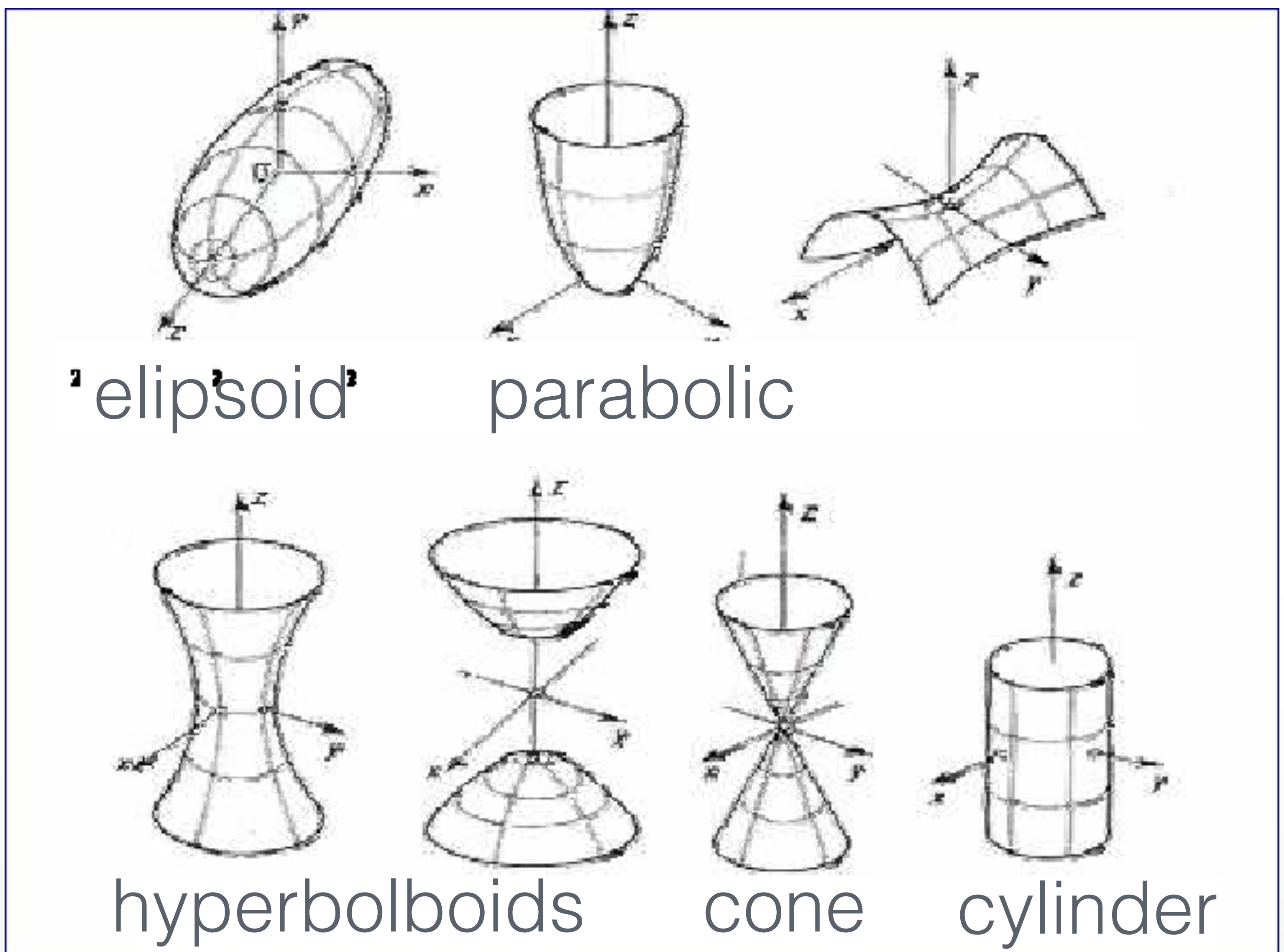
  sphere with radius *r* : $F(x, y, z) = x^2 + y^2 + z^2 - r^2 = 0$

  - terrible for iterating over the surface

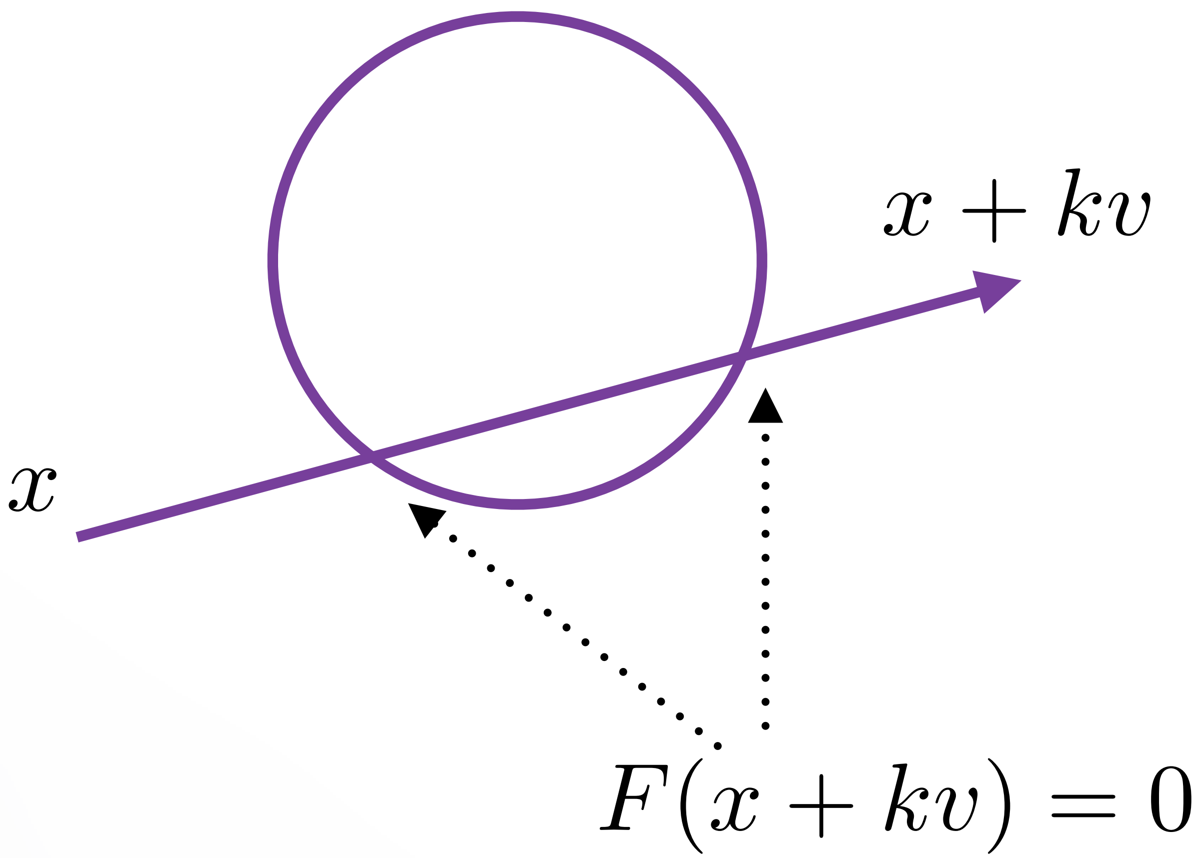
  - great for intersections, inside/outside test

Quadric Surfaces

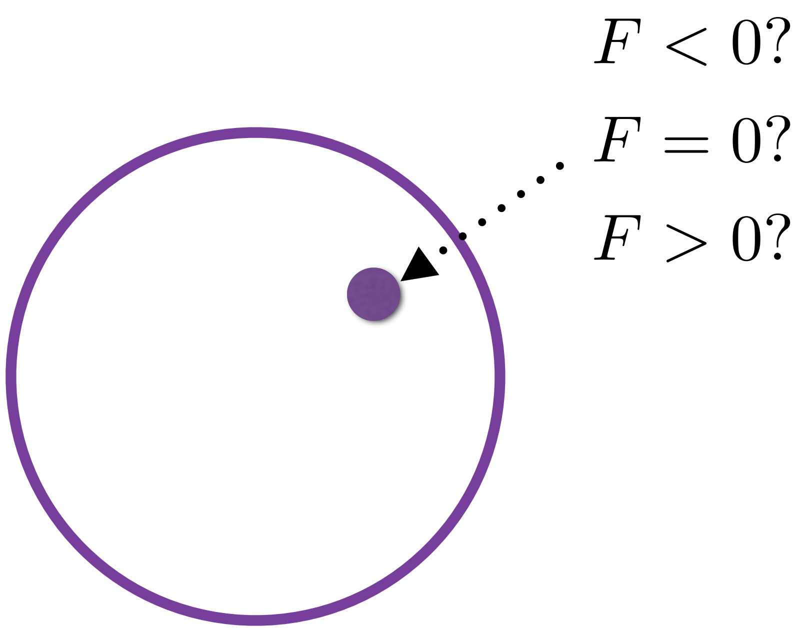
$$
\begin{aligned}
F(x, y, z) &= ax^2 + by^2 + cz^2 \\
&\quad + 2fyz + 2hxy + 2px + 2qy + 2rz + d \\
&= 0
\end{aligned}
$$



What Implicit Functions are Good For



Ray - Surface Intersection Test



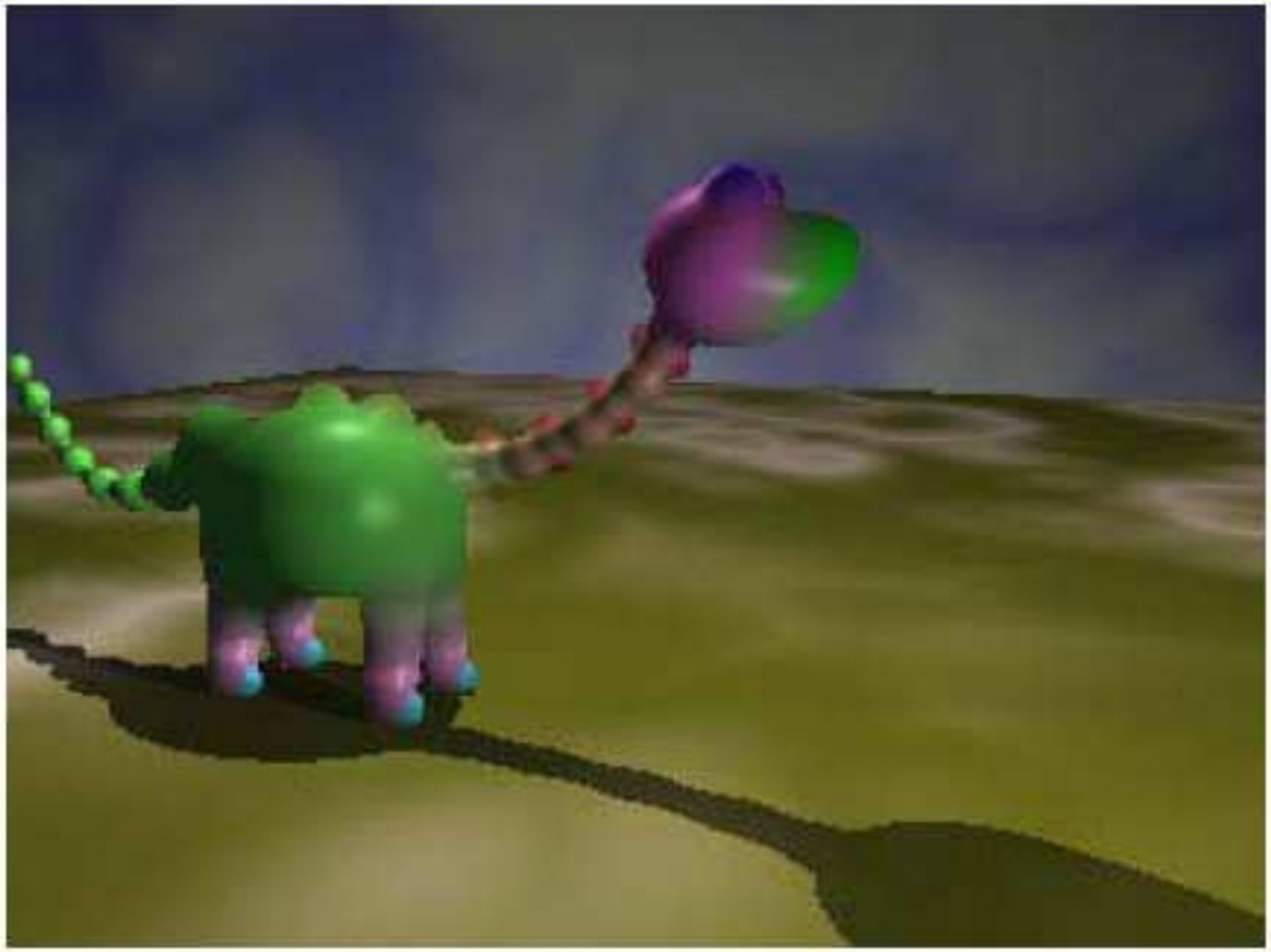
Inside/Outside Test

Surfaces from Implicit Functions

- Constant Value Surfaces are called (depending on whom you ask):
  - constant value surfaces
  - level sets
  - isosurfaces

- Nice Feature: you can add them! (and other tricks)
  - this merges the shapes
  - When you use this with spherical exponential potentials, it's called *Blobs*, *Metaballs*, or *Soft Objects*. Great for modeling animals.

Blobby Models



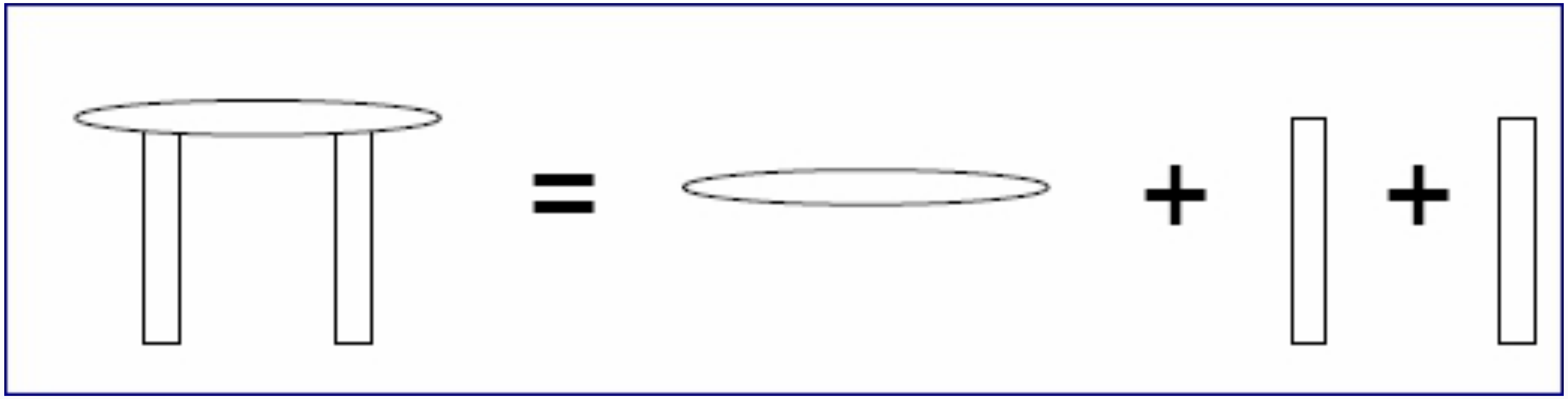
by Brian Wyvill, http://www.cpsc.ucalgary.ca/~blob/

How to draw implicit surfaces?

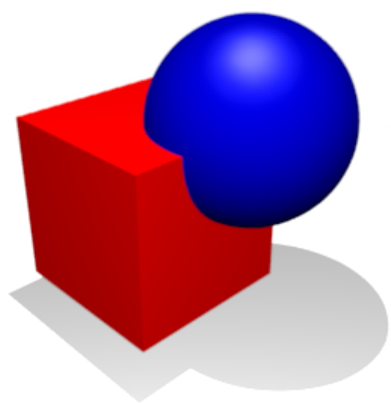
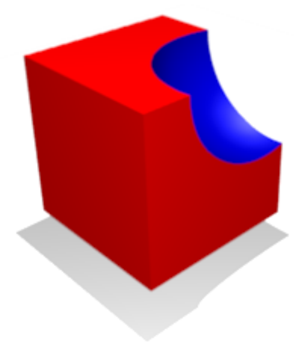
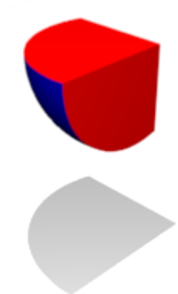
- It’s easy to ray trace implicit surfaces
  - because of that easy intersection test
- Volume Rendering can display them
- Convert to polygons: the Marching Cubes algorithm
  - Divide space into cubes
  - Evaluate implicit function at each cube vertex
  - Do root finding or linear interpolation along each edge
  - Polygonize on a cube-by-cube basis

Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together

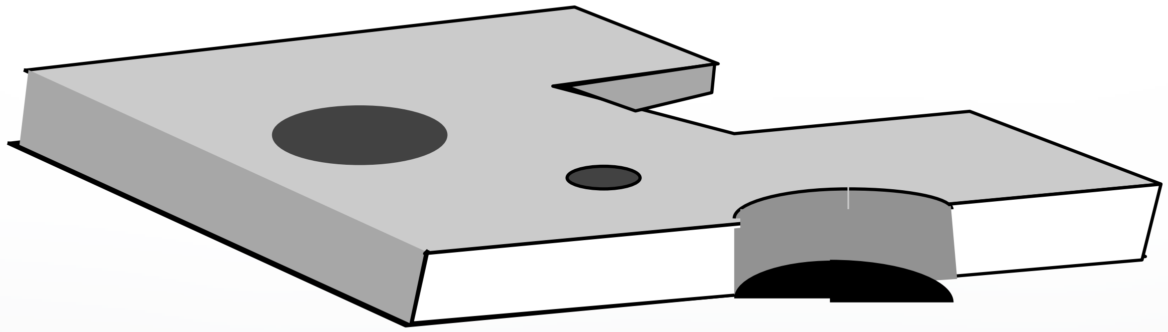


Constructive Solid Geometry (CSG)

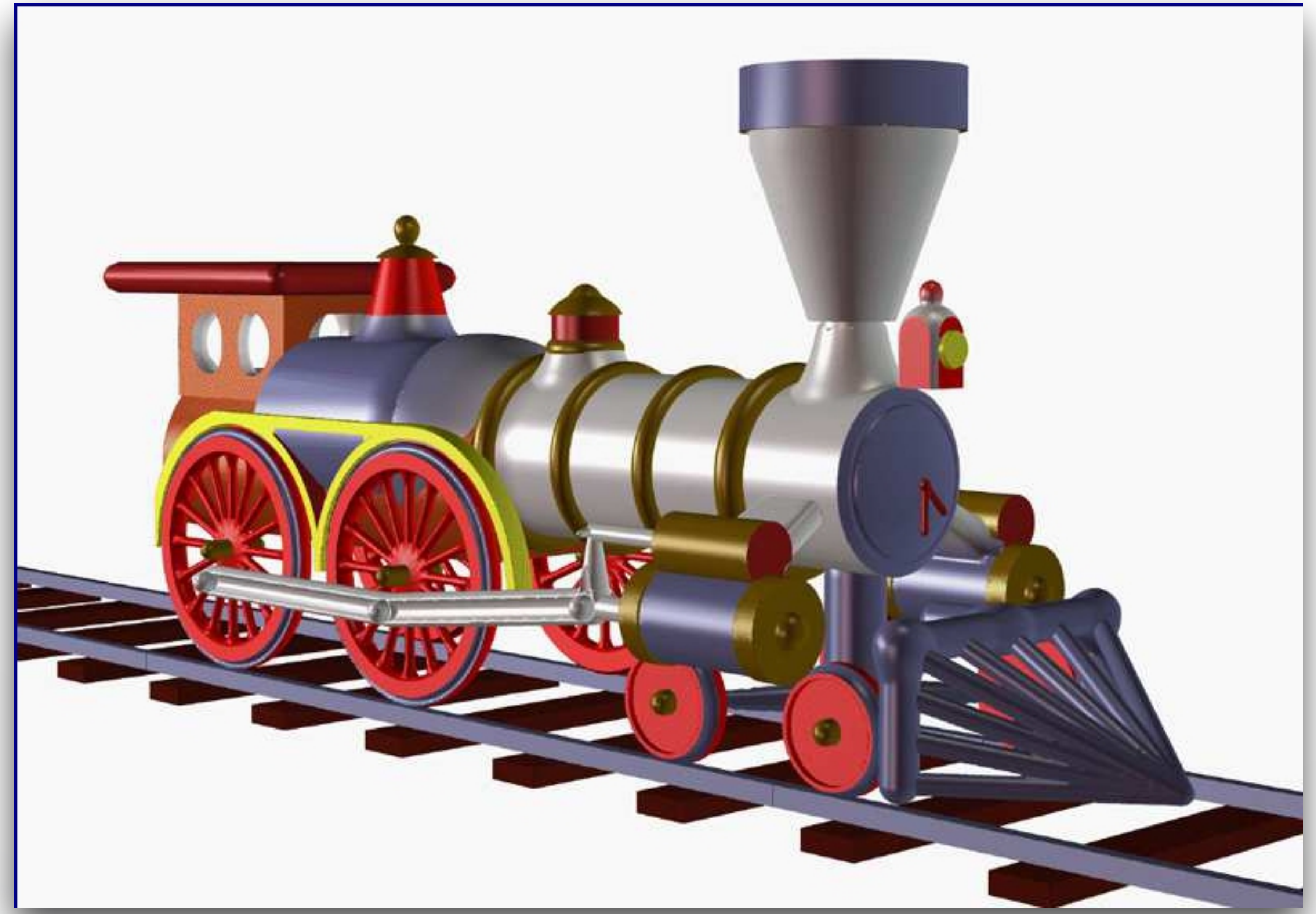
union	difference	intersection
		
the merger of two objects into one	the subtraction of one object from another	the portion common to both objects

Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together
- This is sensible for objects that are actually made that way (human-made, particularly machined objects)



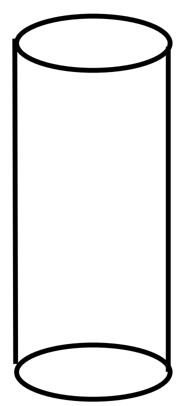
A CSG Train



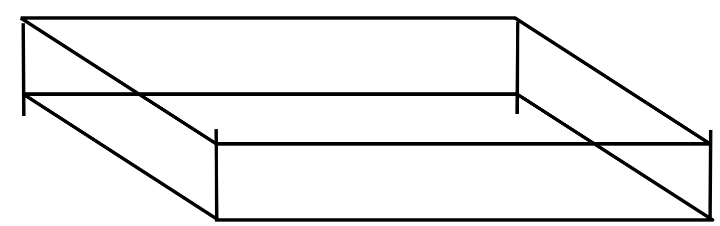
Negative Objects

- Use point-by-point boolean functions
  - remove a volume by using a negative object
  - e.g. drill a hole by subtracting a cylinder

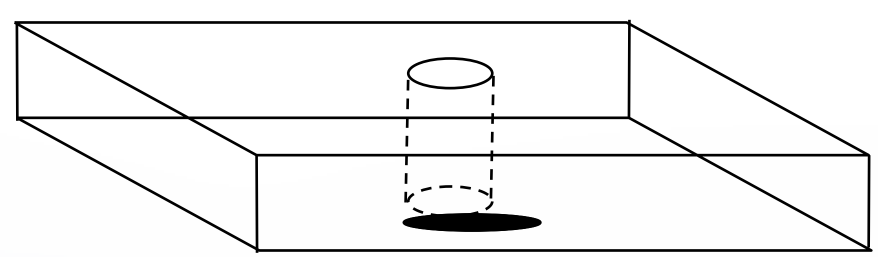
Subtract



From



To get



Inside(BLOCK-CYL) = Inside(BLOCK) And Not(Inside(CYL))

Set Operations

- **UNION:** Inside(A) || Inside(B)

  Join A and B

- **INTERSECTION:** Inside(A) && Inside(B)

  Chop off any part of A that sticks out of B

- **SUBTRACTION:** Inside(A) && (! Inside(B))

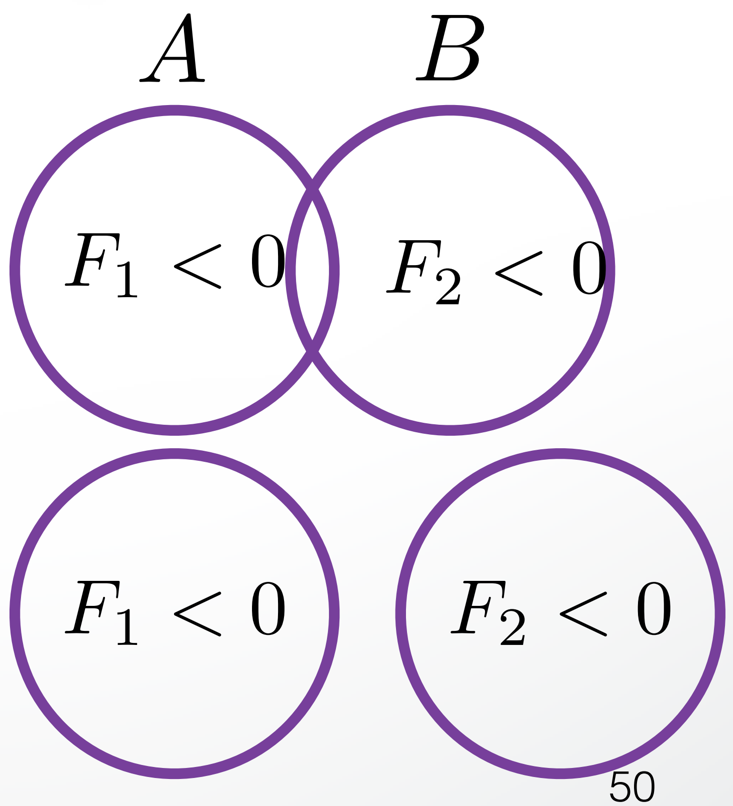
  Use B to Cut A

Examples:

- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.

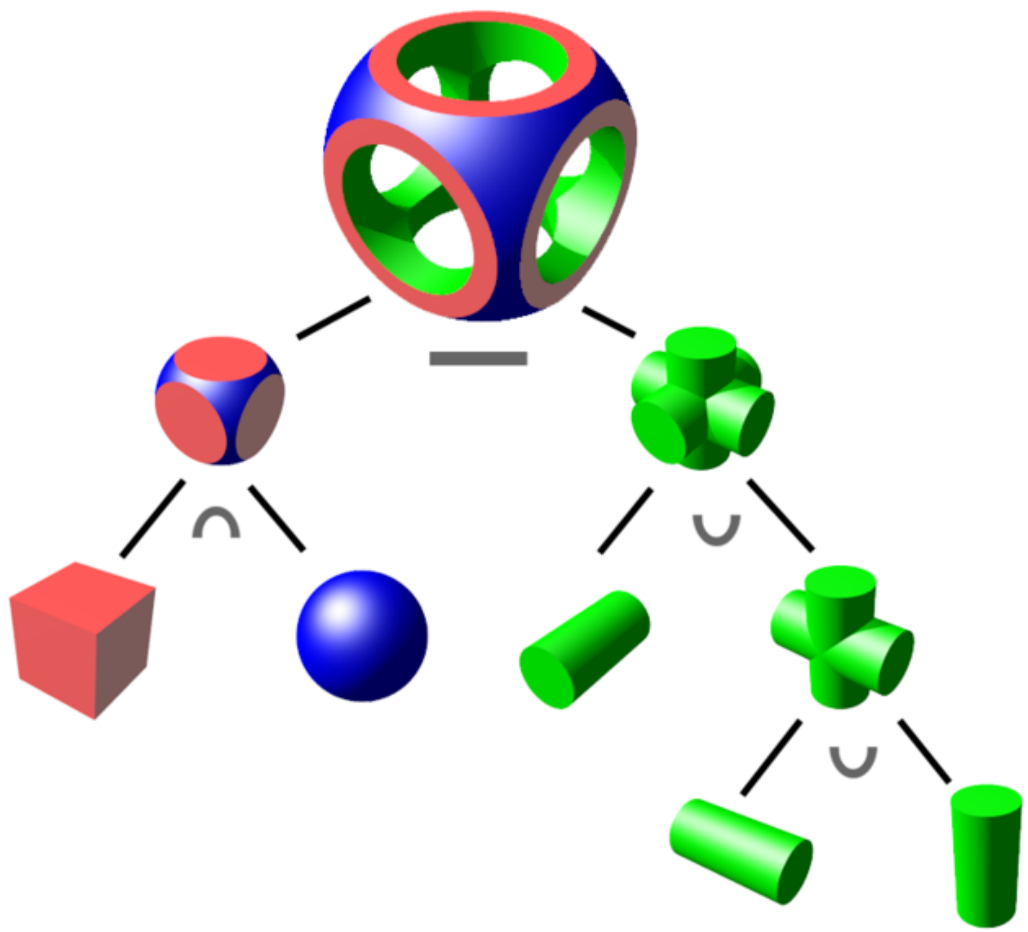
Implicit Functions for Booleans

- Recall the implicit function for a solid: F(x,y,z)<0
- Boolean operations are replaced by arithmetic
  - MAX replaces And (intersection)
  - MIN replaces OR (union)
  - MINUS replaces NOT(unary subtraction)
- Thus
  - F(Intersect(A,B)) = MAX(F(A),F(B))
  - F(Union(A,B)) = MIN(F(A),F(B))
  - F(Subtract(A,B)) = MAX(F(A), -F(B))



CSG Trees

- Set operations yield tree-based representation



Source: Wikipedia

Implicit Surfaces

- Good for smoothly blending multiple components
- Clearly defined solid along with its boundary
- Intersection test and Inside/outside test are easy
- Need to polygonize to render --- expensive
- Interactive control is not easy
- Fitting to real world data is not easy
- Always smooth

Summary

- Polygon Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry

http://cs420.hao-li.com

Thanks!