13.1 Physically Based Simulation I

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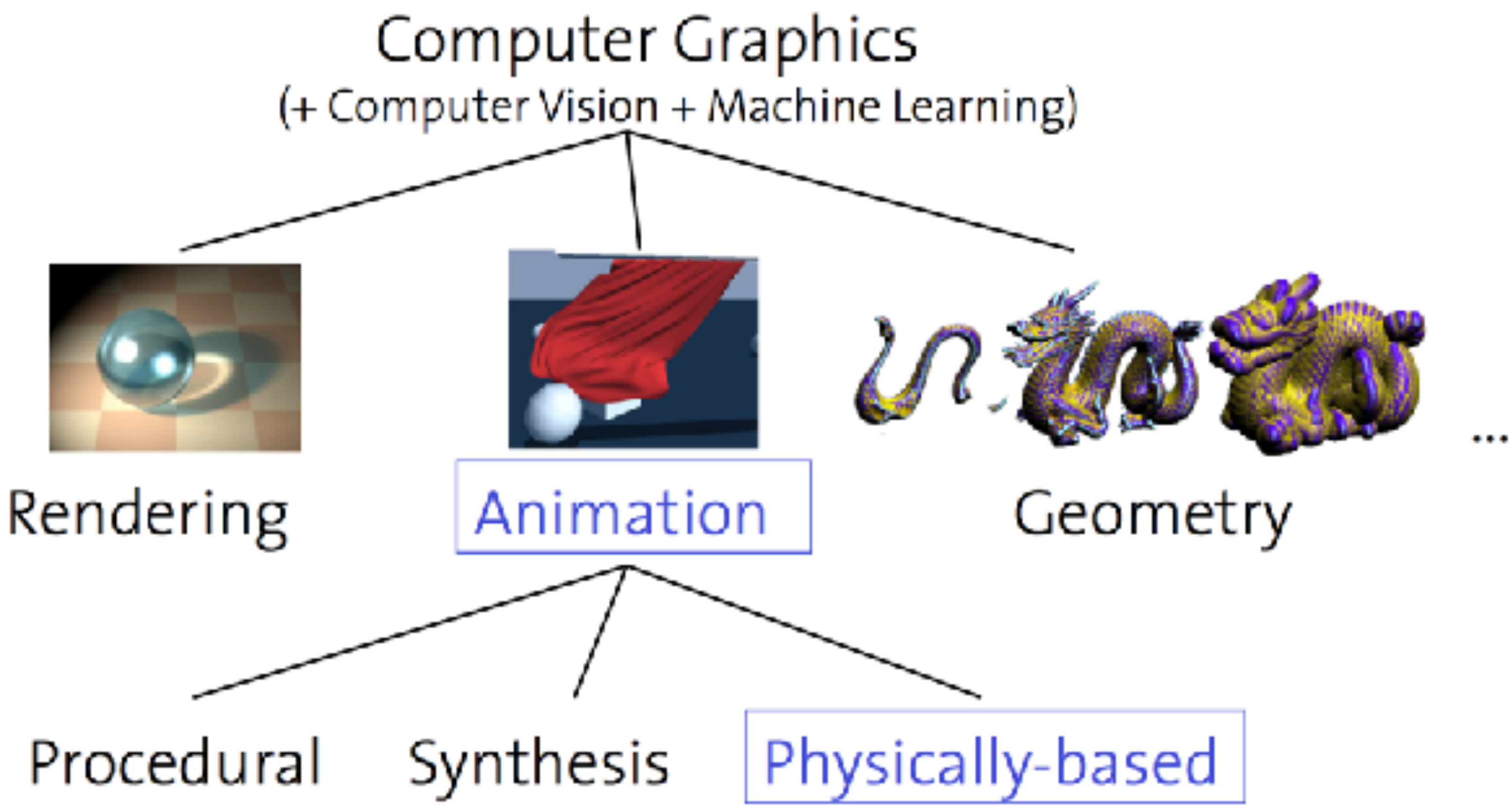
Visual Computing



Animation

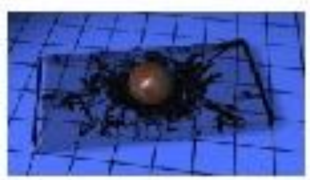
- Animation from *anima* (lat.) = *soul, spirit, breath of life*
- Bring images to life!
- Examples
  - Character animation (humans, animals)
  - Secondary motion (hair, cloth)
  - Physical world (rigid bodies, water, fire)

Animation Techniques

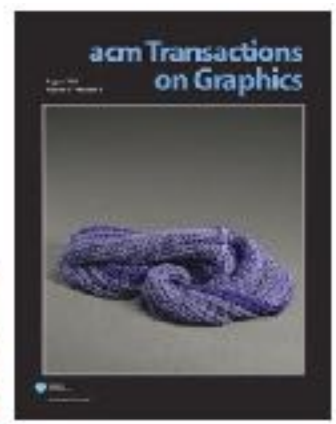
- For character animation
  - Keyframing
  - Motion capturing / motion synthesis
- For secondary motion, physical effects
  - Procedural
  - Simulation (physically based animation)

Physics in Computer Graphics

- Very common
- Computer Animation, Modeling (computational mechanics)
- Rendering (computational optics)

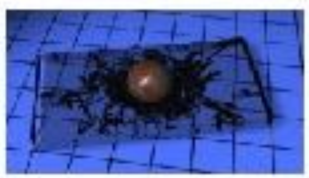





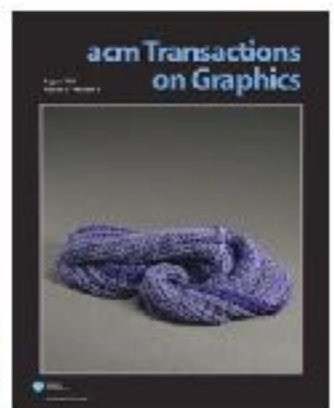


Physics in Computer Animation

- Fluids
- Smoke
- Deformable strands (rods)
- Cloth
- Solid 3D deformable objects .... and many more!

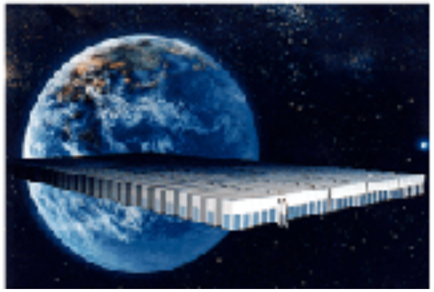







Physical Simulation

- Equations known for a long time
  - Motion (Newton, 1660) $d/dt(m\mathbf{v}) = \mathbf{f}$
  - Elasticity (Hooke, 1670) $\boldsymbol{\sigma} = \mathbf{E}\boldsymbol{\varepsilon}$
  - Fluids (Navier, Stokes, 1822) $\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -k\nabla\rho + \rho\mathbf{g} + \mu\nabla^2\mathbf{v}$
- Simulation made possible by computers
  - 1938: Zuse 1, 0.2 flops,
  - 2008: Roadrunner, 122k cores, 1026 teraflops



Scientific Goals and Challenges

- Goal of scientific computations
  - Reproduction of physical phenomena
  - Substitute for real experiments
- Goal of physically-based animation
  - Imitation of physical phenomena
  - Visually plausible behavior
  - As much realism as possible within performance and stability constraints

→ **Different goals require different methods/ representations...**

Offline Physics

- Special effects (film, commercials)
- Large models:
  millions of particles / tetrahedra / triangles
- Use computationally expensive rendering
  (global illumination)
- Impressive results
- Many seconds of computation time per frame

Real-time Physics

- Interactive systems:
  computer games
  virtual medicine (surgical simulation)

- Must be fast (30 fps, preferably 60 fps for games)
  Only a small fraction of CPU time devoted to physics!

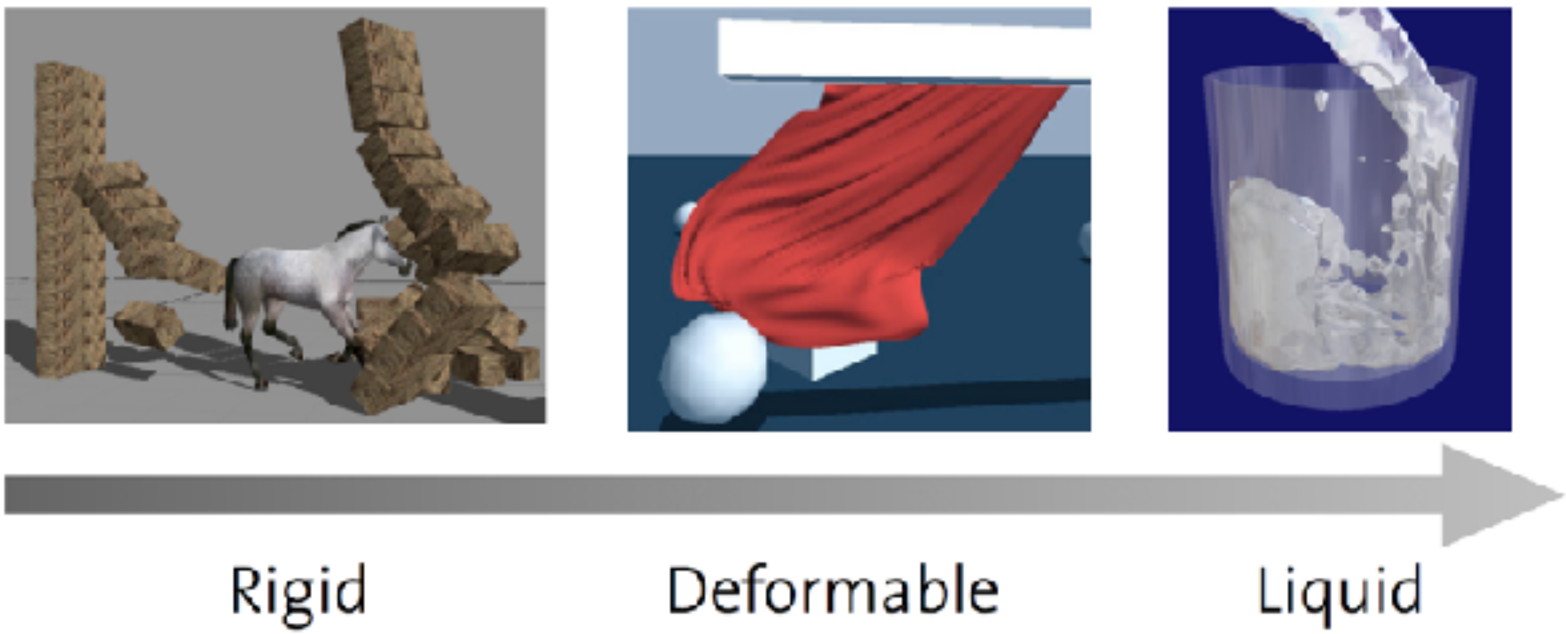
- Has to be stable, regardless of user input

Flight/car Simulators

Surgery trainers

3D Games

Examples



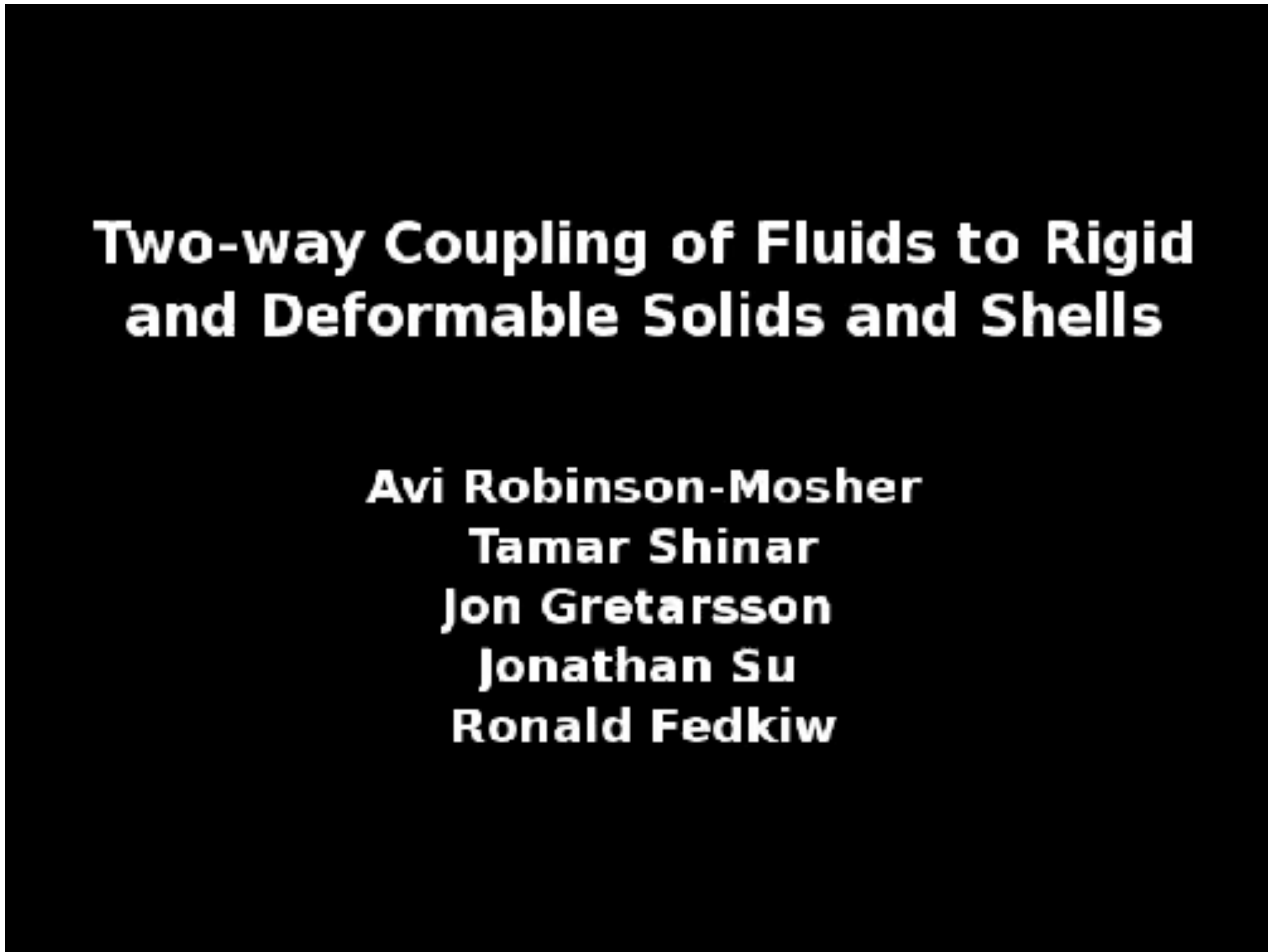
Fluids

Enright, Marschner, Fedkiw, SIGGRAPH 2002

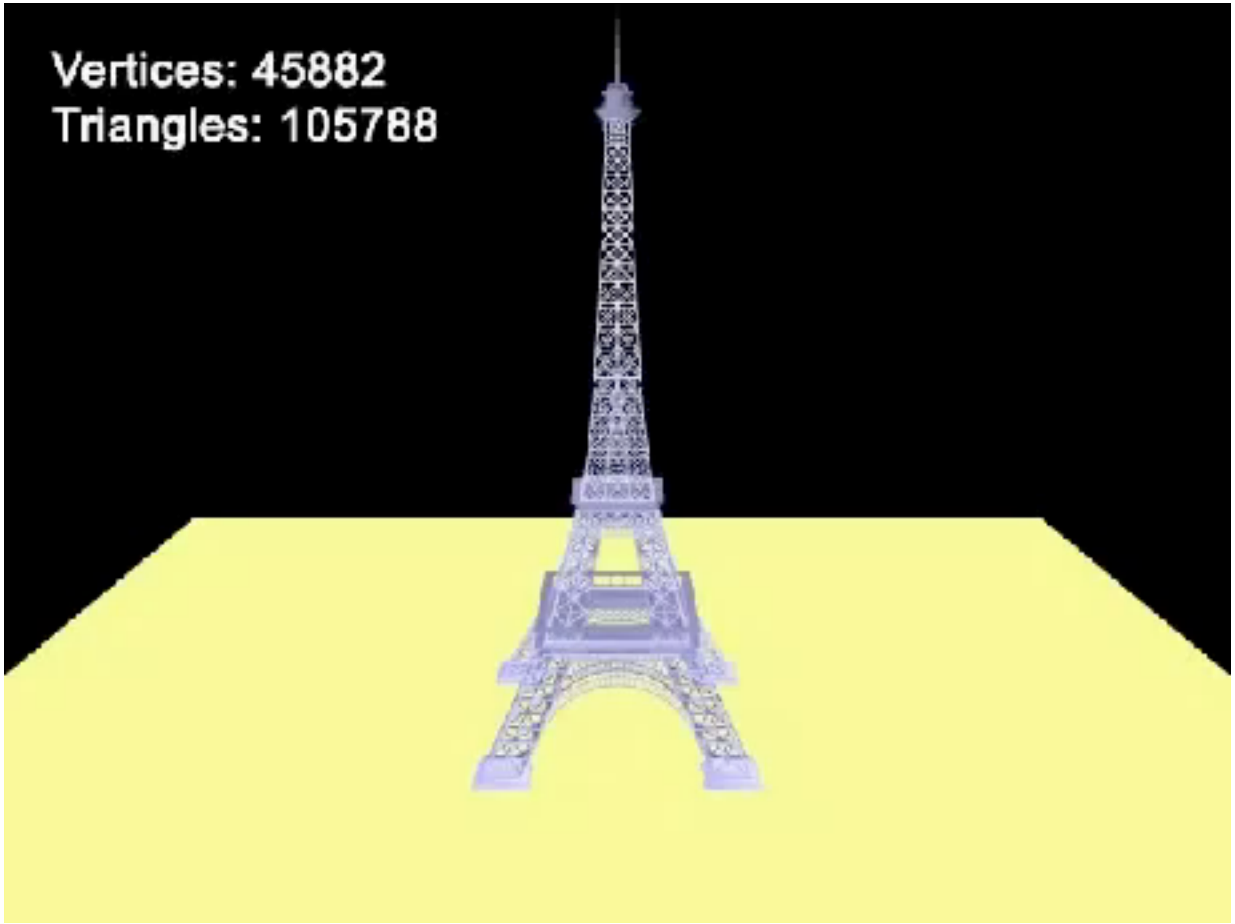
Fluids and Rigid Bodies

Fluids with Deformable Solid Coupling

[Robinson-Mosher,
Shinar,
Gretarsson,
Su, Fedkiw,
SIGGRAPH 2008]

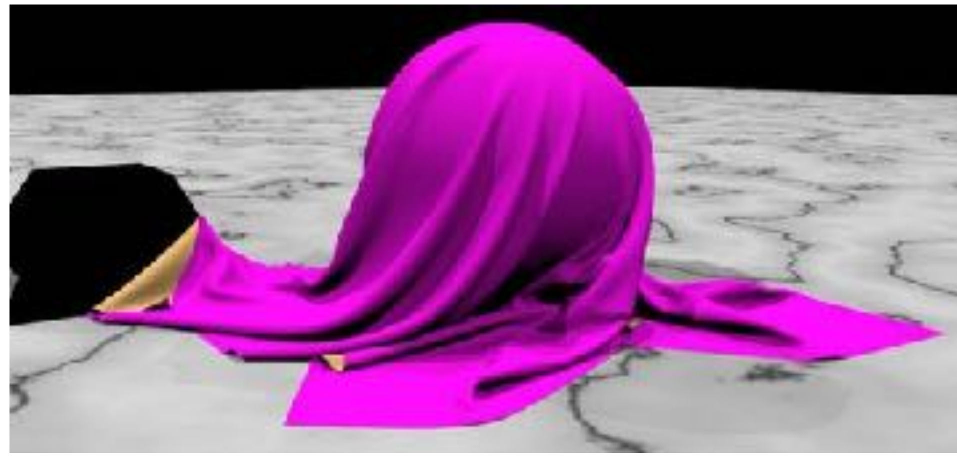


Deformations



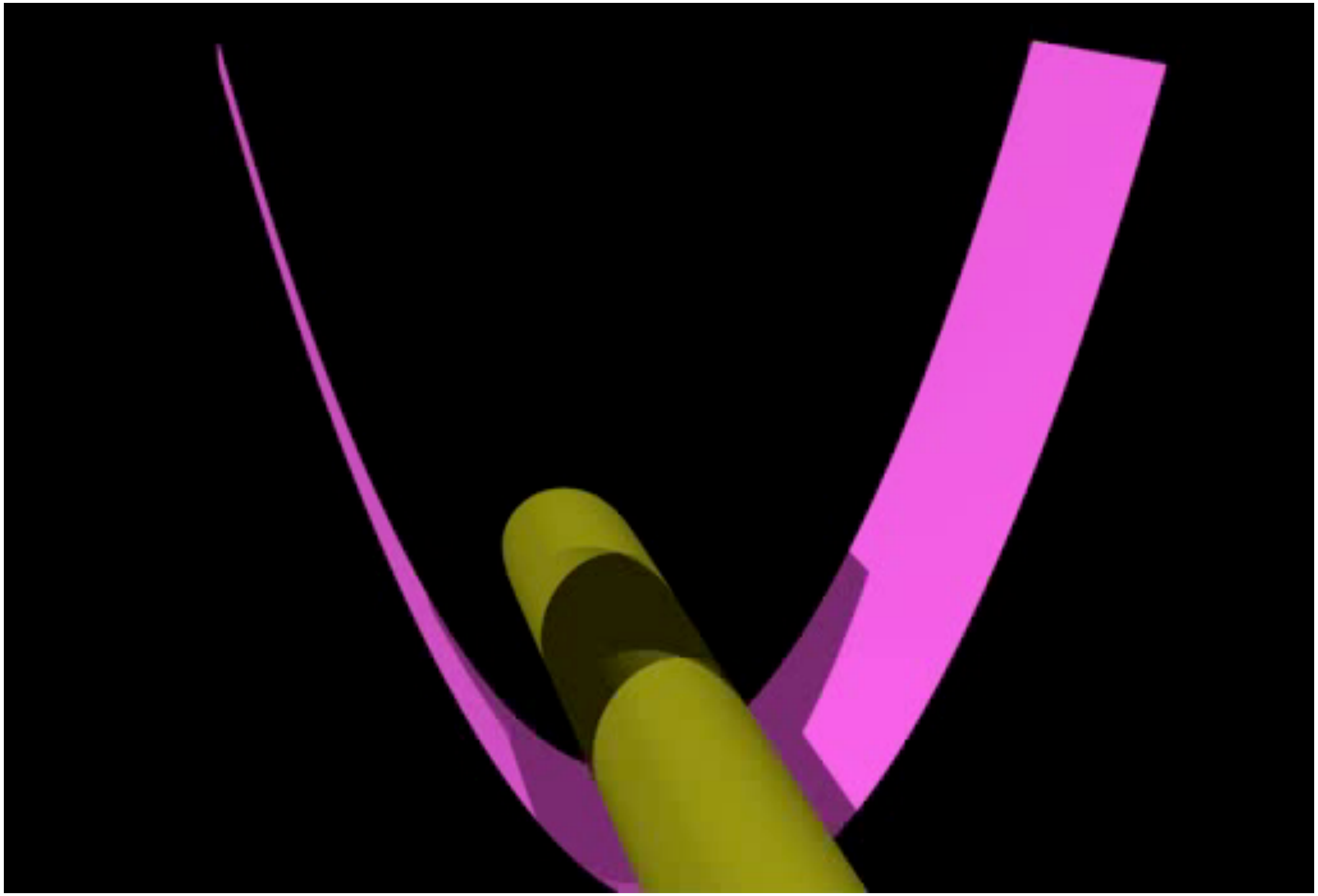
[Barbic and James,
SIGGRAPH 2005]

Cloth



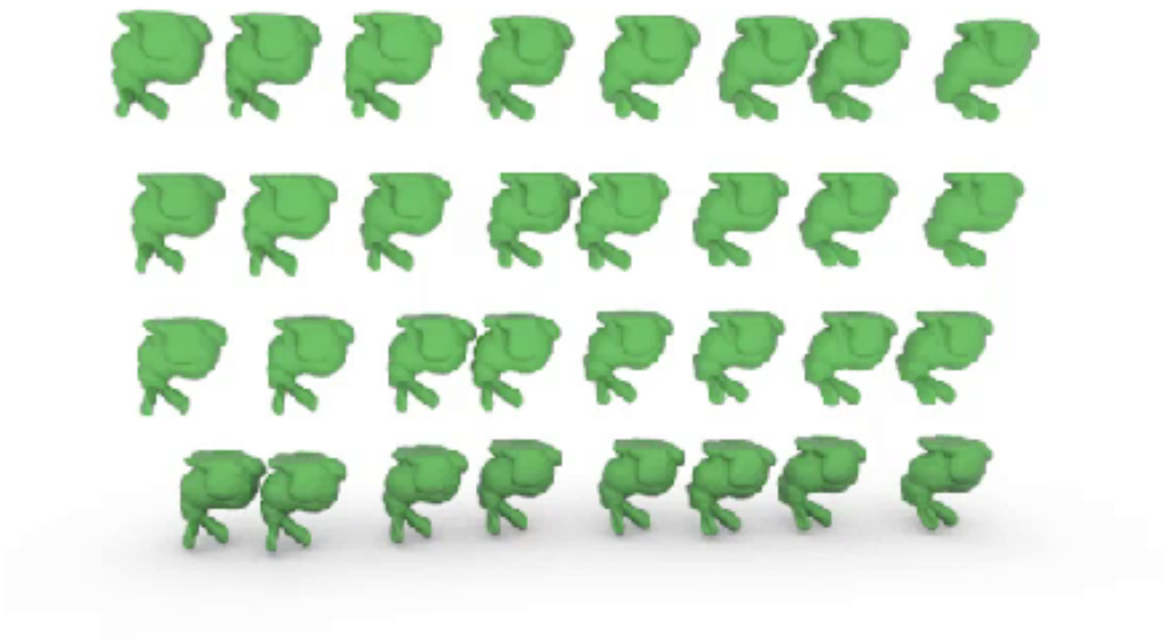
Source:
ACM SIGGRAPH

Cloth (Robustness)



[Bridson, Fedkiw, Anderson, ACM SIGGRAPH 2002

Multibody Dynamics + Self-collision Detection



[Barbic and James, SIGGRAPH 2010]

Surgical Simulation

[James and Pai,
SIGGRAPH 2002]

Multibody Dynamics

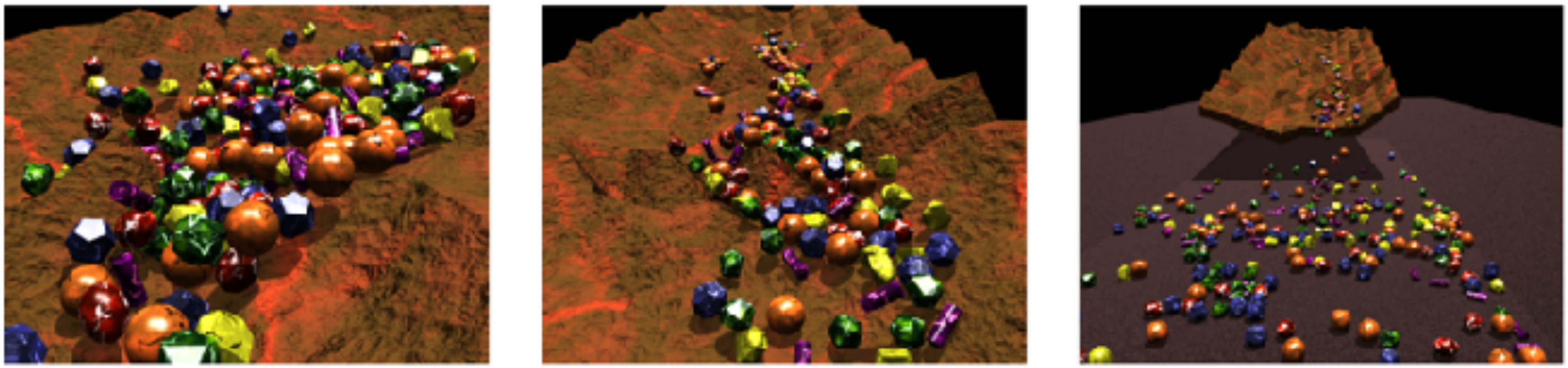
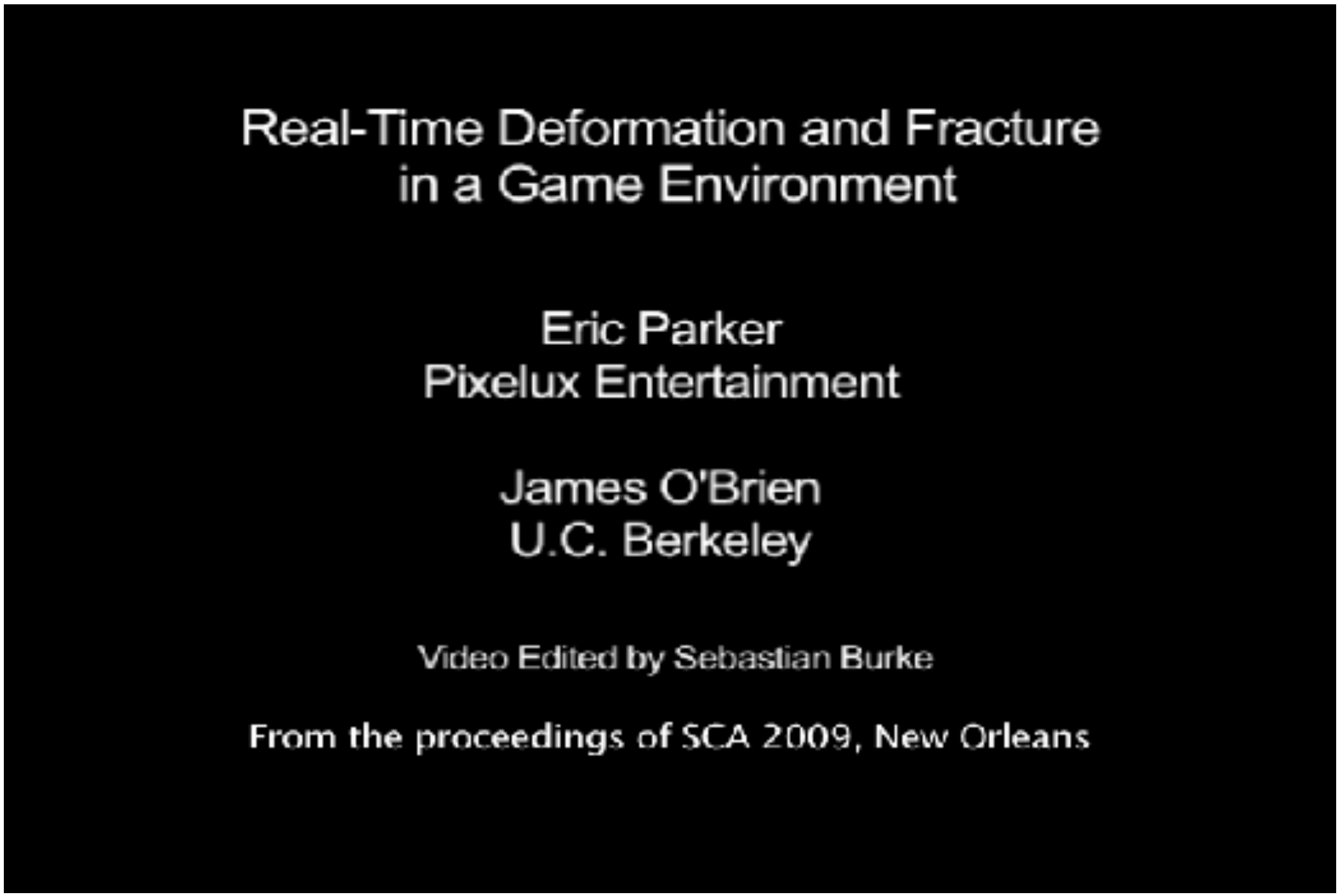


Figure 1: *Avalanche:* 300 rocks tumble down a mountainside.

Physics in Games



[Parker and James,
Symposium on Computer Animation 2009]

Sound Simulation (Acoustics)

[James, Barbic, Pai, SIGGRAPH 2006]

Techniques

- Particle systems
  - Fire, smoke, water ...
- Mass-spring systems
  - Deformable objects, cloth ...
- Rigid body simulation
  - Cars, airplanes, furniture ...
- Grid based methods
  - Water, smoke, airflow ...
- Finite Elements
  - Accurate deformable objects ...

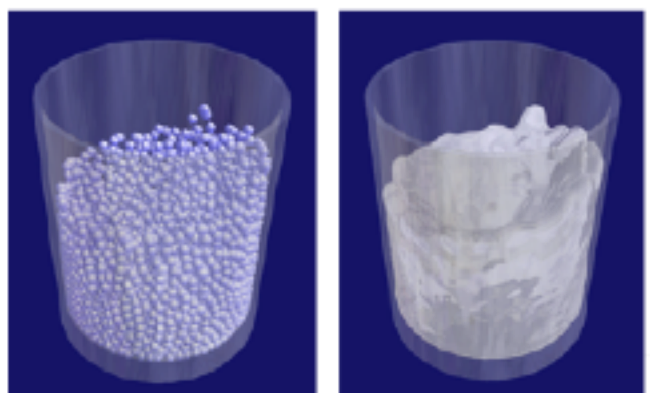
Particle System

Snow, dust, sand

Fire

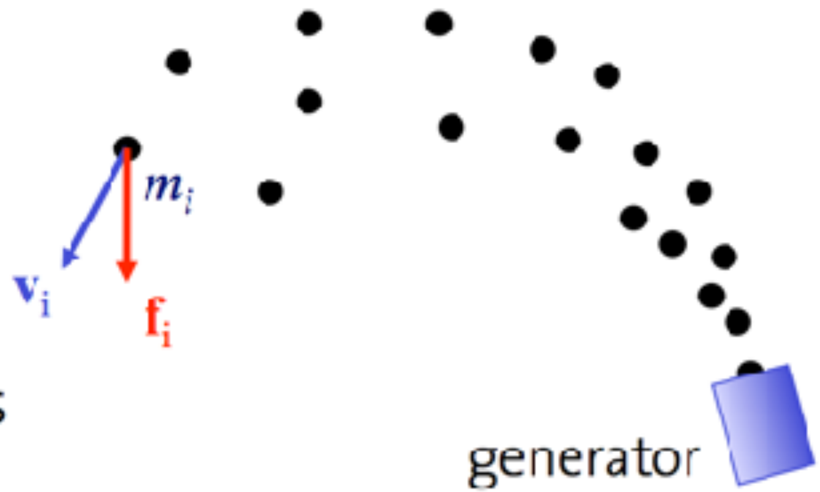
Smoke

Water



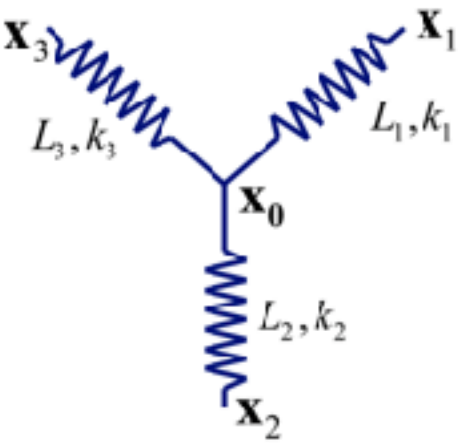
Particle System

- Collection of many small simple particles
- Particle motion influenced by forces
- Generated by emitters
- Deleted when lifetime reached or out of scene



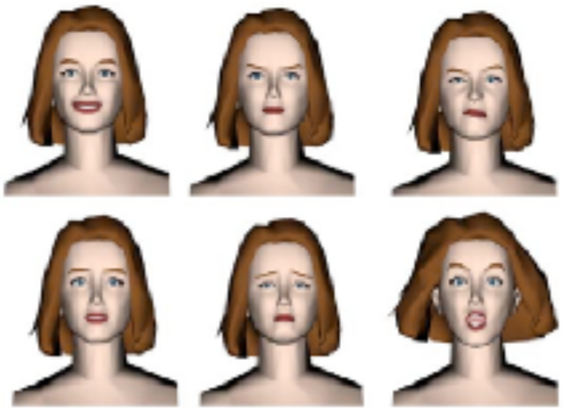
Mass-Spring Systems

- Particle system + springs
- Special interaction force
- Issues:
  - Where to put springs
  - Choice of stiffnesses
  - Collision detection
  - Collision response
  - Stability (time step or stiffness too high)



Applications

Facial animation

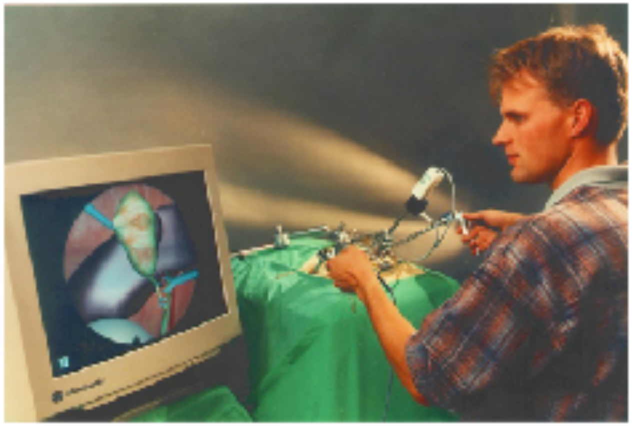


Thalmann

Cloth simulation

Strasser

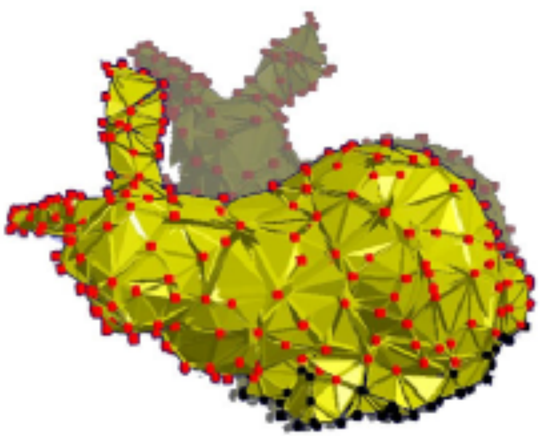
Surgery simulation



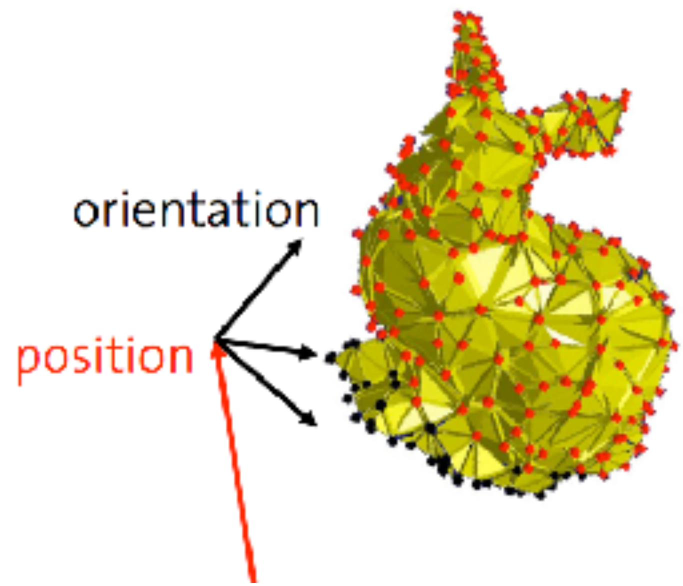
Kuehnapfel

Rigid Body Simulation

- Deformable objects have many degrees of freedom
- Each vertex is simulated separately



- A rigid body only has 6 degrees of freedom
- Faster simulation possible



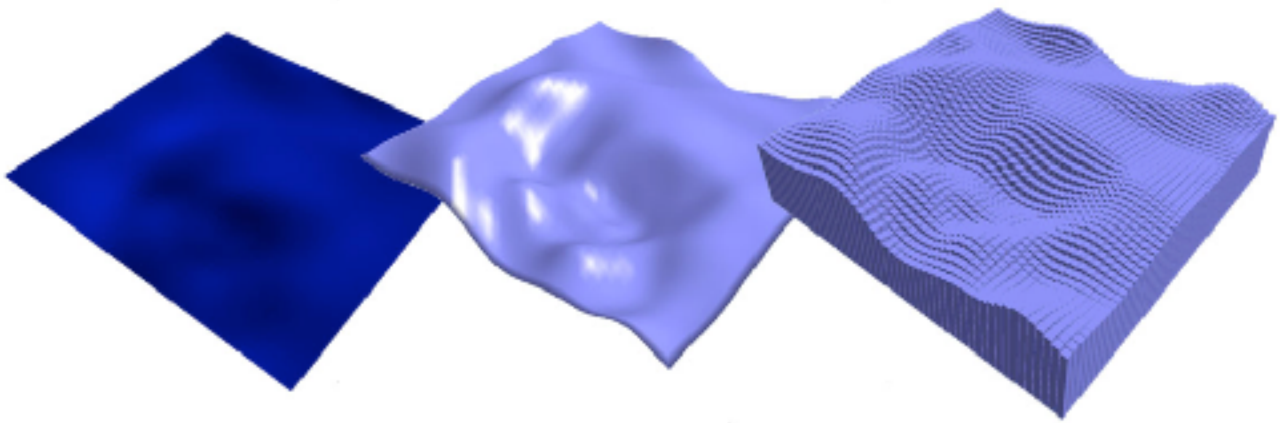
Challenges

- Collision detection
- Collision response for complex configurations
- Constraints (joints)

Applications

- Robotic simulations
- 3D computer games

Grid-Based Methods



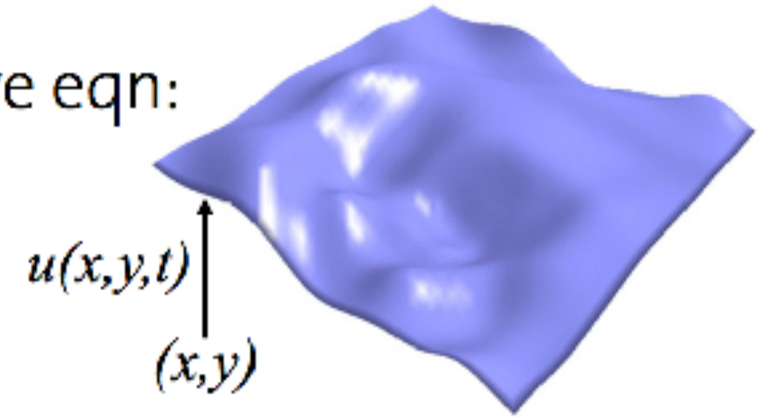
- Basic idea:
  - Solve partial differential equation on (regular) grid
  - Replace differentials by finite differences

Example: Fluid Surface

- Water surface defined as height $u(x,y,t)$ at location x,y at time t

- Dynamics given by 2D wave eqn:

$$\frac{\partial^2}{\partial t^2}u = c^2\left(\frac{\partial^2}{\partial x^2}u + \frac{\partial^2}{\partial y^2}u\right)$$

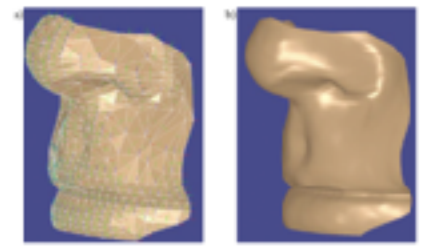


- Discretization:

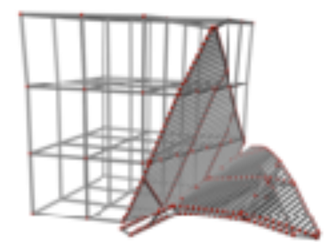
$$v^{t+1}[i,j] = v^t[i,j] + \Delta t\, c^2\, \frac{u^t[i+1,j]+u^t[i-1,j]+u^t[i,j+1]+u^t[i,j-1]-4u^t[i,j]}{h^2}$$

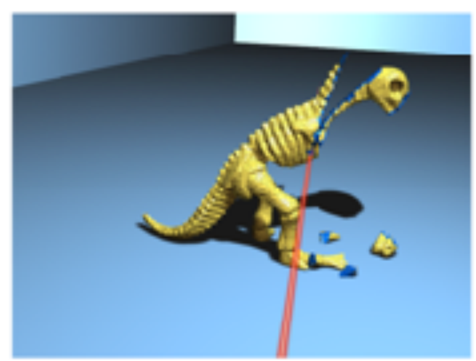
$$u^{t+1}[i,j] = u^t[i,j] + \Delta t\, v^{t+1}[i,j]$$

FEM Simulation



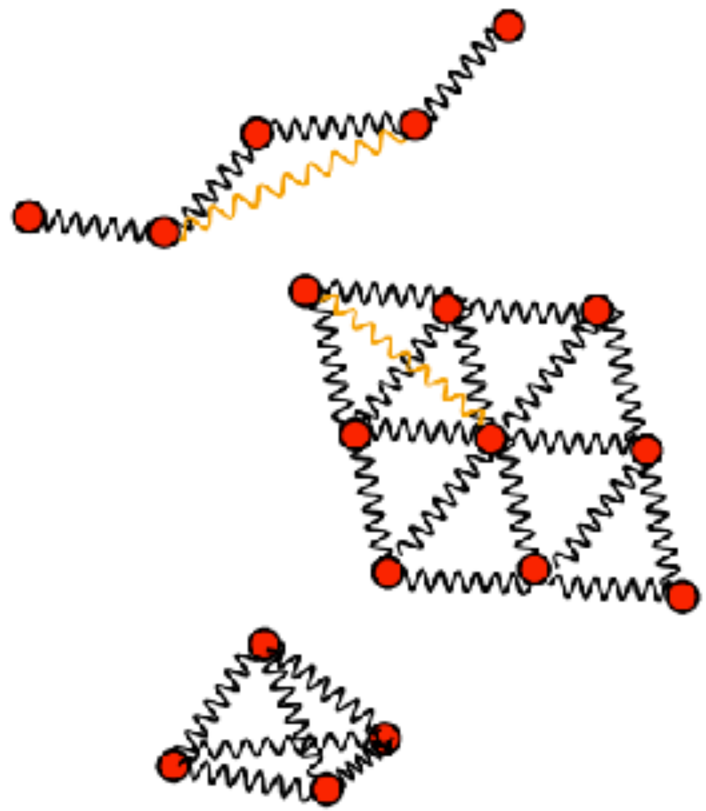
- Discretize equations from continuum mechanics
- Solve (more) accurately
- Independent of tesselation
- Volumetric meshes





Case Study: Mass-spring Systems

- Mass particles connected by elastic springs
- One dimensional: rope, chain
- Two dimensional: cloth, shells
- Three dimensional: soft bodies



Source:Matthias Mueller, SIGGRAPH

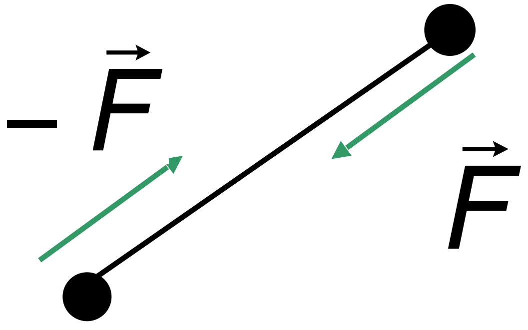
Newton's Laws

- Newton's 2nd law:

$$\vec{F} = m\vec{a}$$

- Gives acceleration, given the force and mass

- Newton's 3rd law: If object A exerts a force F on object B, then object B is at the same time exerting force -F on A

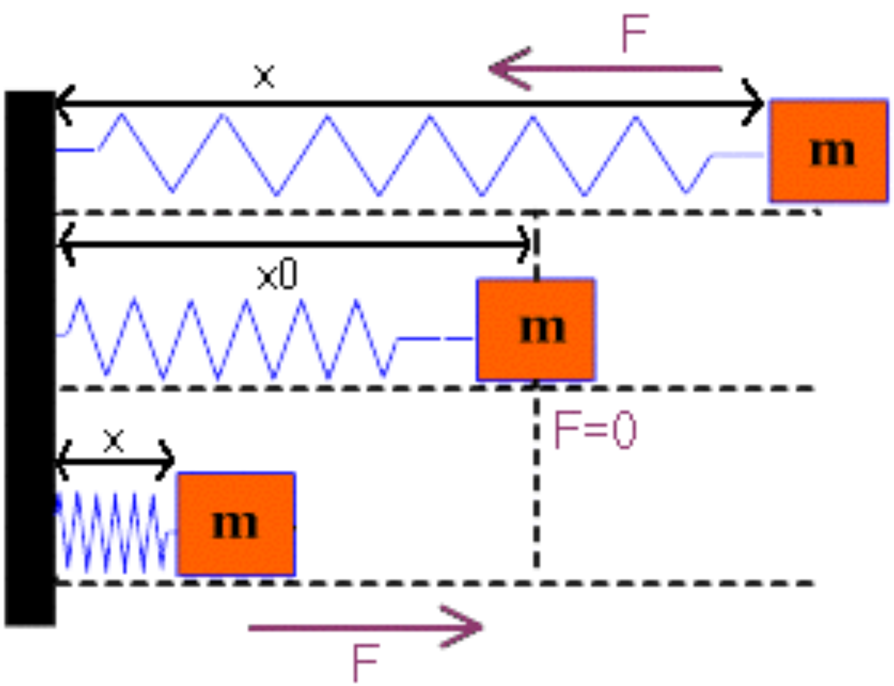


Single spring

- Obeys the *Hook's law*:

  $$F = k\,(x - x_0)$$

- x_0 = rest length
- k = spring elasticity (*stiffness*)
- For $x<x_0$, spring wants to extend
- For $x>x_0$, spring wants to contract



Hook's law in 3D

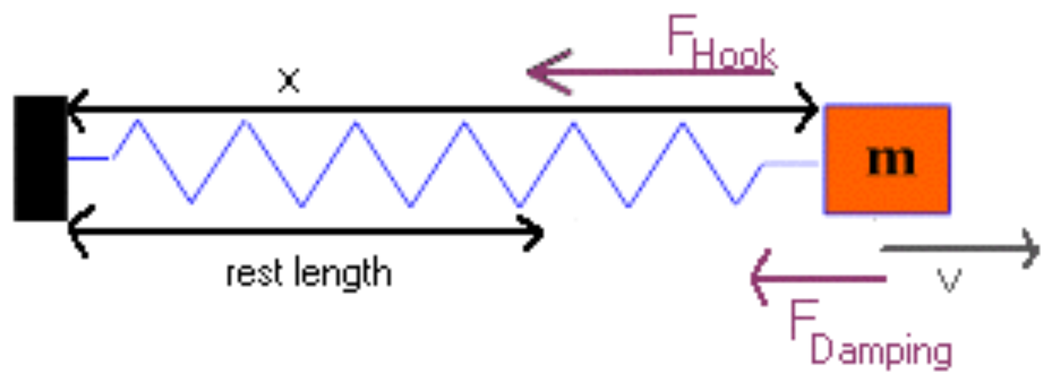
- Assume A and B two mass points connected with a spring.
- Let L be the vector pointing from B to A
- Let R be the spring rest length
- Then, the elastic force exerted on A is:

$$\vec{F} = -k_{Hook}(|\vec{L}| - R)\frac{\vec{L}}{|\vec{L}|}$$

Damping

- Springs are not completely elastic
- They absorb some of the energy and tend to decrease the velocity of the mass points attached to them
- Damping force depends on the velocity:

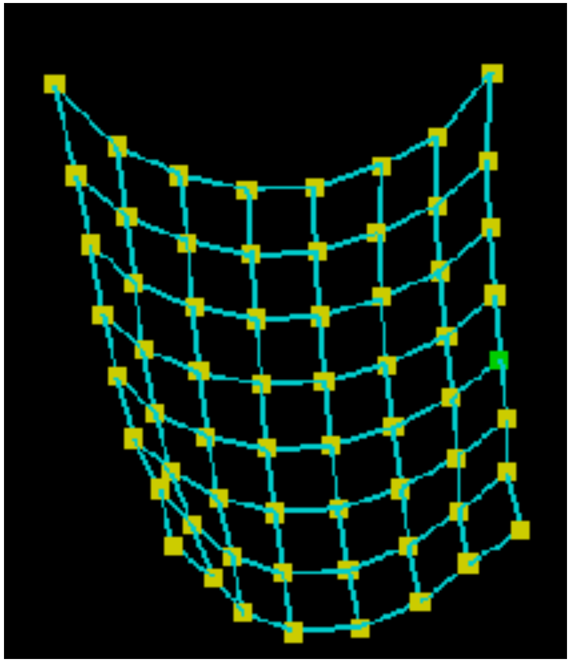
$$\vec{F} = -k_d \vec{v}$$



- k_d = damping coefficient
- k_d different than k_{Hook} !!

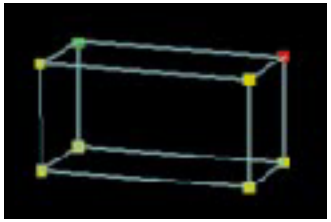
A network of springs

- Every mass point connected to some other points by springs
- Springs exert forces on mass points
  - Hook’s force
  - Damping force
- Other forces
  - External force field
    - Gravity
    - Electrical or magnetic force field
  - Collision force

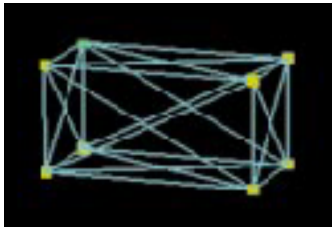


Network organization is critical

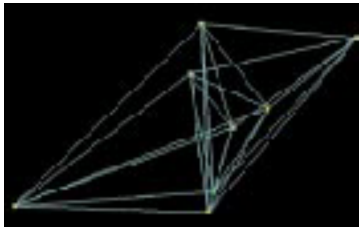
- For stability, must organize the network of springs in some clever way



Basic network

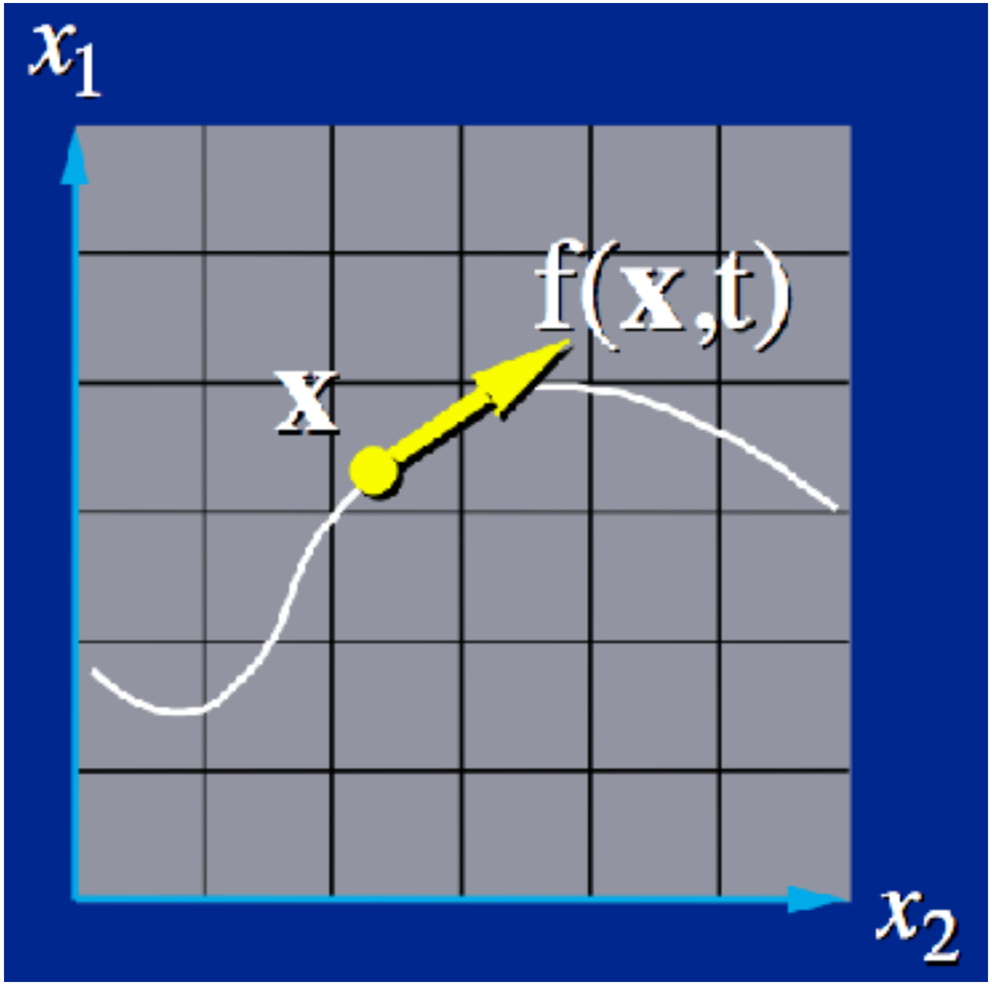


Stable network



Network out of control

Time Integration

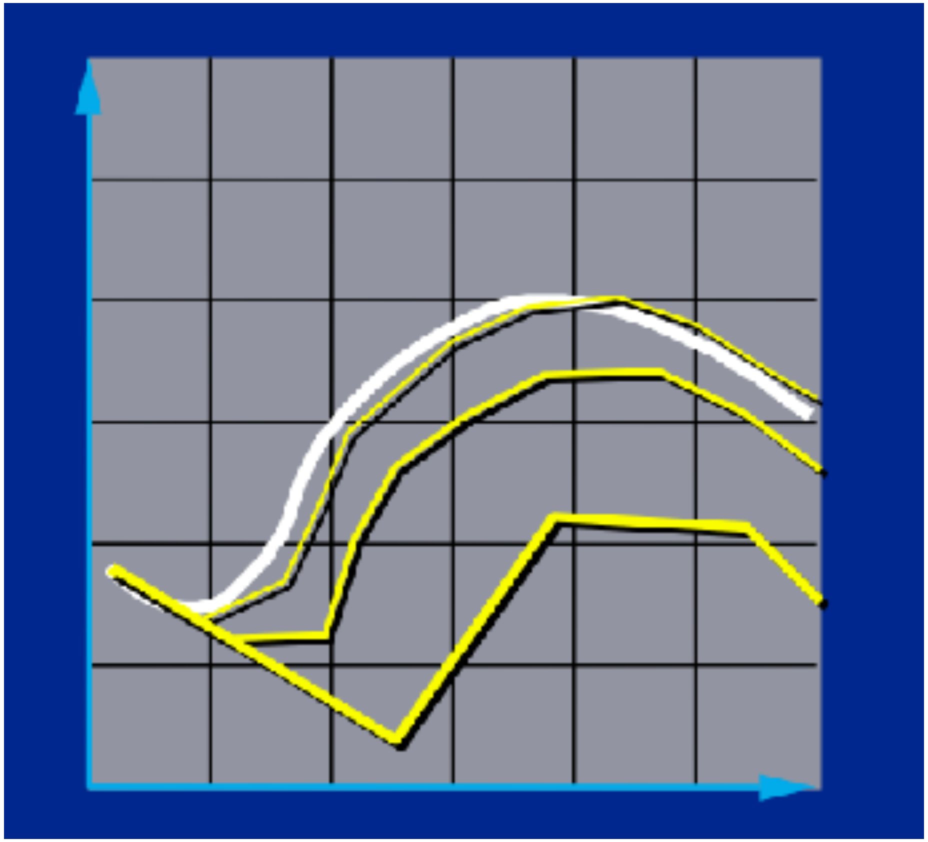


Source:Andy Witkin, SIGGRAPH

Physics equation:
x' = f(x,t)

x=x(t) is particle trajectory

Euler Integration

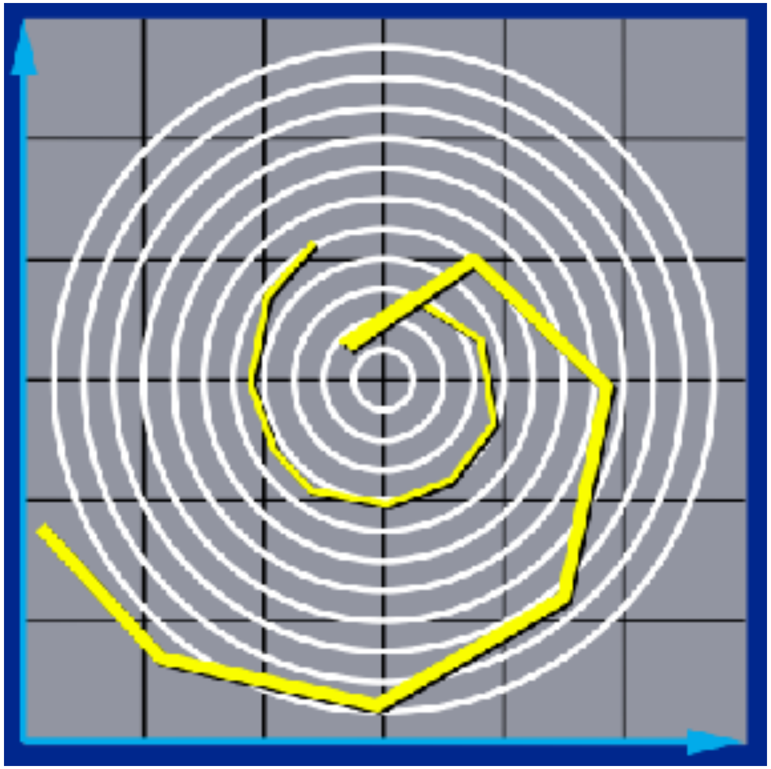


Simple,
but inaccurate.

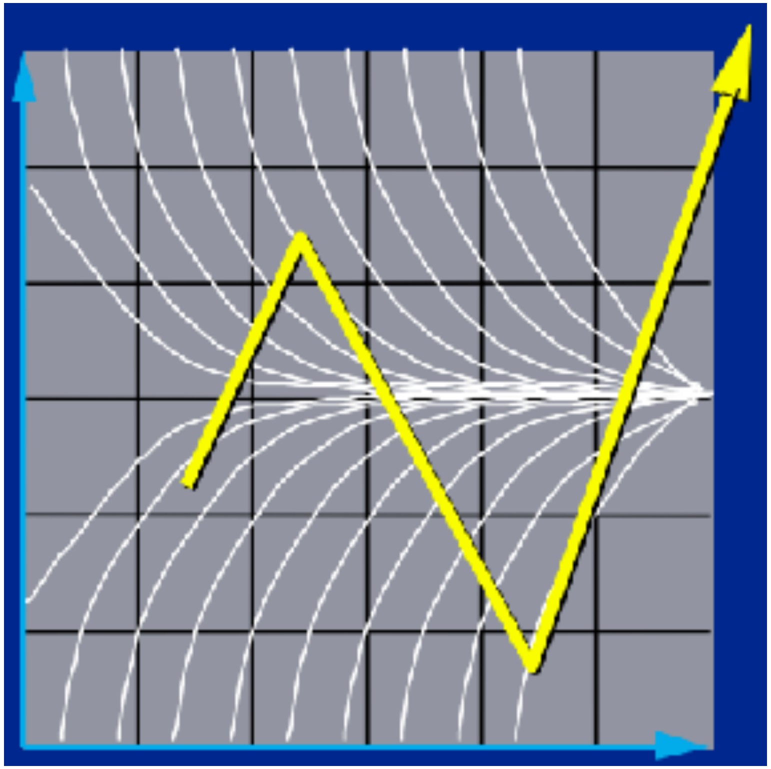
Unstable with
large timesteps.

Source:Andy Witkin, SIGGRAPH

Inaccuracies with explicit Euler



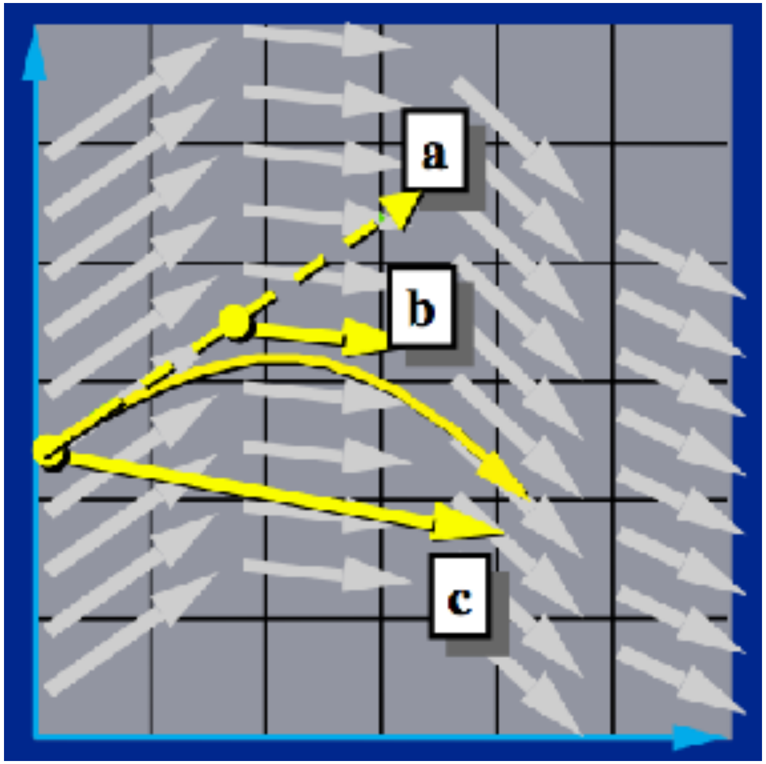
Gain energy



“Blow-up”

Source:Andy Witkin, SIGGRAPH

Midpoint Method



Source:Andy Witkin, SIGGRAPH

Improves stability

1. Compute Euler step
   $\Delta x = \Delta t\ f(x, t)$

2. Evaluate f at the midpoint
   $f_{mid} = f((x+\Delta x)/2, (t+\Delta t)/2)$

3. Take a step using the midpoint value
   $x(t+ \Delta t) = x(t) + \Delta t\ f_{mid}$

Many more methods

- Runge-Kutta (4th order and higher orders)

- Implicit methods
  - sometimes unconditionally stable
  - very popular (e.g., cloth simulations)
  - a lot of damping with large timesteps

- Symplectic methods
  - exactly preserve energy, angular momentum and/or other physical quantities
  - Symplectic Euler

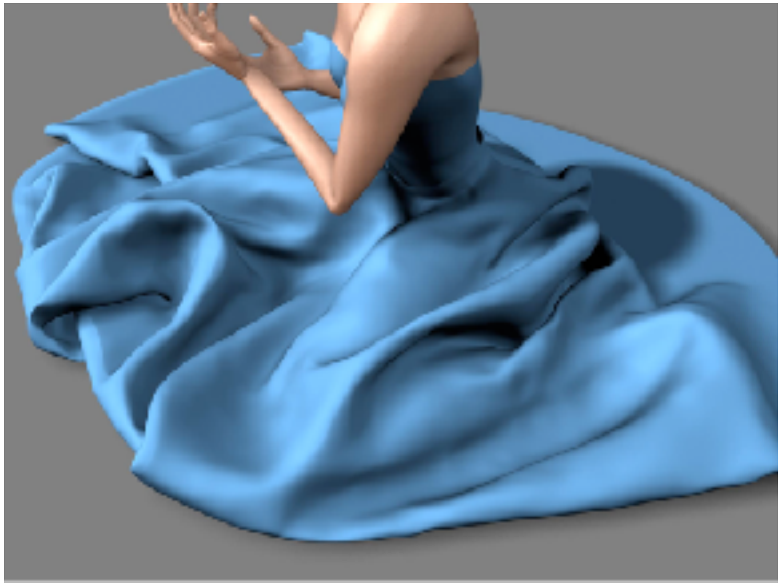
Cloth Simulation

- Stretch
- Shear
- Bend

[Baraff and Witkin, SIGGRAPH 1998]

Challenges

- Complex Formulas
- Large Matrices
- Stability
- Collapsing triangles
- Self-collision detection

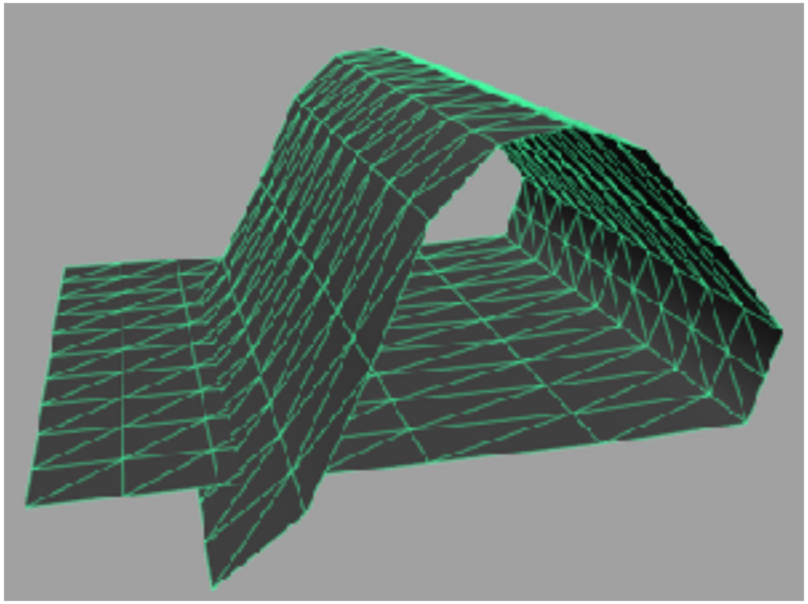


[Govindaraju et al. 2005]

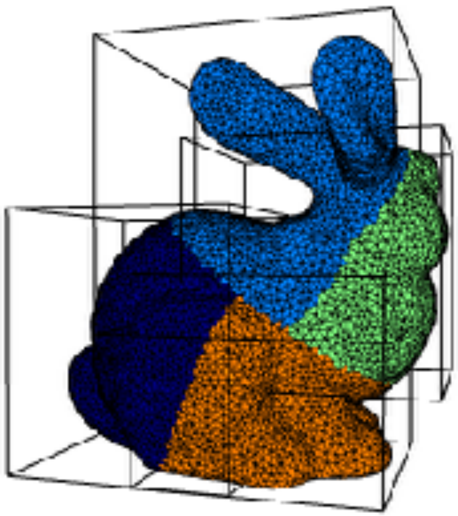
Self-collisions: definition

Deformable model is
self-colliding iff

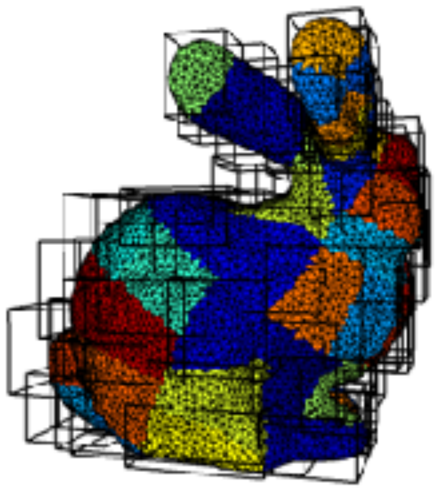
there exist non-neighboring
intersecting triangles.



Bounding volume hierarchies



AABBs
Level 1



AABBs
Level 3

[Hubbard 1995]

[Gottschalk et al. 1996]

[van den Bergen 1997]

[Bridson et al. 2002]

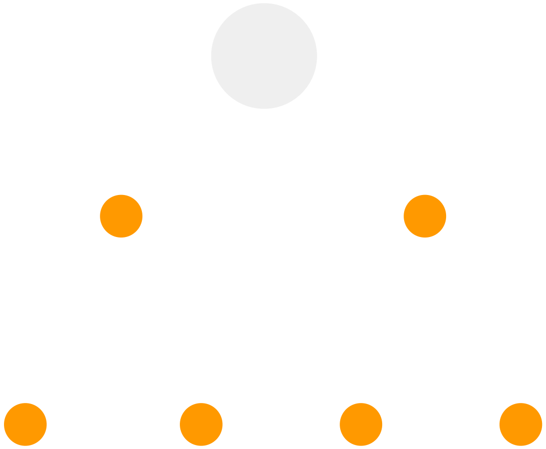
[Teschner et al. 2002]

[Govindaraju et al. 2005]

Bounding volume hierarchy

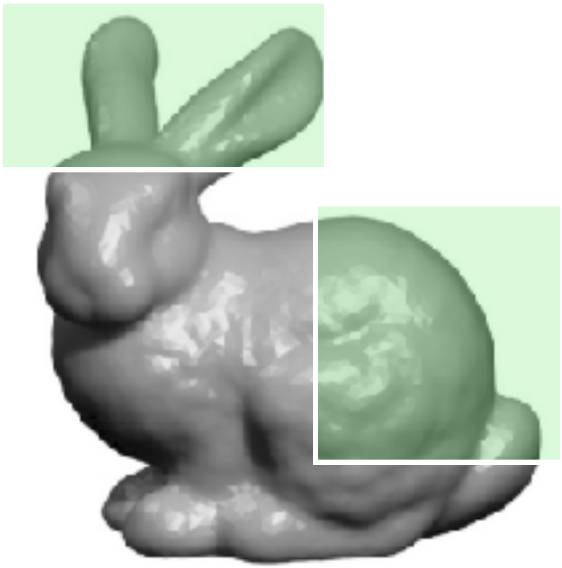
root

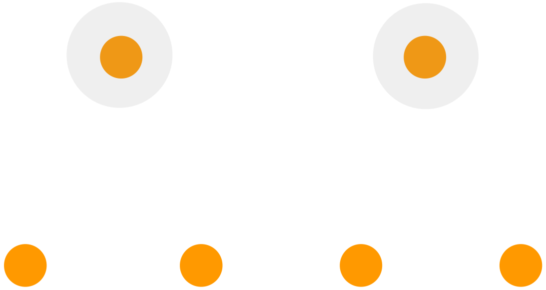
root



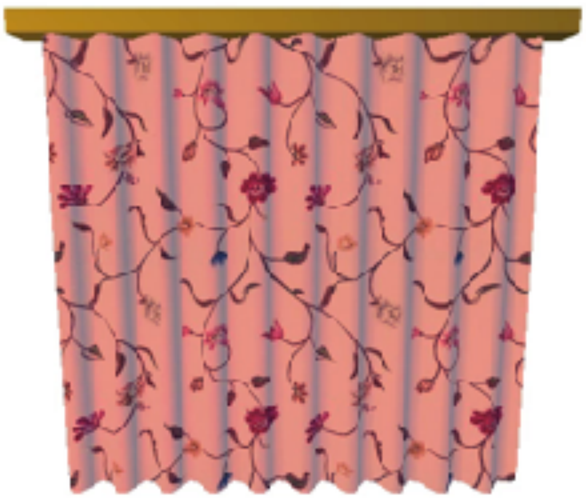
Bounding volume hierarchy

V





Real-time cloth simulation



Source:
Andy Pierce

Model	Triangles	FPS	% Forces + Stiffness Matrix	% Solver
Curtain	2400	25	67	33

http://cs420.hao-li.com

Thanks!